Declining Trends in the Real Interest Rate and Inflation: Role of Aging*

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Abstract

This paper explores a causal link between aging of the labor force and declining trends in the real interest rate and inflation in Japan. We develop a new Keynesian search/matching model that features heterogeneities in age and firm-specific skill levels. Using the model, we examine the long-run implications of the sharp drop in labor force entry in the 1970s. We show that the changes in the demographic structure driven by the drop induce significant low-frequency movements in per-capita consumption growth and the real interest rate. It also leads to similar movements in the inflation rate when the monetary policy rule follows the standard Taylor rule, failing to recognize the time-varying nature of the natural rate of interest. The model suggests that aging of the labor force accounts for roughly 40% of the declines in the real interest rate observed between the 1980s and 2000s in Japan.

JEL Classification: E24, E31, E52
Keywords: aging, natural rate, deflation, Japan

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1 Introduction

Japan's labor force is rapidly aging. Since the early 1980s, the average age of its workforce has risen roughly three years. Moreover, the *level* of the labor force has been falling to date since the late 1990s. At the same time, the Japanese economy has experienced a prolonged slowdown in growth. The real interest rate and the inflation rate have also followed gradual declining trends since the 1980s and, as is well known, Japan has experienced small but persistent deflation in the last 15 years. In this paper, we explore a causal link between the labor force aging and the low-frequency declining trends in important macroeconomic variables over the last three and half decades. For this purpose, we utilize a New Keynesian search/matching framework that incorporates heterogeneities in the worker's age and skill.

The main driver of the Japan's workforce aging is (the end of) the baby boom. The birth rate fell precipitously in the 1950s and this decline translated into a similarly sharp drop in the entry rate of young workers into the labor force in the 1970s. We argue that the sharp drop in the pace of labor force entry in the 1970s has caused predictable low-frequency movements in key macroeconomic variables.[1] This claim is based on our quantitative experiment where we feed the model an exogenous path of the labor-force entry rate that mimics the behavior of the empirical data, and then trace the economy's transition path to the new steady state. The transition to the new steady state takes a long time and accompanies the low frequency variations of the endogenous variables.

There are two key features in our model. First, the model incorporates the empirical regularity that older workers are more productive than young workers. In the model, a worker enters the labor force as a young worker with no experience. The worker becomes old (and "experienced" at the same time). Experienced workers enjoy higher productivity. Second, we incorporate into the model what we believe is the salient feature of the Japanese labor market, namely, the importance of the firm specific skill. The (old) worker's skill associated with his experience in the labor market tends to be firm specific, and therefore, when the worker loses his job, he could lose the productivity premium associated with being an experienced worker. When an old worker indeed loses his experience premium, he needs to look for a job in the market where young (inexperienced) workers also look for their entry-level jobs.

These labor market features are embedded in a standard new Keynesian model where the monopolistically competitive firms sell their goods to the household, subject to price adjustment frictions. The firm produces, combining different types labor (as described above). We completely abstract away from the consumption/saving heterogeneity facing workers with different age and employment statuses, by assuming that all sources of incomes are pooled and consumed equally across the existing household members. One may view this assumption being odd in a paper where declining trends in interest rates and inflations are its key focus. However, we make this assumption deliberately to highlight the key mechanism of our

[1]The birth rate has kept falling, long after the baby boom has ended. For example, the birth rate (total fertility rate) reached the bottom in 2005 at around 1.3. This steady decline in the birth rate has received much attention as a fact exemplifying the aging of the Japanese society. However, the baby boom is a much more quantitatively important factor for the gradual progress of the labor force aging observed in the data.

paper, namely, skill (productivity) heterogeneity in the workforce. Evolution of the demographic structure leads to the changes in the aggregate skill mix of the workforce, resulting in the low frequency movements in aggregate labor productivity, per-capita consumption, the real interest rate, the unemployment rate, and other endogenous variables.

The equilibrium transition path of the economy replicates important low-frequency features of the Japanese economy observed between 1970 and 2010. According to the model, the first decade of this 40-year period is characterized by rising per-capita consumption, real wage, and labor productivity. The real interest rate is also increasing in this period. This period, however, is followed by a long period of a gradual slowdown of the economy, characterized by lower labor productivity growth, per-capita consumption growth, and the real interest rate. In the data, per-capita consumption growth and the real interest rate fell 1.3 percentage points and 2.3 percentage points, respectively, between 1980s (1980-1989) and 2000s (2000-2009). Our experiment indicates that the workforce aging (driven by the decline in labor force entry in the 1970s) accounts for roughly 40% of these variations. The economy exhibits the low frequency swing for the following reasons. First, the initial phase is characterized by the situation where more and more young workers are gaining experience and thus per-capita consumption is growing faster. However, as more and more workers reach a high productivity state (i.e., become experienced), there is simply less room to grow further. This together with fewer entry flows of young workers implies a slowdown of aggregate growth. Further, the firm-specific nature of the worker's skill plays an important role in the later stage of the aging process. That is, aging of the workforce implies that there are a greater number workers who have lost their firm-specific skill. This puts further downward pressures on growth and the real interest rate.

To draw implications for inflation dynamics, we assume that the monetary authority follows the simple Taylor rule in setting the nominal interest rate. Under this assumption, inflation exhibits a similar and significant low frequency swing over the 40-year period. It follows a steady declining trend after an initial increase. Moreover, the model predicts, under our benchmark calibration, that the economy starts experiencing deflation (albeit at a small rate) 35 years after labor force entry begins to drop. This timing corresponds to the mid 2000s. In this model, the natural rate of interest is time-varying because of the skill heterogeneity and the demographic transition. When the monetary authority fails to internalize this variation in setting the nominal interest rate (for whatever reason), inflation falls along with aging of the workforce.

Related literature. The declining trends in real interest rates in the U.S. have received much attention lately. For example, the secular stagnation hypothesis posits that a decline in the birth rate eventually leads to the oversupply of saving and decrease in aggregate demand, resulting in the lower real interest rate (Eggertsson and Mehrotra (2014)). Another related but different explanation is the one based on "global savings glut" by Bernanke (2005). It emphasizes the role of global forces in causing the decline in the real interest rate, namely larger international capital flows into the US. Moreover, Ferrero (2010) associate the larger capital flows into the US with the demographic structure of capital suppliers (i.e., China and

Japan).[2] Note that in this paper, we focus on the closed-economy channel that arises due to worker heterogeneity. But what role this channel plays in the global context is an important future research topic.

