Cross-Sectional Asset Pricing with Individual Stocks: Betas versus Characteristics*

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Abstract

We develop a methodology for bias-corrected return-premium estimation from cross-sectional regressions of individual stock returns on betas and characteristics. Over the period from July 1963 to December 2013, there is some evidence of positive beta premiums on the profitability and investment factors of Fama and French (2014), a negative premium on the size factor and a less robust positive premium on the market, but no reliable pricing evidence for the book-to-market and momentum factors. Firm characteristics consistently explain a much larger proportion of variation in estimated expected returns than factor loadings, however, even with all six factors included in the model.

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A fundamental paradigm in finance is that of risk and return: riskier assets should earn higher expected returns. It is the systematic or nondiversifiable risk that should be priced, and under the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) this systematic risk is measured by an asset's market beta. While Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) do find a significant positive cross-sectional relation between security betas and expected returns, more recently Fama and French (1992) and others find that the relation between betas and returns is negative, though not reliably different from zero. This calls into question the link between risk and expected returns.

There is also considerable evidence of cross-sectional patterns (so-called anomalies) in stock returns that raises doubts about the risk-return paradigm. Specifically, price momentum, documented by Jegadeesh and Titman (1993), represents the strong abnormal performance of past winners relative to past losers. The size and book-to-market effects have been empirically established by, among others, Fama and French (1992). In particular, small market capitalization stock returns have historically exceeded big market capitalization stock returns, and high book-to-market (value) stocks have outperformed their low book-to-market (growth) counterparts. Brennan, Chordia, and Subrahmanyam (1998) find that investments based on anomalies result in reward-to-risk (Sharpe) ratios that are about three times as high as that obtained by investing in the market, too large it would seem, to be consistent with a risk-return model (also see MacKinlay (1995)).

The behavioral finance literature points to psychological biases on the part of investors to explain the breakdown of the risk-return relationship. In contrast, Fama and French (1993) propose a three-factor model that includes risk factors proxying for the size- and value-effects, in addition to the market excess-return factor, Mkt. The size factor, SMB, is a return spread between small firms and big firms, while the value factor, HML, is a return spread between high and low book-to-market stocks. There is controversy in the literature as to whether these two additional factors are really risk factors, however, i.e., whether the factors can be viewed as hedge portfolios in an intertemporal CAPM along the lines of Merton (1973). Greater still, we suspect, is skepticism about a risk-based interpretation of the momentum factor MOM. This (winner-loser) spread factor is often included in a four-factor model along with the three Fama and French (1993) factors, e.g., Carhart (1997) and Fama and French (2012). More recently, Fama and French (2014) have proposed a five-factor model that adds CMA (conservative minus aggressive investment) and RMW (robust minus weak profitability) factors to the original three.[1]

While some researchers are inclined to view expected return variation associated with factor loadings (betas) as due to risk, and variation captured by characteristics like book-to-market as due to mispricing, we believe that a more agnostic perspective on this issue is appropriate. One reason is that the betas on an ex-ante efficient portfolio (a potential "factor") will *always* fully "explain" expected returns as a mathematical proposition (see Roll (1977)), whatever the nature of the underlying economic process. This makes it difficult to infer that a beta effect is truly driven by economic risk unless there is evidence that the factor correlates with some plausible notion of aggregate marginal utility in an intertemporal CAPM or other economic setting.

For the usual spread factors, it is also important to recognize that there is a mechanical relation between, say, the book-to-market ratio and loadings on HML: a weighted average of the loadings for stocks in the high book-to-market portfolio must exceed that for stocks in the low book-to-market portfolio.[2] Therefore, the relation between loadings and expected returns can be mechanical as well. In fact, Ferson, Sarkissian, and Simin (1998) construct an example in which expected returns are determined entirely by a characteristic, but one that is nearly perfectly correlated with loadings on the associated spread factor. In general, though, there need not be a simple relation between loadings and characteristics at the individual stock level. For example, at the end of 2013, Comcast's book-to-market ratio of 3.4 placed it at the 99th percentile, extreme value territory, while its negative loading on the HML factor was at the 30th percentile, suggestive of a growth tilt. Empirically, we find relatively low correlations (less than 0.5) between characteristics and the corresponding loadings, even adjusting for estimation-error noise. Therefore, it is legitimate to ask whether the underlying firm characteristics or the factor loadings do a better job of tracking expected returns in the cross-section. Answering this question is the main objective of our paper.

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[1] See related work by Haugen and Baker (1996), Titman, Wei, and Xie (2004) and Cooper, Gulen, Schill (2008) and Hou, Xue, and Zhang (2014) among others.

[2] The regression of HML on the Fama-French factors must produce a perfect fit, with a loading of one on itself and zero on the other factors. Since the HML loading equals the difference between the value (H) and growth (L) portfolio loadings, that difference must equal one. But, of course, each of these portfolio loadings is a weighted average of the loadings for the stocks in the portfolio.

While the economic interpretation of beta pricing can be unclear, determining the underlying causation for the cross-sectional explanatory power of a characteristic can likewise be challenging. For one thing, it is hard to rule out the possibility that the significance of a stock characteristic reflects the fact that it happens to line up well with the betas on some omitted risk factor. But we need not think solely in terms of risk. For example, Fama and French (2014) use observations about the standard discounted cash flow valuation equation to derive predictions about the relation between expected returns and stock characteristics: market equity, the book-to-market ratio, and the expected values of profitability and investment.[3] This approach is more in the spirit of an implied cost of capital and, as they note, the predictions are the same whether the price is rational or irrational.[4]

Understanding what determines observed pricing patterns is undoubtedly important, but it is not the focus of this paper. Whatever the appropriate economic interpretation, important gaps remain in our knowledge about the relevant *empirical* relations. We fill some of those gaps. Whereas Fama and French (1993) and Davis, Fama, and French (2000) argue that it is factor loadings that explain expected returns, Daniel and Titman (1997) contend that it is characteristics. On the other hand, Brennan, Chordia, and Subrahmanyam (1998) present evidence that firm characteristics explain *deviations* from the three-factor model, whereas Avramov and Chordia (2006) find that size and book-to-market have no incremental effect (momentum and liquidity do) when the model's loadings are time varying. However, despite the considerable literature on this subject, we know of no study that directly evaluates how much of the cross-sectional variation in expected returns is accounted for by betas and how much by characteristics in a head-to-head competition. The main goal of this paper is to provide evidence on this issue using appropriate econometric methods.

A number of methodological issues arise in this setting. Indeed, the lack of a consensus on the betas versus characteristics question stems, in part, from issues of experimental design. For example, Brennan Chordia, and Subrahmanyam and Avramov and Chordia work with individual stocks and employ *risk-adjusted* returns as the dependent variable in their cross-sectional regressions (CSRs). In computing the risk-adjustment, the prices of risk for the given

[3] Similarly, Liu, Whited and Zhang (2009) relate expected returns to stock characteristics in a framework based on q-theory.
[4] While linear functions of the lagged values of profitability and investment may serve as rough proxies for the required expectations, a justification for substituting the corresponding factor loadings for the characteristics in this discounted cash flow (or the related q-theoretic) context has, to our knowledge, yet to be articulated.

factors are constrained to equal the factor means and the zero-beta rate is taken to be the risk-free rate. A virtue of this approach is that the well-known errors-in-variables (EIV) problem is avoided since the betas do not serve as explanatory variables. However, while this can be useful for the purpose of model testing, the relative contributions of loadings and characteristics cannot be inferred from such an experiment.

Unlike these papers, we do not impose restrictions on the prices of risk or document patterns of model misspecification. Rather, we evaluate the role of loadings and of characteristics in the cross-sectional return relation that best fits the data when both are included as explanatory variables. Since (excess) returns, not risk-adjusted returns, serve as the dependent variable, in this context, it is important to address the EIV problem. Typically, in asset pricing empirical work, stocks are grouped into portfolios to improve the estimates of beta and thereby mitigate the EIV problem. However, the particular method of portfolio grouping can dramatically influence the results (see Lo and MacKinlay (1990) and Lewellen, Nagel, and Shanken (2010)). Using individual stocks as test assets avoids this somewhat arbitrary element.

Ang, Liu, and Schwarz (2010) also advocate the use of individual stocks, but from a statistical efficiency perspective, arguing that greater dispersion in the cross-section of factor loadings reduces the variability of the risk-premium estimator. Simulation evidence in Kim (1995) indicates, though, that mean-squared error is higher with individual stocks than it is with portfolios, due to the greater small-sample bias, unless the risk premium estimator is corrected for EIV bias.[5] In this paper, we employ EIV corrections that build on the early work of Litzenberger and Ramaswamy (1979), perhaps the first paper to argue for the use of individual stocks, and extensions by Shanken (1992). We also correct for a potential bias that can arise when characteristics are time-varying and influenced by past returns, as is the case for size and several other characteristics. This influence can induce cross-sectional correlation between characteristics and the measurement errors in betas, a complication that, to our knowledge, has not previously been considered.

We conduct our tests for a comprehensive sample of NYSE, AMEX, and NASDAQ stocks over the period 1963-2013. The independent variables in our CSRs consist of loadings as

[5] Ang, Liu, and Schwarz (2010) use an MLE framework with constant betas to develop analytical formulas for EIV correction to standard errors, but they do not address the bias in the estimated coefficients. Also, they seem to implicitly assume that the factor mean is known, which might explain the huge *t*-statistics that they report (see Jagannathan and Wang (2002) for a similar critique in the context of SDF models).

well as firm characteristics. The asset pricing model betas examined in the paper are those of the CAPM, the Fama-French three- and five-factor models, and models that include a momentum factor along with the Fama-French factors. The firm characteristics that we examine are the "classic" characteristics firm size, book-to-market ratio, and past six-month returns, and the additional characteristics investment and the ratio of operating profitability to book equity.

The results point to some evidence of a positive beta premium on the profitability (RMW) and investment factors (CMA), a negative premium on the size factor (SMB), and a less robust positive premium on the market (multifactor, not CAPM beta), but no evidence for the book-to-market (HML) or the momentum (MOM) factors. Also, the estimated zero-beta rates exceed the risk-free rate by at least 6 percentage points (annualized), even with the additional factors and characteristics in the models. Our main finding is that firm characteristics consistently explain a much larger fraction of the variation in estimated expected returns than factor loadings, even in the case of the six-factor model that includes the Fama-French five-factor model augmented by the momentum factor. Moreover, all of the characteristics are reliably different from zero, with the familiar signs.

The rest of the paper is organized as follows. The next section presents the methodology. Section II provides simulation evidence on the finite-sample behavior of the EIV correction that we employ. Section III presents the data and Section IV discusses the results. Section V explores the impact of time-varying premia. Section VI concludes.

I. Methodology

We run CSRs of individual stock returns on their factor loadings and characteristics, correcting for the biases discussed above.

I.A. Underlying model

Time-series regression

Let F_t be a $k \times 1$ vector of factors. The factors may be traded portfolio return spreads, but we do not impose the restriction that their price of risk is equal to the factor mean. Incorporating this restriction makes sense when testing the null hypothesis that an asset pricing model provides an exact description of expected returns. Here, however, we do not presume that the models hold. Rather, our focus is on the competition between factor loadings and characteristics in accounting for empirically observed variation in expected returns with unconstrained cross-sectional coefficients.

Traditionally, factor loadings/betas are estimated through time-series regressions of excess stock returns on the factors:

$$R_{it} = B_{0i} + B_i F_t + \varepsilon_{it}. \tag{1}$$

This regression can be estimated using the entire sample (Black, Jensen, and Scholes (1972)) or rolling windows (Fama and MacBeth (1973)). Rolling betas are intended to capture the time-variation in betas. In this paper we use two years of past daily stock returns to estimate the betas. Two years is a compromise between a shorter period such as one year, which leads to greater estimation errors, and a longer period (say five years) which involves the use of stale information and only permits very slow time variation in the loadings.[6] Later, we examine the robustness of our conclusions to employing, instead, betas that vary from month to month with the values of the stock characteristics.

To account for non-synchronous trading we follow Dimson (1979) in exploring an additional lag of the factors as follows. We first compute the betas and the standard errors for each stock, each month using OLS with at least 400 days of data over the previous two years. Betas are the sum of the contemporaneous and the lagged (if the lagged factor is included) coefficient estimates. To avoid introducing too much noise in the estimation, however, we use the usual OLS estimator (without lags) unless the *t*-statistics on the lagged term is greater than one. We return to this issue below in the context of EIV correction.

A typical asset-pricing relation would specify the expected excess returns in terms of loadings and factor risk premia. Allowing for the possibility that the zero-beta rate is different from the risk-free rate, the asset pricing restriction using time-varying betas can be written as:

$$E_{t-1}\left(R_{it}\right) = \gamma_0 + B_{it-1}\gamma_1, \tag{2}$$

where γ_0 is the excess zero-beta rate over the risk-free rate, and γ_1 is a $k \times 1$ vector of beta premia. As in the more traditional empirical asset pricing literature, we initially consider models

[6] In a previous version, we estimated betas using a conditional model over the entire time series of monthly returns for each stock, with conditioning variables that included macroeconomic variables as well as firm-level attributes. We became concerned, however, about possible look-ahead biases arising from the use of data that follows a cross-sectional regression to estimate the betas that serve as explanatory variables in that regression. This remains a topic for future research. An appendix addressing bias-correction when the month of the CSR is included in the beta estimation period is available upon request.

with constant beta premiums. Documenting the patterns in unconditional return premiums is of interest even if the conditional premiums do vary over time and, in any event, will facilitate comparison with most of the work in this area. As we discuss below, some difficult econometric issues arise when using individual stocks in the cross-sectional analysis and the statistical challenges are greater still with time-varying premiums. In light of this, evaluating the robustness of our results to different specifications seems well advised and so, later in the paper, we explore the impact of relaxing the constant-premium assumption.