A recent paper by Ferrero and Carvalho (2014) also relates persistent deflation in Japan to aging of the economy as in our paper. Their paper focuses on the role of consumption/saving heterogeneity in the aging economy and the driving force in their model is a longer life expectancy. Our paper emphasizes the role of skill heterogeneity.[3] Ikeda and Saito (2014) develops a model in which the level of the real interest rate is influenced by population growth in the presence of financial frictions. Katagiri (2012) develops a model with labor market frictions with sectoral shifts and explores the effects of an unexpected shock to the population growth forecast. Such a shock results in the change in the demand structure, and thus causes the real interest rate and the inflation rate to change. Katagiri et al. (2014) aim to understand the link between demographics and inflation from a politico-economic perspective using an overlapping generations model where the aggregate price level is pinned down by the fiscal theory of the price level. None of the existing papers recognizes the "productivity channel" stressed in this paper: the evolution of the demographic structure leads to the variations in the average skill mix in the economy and thus aggregate labor productivity.

There are a few papers that explore the cross-sectional empirical relationship between aging and inflation. For example, Yoon et al. (2014) show that higher population growth leads to higher inflation, while Juselius and Takats (2015) find a negative correlation between the share of working age population and the inflation rate. The empirical evidence in both of these papers is consistent with the quantitative results in this paper. Last but not least, Feyrer (2007) provides the strong empirical evidence, supporting the main idea of this paper. In his cross-country panel regression, he finds a strong correlation between the age structure of the workforce and aggregate productivity growth.

The paper is organized as follows. The next section presents some basic facts concerning aging of the Japanese labor force and also reviews wage and inflation behavior in the last two decades. Section 3 develops the model, which is calibrated in Section 4. Section 5 presents the main results of the paper. Section 6 concludes the paper by discussing some implications of our findings.

2 Facts

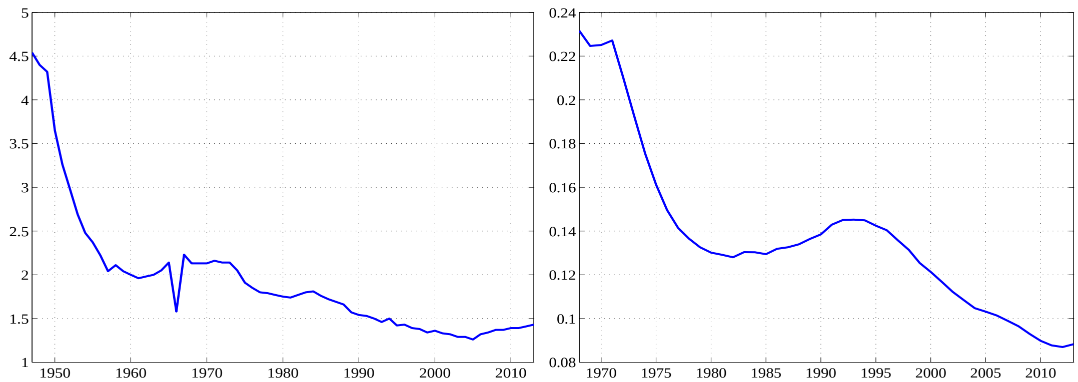
In this section, we present important macro facts for Japan pertaining to the purpose of our paper. We view the Japanese demographic structure as being an important driving factor of the low frequency movements not only in labor market variables but also in other variables such as interest rates and inflation.

[2] Ferrero (2010) concludes that "the international demographic transition is crucial for large US external imbalances to be consistent with the persistent decline of world real interest rates observed in the data."

[3] In the policy sphere, Shirakawa (2012) points out the possibility of the link between deflation and aging, although he does not take a stand on the structural force that underlies this link.

Figure 1: Labor Force Entry in Japan

(a) Birth Rate (b) Entry into the Labor Force



Notes: The birth rate equals "the total fertility rate" which gives the number of children that are born to each woman in her childbearing years (15-49 years old). Panel (b) plots the share of workers between 15 and 24 in the total labor force (workers between 15 and 64).

2.1 Aging of the Labor Force

Panel (a) of Figure 1 plots the birth rate, more specifically, the total fertility rate, starting at 1948.[4] The effect of changes in the birth rate shows up 15-20 years later in the labor force. To see this, Panel (b) plots the share of workers between 15 and 24 years old in the total labor force.[5] We view this share as an approximation to "entry" into the labor force. One can clearly see in Panel (a) that the birth rate was very high in the late 1940s, but fell dramatically in the following 10 years or so. This corresponds to the end of the (first) baby boom in Japan. The birth rate from then on was roughly flat at around 2 until the early 1970s. Since the early 1970s, however, the birth rate kept falling until 2005 when it hit the historical low at around 1.3. Panel (b) attempts translating the behavior of the birth rate into the entry flow into the labor force by plotting the share of workers between 15-24 years old in the total labor force. This series behaves similarly to the birth rate with roughly a 15- to 20-year lag: It fell sharply in the 1970s, recovered temporarily around the early 1990s, and, since then, has been declining steadily. In our quantitative exercise below, this latter fact, the dramatic decline in the labor force entry in the 1970s associated with the end of the baby boom, is taken to be the exogenous force.[6]

In Panel (a) of Figure 2, we compute the average age of the labor force. One can see that the average age has increased from 39 years old to 42 years old since 1980, demonstrating the

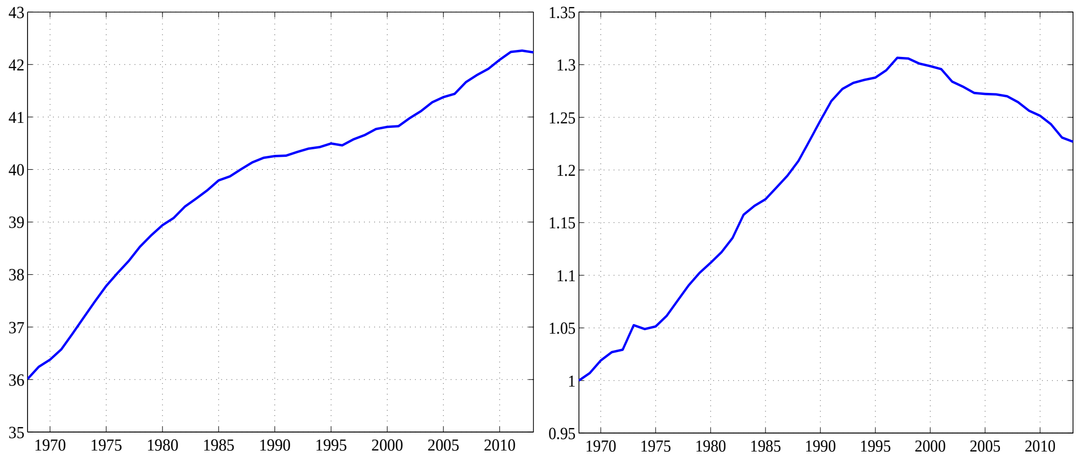
[4] The total fertility rate gives the number of children that are born to each woman in her childbearing years, which are usually defined between 15 and 49 years old. We use the term the birth rate to mean the total fertility rate throughout the paper.