Cross-sectional regression

The factor prices of risk are traditionally estimated using a two-pass procedure. We use the Fama and MacBeth (1973) methodology with betas, as well as firm characteristics, as explanatory variables in monthly CSRs. For each month t, given N_t active stocks, define $\hat{B}_{t-1}$ to be the $N_t \times k$ matrix of estimated betas. In addition, let zcs_{it-1} be a $1 \times k_2$ vector of stock characteristics and Zcs_{t-1} the $N_t \times k_2$ matrix of these characteristics. Define the matrix of independent variables, $\hat{X}_t$, in the month t CSR as:

$$\hat{X}_t \equiv [1_{N_t} : \hat{B}_{t-1} : Zcs_{t-1}]. \tag{3}$$

Each month, estimates of the return premia, γ_1 on factor loadings and γ_2 on characteristics, are calculated by running a CSR of excess stock returns R_t on $\hat{X}_t$. Specifically, the cross-sectional coefficients $\hat{\Gamma}_t \equiv \left(\hat{\gamma}_{0t}, \hat{\gamma}'_{1t}, \hat{\gamma}'_{2t}\right)'$, are estimated using OLS as:

$$\hat{\Gamma}_t = \hat{A}_t R_t, \text{ where } \hat{A}_t \equiv (\hat{X}'_t \hat{X}_t)^{-1} \hat{X}'_t \tag{4}$$

is a $\left(1 + k + k_2\right) \times N_t$ matrix. The time-series average of these estimates yields the overall estimate of Γ. The usual asset-pricing null hypothesis of expected return linearity in the loadings implies that the return premium on characteristics, γ_2, is zero. In principle, the average zero-beta rate in excess of the risk-free rate, γ_0, can be different from zero. Employing OLS on individual stocks, rather than a more complicated weighted estimator or portfolio-based approach, is consistent with our aim of evaluating the relative contributions of loadings and characteristics to the expected return for a typical stock.[7]

[7] We will explore the possible benefits of weighted-least squares estimators in future research.

I.B. Errors-in-variables problem

The literature has largely followed the lead of Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) in dealing with the EIV problem by using portfolios as test assets for two-pass estimation. As a result, there are relatively few studies that work with individual stocks in cross-sectional tests. Fama and French (1992) use individual assets but compute factor loadings from test portfolios. This procedure effectively amounts to running CSRs on test portfolios, despite using individual stocks in the second-stage regressions (see Ang, Liu, and Schwartz (2010)).

Bias-corrected coefficients

As is well known, EIV leads to a bias in the estimated coefficients, toward zero when the factors are orthogonal. In our empirical work, we find that the corrections for this bias are sometimes substantial. Our approach builds on Theorem 5 in Shanken (1992), now allowing for heteroskedasticity of ε_{it} conditional on F_t. Leaving some details to Appendix A, the EIV-corrected OLS coefficients are given by:

$$\hat{\Gamma}_t^{\mathrm{EIV}} = \left(\hat{X}_t' \hat{X}_t - \sum_{i=1}^{N_t} M' \hat{\Sigma}_{\hat{B}_{it-1}} M \right)^{-1} \hat{X}_t' R_t \,, \tag{5}$$

where M is a $k \times (1+k+k_2)$ matrix defined as:

$$M = \begin{bmatrix} 0_{k\times 1} & I_{k\times k} & 0_{k\times k_2} \end{bmatrix},$$

and $\hat{\Sigma}_{\hat{B}_{it-1}}$ is the $k \times k$ White (1980) heteroskedasticity-consistent covariance matrix for the OLS time-series estimate of $\hat{B}_{it-1}$. M serves to insert zeros where needed, as the EIV correction only affects the $k \times k$ term $\hat{B}_{t-1}' \hat{B}_{t-1}$. In Section II, we present evidence from simulations indicating that this correction substantially reduces the bias and mean-squared error of the CSR estimator.

In addition to the standard attenuation bias, there may be an additional EIV bias that involves time-varying characteristics. The idea is that returns realized prior to the month of a CSR influence the estimated betas as well as the price-related characteristics (like size or book-to-market) that serve as explanatory variables in that regression. This can induce correlation between the characteristics and the estimation errors in beta that results in a bias if returns are conditionally heteroskedastic or higher-order return moments are time-varying conditional on the factors. This bias is analyzed and a correction is developed in Appendix A.

Finite-sample issues

As discussed above, EIV correction entails subtracting the estimated covariance matrix of the beta estimation errors from $\hat{B}_{t-1}'\hat{B}_{t-1}$, in an attempt to better approximate the matrix $B_{t-1}'B_{t-1}$. There is some chance, however, that this "correction" will overshoot and the diagonal elements will become negative or, more generally, that the estimate of $B_{t-1}'B_{t-1}$ will not be positive definite. This complicates analysis of the finite-sample properties of $\hat{\Gamma}_t^{\text{EIV}}$, but does not seem to be recognized in the literature. Furthermore, problems of this sort are not limited to the form of EIV correction adopted here.

Shanken (1992) shows that simultaneous maximum-likelihood (ML) estimation of betas and return premia provides another means of correcting for EIV bias in a limiting (large N) sense. But Chen and Kan (2004) prove the surprising result that, despite its appealing asymptotic properties, this ML estimator does not have finite moments. To deal with the possibility that the estimator will occasionally produce extreme results, Shanken and Zhou (2007) propose a "truncated" version of ML. With portfolios as test assets, this involves setting the estimator equal to the GLS estimator when the absolute value of the ML estimator is more than some multiple of the GLS estimator (2x and 5x multiples are used). This modified estimator is "virtually unbiased" in their simulations.[8]

More recently, Jegadeesh and Noh (2014) and Pukthuanthong and Roll (2014) have adopted instrumental-variable approaches to dealing with the EIV problem in CSR estimation. The idea is that betas estimated from one subset of the data can serve as instruments for the betas in another subset. The latter paper reports some puzzling results which are attributed to the "weak instrument" problem that can arise with instrumental-variables estimation. The authors note, further, that this problem is "well-known to potentially produce nonsensical results."[9]

Thus, it appears that finite-sample complications are inherent in methods directed at dealing with this sort of EIV bias. Given that the use of individual stocks in CSR analysis is

[8] As they note, estimation of the mean of the usual ML estimator through simulations is problematic when the population mean does not exist. In particular, the simulation averages need not converge in this case.
[9] We have conducted some exploratory simulations comparing the approach adopted here with that of Jegadeesh and Noh (2014). Without going into details, the estimators exhibit similar performance.

highly desirable, these econometric challenges deserve more attention than they have received. We take several exploratory pragmatic steps here in an attempt to deal with these issues. First, to reduce the likelihood of overshooting due to outliers, we Winsorize each element of the matrix subtracted in (5) at the 1% and 99% levels; i.e., the terms for stocks that are more extreme than those levels are replaced with the values at those percentiles. Then we adopt a variant of the approach in Shanken and Zhou (2007). Specifically, we set the EIV-corrected estimator for a given month equal to the OLS CSR estimator whenever the matrix in parentheses in (5) fails to be positive definite. Moreover, since this "denominator" may produce "nonsensical" results even if it is positive definite, but close to zero, we do the following. If the absolute value of the difference between any corrected beta-premium and the corresponding factor realization for a given month exceeds 20%, we again switch to the OLS estimator. Our implementation of the Dimson (1979) approach reduces the occurrence of these "troublesome" months.

I.C. Relative contribution of betas and characteristics

Our main goal is to calculate measures of the relative contribution that loadings or characteristics make toward a combined model's ability to explain cross-sectional expected return variation. We approach this problem in the following way.

We first compute time-series averages of the premia, $\bar{\hat{\gamma}}$, for the factor loadings as well as the characteristics. The motivation is that we are interested in the explanatory power of the model based on the true return premia and the average estimates will better approximate that ideal than the individual monthly estimates. Using these average return premia, we calculate the expected return each month as:

$$E_{t-1}\left[R_t\right] = \bar{\hat{\gamma}}_0 + E_{t-1}^{\text{beta}}\left[R_t\right] + E_{t-1}^{\text{char}}\left[R_t\right],$$

where (6)

$$E_{t-1}^{\text{beta}}\left[R_t\right] = \hat{B}_{t-1}\bar{\hat{\gamma}}_1, \text{ and } E_{t-1}^{\text{char}}\left[R_t\right] = Zcsr_{t-1}\bar{\hat{\gamma}}_2.$$

We then calculate the cross-sectional variance of expected return $E_{t-1}\left[R_t\right]$ using the fitted values in equation (6). Likewise, we compute variances for each component of measured expected return.

A complication arises, however. To see this, note that the cross-sectional variance of the beta-based component of expected returns can be written as $\bar{\hat{\gamma}}_1'\hat{B}_{t-1}'\hat{B}_{t-1}\bar{\hat{\gamma}}_1/N_t - \left(1'\hat{B}_{t-1}\bar{\hat{\gamma}}_1/N_t\right)^2$. As in the CSR context, estimation error in the $k \times k$ term $\hat{B}'_{t-1}\hat{B}_{t-1} / N_t$ gives rise to a systematic bias. Here, it causes cross-sectional variation in the true loadings to be overstated. Fortunately, however, a correction to the variance estimator can be obtained using the same "trick" employed in equation (5), i.e., we subtract the average over i of $\hat{\Sigma}_{\hat{B}_{it-1}}$ from $\hat{B}'_{t-1}\hat{B}_{t-1} / N_t$. Insofar as the average estimation error should be close to zero, a similar correction is not needed for the second term.

The ratio of the variance of expected returns computed using the beta component, $E_{t-1}^{\text{beta}}\left[R_t\right]$, to the variance of expected returns, $E_{t-1}\left[R_t\right]$, based on the full model, gives the contribution that factor loadings make to the explanatory power of the full model in month t. Similarly, the ratio of the variance of expected returns computed using the characteristics component, $E_{t-1}^{\text{char}}\left[R_t\right]$, to the variance of expected returns, $E_{t-1}\left[R_t\right]$, gives the contribution of characteristics to the explanatory power of the model. These ratios are averaged over all months to obtain a more precise aggregate measure. Note that, without the EIV correction discussed above, the role of loadings in the model, relative to that of characteristics, would be exaggerated. Also, keep in mind that the ratios need not add up to one because of covariation between the two components of expected return.[10]

It is important to note that sampling error in our estimates of the relative contributions of betas and characteristics is induced by the fact that we use estimates of the return premia.[11] Deriving analytical formulas for the standard errors of the relative contributions appears to be infeasible, as these ratios are highly non-linear and time-dependent functions of $\bar{\hat{\gamma}}$. However, we can use our knowledge of the (approximately) normal distribution of $\bar{\hat{\gamma}}$ and apply a version of the bias-corrected bootstrap methodology developed in Section 6 of Efron (1987). This approach allows for non-normality of the contribution estimator and the likely dependence of its variance on the true value of the contribution measure. The procedure is as follows. We draw one million normally distributed return premia with moments matched to the average $\hat{\gamma}$'s and their

[10] A similar issue arises when decomposing returns into cash flow news and expected return news, as in Campbell (1991).

[11] Although estimation error in $\hat{\gamma}$ should be the primary source of sampling variability, there is some additional variation due to the fact that betas are estimated with error. Similar to use of the Fama-MacBeth procedure in computing standard errors for the EIV-corrected return premia, our procedure for conducting inference on the contribution measures does not reflect this additional variation.

covariance matrix. The calculations in equation (6) are then repeated for these return premia. In this way, we obtain an empirical distribution for the corresponding difference of relative contributions. A confidence interval for the true contribution difference is then obtained using Efron's method.[12]

II. Simulation Evidence

In order to gauge the statistical properties of our bias-corrected estimator of Γ for the sample sizes employed in empirical work, we resort to simulations. A simple data-generating process is posited, in which returns are governed by a factor model with constant betas:

$$R_{it} = B_i F_t + \varepsilon_{it}. \tag{7}$$

We consider a five-factor Fama and French (2014) model with the factors RMW and CMA in addition to the three factors from Fama and French (1993). At the beginning of the simulation, for each stock, the five-factor betas for the market, SMB, HML, RMW, and CMA are drawn from $N(0.9, 0.4)$, $N(0.8, 0.6)$, $N(0.2, 0.6)$, $N(-0.1, 0.6)$, and $N(0.0, 0.6)$ distributions, respectively. These parameters are based on the distribution of betas estimated with actual data.

To incorporate conditional heteroskedasticity, for each stock we need a function that will map the realized factors for a given month into a corresponding residual variance. This is implemented as follows. We model the residual variance as a linear function of deviations of market return from its mean, and the square of these deviations. The coefficient vectors for this function are drawn at the beginning of the simulation from a normal distribution with mean and standard deviation matched to those from real data (obtained by running a time-series regression of the squared five-factor model residuals on the market return and the squared market return). The simulation then proceeds with the following steps.

First, factor realizations are drawn from a normal distribution with moments matched to the sample moments of daily returns from July 1963 to December 2013. For each simulated stock and each day, we randomly draw a residual return from a normal distribution with mean zero and variance dependent on the market factor return, as just discussed. The actual stock return for the day is computed, as in equation (7), from the stock's betas, the realized factors, and

[12] The "acceleration constant" used in this approach is estimated as in equation (14.15) of Efron and Tibshirani (1993). The assumptions in Efron (1987) may not hold in the extreme case that all variation is explained by either betas or characteristics. These hypotheses can be tested directly, however, using standard CSR methods.

the residual return. Given the resulting time series of simulated returns, betas are estimated with rolling regressions using the past two years of daily data. The data are then aggregated to monthly frequency and the estimated betas are used in monthly second-pass cross-sectional regressions, both with and without correction for biases, as described in Section I.B. For the corrected estimator, we switch to OLS if non-positive-definite issues or outliers are encountered, as discussed earlier. The simulation findings are very close, and conclusions are the same, whether we Winsorize or not.

Simulation results are presented relative to the true coefficient and also relative to the ex-post price of risk (true coefficient + average factor surprise). The latter is just the simulated time-series factor mean here since the true coefficient is taken to be the factor expected value. The average bias and root-mean-square error (RMSE) of the estimators is shown from both perspectives. The ex-post perspective is informative in that it largely removes the component of estimation variance due to the factor surprise, which is not eliminated by EIV correction. Thus, it highlights the impact of EIV correction on the residual component of variability. On the other hand, the ex-ante evaluation relative to the true coefficient gives the bottom line for estimation performance, which is influenced by both forms of variability. RMSE is not empirically observable, however, and so it is of interest to compare the ex-ante RMSE to the Fama and MacBeth (1973) standard error (FMSE), which estimates variability relative to the true coefficient.