[5] We discard workers who are 65 or above from the total labor force, so that the analysis is not influenced by the fact that a larger number of 65+ workers are in the labor force.

[6] Our exercise does not separately consider the smaller hump in the labor force entry in the late 1980s and early 1990s.

Figure 2: The Average Age and Labor Force Size

(a) Average Age (b) Labor Force Size (1968=100)



significant aging of the Japanese labor force over time. Another consequence of the slowdown in labor force entry is that it eventually leads to decline in the labor force (in the absence of quantitatively meaningful immigration). Panel (b) shows that the labor force started shrinking in the late 1990s and continued that trend to date. Considering that none of the large developed economies have experienced such a sustained decline in the labor force, it represents a serious nature of aging in Japan.

2.2 Wages and Prices

Next, Figure 3 presents hourly nominal and real wages since 1990. Both nominal and real wages increased in the first half of the 1990s. However, since the late 1990s, both series have been on a downward trend. Specifically, nominal wage fell roughly 7% between 1996 and 2013. Real wage declined less because of the decline in the price level. However, deflation did not overturn the decline in real wage itself. The declines in both real and nominal wages are also quite extraordinary from the international perspective. In Appendix, we consider the comparison to the US.

Lastly, Figure 4 shows the price level and the inflation rate measured by the CPI, starting at 1978. Not surprisingly, the behavior of the price level and nominal wage is similar to each other. As is well known, Japan has suffered deflation since the late 1990s, as shown in Panel (b).

2.3 Real Interest Rate and Consumption Growth

Figure 5 presents the annual series of the real interest rate and per-capita consumption growth between 1980 and 2013. To summarize the long-run trend of the two variables, Table 1 computes the average levels of the two variables for each of the three decades from 1980.

Figure 3: Wages in Japan (1996=100)

(a) Nominal Hourly Wage (b) Real Hourly Wage

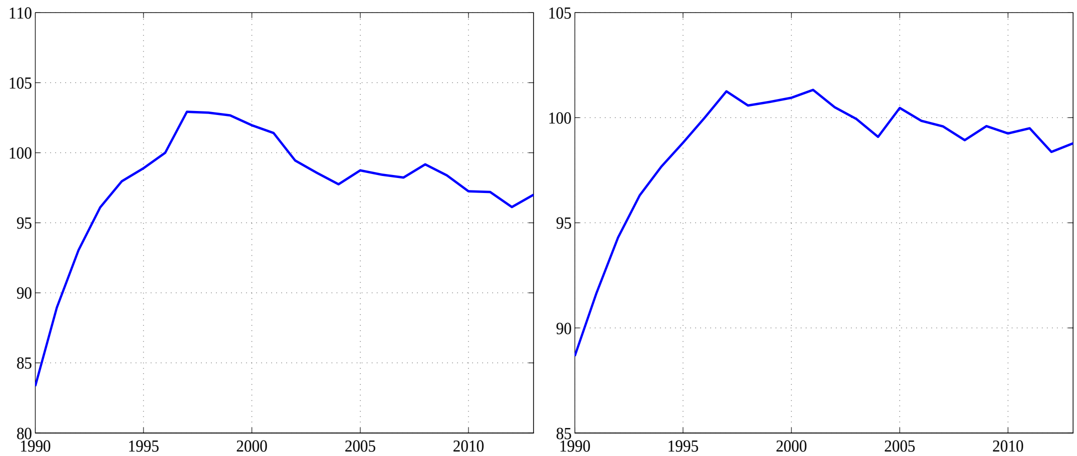
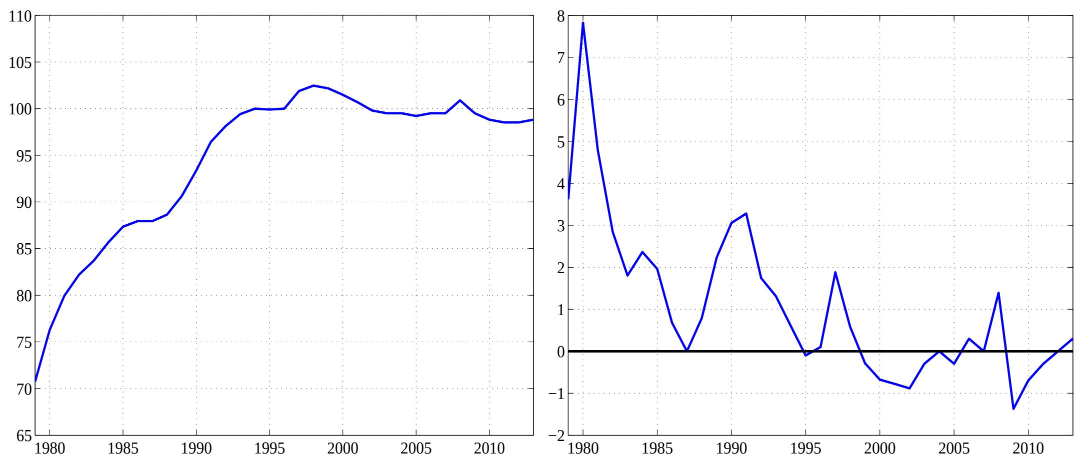


Figure 4: Price Level and Inflation Rate in Japan

(a) Price Level (CPI; 1996=100) (b) Inflation Rate

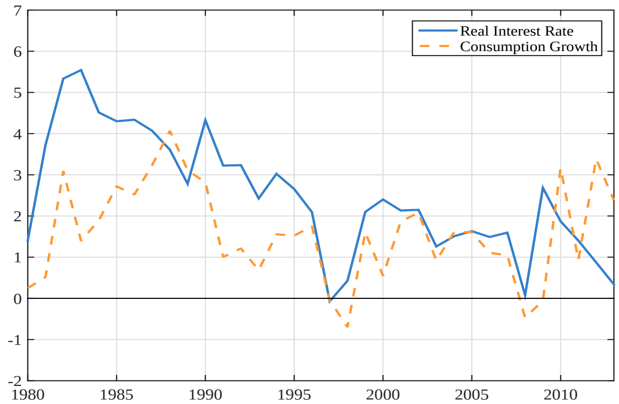


One can see that the real interest rate averaged roughly 4% in the 1980s, but fell considerably in the 1990s, averaging 2.3%. The gradual decline in the real interest rate has persisted into the 2000s with the average level of 1.7% in that decade.[7] Per-capita consumption growth exhibits a similar, declining low frequency trend: It grew 2.3% in the 1980s and fell more than 1 percentage points over the past three decades.