Table 1 reports the average bias and RMSE of the estimators, as well as average FMSEs, in percent per month across 1,000 simulations. For reference, the true (simulated) risk premia for the five factors Mkt, SMB, HML, RMW, and CMA equal 0.50%, 0.15%, 0.38%, 0.29%, and 0.28% per month, the respective factor means over the original sample. Consistent with standard EIV analysis, the estimated risk premium without bias correction is always lower, on average, than the true value for each of the five risk premium estimates. For instance, the OLS bias is over 0.12% in the case of the market premium. This negative attenuation bias is not eliminated, or even reduced, as the number of stocks increases. The bias in the corrected risk-premium estimator is close to zero in magnitude (less than 0.01) in each case.

Next, we turn to the RMSEs calculated relative to the factor means. For each factor premium, there is a sharp decline in the EIV-corrected RMSE as the number of stocks increases. This is largely due to a drop in the residual component of variability, as there is very little bias.

The reduction in RMSE is nearly by half for the market factor and more than that for the other factors. We do not see comparable declines for OLS because of the persistent bias. More importantly, the proportionate declines are also much more modest when the EIV-corrected estimator is evaluated relative to the true risk premium. This makes sense since the factor component of variability is substantial and does not systematically decline as the cross-section of stocks increases. We also note that RMSE is almost always lower with EIV correction than without, but the difference is much larger for RMW and CMA than the other factors.

As is well known, FMSEs are consistent (for large T) when there is no measurement error in the betas. Shanken (1992) provides an asymptotic correction to account for measurement error for the case of constant betas and Jagannathan and Wang (1998) develop extensions to deal with non-iid residuals. However, the FMSEs have been found to provide a good approximation to asymptotic standard errors when, as is the case here, traded factors are employed, e.g., Kan, Robotti, and Shanken (2013). In our simulations, the FMSEs are always quite close to the (finite-sample) RMSEs for the EIV-corrected estimator. Insofar as the estimator is close to unbiased, this supports use of the Fama-MacBeth method in calculating standard errors. We adopt this approach throughout the paper.

III. Data

The data consist of monthly returns, size, book-to-market ratio, operating profitability, investment, and lagged six-month returns for a sample of common stocks of NYSE, AMEX, and NASDAQ-listed companies. Book values are from Compustat and are calculated following the procedure described in Fama and French (1992).[13] The rest of the stock data come from CRSP. Factors are downloaded from Ken French's website.

Book-to-market ratio is calculated as the ratio of the most recently available book value of equity divided by the current market capitalization. Operating profitability is the most recently available revenues minus cost of goods sold, minus selling, general and administrative expenses, minus interest expense, all divided by book equity. Investment is defined as the change in total assets from the fiscal year $t-2$ to fiscal year $t-1$, divided by the total assets as of fiscal year $t-1$. All the accounting variables, book value of equity, operating profitability and investment are

[13] Book values from Compustat are supplemented with hand-collected values from Moody's, whenever available (see Davis, Fama, and French (2000) for the exact description of these data). These are available on Kenneth French's website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

assumed to be available six months after the fiscal year-end. For each characteristic, values greater than the 0.99 fractile or less than the 0.01 fractile are set equal to the 0.99 and the 0.01 fractiles each month. We take natural logs of size and book-to-market before using them in time-series or CSRs. The sample spans the period July 1963 to December 2013.

We include only common stocks with share codes 10 or 11 on CRSP. This criterion filters out ADRs, units, American Trust components, closed-end funds, preferred stocks and REITs. Stocks with prices less than one dollar in a month are not included in the CSR for that month (they are included in other months when their prices exceed the one dollar limit). We will present the results for a sample of non-micro-cap stocks as well. Following Fama and French (2008), micro-cap stocks are defined as those with market capitalization lower than the 20th percentile in the cross-section using NYSE breakpoints. The average number of all (non-micro-cap) stocks in the sample is 3,191 or 3,363 (1,493 or 1,551) depending on the specification.

Our CSRs use both factor loadings and characteristics, as independent variables. The firm characteristics are logarithm of market capitalization (Sz), logarithm of the book-to-market ratio (B/M), operating profitability as a fraction of book equity (Profit), and asset growth (Invest) and the logarithm of one plus the last six-month return (Ret6). We lag Ret6 by one additional month to capture the usual momentum effect and avoid short-term reversals.

Table 2 examines the cross-sectional distribution of the first stage factor loading estimates for the Fama and French (2014) factors. We first compute the cross-sectional means, standard deviations and percentiles of the betas each month. Panel A of Table 2 reports the time-series averages of these summary statistics. The standard deviations are corrected for biases due to estimation error in the betas.[14] The cross-sectional average (median) beta for the market factor is 0.94 (0.93); for SMB it is 0.75 (0.68); for HML it is 0.17 (0.19); for RMW it is −0.09 (−0.01) and for CMA it is −0.01 (0.01). The average of the cross-sectional standard deviations is 0.51 for the market beta; 0.75 for the SMB beta; 0.88 for the HML beta; 0.98 for the RMW beta; and 0.97 for the CMA beta.

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[14] Reasoning as in the paragraph below (6), the $k \times k$ cross-sectional second moment matrix of the true betas is approximated by the corresponding matrix of the beta estimates minus the average measurement error covariance matrix. This correction, which is substantial, is used in the standard deviations of betas, as well as the covariances between betas that enter in the cross-sectional correlations in Panel B. In the case of correlation between a characteristic and a beta, a correction for time-varying characteristics is employed in the numerator based on (A.7) and the discussion that follows (this effect turns out to be minimal). It was not clear to us how to correct the percentiles, so we report the raw numbers there. As a result, the 10th and 90th percentiles may be somewhat too extreme.

Panel B of Table 2 presents the time-series averages of the cross-sectional correlations between the different factor loadings and the characteristics. These are also corrected for EIV bias. As expected, the beta for SMB is negatively correlated with firm size, the HML beta is positively correlated with the book-to-market ratio, the profitability beta is positively correlated with operating profits and the investment beta is negatively correlated with investments. The respective correlations are −0.43, 0.33, 0.26, and −0.12 (−0.33, 0.22, 0.16, and −0.08 without EIV correction). Thus, there is considerable independent variation of the characteristics and corresponding factor loadings, permitting identification of their separate effects on expected return.

In addition, size and book-to-market have a correlation of −0.31 and the past six-month return has a correlation of −0.25 with book-to-market. Firm size is positively correlated and book-to-market is negatively correlated with operating profits and investments. The correlation between profitability and investment is 0.27, suggesting that the profitable firms have better opportunities as well as access to internal or external financing.

IV. Cross-sectional Results

We present results for the one-factor CAPM, the Fama and French (1993) three-factor model FF3, the four-factor model which augments the Fama and French (1993) model with the momentum factor (MOM), the five-factor Fama and French (2014) model FF5, and the six-factor model which augments the Fama and French (2014) model with the momentum factor. Separate analysis of these factor models helps in analyzing the additional importance of the various factors. We present the standard Fama and MacBeth (1973) coefficients as well as bias-corrected coefficients side by side in all our results. This facilitates an evaluation of the importance of bias correction to the estimated premia. Finally, we report the Fama and MacBeth (1973) *t*-statistics.

In the next subsection, we present the results for the sample of all stocks and later we will present the results for the sample of non-microcap stocks.

IV.A. All stocks

Since our goal in this paper is to examine the relative contributions of factor loadings and characteristics to expected returns, we will present results for the Fama-MacBeth (1973) regressions that include characteristics along with the betas. However, we have examined the factor models in the absence of the characteristics and first discuss these results.[15] Across all models, from the single-factor CAPM to the six-factor model, the risk premium on the market is negative and statistically insignificant. The risk premiums on SMB, MOM, and RMW are also statistically indistinguishable from zero. This contrasts with the significance of the corresponding means for these factors in Fama and French (2014) and suggests that the associated expected return relation is violated for these models. The risk premium on HML is positive and significant in FF3 and FF5, but is no longer significant when MOM is included in the four- and six-factor models. For instance, the risk premium estimate for HML is 0.35% per month in FF3. The risk premium on CMA is positive and significant. In FF5 this risk premium is 0.27% and it is 0.23% per month in the six-factor model.

Panel A of Table 3 reports results for the factor models when the characteristics Sz, B/M, and Ret6 are included in the Fama-MacBeth regressions. Panel B of Table 3 adds the firm-level characteristics Profit and Invest. With uncorrelated factors, estimation error in the betas would bias all of the estimated risk premiums toward zero. While there is some correlation between the factors, we find nonetheless that correcting the EIV bias generally increases the risk premium estimates, sometimes by over 100%.

Consider first the results in Panel A. In the one-factor and FF3 models, the market beta is not priced when the characteristics are included in the CSRs.[16] In the case of FF5, the market risk premium is 0.42% per month with a *t*-statistic of 2.16. For comparison, the sample average market excess return is 0.50% per month. The beta premium on SMB is negative across all factor models despite its positive sample mean. For instance, in FF3, the premium is −0.29% per month with a *t*-statistic of −2.21. The negative premium may seem odd, but it is important to note that this premium captures the *partial* effect on return of the SMB beta, controlling for the size characteristic and the other variables (similarly for the other factors). With nonzero characteristic premiums, the usual restriction that the beta premiums equal the factor means need not hold under the cross-sectional model.

Unlike the case where the firm-level characteristics were not included in the regressions, the beta premium for HML is now no longer significant, possibly due to competition between the HML beta and the book-to-market ratio. The beta premiums on RMW and CMA are both

[15] These results are available upon request.
[16] For conciseness, we refer to FF3 or FF5 to identify the factors, but the models always include characteristics as well from this point on.

significant, with respective estimates of 0.31 (t-statistic=2.46) and 0.22 (t-statistic=2.33) in FF5 and estimates of 0.26 (t-statistic=2.34) and 0.18 (t-statistic=2.00) in the case of the six-factor model.

The intercepts in second-pass regressions are around 6% to 8% per year, with t-statistics of about four or more. Since characteristics are measured as deviations from NYSE means, the intercepts can be interpreted as the expected return on a zero-beta portfolio with weighted characteristics equal to the NYSE average. Such large differences between the zero-beta rate and the risk-free rate, common in the literature going back to Black, Jensen and Scholes (1972) and Fama and MacBeth (1973), are hard to fully reconcile with more general versions of the CAPM that incorporate restrictions on borrowing.[17]

The premia on firm characteristics are also noteworthy—as usual, large firms earn lower returns, value firms earn higher returns, and firms with higher past returns continue to earn higher returns and the estimates are statistically significant. In economic terms, for the bias-corrected six-factor model, a one standard deviation increase in firm size decreases monthly returns by 28 basis points, a one standard deviation increase in the book-to-market ratio leads to an increase in returns of 24 basis points per month, and a standard deviation increase in the past six month returns raises returns by 43 basis points per month.

The CAPM and FF3 results for the firm characteristics are similar to those in Brennan, Chordia, and Subrahmanyam (1998) and imply rejection of those beta-pricing models. However, Brennan, Chordia, and Subrahmanyam relate *beta-adjusted* returns to characteristics, with risk premiums restricted to equal the factor means and the zero-beta rate equal to the riskless rate. In contrast, we let the loadings and characteristics compete without constraints on the risk premia or the zero-beta rate. What we learn from the new results is that the premia on firm characteristics (specifically Sz, B/M, and Ret6) remain significant even without those constraints and the addition of the factors RMW, CMA and MOM.

There is a controversy in the literature about the interpretation of the size- and value-effects. Fama and French (1993) and Davis, Fama, and French (2000) argue that these empirical phenomena point to the existence of other risk factors, proxied for by SMB and HML. In other words, these studies claim that factor loadings explain cross-sectional variation in expected returns. Daniel and Titman (1997), on the other hand, show that portfolios of firms with similar

[17] See also Frazzini and Pedersen (2013) who show that high zero-beta returns are obtained for most countries.

characteristics but different loadings on the Fama and French factors have similar average returns. They conclude from this finding that it is characteristics that drive cross-sectional variation in expected returns. None of the studies, however, runs a direct horse race between these two competing hypotheses. Our approach using individual stocks is designed to directly address this controversy. We allow both factor loadings and characteristics to jointly explain the cross-section of returns.

The average cross-sectional adj-R^2 values (not reported) are higher when the characteristics are included as independent variables in the cross-sectional regressions than when they are not. This might seem to provide prima-facie evidence about the additional explanatory power of characteristics (beyond market beta) in the cross-section of returns. However, one cannot draw conclusions about the relative explanatory power of characteristics and betas by comparing these adj-R^2s. To see this, consider a scenario in which the *ex-post* coefficient on an explanatory variable is positive (+x, for instance) and significant in half the sample and negative (−x, for instance) and significant in the other half. The computed average of the cross-sectional adj-R^2s could be high even though the coefficient is zero on average and carries no *ex-ante* premium.

To address these problems with adj-R^2s, it is common in the literature to report the adj-R^2 from a single regression of *average* returns on unconditional betas for a set of test asset portfolios (see Kan, Robotti, and Shanken (2013)). This is problematic in our context, as our regressions are for individual stocks with an unbalanced panel dataset. One approach would be to report the adj-R^2 for a regression of average returns on average betas and average characteristics. However, a momentum characteristic averaged over time would display minimal cross-sectional variation and, therefore, its highly significant explanatory power for expected returns would essentially be neglected by such an adj-R^2 measure. For these reasons, we do not report adj-R^2s for our regressions. Instead, we report measures of the relative contributions of loadings or characteristics make toward explaining the variation in expected returns, as discussed in Section I.C.

The last four rows of Table 3 present the contributions made by factor loadings and characteristics, followed by the contribution differences and a 95% bootstrap confidence interval for the latter computed following the procedure in Section I.C.[18] Focusing on the bias-corrected coefficients, we find that the CAPM beta explains only 0.8% and the characteristics explain 104.2% of the cross-sectional variation; in the case of FF3, the betas explain 12% and the characteristics explain 110%; with the four-factor model, the betas explain about 11% and the characteristics 109%; with FF5, it is betas 31% and characteristics 97%; and for the six-factor model, betas explain 24% and characteristics 102%.[19] Clearly, the characteristics explain an overwhelming majority of the variation in expected returns. This is confirmed by the 95% confidence intervals, which, in each case, indicate that the difference is significantly positive at the 5% level. The best showing for beta is in FF5, but even there the point estimate of the difference is 67% and the confidence interval indicates a difference of at least 36%.

The findings when we include the additional firm-level characteristics Profit and Invest in Panel B of Table 3 are very similar. The risk premium on the market beta is not significantly different from zero in the CAPM and the four-factor model, but it is significant in the other cases. For instance, the market risk premium is 0.47% in FF5. The premiums for SMB are still negative, but significant at the 5% level only in the case of FF5. The premium on RMW remains significant in the five- and six-factor models, but the premiums on HML, MOM and CMA are never reliably different from zero. As compared to Panel A, the CMA beta loses its significance, probably due to competition with the corresponding characteristic Invest.

Even with the additional factor loadings included, the characteristic premiums for size, book-to-market, past six-month return, profitability and investment growth are all consistent with the prior literature and highly significant. In economic terms, for the six-factor model, a one standard deviation increase in Sz, B/M, Ret6, Profit, and Invest increases returns by −31, 23, 40, 21, and −22 basis points per month, respectively. Once again, the characteristics explain most of the variation in expected returns for this specification. Similarly, the bootstrap confidence intervals are consistent with a significantly larger fraction of the variation in returns being explained by characteristics as compared to the factor loadings.

IV.B. Non-microcap stocks

[18] A comparison between our results and those in Daniel and Titman (1997) is complicated by the fact that we use past returns as an additional characteristic in our cross-sectional regressions.

[19] Recall that the total percent explained can differ from 100% because of correlation between the components of expected returns due to betas and due to characteristics.

Next, we turn our attention to non-microcap stocks which, following Fama and French (2008), includes all stocks whose market capitalization is larger than that of the 20^{th} percentile of NYSE stocks. Table 4 shows the second-stage CSR beta-premium estimates for the different models as well as the characteristic premiums. Panel A presents the results with the characteristics Sz, B/M, and Ret6 included in the regressions and Panel B includes Profit and Invest along with the characteristics in Panel A. The bias-corrected beta premiums for the market, HML and MOM are not statistically significant in either of the two Panels. However, the premium on SMB is significantly negative in the four-, five-, and six-factor models in Panel A and in the four- and six-factor models in Panel B of Table 4. The premiums on the RMW and CMA betas are generally significant in Panel A of Table 4, but in Panel B only the premium of RMW is significant and that only in the six-factor model. This suggests that the factors RMW and CMA are robustly priced only in the absence of the firm-level characteristics Profit and Invest. All of the characteristic premiums, i.e., those for size, book-to-market, past return, profitability and investment, are statistically and economically significant. The bias-corrected estimates all have *t*-statistics greater than two (and often much larger) in both panels of Table 4.

The economic magnitudes and statistical significance reported thus far indicate that both factor loadings and characteristics matter for non-microcap stocks. But how much variation does each explain? Note, first, that the contribution of factor loadings to the variation in expected returns, as shown in Table 4, increases with the number of factors in the asset pricing models. This contrasts with the all-stock results, where the contribution of betas declined with the addition of MOM to FF5. However, as in Table 3, the contribution of characteristics far exceeds that of the factor loadings in all cases presented in Panels A and B of Table 4. The corresponding differences are statistically significant except for FF5 in Panel A, where the difference of 39.7% is not quite distinguishable from zero at the 5% level, given the wide confidence interval.

IV.D. Additional robustness checks

Recall that, in implementing EIV correction, we to switch to OLS estimation in a given month if the "correction" leads to an $X'X$ estimate that is not positive definite or if the premium estimator is an "outlier," i.e., differs from the factor realization by more than 20%. These issues are encountered only with four or more factors and occur in at most nine months with less than six factors. For the six-factor model, there are 23 not-positive-definite months and eight outliers.

We have also explored 10% and 50% outlier criteria. Not surprisingly, there are many more outliers with 10%, but our main conclusion, that characteristics explain much more variation in expected returns than betas is not sensitive to the treatment of outliers. Individual beta-premium coefficients are occasionally materially affected, however. For example, the premium for RMW in the five-factor model with all characteristics goes from 0.24 (*t*-statistic=2.01) to 0.16 (*t*-statistic=1.45) with a 10% outlier cutoff. There is a larger change for the MOM beta premium in the six-factor model, but none of the estimates is statistically significant.

We have also conducted the analysis without including the correction, described in the appendix, for time-varying characteristics. While the tenor of the results is unchanged, the impact on the magnitude of return premia is occasionally non-trivial (around 30% up or down).

Finally, a conditional time-series regression framework for estimating betas with monthly returns has also been explored. Here, each individual stock beta is allowed to vary as a linear function (for simplicity) of the corresponding characteristic and each stock alpha is a linear function of all the characteristics, similar to the approach in Shanken (1990). Thus, the beta on SMB depends on size, the beta on HML depends on book-to-market, etc. Details are provided in Appendix C. This approach is appealing (in principle), since it directly addresses the possibility that, with betas *assumed* to be constant, the appearance of significant pricing of a characteristic such as size may actually be a reflection of the premium for a time-varying SMB beta.[20] In practice, however, we encountered the not-positive definite problem with greater frequency and found no evidence of beta pricing other than a *t*-statistic of 2.0 on the RMW beta in the six-factor specification.[21] Again, characteristics dominate.

V. Time-Varying Premia

In this section, we consider the possibility that the expected return premia for loadings or cross-sectional characteristics are time varying and we examine the impact that this has on our measures of the relative contributions to cross-sectional expected-return variation.[22] Following Ferson and Harvey (1991), we estimate changing premia via time-series regressions of the monthly CSR estimates on a set of predictive variables. The idea is that the premium estimate for

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[20] See related work by Ferson and Harvey (1998), Lewellen (1999), and Avramov and Chordia (2006)

[21] Concerned about the possibility of noise related to the large number of parameters that must be estimated in these time-series regressions for individual stocks, we also tried zeroing-out estimates of the interaction terms with *t*-statistics less than one. This made little difference in the results.

[22] Gagliardinia, Ossola, and Scaillet (2011) also consider time-varying premia in large cross sections.

a given month is equal to the true conditional premium plus noise. Therefore, regressing that series on relevant variables known at the beginning of each month identifies the expected component.

As predictive variables, x, we use the payout ratio for the S&P 500 defined as the sum of dividends and repurchases divided by price (Payout), the term spread (Term) and the default spread (Def). These variables have frequently been used in predictive regressions for aggregate stock and bond returns, e.g., Fama and French (1989) and Boudoukh, Michaely, Richardson, and Roberts (2007). Thus, the time-series regression of the γ coefficients is:

$$\hat{\gamma}_t = c_0 + c_1' x_{t-1} + \xi_t. \tag{8}$$

Using this time-series regression, each month we calculate the fitted values of the prices of risk and characteristics as $\hat{\gamma}_{t-1}^{fit} = \hat{c}_0 + \hat{c}_1' x_{t-1}$. We then calculate the relative contributions as detailed in Section I.C using the fitted values $\hat{\gamma}_{t-1}^{fit}$, rather than average values, as the expected premia:

$$E_{t-1}[R_t] = \bar{\hat{\gamma}}_0 + E_{t-1}^{\text{beta}}[R_t] + E_{t-1}^{\text{char}}[R_t],$$

where

$$E_{t-1}^{\text{beta}}[R_t] = \hat{B}_{t-1}\hat{\gamma}_{1t-1}^{fit}, \text{and } E_{t-1}^{\text{char}}[R_t] = Zcs_{t-1}\hat{\gamma}_{2t-1}^{fit}. \tag{9}$$

To accommodate the time-varying premiums, the bootstrap procedure in Section I.C is modified as follows. Rather than repeatedly sample (unconditional) premium coefficients, we sample values of the coefficients in equation (8) from a multivariate normal distribution with mean vector equal to the coefficient estimates and covariance matrix equal to the White (1980) heteroskedasticity-consistent asymptotic covariance matrix for those estimates. The sampled coefficient values are then combined with the historical values of the predictive variables to obtain corresponding conditional return premia that are used to recompute values of the contribution numbers based on equation (9).

Table 5 presents evidence on the predictability of the premia, followed by results on the contribution of betas and characteristics to expected-return variation. Given the large number of predictive coefficients, we focus on F-statistics for the relevant *joint* hypotheses. For example, in the case of FF5, we separately regress each of the five time-series of monthly beta-premium estimates on the predictor variables (Payout, Term, and Def) and compute the joint F-statistic across the five regressions (15 slope coefficients). Similarly, we regress each of the characteristic premiums on the predictor variables and obtain the joint F-statistic across the three or five regressions depending on the number of characteristics in the model.

Panel A1 presents the results for all stocks with the characteristics Sz, B/M, and Ret6, while Panel A2 of Table 5 includes the characteristics Profit and Invest as well. Focusing on the bias-corrected estimates of the premiums in Panel A1, we see that the p-values for the F-statistics on the beta premiums are all above 0.2 while for the characteristic premiums they are at most 0.03. This points to time variation in the characteristic premiums but not in the beta premiums. Even so, compared to Panel A of Table 3, the contribution of betas to the variation in expected returns increases while the contribution of characteristics decreases for each model. It is still the case, however, that the contribution of characteristics significantly exceeds that of the betas for every model.

When we add the characteristics Profit and Invest in Panel A2, once again the F-statistics reject the null of no predictability for the characteristic premiums but not the beta premiums, yet the beta contribution increases. Not surprisingly, as compared to Panel A1, with additional characteristics the contribution of betas to the variation in expected returns decreases, while that for characteristics increases. For example, with six factors, the contribution difference increases to 58.3% from 46.1% with the additional characteristics. The confidence intervals show that the contribution of characteristics significantly exceeds that of the factor loadings.

Panels B1 and B2 of Table 5 are the counterparts to Panels A1 and A2, but for the non-micro-cap stocks. In both panels, we again reject the null of no predictability for the characteristic premiums, but not the beta premiums. Compared to Panels A and B of Table 4, the corresponding contribution of the betas to the variation in expected returns is higher and that of the characteristics is lower for all but one of the models. Nonetheless, the contribution of characteristics significantly exceeds that of the factor loadings in all cases.

It is curious that the relative contribution of betas increases despite the evidence of time-variation in the characteristic premia, but not the beta premia. To better understand this, in Appendix B we derive an expression for the difference of the average beta-related components of expected return with and without time-varying premia (likewise for the characteristic component).[23] Let $\Sigma_{\hat{B}_t}$ be the $k \times k$ EIV-corrected cross-sectional covariance matrix of the betas

[23] Although the relative contribution was defined in Section I.C as the average of the monthly contribution ratios, we have also explored using the ratio of the time-series averages of the components, with similar results.

with (i, j) element $\Sigma_{\hat{B}_t}^{ij}$. The beta premium for month t is $\hat{\gamma}_{1t}^{fit}$ and $\bar{\bar{\gamma}}_1$ is the time-series average. Then the contribution of beta to expected return in month t with time-varying premia is the cross-sectional variance $C_t^{TV}\left(\hat{B}_t\right)=\hat{\gamma}_{1t}'^{fit}\Sigma_{\hat{B}_t}\hat{\gamma}_{1t}^{fit}$ and the average contribution over time is (see the appendix):

$$C^{TV}\left(\hat{B}\right)=\sum_i\sum_j \operatorname{cov}\left(\Sigma_{\hat{B}_t}^{ij},\hat{\gamma}_{1t}^{i,fit}\hat{\gamma}_{1t}^{j,fit}\right)+\sum_i\sum_j E\left(\Sigma_{\hat{B}_t}^{ij}\right)\operatorname{cov}\left(\hat{\gamma}_{1t}^{i,fit},\hat{\gamma}_{1t}^{j,fit}\right)+C\left(\hat{B}\right) \tag{10}$$

where the last term, $C\left(\hat{B}\right)$, is the time-series average of $\bar{\bar{\gamma}}_1'\Sigma_{\hat{B}_t}\bar{\bar{\gamma}}_1$, the month t contribution of beta to expected return under constant premia.

Thus, the difference between contributions with time-varying or constant premiums involves two terms. Focusing on the case i=j, the first term depends on comovement over time between the cross-sectional variances of the betas and the *magnitude* of the corresponding beta premiums and can be positive or negative. The second term depends on products of the average beta variances and the time-series variances of the premiums, which unambiguously increase the average contribution of betas. A similar analysis applies if the characteristic vector replaces the betas in (10).

We find that the first covariance term in (10) is small for both betas and characteristics. Given this, if beta premia are constant and characteristic premia time varying, we would expect the *relative* contribution of characteristics to increase and that of betas to decline, contrary to the findings above. However, while the variability of the beta premia is low statistically, i.e., is not sufficient to reject the constant premia hypothesis, the *sample* variability is not zero. In fact, it is large enough to more than double the very low (absolute) contribution of betas in the base-case analysis, the term $C\left(\hat{B}\right)$. On the other hand, the variability in characteristic premia, while far more substantial in absolute terms, raises the already large characteristic contribution to expected returns proportionately less, by about 50%. The result is that the contribution of betas *relative* to that of characteristics increases. But characteristics still dominate betas in explaining cross-sectional expected return variation.

Thus far, we have not considered any small-sample bias of the sort analyzed by Stambaugh (1999). This bias arises when a predictor is autocorrelated and innovations in the predictor are contemporaneously correlated with return surprises. To explore this issue, we use techniques for multiple predictors developed by Amihud and Hurvich (2004) and Amihud, Hurvich, and Wang (2008). The former approach is simpler and assumes that the best forecast of each predictor only requires its own lagged value; correlation between predictors comes only through the forecast errors in this case. The more general approach relaxes this restriction, but at the cost of being a first-order, rather than second-order, correction.