In the model presented below, these two variables are linked by a single consumption Euler equation given our large-family assumption even though the model features heterogeneity in the production side. Under that assumption, we examine how much of the variations

[7]Obviously, the level of the real interest rate changes depending on the nominal interest rate series we use. However, the size of the decline is insensitive to the alternative measures of nominal rates.

Figure 5: Real Interest Rate and Consumption Growth



Notes: Real interest: The Treasury bill rate minus the realized inflation rate measured by the CPI. The Treasury bill rate is taken from International Financial Statistics by the IMF.

in these two variables can be accounted for by change in the demographic structure which is ultimately driven by the fall in the labor force entry which took place in the late 1960s through the 1970s.

3 Model

This section lays out the model. The model incorporates what we believe are salient features of the Japanese labor market, aging and firm-specific human capital. The economy consists of three types of agents: households, firms, and a monetary authority. Firms produce and sell differentiated goods to the household. The goods market is characterized by the monopolistic competition and price stickiness as in the standard new Keynesian models. Labor is the only input for production; and there are three types of workers, as specified below. Hiring is subject to search frictions. The central bank sets the nominal interest rate following the Taylor-type rule.

3.1 Labor Market with Heterogeneous Workers

In the economy, there are young and old workers. A mass of ϕ young workers are born every period and enter the labor market as jobless. The key quantitative experiment below entails tracing the effects of a large decline in this parameter. Young workers become old with probability μ every period. Old workers die with probability d every period.[8] When young workers become old, they also become "experienced," having higher labor productivity by a

[8]See Cheron et al. (2013) and Esteban-Pretel and Fujimoto (2012) for search/matching models with a more explicit demographic structure.

Table 1: Long-Run Levels of Real Interest Rate and Consumption Growth

	Real Interest Rate (%)	Consumption Growth (%)
1980−1989	3.96	2.28
1990−1999	2.34	1.14
2000−2009	1.69	1.03
Difference	−2.27	−1.26

Notes: See notes to Figure 5. "Difference" refers to the difference between the levels in the 2000s and the 1980s.

factor of $1+\gamma$. Jobs are subject to exogenous job destruction risks. When jobs are destroyed, workers enter the matching market where they look for a new job opportunity. We capture specificity of human capital by assuming that experienced (i.e., old) workers lose their skills at the time of separation. When they lose their skill, they become "inexperienced," losing their productivity premium γ. Note that if human capital is fully firm specific, job separation results in a complete loss of skills. With probability $1-\delta$, the worker remains experienced and can be hired as an experienced worker retaining the productivity premium. This structure implies that there are three types of workers in the model: (i) young and inexperienced workers, (ii) old and experienced workers, and (iii) old and inexperienced workers. Note that all young workers are inexperienced and thus the terms "young" and "inexperienced" are equivalent in the model. Note also that all experienced workers are old workers. However, there are old workers that used to be experienced but are currently inexperienced due to the skill depreciation that occurred at the time of job loss.

We assume that the matching market is divided by the skill level, meaning that experienced workers and inexperienced workers look for a job in two separate labor markets. In other words, firms hire workers separately for different types of jobs: (i) jobs that require previous experience and thus are suitable only for experienced (and hence old) workers, and (ii) entry-level jobs for which any workers can be employed. We call the matching market for the first type of job "E-matching market" and for the latter type of job "I-matching market." Each jobless worker finds a job with probability, either $f^e(\theta_t^e)$ or $f^i(\theta_t^i)$, depending on whether he undertakes job search in the E-matching market or I-matching market. Job finding probabilities are a function of labor market tightness in the respective matching market (θ_t^e and θ_t^i).[9]

3.1.1 Timing of Events

We adopt the following timing of events in each period.

1. Demographic transitions occur.

[9] We exclude the possibility that old workers look for a job in the I-matching market. Such an incentive does not exist under plausible calibrations. In our quantitative exercises below, we ensure that experienced workers are better off in looking for a job in the E-matching market.

Table 2: Summary of Worker Transitions

State at the End of $t-1$	Transition Probabilities	State at the End of t
$n_{y,t-1}$	$(1-\mu)s^i f^i(\theta_t^i)$	$n_{y,t}$
$n_{y,t-1}$	$\mu s^e f^e(\theta_t^e)$	$n_{o,t}^e$
$n_{y,t-1}$	$(1-\mu)s^i(1-f^i(\theta_t^i))$	$u_{y,t}$
$n_{y,t-1}$	$\mu s^e(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{y,t-1}$	$(1-\mu)(1-s^i)$	$n_{y,t}$
$n_{y,t-1}$	$\mu(1-s^e)$	$n_{o,t}^e$
$u_{y,t-1}$	$(1-\mu)f^i(\theta_t^i)$	$n_{y,t}$
$u_{y,t-1}$	$\mu f^e(\theta_t^e)$	$n_{o,t}^e$
$u_{y,t-1}$	$(1-\mu)(1-f^i(\theta_t^i))$	$u_{y,t}$
$u_{y,t-1}$	$\mu(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{o,t-1}^e$	d	$-$
$n_{o,t-1}^e$	$(1-d)s^e\delta f^i(\theta_t^i)$	$n_{o,t}^i$
$n_{o,t-1}^e$	$(1-d)s^e\delta(1-f^i(\theta_t^i))$	$u_{o,t}^i$
$n_{o,t-1}^e$	$(1-d)s^e(1-\delta)f^e(\theta_t^e)$	$n_{o,t}^e$
$n_{o,t-1}^e$	$(1-d)s^e(1-\delta)(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{o,t-1}^e$	$(1-d)(1-s^e)$	$n_{o,t}^e$
$u_{o,t-1}^e$	d	$-$
$u_{o,t-1}^e$	$(1-d)f^e(\theta_t^e)$	$n_{o,t}^e$
$u_{o,t-1}^e$	$(1-d)(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{o,t-1}^i$	d	$-$
$n_{o,t-1}^i$	$(1-d)s^i f^i(\theta_t^i)$	$n_{o,t}^i$
$n_{o,t-1}^i$	$(1-d)s^i(1-f^i(\theta_t^i))$	$u_{o,t}^i$
$n_{o,t-1}^i$	$(1-d)(1-s^i)$	$n_{o,t}^i$
$u_{o,t-1}^i$	d	$-$
$u_{o,t-1}^i$	$(1-d)f^i(\theta_t^i)$	$n_{o,t}^i$
$u_{o,t-1}^i$	$(1-d)(1-f^i(\theta_t^i))$	$u_{o,t}^i$