Using the general approach, we find that the estimated contribution differences are larger than those presented in Table 5, by amounts ranging from 2 to 7 percentage points. This suggests that the dominance of characteristics may be even stronger than suggested above. Using the diagonal method, the estimated contribution differences are mostly smaller than those in the tables, and occasionally slightly larger. The implied overstatement in the earlier numbers would be at most 10 percentage points (from 95% earlier to 85%) for the non-microcaps and 3 percentage points for all stocks. Such discrepancies would not alter any of our conclusions.

VI. Summary and Conclusions

Despite strong theoretical and practical reasons for conducting asset pricing tests using individual stocks, there are relatively few studies doing so. The flexibility of the two-pass methodology is an advantage over the more general GMM approach in this context. However, the major difficulty in two-pass regressions is to properly account for the bias introduced by imprecisely estimated individual betas. Therefore, we employ bias-corrected coefficient estimators that are adjusted to reflect these estimation errors. Simulations indicate that our correction for the errors-in-variables bias is effective and also reduces the mean-square error in estimating the beta premia.

We document a number of important findings. As in many other studies, the premium for CAPM beta is not reliably different from zero. The premium for the Fama and French (1993) size factor, SMB, is always negative in the presence of the characteristics, and often reliably so. The premiums on the book-to-market factor, HML, and the momentum factor, MOM, are both statistically insignificant. We find significant premiums for the profitability factor, RMW, and the investment factor, CMA. The premium for CMA loses significance in the presence of the investment characteristic, however. The coefficients on all the characteristics – firm size, book-to-market, six-month past return, profitability and investment – are highly significant across all specifications, with the usual signs.

We also offer new results on the "loadings versus characteristics" controversy. The previous literature has tended to focus on whether it is one or the other that ultimately explains differences in expected returns. In contrast, we provide an intuitive and simple way to disentangle the *relative importance* of betas and firm characteristics in explaining the cross-section of expected returns. Regardless of the factor model and whether we allow the premiums to be time-varying, it is mainly the characteristics that contribute to the cross-sectional variation in expected stock returns.

We have focused on the CAPM, the Fama and French (1993) and the Fama and French (2014) models with and without the momentum factor in this paper. It would be of interest to examine the performance and return premia for other asset pricing models as well, using individual stocks.

References

Amihud, Yakov, and Clifford M. Hurvich, 2004, "Predictive Regression: A Reduced-Bias Estimation Method," *Journal of Financial and Quantitative Analysis* 39, 813–841.

Amihud, Yakov, Clifford M. Hurvich, and Yi Wang, 2008, "Multiple-Predictor Regressions: Hypothesis Testing," *Review of Financial Studies* 22, 413–434.

Ang, Andrew, Jun Liu, and Krista Schwarz, 2010, "Using Stocks or Portfolios in Tests of Factor Models," Working paper, Columbia University.

Avramov, Doron, and Tarun Chordia, 2006, "Asset Pricing Models and Financial Market Anomalies," *Review of Financial Studies* 19, 1001–1040.

Black, Fischer, Michael C. Jensen, and Myron Scholes, 1972, "The Capital Asset Pricing Model: Some Empirical Tests," in M. C. Jensen, ed., *Studies in the Theory of Capital Markets*, pp. 79–121. (Praeger, New York).

Brennan, Michael, Tarun Chordia, and Avanidhar Subrahmanyam, 1998, "Alternative Factor Specifications, Security Characteristics and the Cross-Section of Expected Stock Returns," *Journal of Financial Economics* 49, 345–373.

Boudoukh, Jacob, Roni Michaely, Matthew Richardson, and Michael R. Roberts, 2007, "On the Importance of Measuring Payout Yield: Implications for Empirical Asset Pricing," *Journal of Finance* 62, 877–915.

Campbell, John Y., 1991, "A Variance Decomposition for Stock Returns," *Economic Journal* 101, 157–179.

Carhart, Mark, 1997, "On Persistence in Mutual Fund Performance," *Journal of Finance* 52, 57–82.

Cooper, Michael J., Huseyin Gulen, and Michael J. Schill, 2008, "Asset Growth and the Cross-Section of Stock Returns," *Journal of Finance* 63, 1609–1651.

Daniel, Kent, and Sheridan Titman, 1997, "Evidence on the Characteristics of Cross-Sectional Variation in Common Stock Returns," *Journal of Finance* 52, 1–33.

Davis, James, Eugene F. Fama, and Kenneth R. French, 2000, "Characteristics, Covariances, and Average Returns: 1929-1997," *Journal of Finance* 55, 389–406.

Dimson, E., 1979, "Risk Measurement when Shares are Subject to Infrequent Trading," *Journal of Financial Economics* 7, 197–226.

Efron, Bradley, 1987, "Better Bootstrap Confidence Intervals," *Journal of the American Statistical Association* 82, 171–185.

Efron, Bradley and Robert Tibshirani, 1993, *An Introduction to the Bootstrap*, Chapman & Hall.

Fama, Eugene F., and Kenneth R. French, 1989, “Business Conditions and Expected Returns on Stocks and Bonds,” *Journal of Financial Economics* 25, 23–49.

Fama, Eugene F., and Kenneth R. French, 1992, “The Cross-Section of Expected Stock Returns,” *Journal of Finance* 47, 427–465.

Fama, Eugene F., and Kenneth R. French, 1993, “Common Risk Factors in the Returns on Stocks and Bonds,” *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and Kenneth R. French, 2008, “Dissecting Anomalies,” *Journal of Finance* 63, 1653–1678.

Fama, Eugene F., and Kenneth R. French, 2012, “Size, Value, and Momentum in International Stock Returns,” *Journal of Financial Economics* 105, 457–472.

Fama, Eugene F., and Kenneth R. French, 2014, “A Five-Factor Asset Pricing Model,” Working paper.

Fama, Eugene F., and James D. MacBeth, 1973, “Risk, Return and Equilibrium: Empirical Tests,” *Journal of Political Economy* 81, 607–636.

Ferson, Wayne E., and Campbell R. Harvey, 1991, “The Variation of Economic Risk Premiums,” *Journal of Political Economy* 99, 385–415.

Ferson, Wayne E., and Campbell R. Harvey, 1998, “Fundamental Determinants of National Equity Market Returns: A Perspective on Conditional Asset Pricing,” *Journal of Banking and Finance* 21, 1625–1665.

Ferson, Wayne E., and Campbell R. Harvey, 1999, “Conditioning Variables and the Cross-Section of Stock Returns,” *Journal of Finance* 54, 1325–1360.

Frazzini, Andrea, and Lasse Heje Pedersen, 2011, “Betting Against Beta,” Working paper, New York University.

Gagliardinia, Patrick, Elisa Ossola, and Olivier Scaillet, 2011, “Time-Varying Risk Premium in Large Cross-Sectional Equity Datasets,” Working paper, Swiss Finance Institute.

Haugen, Robert A., and Nardin L. Baker, 1996, “Commonality in the Determinants of Expected Stock Returns,” *Journal of Financial Economics* 41, 401–439.

Hou, Kewei, Chen Xue, and Lu Zhang, 2014, Digesting anomalies: An investment approach, Review of Financial Studies.

Jagannathan, Ravi, and Zhenyu Wang, 1998, "An Asymptotic Theory for Estimating Beta-Pricing Models Using Cross-Sectional Regressions," *Journal of Finance* 53, 1285–1309.

Jagannathan, Ravi, and Zhenyu Wang, 2002, "Empirical Evaluation of Asset-Pricing Models: A Comparison of the SDF and Beta Methods," *Journal of Finance* 57, 2337–2367.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance* 48, 65–92.

Kan, Raymond, Cesare Robotti, and Jay Shanken, 2013, "Pricing Model Performance and the Two-Pass Cross-Sectional Regression Methodology," *Journal of Finance* 68, 2617–2649.

Kim, Dongcheol, 1995, "The Errors in the Variables Problem in the Cross-Section of Expected Stock Returns," *Journal of Finance* 50, 1605–1634.

Lewellen, Jonathan W., 1999, "The Time-Series Relations Among Expected Return, Risk, and Book-to-Market.," *Journal of Financial Economics* 54, 5–43.

Lewellen, Jonathan W., Jay Shanken, and Stefan Nagel, 2010, "A Skeptical Appraisal of Asset Pricing Tests," *Journal of Financial Economics* 96, 175–194.

Lintner, John, 1965, "Security Prices, Risk and Maximal Gains from Diversification," *Journal of Finance* 20, 587–616.

Litzenberger, Robert H., and Krishna Ramaswamy, 1979, "The Effect of Personal Taxes. and Dividends on Capital Asset Prices: Theory and Empirical Evidence," *Journal of Financial Economics* 7, 163–196.

Liu, Laura, 2009, Toni Whited, and Lu Zhang, "Investment-Based Expected Stock Returns, Journal of Political Economy 117, 1105–1139.

Lo, Andrew, and A. Craig MacKinlay, 1990, "Data-Snooping Biases in Tests of Financial Asset Pricing Models," *Review of Financial Studies* 3, 431–468.

MacKinlay, A. Craig, 1995, "Multifactor Models Do Not Explain Deviations from the CAPM," *Journal of Financial Economics* 38, 3–28.

Mossin, Jan, 1966, "Equilibrium in a Capital Asset Market," *Econometrica* 34, 768–783.

Roll, Richard, 1977, "A Critique of the Asset Pricing Theory's Tests Part I: On Past and Potential Testability of the Theory," *Journal of Financial Economics* 4, 129–176.

Sharpe, William, 1964, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* 19, 425–442.

Shanken, Jay, 1990, "Intertemporal Asset Pricing: An Empirical Investigation," *Journal of*

Econometrics 45, 99–120.

Shanken, Jay, 1992, "On the Estimation of Beta-Pricing Models," *Review of Financial Studies* 5, 1–33.

Shanken, Jay, and Guofu Zhou, 2007, "Estimating and Testing Beta-Pricing Models: Alternative Methods and Their Performance in Simulations," *Journal of Financial Economics* 84, 40–86.

Stambaugh, Robert F., 1999, "Predictive Regressions," *Journal of Financial Economics* 54 375–421.

Titman, Sheridan, John K.C. Wei, and Feixue Xie, 2004, "Capital Investments and Stock Returns," *Journal of Financial and Quantitative Analysis* 39, 677–700.

White, Halbert, 1980, "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica* 48, 817–838.

Appendix A: Bias correction

We analyze the OLS CSR estimator for month *t*, $\hat{\Gamma}_t = (\hat{X}_t'\hat{X}_t)^{-1}\hat{X}_t'R_t$, where $\hat{X}_t \equiv [1_{N_t} : \hat{B}_{t-1} : Zcs_{t-1}]$, as in equation (3). Let U_{t-1} be the $N_t \times k$ matrix of estimation errors in the betas, $\hat{B}_{t-1}$. Initially, we assume the true betas and characteristics (the matrix *X*) are constant, as in Shanken (1992). Reasoning as in Theorem 5 and Lemma 2 of that paper, first note that since $E\left(U_{t-1}\right) = 0$, we also have $E\left(U_{t-1}'X\right) = 0$, so this term does not systematically deviate from 0. EIV correction to the "denominator" $\hat{X}_t'\hat{X}_t$ then amounts to subtracting an estimate of the expected value of $U_{t-1}'U_{t-1}$ from the $\hat{B}_{t-1}'\hat{B}_{t-1}$ submatrix of $\hat{X}_t'\hat{X}_t$, so as to better approximate the true $X'X$.[24] Let $U_{it-1} = \hat{B}_{it-1} - B_{it-1}$ be the i^{th} row of U_{t-1}. Then

$$E(U_{t-1}'U_{t-1}) = \sum_{i=1}^{N_t} E(U_{it-1}'U_{it-1}) = \sum_{i=1}^{N_t} Cov(U_{it-1}'), \tag{A.1}$$

where $Cov(U_{it-1}')$ is the covariance matrix of the estimator of B_{it-1}. To accommodate conditional heteroskedasticity, we use the White covariance matrix estimator of $\hat{B}_{it-1}$ here.[25] This gives the correction to the denominator of $\hat{\Gamma}_t$.

The numerator $\hat{X}_t'R_t$ does not require a correction with constant *X*. To see why, note that:

$$\hat{X}_t'R_t = X'R_t + \left[0 : U_{t-1} : 0\right]' R_t \ . \tag{A.2}$$

It is convenient to write the return vector as:

$$R_t = X'\Gamma + \varepsilon_t, \tag{A.3}$$

where $\Gamma_t = \left(\gamma_0, \gamma_1', \gamma_2'\right)'$ includes the ex-post price of risk vector, $\gamma_{1t} = \gamma_1 + F_t - E\left(F\right)$.[26] Then

$$[0 : U_{t-1} : 0]'R_t = [0 : U_{t-1} : 0]'X\Gamma_t + [0 : U_{t-1} : 0]'\varepsilon_t. \tag{A.4}$$

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[24] More specifically, the idea is to approximate the large N_t limits of each component of $\left(X_t'X_t/N_t\right)^{-1}\left(X_t'R_t/N_t\right)$. Limiting arguments for the simpler specification in Shanken (1992) require that cross-sectional correlation in the disturbances is not too high, so that laws of large numbers kick in. This allows for the possibility that the factor model disturbances are influenced by additional common factors. For example, a block diagonal residual covariance matrix with (bounded) blocks corresponding to industries or some other shared property is permitted.

[25] We appeal to large-*T* asymptotic results here based on our conditional mean assumption for the return disturbances. For those stocks where we employ the Dimson estimator of beta, we use the variance of the sum of betas as the measurement-error variance matrix.

[26] We can also allow for a vector of deviations from the expected return relation as in Shanken and Zhou (2007), provided the vector is fairly diffuse.

As above, $E\left(U_{t-1}'X\right)=0$ with constant X. Given that the betas are estimated using data before month t and returns and factors are assumed to be independent over time, $E\left(U_{t-1}'X\right)=0$ as well. Thus, no correction is needed for these terms. It follows that the EIV-corrected estimator $\hat{\Gamma}_t^{\mathrm{EIV}}$ is as given in equation (5) of the text.