2. Job destruction occurs. If the worker is old, the worker may lose their skill with probability δ at this point.

3. Job search takes place if the worker is jobless; whether the worker finds a job or not is then determined.

4. Production takes place.

Note that workers who lose their job can possibly find a new job in the same period. This timing assumption is often used in the new Keynesian literature. We follow this convention.[10]

[10]Note also that there are different but equally plausible timing assumptions (for example, we could assume that demographic transitions occur at the end). Those alternative timing assumptions lead to different

Table 2 considers all possibilities. The variables n and u respectively denote employment and unemployment. The subscripts y and o indicate the age group, young and old. The superscripts for the old worker e and i indicate the skill group, experienced and inexperienced. Job destruction probabilities are denoted by s with superscripts i and e, indicating the two skill groups are subject to potentially different job destruction probabilities.

3.1.2 Stock-Flow Relationships

Given the transitions summarized in Table 2, we can write the laws of motion for each type of workers as follows:

$$
\begin{aligned}
n_{y,t} &= (1-\mu)[1-s^i+s^i f^i(\theta_t^i)]n_{y,t-1} + f^i(\theta_t^i)(1-\mu)u_{y,t-1}, && (1)\\
u_{y,t} &= (1-f^i(\theta_t^i))(1-\mu)u_{y,t-1} + s^i(1-f^i(\theta_t^i))(1-\mu)n_{y,t-1} + \phi h_{y,t-1}, && (2)\\
n_{o,t}^e &= (1-d)[1-s^e+(1-\delta)s^e f^e(\theta_t^e)]n_{o,t-1}^e \\
&\quad + [1-s^e+s^e f^e(\theta_t^e)]\mu n_{y,t-1} + f^e(\theta_t^e)\left(\mu u_{y,t-1} + (1-d)u_{o,t-1}^e\right), && (3)\\
u_{o,t}^e &= (1-d)(1-f^e(\theta_t^e))u_{o,t-1}^e \\
&\quad + (1-f^e(\theta_t^e))\mu[s^e n_{y,t-1} + u_{y,t-1}] + (1-d)s^e(1-\delta)(1-f^e(\theta_t^e))n_{o,t-1}^e, && (4)\\
n_{o,t}^i &= (1-d)[1-s^i+s^i f^i(\theta_t^i)]n_{o,t-1}^i + (1-d)f^i(\theta_t^i)\left(s^e\delta n_{o,t-1}^e + u_{o,t-1}^i\right), && (5)\\
u_{o,t}^i &= (1-d)(1-f^i(\theta_t^i))\left(u_{o,t-1}^i + s^i n_{o,t-1}^i + s^e\delta n_{o,t-1}^e\right). && (6)
\end{aligned}
$$

The last term in Equation (2) $\phi h_{y,t-1}$ represents the number of workers entering the labor force, expressed as a product of the entry rate ϕ and the total number of young workers, denoted by $h_{y,t}(= n_{y,t} + u_{y,t})$. Given Equations (1) and (2), the total number of young workers h_y can be written as:

$$h_{y,t} = (1-\mu)h_{y,t-1} + \phi h_{y,t-1}. \tag{7}$$

The number of old experienced workers ($h_{o,t}^e = n_{o,t}^e + u_{o,t}^e$) and old inexperienced workers ($h_{o,t}^i = n_{o,t}^i + u_{o,t}^i$), respectively, evolves according to:

$$
\begin{aligned}
h_{o,t}^e &= (1-d)(h_{o,t-1}^e - s^e\delta n_{o,t-1}^e) + \mu h_{y,t-1}, && (8)\\
h_{o,t}^i &= (1-d)(h_{o,t-1}^i + s^e\delta n_{o,t-1}^e). && (9)
\end{aligned}
$$

Aggregating these two equations, one gets the evolution of the number of the old workers in the economy:

$$h_{o,t} = (1-d)h_{o,t-1} + \mu h_{y,t-1}. \tag{10}$$

Finally, the total number of workers in the economy evolves according to:

$$h_t = h_{t-1} + \phi h_{y,t-1} - d h_{o,t-1}. \tag{11}$$

results. However, we find that our timing assumption results in cleaner algebraic expressions than the other alternatives, and the differences due to alternative timing assumptions are quantitatively unimportant.

Note that the number of job seekers in each of the two matching markets is not equal to us, because those who are separated at the beginning of each period can start looking for a job within the period. Let $\bar{u}_t^i$ and $\bar{u}_t^o$ be the number of job seekers in the I- and E-matching markets, respectively. These variables are written as

$$\bar{u}_t^i = (1-\mu)[s^i n_{y,t-1} + u_{y,t-1}] + (1-d)[\delta s^e n_{o,t-1}^e + s^i n_{o,t-1}^i + u_{o,t-1}^i], \tag{12}$$

$$\bar{u}_t^e = \mu[s^e n_{y,t-1} + u_{y,t-1}] + (1-d)[(1-\delta)s^e n_{o,t-1}^e + u_{o,t-1}^e]. \tag{13}$$

The terms inside the first square brackets in Equation (12) correspond to the number of young workers who were unemployed at the end of the previous period and who have just lost their job at the beginning of this period, respectively. The terms in the second square brackets are the number of old inexperienced job seekers. A similar interpretation applies to Equation (13). We define the share of old inexperienced workers within $\bar{u}_t^i$ as $\omega_{o,t}$:

$$\omega_{o,t} \equiv \frac{(1-d)[\delta s^e n_{o,t-1}^e + s^i n_{o,t-1}^i + u_{o,t-1}^i]}{\bar{u}_t^i}. \tag{14}$$