When $\hat{X}_t$ includes time-varying price-related characteristics, Zcs_{t-1}, like those employed in our empirical application, there is the possibility of additional bias arising from the submatrix $U_{t-1}'Zcs_{t-1}$ of $U_{t-1}'X_t$. This term occurs in both the denominator $\hat{X}_t'\hat{X}_t$ and, by (A.4), the numerator $\hat{X}_t'R_t$ of $\hat{\Gamma}_t$. As earlier, U_{t-1} is $N_t \times k$ and Zcs_{t-1} is $N_t \times k_2$, where k is the number of factors (betas) and k_2 the number of characteristics. To make the analysis tractable, we retain the constant beta assumption and assume, without loss of generality, that *Zcs* consists of a single column, *Z*. Then:

$$U_{t-1}'Z_{t-1} = \sum_{i=1}^{N_t} U_{it-1}'Z_{it-1}\,, \tag{A.5}$$

where U_{it-1} is the i^{th} row of U_{t-1} and Z_{it-1} is the i^{th} element of Z_{t-1}. Since estimation error in the betas can be written as:

$$U_{it-1}' = (F_d'F_d)^{-1}\sum\nolimits_s F_{ds}'\varepsilon_{is}, \tag{A.6}$$

we have

$$U_{it-1}'Z_{it-1} = (F_d'F_d)^{-1}\sum\nolimits_s F_{ds}'\varepsilon_{is}Z_{it-1}\,, \tag{A.7}$$

where s varies over the two years of daily trading days prior to month t (time $t-1$).[27] F_d is the matrix of factor values (k columns) used in beta estimation, expressed as deviations from the factor sample means, and F_{ds} is row s of F_d.

We focus on the term $F_{ds}'\varepsilon_{is}Z_{it-1}$. Note that if Z_{it-1} is influenced by ε_{is}, e.g., past returns affect current market cap, and ε_{is} is heteroskedastic conditional on the contemporaneous factors, then this product need not have mean zero (higher conditional moments might be relevant as well). For simplicity, we model the daily series Z_i as a stationary AR(1) series,

$$Z_{im} - \mu_{iz} = \rho(Z_{im-1} - \mu_{iz}) + \nu_{im}\,, \tag{A.8}$$

[27] In this context and in the relations below, we think of $t-1$ as denoting the last trading day in month $t-1$ so that it can be combined with the daily index s. For example, if we're running a CSR with March (month t) returns, then $t-1$ is the last trading day in February (month $t-1$).

where ν_{im} has mean zero conditional on all past information. Then Z_i can be written as a weighted sum of the current and past independent innovations with geometrically declining weights:

$$Z_{im} - \mu_{iz} = \nu_{im} + \rho_i \nu_{im-1} + \rho_i^2 \nu_{im-2} + \rho_i^3 \nu_{im-3} + \ldots. \tag{A.9}$$

It follows that $E(F'_{ds}\varepsilon_{is}\nu_{im}) = 0$ for $m \neq s$. Now, the time s component of $Z_{it-1} - \mu_{iz}$ is $\rho_i^{t-1-s}\nu_{is}$. Therefore, the expected value of $F'_{ds}\varepsilon_{is}Z_{it-1}$ equals the expected value of $\rho_i^{t-1-s}F'_{ds}\varepsilon_{is}\nu_{is}$ (both conditional on F). Since the disturbance terms are not observed, we replace ε_{is} and ν_{is} by the corresponding OLS residuals from the time-series factor-model regression and the AR(1) equation, respectively. Given equations (A.5)–(A.7), the corresponding correction entails summing the estimate of $\rho_i^{t-s-1}F'_{ds}\varepsilon_{is}\nu_{is}$ over all $s < t$ in the sample for stock i, pre-multiplying the result by $(F'_d F_d)^{-1}$, then summing these terms over all stocks i in the CSR for month t. We execute these steps separately for each column of Zcs. Then we subtract the resulting $k \times k_2$ matrix from the $\hat{B}'_{t-1}Zcs_{t-1}$ block (similarly for the transpose) of $\hat{X}'_t\hat{X}_t$.

We use the autoregressive model for the size and B/M characteristics. In the case of Ret6, we ignore compounding and treat Z_{it-1}, the past six-month return (skipping the most recent month) as the sum of the daily returns R_{is}, and so the expected value of $F'_{ds}\varepsilon_{is}Z_{it-1}$ is the expected value of $F'_{ds}\varepsilon_{is}^2$ (both conditional on F). The correction then involves summing these terms over all days s from the beginning of month $t-7$ to the end of month $t-2$ and otherwise proceeding as above in the autoregressive case. Since the investment and profitability characteristics are not *directly* price-based, implementing an adjustment for possible biases is more challenging and is left to future research.

An additional internet appendix is available for those who are interested in *calculation* of the corrected estimator, rather than its derivation.

Appendix B: Time-varying contributions

To prove (10), following the notation of Section 5, we have

$$C^{TV}\left(\hat{B}\right)=\frac{1}{T}\sum_t C_t^{TV}\left(\hat{B}_t\right)=\frac{1}{T}\sum_t \hat{\gamma}_{1t}'^{fit}\,\Sigma_{\hat{B}_t}\,\hat{\gamma}_{1t}^{fit}=\frac{1}{T}\sum_t\sum_i\sum_j \Sigma_{\hat{B}_t}^{ij}\hat{\gamma}_{1t}^{i,fit}\hat{\gamma}_{1t}^{j,fit}$$
$$=\sum_i\sum_j\frac{1}{T}\sum_t \Sigma_{\hat{B}_t}^{ij}\hat{\gamma}_{1t}^{i,fit}\hat{\gamma}_{1t}^{j,fit}$$
$$=\sum_i\sum_j \operatorname{cov}\left(\Sigma_{\hat{B}_t}^{ij},\hat{\gamma}_{1t}^{i,fit}\hat{\gamma}_{1t}^{j,fit}\right)+\sum_i\sum_j E\left(\Sigma_{\hat{B}_t}^{ij}\right)E\left(\hat{\gamma}_{1t}^{i,fit}\hat{\gamma}_{1t}^{j,fit}\right)$$
$$=\sum_i\sum_j \operatorname{cov}\left(\Sigma_{\hat{B}_t}^{ij},\hat{\gamma}_{1t}^{i,fit}\hat{\gamma}_{1t}^{j,fit}\right)+\sum_i\sum_j E\left(\Sigma_{\hat{B}_t}^{ij}\right)\operatorname{cov}\left(\hat{\gamma}_{1t}^{i,fit},\hat{\gamma}_{1t}^{j,fit}\right)+\sum_i\sum_j E\left(\Sigma_{\hat{B}_t}^{ij}\right)\bar{\hat{\gamma}}_1^{\,i}\bar{\hat{\gamma}}_1^{\,j},$$

where E and cov refer to time-series sample averages and covariances, respectively, and the last term is the average of the month t contributions with constant premiums, $C_t\left(\hat{B}_t\right)=\bar{\hat{\gamma}}_1'\Sigma_{\hat{B}_t}\bar{\hat{\gamma}}_1$.

Appendix C: Bias correction with conditional betas

Let zts_{it} be a $p\times 1$ vector of firm-specific characteristics (macro variables are accommodated by getting rid of the i subscript). The first element of the zts vector is a constant, so $p\geq 1$; let $zts_{(p-1)it}$ denote the corresponding $(p-1)\times 1$ subvector that excludes the constant. It is useful to define F_{it}^* as:

$$F_{it}^*\equiv[zts_{(p-1)it-1}, zts_{i1t-1}F_t',\ \ldots, zts_{ipt-1}F_t']', \tag{C.1}$$

a $(p-1+kp)\times 1\equiv k^*\times 1$ vector of independent variables. Then our time-series model for excess stock returns R_{it} can be compactly represented as:

$$R_{it}=B_{0i}^*+B_i^*F_{it}^*+\varepsilon_{it}, \tag{C.2}$$

where B_{0i}^* is a scalar and B_i^* is a $1\times k^*$ vector of slope coefficients on the *expanded factors*, i.e., the scaled intercept (excluding the constant) and the scaled factors. In effect, we allow for the possibility that the intercept, as well as the betas on each of the factors, vary with (lagged) firm characteristics. We can recover the time-varying betas implied by this model as follows. Define the $k^*\times k$ matrix Zts_{it} as:[28]

$$Zts_{it}=\begin{bmatrix}0_{p-1\times k}\\ I_k\otimes zts_{it}\end{bmatrix}.$$

[28] The submatrix of zeroes captures the fact that the scaled intercept coefficients are not needed here. The $\otimes$ reflects the fact that the beta on each original factor is linear in the same conditioning variables.

Then the $1 \times k$ vector of implied betas B_{it-1} on the original factors is given as a function of the lagged firm characteristics by:

$$B_{it-1} \equiv B_i^* Zts_{it-1} . \tag{C.3}$$

Note that the original time-series model can be rewritten as:

$$R_{it} = (B_{0i}^* + B_{i1:p-1}^* zts_{(p-1)it-1}) + B_{it-1} F_t + \varepsilon_{it} , \tag{C.4}$$

with both the intercept and betas time-varying. Here, $B_{i1:p-1}^*$ is the subvector of B_i^* consisting of the first $p-1$ components.

Note that with betas linearly related to variables that are also included as cross-sectional characteristics, there is an identification issue. The corresponding beta and characteristic premia would not be separately estimable if the time-series relations were the same for each stock, as this would create perfect multicollinearity in the CSRs. Therefore, identification of γ_1 and γ_2 requires some cross-sectional variation in the relevant elements of the $B_i^* s$, which we estimate individually for each stock.

We assume that the return disturbances have zero mean conditional on all information known at the beginning of the month, as well as the contemporaneous factors. Let $U_i^* \equiv \hat{B}_i^* - B_i^*$ be the $1 \times k^*$ vector of estimation errors in the time-series slope coefficients from equation (C.2) for stock i. For each month t, the estimator $\hat{B}_i^*$ (subscript t omitted for simplicity) is obtained for each of the N_t relevant stocks from an OLS time-series regression over the period from $t-60$ to $t-1$. As suggested by (C.3), this estimate is then multiplied by the time $t-1$ matrix of characteristic values to get the conditional beta estimate, $\hat{B}_{it-1}$ ($1 \times k$), to be used in the month t CSR.[29] Let $\hat{B}_{t-1}$ be the $N_t \times k$ stacked matrix of these estimates. U_t is the corresponding matrix of estimation errors, with i^{th} row $U_{it} = \hat{B}_{it-1} - B_{it-1}$. Then we have:

$$U_{it} = \left(\hat{B}_i^* - B_i^*\right) Zts_{it-1} = U_i^* Zts_{it-1} \tag{C.5}$$

and

[29] The returns and (original) factors in this regression go from $t-60$ to $t-1$, while the predictive variables (*Zts*) go from $t-61$ to $t-2$. As mentioned in the paper, we similarly estimate a "recursive" specification using all available (but at least five years) past data in the time-series regressions.

$$E(U_t'U_t) = \sum_{i=1}^{N_t} E(U_{it}'U_{it}) = \sum_{i=1}^{N_t} Zts_{it-1}' E\left(U_{it}'^{*}U_{it}^{*}\right) Zts_{it-1}, \tag{C.6}$$

where

$$E\left(U_{it}'^{*}U_{it}^{*}\right) \equiv \operatorname{cov}\left(U_{it}'^{*}\right) \tag{C.7}$$

is the covariance matrix of the estimator of B_{it}^{*}. To accommodate conditional heteroskedasticity, we use the White estimator of the covariance matrix in equation (C.7). Then, substituting

$$\hat{\Sigma}_{\hat{B}_{it-1}} = Zts_{it-1}' \hat{\Sigma}_{\hat{B}_i^{*}}^{\text{White}} Zts_{it-1}$$

in equation (5) gives the EIV-corrected estimator with conditional betas.[30] The correction for time-varying cross-sectional characteristics in Appendix A can also be modified to accommodate conditional betas.

[30] We have treated the case in which each beta depends on all of the characteristics in *Zts*. If some of the predictive coefficients are constrained to equal zero, as in our application, one need only redefine the set of expanded factors to include only those for which the coefficients are nonzero. The matrix of betas and *Zts* are similarly redefined. One then proceeds as above.

Table 1: Simulation Results

The data generating process is:

$$R_{it} = B_i F_t + \varepsilon_{it} ,$$

where we use five Mkt, SMB, HML, RMW, and CMA factors. Mkt, SMB, and HML are the three Fama and French (1993) factors, and RMW and CMA are the two additional factors in Fama and French (2014). At the beginning of the simulation, the five betas are drawn from $N(0.9,\ 0.4)$, $N(0.8,\ 0.6)$, $N(0.2,\ 0.6)$, $N(-0.1,\ 0.6)$, and $N(0.0,\ 0.6)$ distributions, respectively. Daily factor realizations are drawn from a normal distribution with moments matched to the sample moments from July 1963 to December 2013. The residuals are drawn from a normal distribution with mean zero and conditionally heteroskedastic standard deviation following the procedure described in the text. Betas are estimated from a rolling time-series regressions using the past two years of daily data. The data are then aggregated to monthly frequency and the estimated betas are used in monthly second-pass cross-sectional regressions. The table reports the mean bias, mean Fama-MacBeth standard errors (FMSE), and root mean squared error (RMSE), across 1,000 simulations, in percent per month of the estimated risk premiums, both with and without EIV correction. The bias and RMSE are presented both for the simulation less the factor average in the simulation as well as for simulation less the true factor means. The ex-post risk premia for Mkt, SMB, HML, RMW, and CMA are 0.50%, 0.15%, 0.38%, 0.29%, and 0.28% per month, respectively. These serve as the ex-ante premia in the simulations.