The matching function in each market, indicated by the superscript i or e, takes the following Cobb-Douglas form:

$$m_t^i = \overline{m}^i \left(\bar{u}_t^i\right)^{\alpha} \left(v_t^i\right)^{1-\alpha}, \tag{15}$$

$$m_t^e = \overline{m}^e \left(\bar{u}_t^e\right)^{\alpha} \left(v_t^e\right)^{1-\alpha}, \tag{16}$$

where the left-hand side gives the number of hires in each market, v_t^i and v_t^e are the number job openings posted in I- and E-matching market, respectively, and $\bar{m}^i$ and $\bar{m}^e$ are scale parameters of the two matching functions. The job finding probabilities in the two job search pools are written as

$$f^i(\theta_t^i) = \overline{m}^i \left(\theta_t^i\right)^{1-\alpha},$$
$$f^e(\theta_t^e) = \overline{m}^e \left(\theta_t^e\right)^{1-\alpha},$$

where θ_t^i and θ_t^e represent labor market tightness in the two matching markets and are defined as:

$$\theta_t^i = \frac{v_t^i}{\bar{u}_t^i},$$
$$\theta_t^e = \frac{v_t^e}{\bar{u}_t^e}.$$

Lastly we can also define the job filling probability for each job vacancy posted in the two markets as follows:

$$q^i(\theta_t^i) = \overline{m}^i \left(\theta_t^i\right)^{-\alpha},$$
$$q^e(\theta_t^e) = \overline{m}^e \left(\theta_t^e\right)^{-\alpha}.$$

3.2 Firms

Let us now consider the firm's profit maximization problem. Each firm combines the three types of labor to produce differentiated goods that are sold in the monopolistically competitive market at price $p_t(j)$. The firm's profit maximization is subject to the convex price adjustment cost as in Rotemberg (1982) and search frictions in hiring workers. Regarding the latter, the firm pays κ^i and κ^e per vacancy posted in I- and E-matching markets, respectively.

The following linear production technology is available to the firms, which are indexed by j:

$$y_t(j) = n_{y,t}(j) + n^i_{o,t}(j) + (1+\gamma)n^e_{o,t}(j).$$

where $y_t(j)$ is output of the firm j in period t. Let $\mathbf{s}^f_t(j)$ be the vector of the state variables for the firm j coming into period t. The firm maximizes its value, $\Pi(\mathbf{s}^f_t(j))$, by choosing $p_t(j), n_{y,t}(j), n^i_{o,t}(j), n^e_{o,t}(j), v^i_t(j)$, and $v^e_t(j)$:

$$\begin{aligned}\Pi\big(\mathbf{s}^f_t(j)\big) = \max \left(\frac{p_t(j)}{P_t}\right)^{1-\epsilon} & Y_t - w_{y,t}(j)n_{y,t}(j) - w^i_{o,t}(j)n^i_{o,t}(j) - w^e_{o,t}(j)n^e_{o,t}(j) - T_t \\ & - \kappa^i v^i_t(j) - \kappa^e v^e_t(j) - \frac{\chi}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t + \hat{\beta}_{t,t+1}\Pi\big(\mathbf{s}^f_{t+1}(j)\big), \end{aligned} \tag{17}$$

where Y_t is aggregate output in the economy; P_t is the aggregate consumer price index; χ is the parameter for the price adjustment cost; ϵ is the elasticity of substitution between goods; $\hat{\beta}_{t,t+1}$ is a stochastic discount factor of the representative household, which will be specified below; T_t is a lump sum tax that will be used to finance unemployment insurance benefits; and $w_{y,t}(j)$, $w^i_{o,t}(j)$, and $w^e_{o,t}(j)$ are wages of young workers, old inexperienced workers, and old experienced workers, respectively. Wages are determined though Nash bargaining between each worker and the firm. We will discuss the wage determination process below.

The vector of the state variables $\mathbf{s}^f_t(j)$ includes $p_{t-1}(j)$, $n_{y,t-1}(j)$, $n^i_{o,t-1}(j)$, and $n^e_{o,t-1}(j)$.[11] Note also that in writing the first term on the right-hand side of (17), we used the demand function for the good j:

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

The firm's profit maximization is subject to the following constrains:

$$\left[\frac{p_t(j)}{P_t}\right]^{-\epsilon} Y_t = n_{y,t}(j) + n^i_{o,t}(j) + (1+\gamma)n^e_{o,t}(j), \tag{18}$$

$$n_{y,t}(j) = (1-\mu)(1-s^i)n_{y,t-1}(j) + (1-\omega_{o,t})q^i(\theta^i_t)v^i_t(j), \tag{19}$$

$$n^i_{o,t}(j) = (1-d)(1-s^i)n^i_{o,t-1}(j) + \omega_{o,t}q^i(\theta^i_t)v^i_t(j), \tag{20}$$

$$n^e_{o,t}(j) = (1-d)(1-s^e)n^e_{o,t-1}(j) + (1-s^e)\mu n_{y,t-1}(j) + q^e(\theta^e_t)v^e_t(j). \tag{21}$$

[11] The price level drops out of the list in the end but we include it here for completeness.

Recall that $\omega_{o,t}$ is the share of old workers in the I-matching market. When the firm is posting a vacancy in the I-matching market, the probability of finding any worker is given by $q^i(\theta_t^i)$, and conditional on that, he is an old worker with probability $\omega_{o,t}$, which is defined in (14), and a young worker with probability $1-\omega_{o,t}$.

The firm's decision is characterized by the following first-order conditions. At this point, we impose the symmetry of the equilibrium and drop the index j. First, the following condition characterizes the evolution of the inflation rate in the presence of the price adjustment cost as in the standard new Keynesian model:

$$1-\epsilon-\chi\left(\pi_t-1\right)\pi_t+\hat{\beta}_{t,t+1}\chi\left(\pi_{t+1}-1\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}+\tau_t\epsilon=0, \tag{22}$$

where τ_t is the Lagrange multiplier associated with constraint (18) and π_t represents the gross inflation rate, defined by $\frac{P_t}{P_{t-1}}$. The multiplier τ_t can be interpreted as the marginal cost for the firm as in the standard new Keynesian model. The following two equations govern job creation (vacancy posting) in the two matching markets:

$$\omega_{o,t}J_{o,t}^i+(1-\omega_{o,t})J_{y,t}=\frac{\kappa^i}{q^i(\theta_t^i)}, \tag{23}$$

$$J_{o,t}^e=\frac{\kappa^e}{q^e(\theta_t^e)}, \tag{24}$$

where $J_{y,t}$, $J_{o,t}^i$ and $J_{o,t}^e$ are the Lagrange multipliers associated with constraints (19), (20), and (21), respectively and can be interpreted as the marginal gain to the firm from adding each type of labor by one unit. Equations (23) and (24) equate the marginal cost of posting a vacancy (RHS) and the marginal gain (LHS) in the I-matching market and the E-matching market, respectively. The LHS of Equation (23) indicates that the gain from posting a vacancy in the I-matching market is influenced by the composition of the matching market. In particular, if $J_{y,t}>J_{o,t}^i$ holds, then a higher $\omega_{o,t}$ would lower the value of LHS and thus, in equilibrium, market tightness θ_t^i decreases. Lastly, the marginal value of each type of job evolves according to:

$$J_{y,t}=\tau_t-w_{y,t}+\hat{\beta}_{t,t+1}\Big[(1-\mu)(1-s^i)J_{y,t+1}+(1-s^e)\mu J_{o,t+1}^e\Big], \tag{25}$$

$$J_{o,t}^e=(1+\gamma)\tau_t-w_{o,t}^e(j)+(1-d)(1-s^e)\hat{\beta}_{t,t+1}J_{o,t+1}^e, \tag{26}$$

$$J_{o,t}^i=\tau_t-w_{o,t}^i+(1-d)(1-s^i)\hat{\beta}_{t,t+1}J_{o,t+1}^i. \tag{27}$$

The interpretation of these three equations is straightforward. First, a job filled by a young worker yields a flow payoff of $\tau_t-w_{y,t}$. The worker becomes an old and experienced worker with probability μ, while he continues to be inexperienced with probability $1-\mu$. Next, if the worker is experienced, the flow payoff is $(1+\gamma)\tau_t-w_{o,t}^e$. When either the worker exits from the labor force with probability d or the job is destroyed with probability s_o^e, the firm obtains zero due to free entry; otherwise the value is $J_{o,t+1}^e$. Lastly, the flow payoff for the firm when the job is filled by an old but inexperienced worker is given by $\tau_t-w_{o,t}^i$. When either the worker exits from the labor force with probability d or the job is destroyed with probability s_o^i, the firm obtains zero due to free entry; otherwise the value is $J_{o,t+1}^i$.

3.3 Household

The household sector consists of different types of workers in terms of their age, their skill level, and their labor market status. We assume that the representative household pools incomes of all members and allocates consumption across its members equally. The household is also assumed to be fully altruistic towards its future members as well. These assumptions imply that the member's age, skill, and labor market status do not matter for his consumption.[12]

The household maximizes the following value function $V(.)$:

$$V(\mathbf{s}_t^h) = \max h_t u\left(c_t\right) + \beta V(\mathbf{s}_{t+1}^h), \tag{28}$$

where c_t is the per-capita consumption index given by:

$$c_t \equiv \left(\int_0^1 c_t(j)^{1-\frac{1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

with $c_t(j)$ representing the quantity of good j consumed by each member of the household. A continuum of goods exists over the interval $[0,1]$. $\mathbf{s}_t^h$ is the vector of the state variables for the household. Recall that h_t represents population in the economy and thus Equation (28) implies that the household maximizes total welfare of the dynasty into the indefinite future, with the future periods discounted by a common discount factor β per period. All employed workers supply labor inelastically with the same constant disutility level (regardless of their types). We also assume that jobless workers look for a new job with the same constant disutility level, again regardless of their types. Accordingly, hours of work and search do not enter the value function above.[13]

Each member of the household allocates his consumption expenditures among the different goods by maximizing c_t for any given level of expenditures $\int_0^1 p(j)c(j)dj$, resulting in the demand equations:

$$c_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\epsilon} c_t \tag{29}$$

for all $j \in [0,1]$ where $P_t \equiv \left(\int_0^1 p_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$. Under this situation, we can write the household budget constraint as:

$$h_t c_t + \frac{B_{t+1} h_t}{P_t} = (1 + i_{t-1}) \frac{B_t h_{t-1}}{P_t} + w_{y,t} n_{y,t} + w_{o,t}^e n_{o,t}^e + w_{o,t}^i n_{o,t}^i + b_y u_{y,t} + b_o^e u_{o,t}^e + b_o^i u_{o,t}^i + d_t,$$

where B_t represents the per-capita bond holdings that pay nominal interests; b_y, b_o^e, and b_o^i represent unemployment insurance benefits that each type of jobless worker receives at the

[12] We can extend our model by allowing for heterogeneity in the consumption side of the model as in Fujiwara and Teranishi (2008) and Ferrero and Carvalho (2014). Our intension of abstracting away from it is to focus on productivity heterogeneity and isolate its effects in this paper.

[13] This specification is adopted only for the purpose of focusing on the margins we would like to highlight in this paper. We can easily adopt a different formulation, for example, in which unemployed workers enjoy some utility from leisure.

end of the period; and d_t represents a dividend from the firm that the household owns. The state variables for the household $\mathbf{s}_t^h$ consists of $n_{y,t-1}$, $n_{o,t-1}^e$, $n_{o,t-1}^i$, $h_{o,t-1}^e$, and B_t.

The conditions with respect to consumption and bond holdings result in the following consumption Euler equation:

$$u'(c_t) = \beta \frac{1+i_t}{\pi_{t+1}} u'(c_{t+1}).$$

Note that the stochastic discount factor $\hat{\beta}_{t,t+1} \equiv \frac{u'(c_{t+1})}{u'(c_t)}$ was used in discounting the future profit flows in the firm's problem.