Number of stocks	Actual – Factor Mean				Actual – Truth					
	Bias		RMSE		Bias		RMSE		FMSE	
	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV
	Premia on B_{Mkt}									
500	–0.1287	0.0057	0.1503	0.0878	–0.1310	0.0033	0.2207	0.2029	0.1827	0.2092
1,000	–0.1210	0.0050	0.1363	0.0688	–0.1126	0.0134	0.2073	0.1967	0.1772	0.1999
2,500	–0.1310	0.0029	0.1402	0.0521	–0.1326	0.0013	0.2172	0.1953	0.1724	0.1952
5,000	–0.1376	–0.0008	0.1454	0.0462	–0.1415	–0.0047	0.2211	0.1938	0.1706	0.1940
	Premia on B_{SMB}									
500	–0.0682	0.0020	0.0869	0.0651	–0.0708	–0.0006	0.1188	0.1192	0.0923	0.1150
1,000	–0.0612	0.0021	0.0730	0.0458	–0.0614	0.0019	0.1069	0.1067	0.0865	0.1058
2,500	–0.0655	0.0005	0.0723	0.0324	–0.0620	0.0039	0.1033	0.1000	0.0832	0.1008
5,000	–0.0662	0.0008	0.0719	0.0269	–0.0666	0.0004	0.1048	0.0982	0.0818	0.0993
	Premia on B_{HML}									
500	–0.0620	–0.0016	0.0822	0.0684	–0.0570	0.0034	0.1055	0.1133	0.0882	0.1122
1,000	–0.0803	0.0033	0.0905	0.0505	–0.0847	–0.0011	0.1197	0.1064	0.0819	0.1023
2,500	–0.0792	0.0009	0.0853	0.0335	–0.0742	0.0059	0.1076	0.0961	0.0786	0.0968
5,000	–0.0812	0.0003	0.0858	0.0275	–0.0841	–0.0025	0.1136	0.0941	0.0773	0.0950
	Premia on B_{RMW}									
500	–0.1506	0.0059	0.1601	0.0807	–0.1483	0.0083	0.1625	0.1041	0.0668	0.1045
1,000	–0.1335	0.0031	0.1384	0.0457	–0.1320	0.0046	0.1439	0.0810	0.0585	0.0827
2,500	–0.1290	0.0032	0.1326	0.0335	–0.1352	–0.0030	0.1453	0.0756	0.0529	0.0754
5,000	–0.1365	0.0006	0.1391	0.0269	–0.1314	0.0057	0.1410	0.0722	0.0514	0.0721
	Premia on B_{CMA}									
500	–0.0867	0.0051	0.0990	0.0764	–0.0873	0.0045	0.1092	0.1015	0.0650	0.0998
1,000	–0.0930	0.0046	0.1009	0.0541	–0.0956	0.0020	0.1105	0.0813	0.0576	0.0852
2,500	–0.0854	0.0040	0.0905	0.0348	–0.0828	0.0066	0.0982	0.0749	0.0529	0.0738
5,000	–0.0911	0.0016	0.0952	0.0268	–0.0906	0.0022	0.1039	0.0704	0.0512	0.0703

Table 2: Descriptive statistics

We estimate the 5-factor model where the factors are Mkt, SMB, HML, RMW, and CMA. Mkt, SMB, and HML are the three Fama and French (1993) factors, and RMW and CMA are the two additional factors in Fama and French (2014). Betas are estimated from rolling time-series regressions using past two years of daily data; we require at least 400 observations to estimate beta. Panel A presents descriptive statistics on these betas. We first take cross-sectional means, standard deviations, and percentiles on these betas each month and then report time-series averages of these statistics. Panel B reports the time-series average of the cross-sectional correlations between betas and other stock characteristics. The cross-sectional variables include size (Sz), book-to-market (B/M), last six-month return (Ret6), operating profitability (Profit), and investment (Invest). Sz is the logarithm of market capitalization. B/M is the logarithm of the ratio of most recently available book-value (assumed available six months after fiscal year-end) divided by the current market capitalization. Profit is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Invest is the change in total assets divided by current total assets. For each characteristic, values greater than the 0.99 fractile or less than the 0.01 fractile are set equal to the 0.99 and the 0.01 fractiles each month. The sample period is July 1963 to December 2013.

Panel A: Cross-sectional statistics

	B_{Mkt}	B_{SMB}	B_{HML}	B_{RMW}	B_{CMA}
Mean	0.944	0.754	0.173	−0.085	−0.009
Sdev	0.512	0.750	0.879	0.982	0.968
10%	0.299	−0.118	−0.857	−1.271	−1.115
25%	0.593	0.235	−0.295	−0.569	−0.499
Median	0.930	0.684	0.186	−0.011	0.008
75%	1.273	1.201	0.655	0.482	0.498
90%	1.599	1.729	1.171	0.993	1.072

Panel B: Correlations

	B_{SMB}	B_{HML}	B_{RMW}	B_{CMA}	Sz	B/M	Ret6	Profit	Invest
B_{Mkt}	0.430	0.047	−0.068	−0.058	0.276	−0.200	−0.038	0.024	0.115
B_{SMB}		0.119	0.043	0.011	−0.429	−0.041	−0.047	−0.147	−0.002
B_{HML}			0.422	−0.244	−0.057	0.327	0.026	0.034	−0.103
B_{RMW}				0.111	0.040	0.068	0.060	0.255	0.019
B_{CMA}					−0.093	0.007	0.031	−0.043	−0.120
Sz						−0.307	0.143	0.252	0.140
B/M							−0.252	−0.095	−0.174
Ret6								0.060	−0.050
Profit									0.268

Table 3: Cross-sectional regression of all stocks

This table presents the time-series averages of γ coefficients from the following individual stock cross-sectional OLS regression:

$$R_{it} - R_{ft} = \gamma_{0t} + \gamma_{1t}\hat{B}_{it-1} + \gamma_{2t}' zcs_{it-1} + u_{it}.$$

The factor models are:

1-factor :	Mkt
3-factor :	Mkt, SMB, HML
4-factor :	Mkt, SMB, HML, MOM
5-factor :	Mkt, SMB, HML, RMW, CMA
6-factor :	Mkt, SMB, HML, MOM, RMW, CMA

Mkt, SMB, and HML are the three Fama and French (1993) factors, MOM is the momentum factor, and RMW and CMA are the two additional factors in Fama and French (2014). Only stocks with price greater than $1 at the end of month *t* are used in the regression at time *t*. The first row is the coefficient (multiplied by 100) and the second row is *t*-statistic. For reach factor model, we report bias uncorrected coefficients from a regular OLS regression and coefficients corrected for EIV-bias following the procedure described in the text. Betas are estimated from rolling time-series regressions using past two years of daily data; we require at least 400 observations to estimate beta. The cross-sectional variables (zcs_{it}) include size (Sz), book-to-market (B/M), and the last six-month return (Ret6) in Panel A. We add operating profitability (Profit) and investment (Invest) to this list in Panel B. Sz is the logarithm of market capitalization. B/M is the logarithm of the ratio of most recently available book-value (assumed available six months after fiscal year-end) divided by the current market capitalization. Profit is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Invest is the change in total assets divided by current total assets. For each characteristic, values greater than the 0.99 fractile or less than the 0.01 fractile are set equal to the 0.99 and the 0.01 fractiles each month. The last rows in each panel report the fraction of cross-sectional variation in expected returns given by betas and characteristics (the numbers do not add up to 100 because of covariation). Numbers in parenthesis below the differences give the 95% confidence intervals (please refer to the text for further details). The sample period is July 1963 to December 2013.

	1-factor		3-factor		4-factor		5-factor		6-factor	
	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV
Panel A : With fewer characteristics in CSR										
Cnst	0.694	0.674	0.676	0.600	0.679	0.643	0.666	0.537	0.662	0.589
	(5.31)	(5.20)	(5.37)	(4.89)	(5.47)	(5.26)	(5.35)	(4.23)	(5.37)	(4.80)
B_{Mkt}	0.062	0.081	0.105	0.274	0.099	0.221	0.119	0.418	0.117	0.329
	(0.38)	(0.39)	(0.74)	(1.49)	(0.71)	(1.23)	(0.85)	(2.16)	(0.84)	(1.79)
B_{SMB}			−0.097	−0.293	−0.099	−0.280	−0.086	−0.319	−0.081	−0.292
			(−1.52)	(−2.21)	(−1.58)	(−2.10)	(−1.40)	(−2.44)	(−1.33)	(−2.18)
B_{HML}			0.067	0.094	0.045	0.037	0.046	−0.059	0.024	−0.084
			(0.87)	(0.76)	(0.61)	(0.29)	(0.63)	(−0.43)	(0.33)	(−0.67)
B_{MOM}					−0.046	−0.115			−0.038	0.090
					(−0.43)	(−0.51)			(−0.37)	(0.42)
B_{RMW}							0.093	0.314	0.097	0.258
							(1.64)	(2.46)	(1.72)	(2.34)
B_{CMA}							0.073	0.220	0.069	0.182
							(1.43)	(2.33)	(1.38)	(2.00)
Sz	−0.087	−0.091	−0.106	−0.158	−0.103	−0.143	−0.104	−0.168	−0.102	−0.154
	(−2.15)	(−2.17)	(−3.01)	(−4.38)	(−2.95)	(−4.05)	(−3.01)	(−4.68)	(−2.96)	(−4.36)
B/M	0.318	0.315	0.297	0.260	0.299	0.268	0.297	0.272	0.298	0.285
	(6.04)	(6.24)	(6.34)	(5.85)	(6.66)	(6.20)	(6.55)	(6.23)	(6.77)	(6.69)
Ret6	1.579	1.585	1.595	1.574	1.586	1.566	1.570	1.551	1.562	1.551
	(9.85)	(10.24)	(10.31)	(10.37)	(10.47)	(10.72)	(10.28)	(10.33)	(10.38)	(10.76)
% Betas	0.5	0.8	3.5	12.0	3.2	10.6	9.0	30.5	8.8	23.6
% Chars	103.1	104.2	99.7	110.0	100.7	109.1	93.4	97.3	94.0	101.8
% Diff	102.5	103.4	96.2	98.1	97.5	98.5	84.4	66.8	85.1	78.3
	(97.5, 104.9)	(96.9, 106.4)	(77.7, 108.1)	(68.6, 117.8)	(85.0, 108.8)	(77.6, 118.8)	(54.9, 104.2)	(35.8, 103.6)	(62.0, 104.0)	(54.5, 109.0)

	1-factor		3-factor		4-factor		5-factor		6-factor	
	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV
Panel B : With all characteristics in CSR										
Cnst	0.641	0.595	0.633	0.531	0.633	0.585	0.623	0.498	0.619	0.543
	(4.90)	(4.58)	(5.04)	(4.32)	(5.10)	(4.75)	(5.01)	(3.96)	(5.01)	(4.35)
B_{Mkt}	0.154	0.202	0.177	0.365	0.178	0.324	0.191	0.469	0.191	0.421
	(0.96)	(0.98)	(1.25)	(1.95)	(1.27)	(1.77)	(1.36)	(2.36)	(1.37)	(2.21)
B_{SMB}			−0.066	−0.247	−0.069	−0.248	−0.058	−0.270	−0.055	−0.267
			(−1.04)	(−1.83)	(−1.12)	(−1.82)	(−0.95)	(−1.99)	(−0.92)	(−1.89)
B_{HML}			0.034	0.073	0.014	0.012	0.015	−0.073	−0.004	−0.131
			(0.45)	(0.58)	(0.19)	(0.09)	(0.21)	(−0.51)	(−0.05)	(−1.00)
B_{MOM}					0.004	0.049			0.004	0.229
					(0.03)	(0.21)			(0.04)	(0.99)
B_{RMW}							0.069	0.244	0.070	0.240
							(1.25)	(2.01)	(1.27)	(2.00)
B_{CMA}							0.049	0.166	0.046	0.136
							(0.95)	(1.72)	(0.91)	(1.44)
Sz	−0.105	−0.110	−0.118	−0.168	−0.117	−0.158	−0.117	−0.172	−0.116	−0.168
	(−2.80)	(−2.82)	(−3.57)	(−4.76)	(−3.56)	(−4.56)	(−3.57)	(−4.79)	(−3.55)	(−4.70)
B/M	0.284	0.286	0.266	0.232	0.271	0.250	0.269	0.252	0.271	0.272
	(5.54)	(5.77)	(5.76)	(5.21)	(6.12)	(5.76)	(5.96)	(5.71)	(6.18)	(6.28)
Ret6	1.447	1.460	1.464	1.449	1.455	1.427	1.451	1.442	1.438	1.435
	(8.93)	(9.31)	(9.30)	(9.36)	(9.45)	(9.54)	(9.34)	(9.41)	(9.41)	(9.67)
Profit	0.709	0.704	0.686	0.642	0.691	0.646	0.678	0.600	0.681	0.602
	(7.00)	(7.04)	(7.06)	(6.83)	(7.21)	(6.99)	(7.12)	(6.04)	(7.23)	(6.28)
Invest	−1.245	−1.262	−1.226	−1.211	−1.209	−1.189	−1.188	−1.138	−1.176	−1.155
	(−11.23)	(−11.26)	(−11.45)	(−10.81)	(−11.46)	(−10.68)	(−11.28)	(−10.08)	(−11.25)	(−9.99)
% Betas	2.5	3.5	3.0	9.5	2.8	8.9	5.6	20.1	5.4	20.1
% Chars	105.8	107.9	103.4	111.8	104.8	114.7	98.8	103.3	99.7	107.6
% Diff	103.3	104.4	100.4	102.3	102.0	105.8	93.3	83.1	94.3	87.6
	(102.3, 105.1)	(103.0, 106.5)	(91.8, 108.7)	(86.5, 117.6)	(97.8, 110.3)	(97.1, 121.3)	(77.9, 106.2)	(62.9, 109.7)	(84.1, 106.8)	(70.5, 113.0)

Table 4: Cross-sectional regression of non-microcap stocks

This table presents the time-series averages of γ coefficients from the following individual stock cross-sectional OLS regression:

$$R_{it} - R_{ft} = \gamma_{0t} + \gamma_{1t}\hat{B}_{it-1} + \gamma'_{2t} zcs_{it-1} + u_{it}.$$

The factor models are:

1-factor :	Mkt
3-factor :	Mkt, SMB, HML
4-factor :	Mkt, SMB, HML, MOM
5-factor :	Mkt, SMB, HML, RMW, CMA
6-factor :	Mkt, SMB, HML, MOM, RMW, CMA

Mkt, SMB, and HML are the three Fama and French (1993) factors, MOM is the momentum factor, and RMW and CMA are the two additional factors in Fama and French (2014). Only stocks with price greater than \$1 at the end of month *t* and which have market capitalization above the 20[th] percentile of NYSE market capitalization at time *t* are used in the regression at time *t*. The first row is the coefficient (multiplied by 100) and the second row is *t*-statistic. For reach factor model, we report bias uncorrected coefficients from a regular OLS regression and coefficients corrected for EIV-bias following the procedure described in the text. Betas are estimated from rolling time-series regressions using past two years of daily data; we require at least 400 observations to estimate beta. The cross-sectional variables (zcs_{it}) include size (Sz), book-to-market (B/M), and the last six-month return (Ret6) in Panel A. We add operating profitability (Profit) and investment (Invest) to this list in Panel B. Sz is the logarithm of market capitalization. B/M is the logarithm of the ratio of most recently available book-value (assumed available six months after fiscal year-end) divided by the current market capitalization. Profit is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Invest is the change in total assets divided by current total assets. For each characteristic, values greater than the 0.99 fractile or less than the 0.01 fractile are set equal to the 0.99 and the 0.01 fractiles each month. The last rows in each panel report the fraction of cross-sectional variation in expected returns given by betas and characteristics (the numbers do not add up to 100 because of covariation). Numbers in parenthesis below the differences give the 95% confidence intervals (please refer to the text for further details). The sample period is July 1963 to December 2013.

	1-factor		3-factor		4-factor		5-factor		6-factor	
	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV
Panel A : With fewer characteristics in CSR										
Cnst	0.787	0.790	0.749	0.721	0.791	0.826	0.730	0.682	0.761	0.768
	(5.26)	(5.25)	(5.48)	(5.38)	(5.81)	(5.98)	(5.50)	(5.19)	(5.72)	(5.56)
B_{Mkt}	−0.017	−0.021	0.064	0.143	0.046	0.117	0.098	0.245	0.082	0.208
	(−0.10)	(−0.11)	(0.41)	(0.77)	(0.30)	(0.64)	(0.63)	(1.31)	(0.54)	(1.11)
B_{SMB}			−0.137	−0.252	−0.171	−0.355	−0.124	−0.260	−0.148	−0.328
			(−1.77)	(−1.82)	(−2.30)	(−2.60)	(−1.71)	(−1.95)	(−2.09)	(−2.45)
B_{HML}			0.131	0.186	0.094	0.113	0.086	0.086	0.053	0.021
			(1.51)	(1.63)	(1.13)	(1.00)	(1.06)	(0.73)	(0.66)	(0.18)
B_{MOM}					−0.031	0.029			−0.026	0.052
					(−0.24)	(0.14)			(−0.21)	(0.26)
B_{RMW}							0.141	0.229	0.157	0.277
							(2.21)	(2.31)	(2.48)	(2.81)
B_{CMA}							0.101	0.203	0.089	0.175
							(1.62)	(2.17)	(1.49)	(1.93)
Sz	−0.083	−0.084	−0.116	−0.152	−0.129	−0.190	−0.110	−0.153	−0.122	−0.184
	(−2.06)	(−2.07)	(−3.65)	(−4.18)	(−4.09)	(−5.25)	(−3.50)	(−4.25)	(−3.90)	(−5.11)
B/M	0.189	0.186	0.138	0.097	0.150	0.127	0.151	0.130	0.154	0.144
	(3.27)	(3.31)	(2.78)	(2.01)	(3.24)	(2.87)	(3.17)	(2.79)	(3.37)	(3.25)
Ret6	1.316	1.311	1.317	1.305	1.325	1.339	1.331	1.318	1.316	1.322
	(6.47)	(6.53)	(6.74)	(6.74)	(7.24)	(7.55)	(7.09)	(7.12)	(7.28)	(7.53)
% Betas	0.1	0.1	14.5	25.8	14.1	32.4	25.7	39.8	26.5	44.4
% Chars	99.4	99.2	90.8	97.1	99.1	117.3	77.7	79.5	82.5	91.8
% Diff	99.3	99.1	76.3	71.2	84.9	84.9	52.0	39.7	56.0	47.4
	(86.4, 102.0)	(82.5, 102.3)	(34.6, 106.6)	(20.4, 113.1)	(54.2, 109.6)	(44.9, 120.9)	(7.0, 95.0)	(−1.9, 92.6)	(14.1, 96.7)	(9.1, 99.5)

	1-factor		3-factor		4-factor		5-factor		6-factor	
	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV
Panel B : With all characteristics in CSR										
Cnst	0.715	0.699	0.693	0.661	0.733	0.765	0.682	0.633	0.712	0.736
	(4.76)	(4.62)	(5.05)	(4.90)	(5.36)	(5.46)	(5.09)	(4.75)	(5.29)	(5.40)
B_{Mkt}	0.092	0.108	0.135	0.206	0.123	0.195	0.159	0.296	0.147	0.251
	(0.54)	(0.54)	(0.85)	(1.09)	(0.80)	(1.04)	(1.03)	(1.56)	(0.97)	(1.34)
B_{SMB}			−0.095	−0.186	−0.131	−0.301	−0.089	−0.209	−0.112	−0.291
			(−1.22)	(−1.29)	(−1.76)	(−2.15)	(−1.21)	(−1.49)	(−1.57)	(−2.11)
B_{HML}			0.082	0.127	0.045	0.053	0.048	0.050	0.014	−0.017
			(0.95)	(1.10)	(0.54)	(0.47)	(0.59)	(0.42)	(0.18)	(−0.14)
B_{MOM}					0.015	0.090			0.013	0.080
					(0.12)	(0.44)			(0.10)	(0.40)
B_{RMW}							0.109	0.191	0.125	0.249
							(1.72)	(1.89)	(1.99)	(2.49)
B_{CMA}							0.076	0.175	0.066	0.145
							(1.22)	(1.83)	(1.10)	(1.57)
Sz	−0.101	−0.102	−0.120	−0.148	−0.135	−0.191	−0.116	−0.151	−0.128	−0.188
	(−2.58)	(−2.58)	(−3.83)	(−3.98)	(−4.31)	(−5.14)	(−3.72)	(−4.03)	(−4.12)	(−5.01)
B/M	0.185	0.185	0.145	0.111	0.161	0.145	0.155	0.131	0.159	0.141
	(3.04)	(3.09)	(2.67)	(2.03)	(3.17)	(2.90)	(2.99)	(2.56)	(3.18)	(2.83)
Ret6	1.272	1.269	1.270	1.257	1.279	1.297	1.282	1.259	1.265	1.269
	(6.25)	(6.33)	(6.48)	(6.47)	(6.96)	(7.28)	(6.79)	(6.76)	(6.96)	(7.15)
Profit	0.659	0.646	0.611	0.558	0.622	0.568	0.554	0.428	0.556	0.414
	(5.04)	(4.94)	(4.87)	(4.42)	(5.15)	(4.76)	(4.72)	(3.57)	(4.84)	(3.50)
Invest	−0.985	−1.003	−0.916	−0.890	−0.899	−0.856	−0.856	−0.802	−0.841	−0.752
	(−7.16)	(−7.34)	(−7.03)	(−6.74)	(−7.09)	(−6.59)	(−6.80)	(−5.99)	(−6.75)	(−5.63)
% Betas	1.6	2.0	6.4	12.1	6.0	18.4	13.7	25.8	13.8	30.9
% Chars	102.8	103.5	94.8	97.0	102.0	115.1	83.4	80.0	87.7	92.1
% Diff	101.2	101.5	88.4	84.9	96.0	96.7	69.7	54.2	73.9	61.2
	(99.8, 103.6)	(100.0, 104.2)	(65.6, 107.2)	(54.0, 112.6)	(85.8, 111.0)	(79.1, 120.5)	(39.7, 100.8)	(22.1, 98.9)	(50.5, 102.7)	(32.9, 105.5)

Table 5: Time variation in prices of risk and characteristics

This table presents the results from a time-series regression of γ coefficients on macro variables:

$$\hat{\gamma}_t = c_0 + c_1' x_{t-1} + \xi_t.$$

The macro variables (x) are payout ratio of the S&P500 index (Pay), default spread (difference between BAA- and AAA-rated bonds, Def), and term spread (difference between long-term government bond yield and 3-month Treasury-bill rate, Term). The factor models are:

1-factor :	Mkt
3-factor :	Mkt, SMB, HML
4-factor :	Mkt, SMB, HML, MOM
5-factor :	Mkt, SMB, HML, RMW, CMA
6-factor :	Mkt, SMB, HML, MOM, RMW, CMA

Mkt, SMB, and HML are the three Fama and French (1993) factors, MOM is the momentum factor, and RMW and CMA are the two additional factors in Fama and French (2014). Panel A shows results from the sample of all stocks (the same sample as in Table 3) while Panel B shows the results from the sample of non-micro-cap stocks (the same sample as in Table 4). Each Panel is further subdivided into two panels where the top panel uses fewer characteristics in cross-sectional regression while the bottom panel uses all the characteristics in the cross-sectional regression. The top half of each panel reports only the F-statistic for the joint significance of all the coefficients c_1 in the third-stage time-series predictive regression. This F-statistic is reported separately for γ premia from betas and characteristics (p-value in parentheses). We use the fitted values from these third-stage time-series regressions to calculate the contributions to cross-sectional variation in expected returns made by betas and characteristics. These fractions are reported in the bottom part of each panel (the numbers do not add up to 100 because of covariation). Numbers in parenthesis below the differences give the 95% confidence intervals (please refer to the text for further details). The sample period is July 1963 to December 2013.

	1-factor		3-factor		4-factor		5-factor		6-factor	
	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV
Panel A1 : All stocks, With fewer characteristics in CSR										
Fstat Betas	0.63	0.57	0.97	0.79	1.01	1.04	1.07	0.86	1.36	1.23
	(0.59)	(0.63)	(0.46)	(0.63)	(0.44)	(0.41)	(0.38)	(0.62)	(0.15)	(0.23)
Fstat Chars	2.42	2.12	3.59	4.32	3.58	3.89	3.6	4.12	3.64	3.91
	(0.01)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
% Betas	7.2	8.6	13.0	21.7	14.8	24.4	19.6	39.0	21.2	38.2
% Chars	95.3	94.9	96.8	108.0	93.1	97.5	89.0	89.9	85.5	84.3
% Diff	88.0	86.2	83.8	86.3	78.3	73.1	69.4	51.0	64.3	46.1
	(72.1, 102.3)	(68.1, 102.8)	(71.4, 103.5)	(74.7, 111.7)	(70.2, 100.8)	(64.8, 106.1)	(53.6, 96.5)	(33.3, 88.7)	(53.1, 93.2)	(33.4, 83.6)

	1-factor		3-factor		4-factor		5-factor		6-factor	
Panel A2 : All stocks, With all characteristics in CSR										
Fstat Betas	0.44	0.37	1.03	0.91	1.01	1.06	1.06	1.02	1.33	1.36
	(0.72)	(0.77)	(0.41)	(0.51)	(0.44)	(0.39)	(0.39)	(0.44)	(0.16)	(0.15)
Fstat Chars	2.15	1.98	2.86	3.38	2.88	3.19	2.91	3.37	2.92	3.3
	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
% Betas	5.3	6.5	9.5	18.1	11.2	21.3	14.2	28.7	15.7	33.6
% Chars	99.5	99.9	100.7	111.3	97.5	104.3	95.4	100.4	92.0	91.8
% Diff	94.2	93.4	91.2	93.2	86.3	83.0	81.2	71.7	76.3	58.3
	(84.9, 102.9)	(83.0, 103.9)	(84.8, 106.1)	(86.8, 112.9)	(81.7, 101.4)	(77.3, 106.2)	(72.4, 100.7)	(62.4, 102.5)	(70.2, 97.8)	(46.6, 91.9)

	1-factor		3-factor		4-factor		5-factor		6-factor	
	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV	OLS	EIV
Panel B1 : Non micro-cap stocks, With fewer characteristics in CSR										
Fstat Betas	0.33	0.32	0.78	0.63	0.92	1.11	1.1	1.05	1.41	1.39
	(0.80)	(0.81)	(0.63)	(0.77)	(0.53)	(0.35)	(0.35)	(0.40)	(0.12)	(0.13)
Fstat Chars	2.06	2.04	3.21	3.48	2.79	2.83	2.96	3.37	2.71	2.82
	(0.03)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
% Betas	7.4	8.1	22.2	34.2	26.2	46.4	35.1	49.3	38.0	56.0
% Chars	94.1	93.6	88.9	96.5	84.7	91.7	74.3	75.0	69.8	69.6
% Diff	86.7	85.5	66.8	62.3	58.5	45.3	39.2	25.7	31.8	13.6
	(76.5, 100.9)	(74.7, 100.9)	(56.7, 100.0)	(50.7, 100.2)	(53.4, 93.2)	(35.9, 90.4)	(24.4, 82.4)	(12.7, 76.4)	(21.0, 75.6)	(1.4, 59.1)

	1-factor		3-factor		4-factor		5-factor		6-factor	
Panel B2 : Non micro-cap stocks, With all characteristics in CSR										
Fstat Betas	0.18	0.16	0.65	0.58	0.86	1.08	1.09	1.05	1.39	1.43
	(0.91)	(0.92)	(0.75)	(0.82)	(0.59)	(0.37)	(0.37)	(0.40)	(0.13)	(0.11)
Fstat Chars	1.92	1.96	3.11	3.34	2.79	2.8	3.11	3.36	2.93	2.85
	(0.02)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
% Betas	4.2	4.7	12.5	21.4	17.7	34.9	23.6	37.5	27.0	45.5
% Chars	99.3	99.5	94.3	98.9	89.4	92.7	82.0	79.9	76.4	72.6
% Diff	95.1	94.8	81.8	77.5	71.7	57.8	58.4	42.4	49.4	27.0
	(92.1, 102.5)	(92.1, 103.0)	(79.8, 101.5)	(74.1, 103.4)	(70.6, 94.1)	(52.7, 93.4)	(50.9, 96.5)	(34.1, 81.9)	(44.2, 78.5)	(16.6, 71.8)