3.3.1 Marginal Values of Employment

We assume that wages are determined through Nash bargaining between each individual worker and the firm. For this purpose, we need to compute the marginal value of each type of employment. Note that evolutions of the marginal values take into account the following laws of motion:

$$n_{y,t} = (1-\mu)[1 - s^i + s^i f^i(\theta_t^i)]n_{y,t-1} + f^i(\theta_t^i)(1-\mu)(h_{y,t} - n_{y,t-1}), \tag{30}$$

$$\begin{aligned} n_{o,t}^e = (1-d)[1 - s^e + (1-\delta)s^e f^e(\theta_t^e)]n_{o,t-1}^e + [1 - s^e + s^e f^e(\theta_t^e)]\mu n_{y,t-1} \\ + f^e(\theta_t^e)[\mu(h_{y,t-1} - n_{y,t-1}) + (1-d)(h_{o,t-1}^e - n_{o,t-1}^e)], \end{aligned} \tag{31}$$

$$n_{o,t}^i = (1-d)[(1 - s^i + s^i f^i(\theta_t^i))n_{o,t-1}^i + f^i(\theta_t^i)(s^e \delta n_{o,t-1}^e + h_{o,t-1} - h_{o,t-1}^e - n_{o,t-1}^i)], \tag{32}$$

$$h_{o,t}^e = (1-d)(h_{o,t-1}^e - s^e \delta n_{o,t-1}^e) + \mu_{t-1} h_{y,t-1}, \tag{33}$$

where we substituted out $u_{y,t-1}$, $u_{o,t-1}^e$, $u_{o,t-1}^i$, and $h_{o,t-1}^i$ from (1), (3), (5), and (8). To simplify the notation, let us introduce the following four variables:

$$M_{y,t} \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{y,t-1}}, M_{o,t}^e \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{o,t-1}^i}, M_{o,t}^i \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{o,t-1}^i}, \text{and } D_t \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial h_{o,t-1}^e}.$$

Given (30) through (32), the evolutions of these four variables are written as follows:

$$\begin{aligned} M_{y,t} &= u'(c_t)(w_{y,t} - b_y) \\ &\quad + \beta\Big[(1-\mu)(1-s^i)(1 - f^i(\theta_{t+1}^i))M_{y,t+1} + \mu(1-s^e)(1 - f^e(\theta_{t+1}^e))M_{o,t+1}^e\Big], \end{aligned} \tag{34}$$

$$M_{o,t}^i = u'(c_t)(w_{o,t}^e - b_o^e) + (1-d)(1-s^i)(1 - f^i(\theta_{t+1}^i))\beta M_{o,t+1}^i, \tag{35}$$

$$\begin{aligned} M_{o,t}^e = u'(c_t)(w_{o,t}^e - b_o^e) + (1-d)\beta\Big[(1-s^e)(1 - f^e(\theta_{t+1}^e))M_{o,t+1}^e \\ - \delta s^e \big\{f^e(\theta_{t+1}^e)M_{o,t+1}^e - f^i(\theta_{t+1}^i)M_{o,t+1}^i + D_{t+1}\big\}\Big], \end{aligned} \tag{36}$$

$$D_t = u'(c_t)(b_o^e - b_o^i) + (1-d)\beta\Big[f^e(\theta_{t+1}^e)M_{o,t}^e - f^i(\theta_{t+1}^i)M_{o,t+1}^i + D_{t+1}\Big]. \tag{37}$$

Equations (34) through (35) give the marginal values of each type of employment to the household net of having one more unemployed worker of the corresponding type. In Equation (34), the first term represents the flow surplus of employment for the young worker and the terms that follow capture the future possibilities that the worker stays young with probability $1 - \mu$, while the worker becomes old and experienced with probability μ. A similar interpretation applies to Equation (35). Equation (36) looks somewhat different because of the terms in the curly brackets which capture the possibility of skill loss that occurs with probability $s^e\delta$. The value D_t whose evolution is described in Equation (37) corresponds to the difference in the values of being unemployed as experienced and inexperienced old workers.

3.4 Wages

We assume that each type of worker and the firm engage in Nash bargaining individually. Let $S_{y,t}$ be the joint surplus of the match between the young worker and the firm, i.e., $S_{y,t} = J_{y,t} + \frac{1}{u'(c_t)} M_{y,t}$. This surplus is split between the worker and the firm according to the worker bargaining power η and the firm bargaining power $1 - \eta$:

$$\begin{aligned} (1-\eta) S_{y,t} &= J_{y,t}, \\ \eta S_{y,t} &= \frac{1}{u'(c_t)} M_{y,t}. \end{aligned}$$

Plugging (25) and (34) in $\eta J_{y,t} = (1-\eta)\frac{1}{u'(c_t)} M_{y,t}$ and solving for wage, we obtain the following expression:

$$w_{y,t} = \eta\tau_t + (1-\eta) b_y + \eta\hat{\beta}_{t,t+1}\left[(1-\mu)(1-s^i) f^i(\theta^i_{t+1}) J_{y,t+1} + \mu(1-s^e) f^e(\theta^e_{t+1}) J^e_{o,t+1}\right], \tag{38}$$

Similar algebras applied to the other two types of matches result in the following wage equations:

$$w^i_{o,t} = \eta\tau_t + (1-\eta) b^i_o + (1-d)(1-s^i)\eta\hat{\beta}_{t,t+1} f^i(\theta^i_{t+1}) J^i_{o,t+1} \tag{39}$$

$$\begin{aligned} w^e_{o,t} = \eta(1+\gamma)\tau_t + (1-\eta) b^e_o + (1-d)\hat{\beta}_{t,t+1}\Bigg[&(1-s^e) f^e(\theta^e_{t+1})\eta J^e_{o,t+1} \\ &+ \delta s^e \left\{ f^e(\theta^e_{t+1})\eta J^e_{o,t+1} - f^i(\theta^i_{t+1})\eta J^i_{o,t+1} + \frac{D_{t+1}}{u'(c_{t+1})} \right\} \Bigg]. \end{aligned} \tag{40}$$

3.5 Monetary Policy and Resource Constraint

Imposing zero net supply of the bonds in the aggregate, and assuming that $b_y u_{y,t} + b^e_o u^e_{o,t} + b^i_o u^i_{o,t} = T_t$, one can obtain the following resource constraint:

$$Y_t = h_t c_t + \kappa^i v^i_t + \kappa^e v^e_t + \frac{\chi}{2}(\pi_t - 1)^2 Y_t.$$

Lastly, monetary policy is characterized by a standard Taylor-type rule:

$$1 + i_t = \frac{1}{\beta} + \psi_\pi(\pi_t - 1).$$

where we assume that $\psi_\pi > 1$.

4 Benchmark Calibration

The calibration below intends to replicate the Japanese economy at the start of the aging process, assuming that the economy is in the steady state at that point. In the quantitative exercises below, we compute the transition path of the economy when the labor force entry ϕ follows the path of its empirical counterpart. One period in the model corresponds to one quarter in the actual economy.

4.1 Demographic Transitions

We assume that each worker in the model spends on average 20 years as a "young" worker and 25 years as an "old" worker. What we have in mind as a career of a typical worker is that he enters the labor force when he is 20 years old, accumulates his experience over the next 20 years, then becomes an experienced worker when he is 40 years old, spends the rest of his career as an old worker, and then retires (or die) at the age of 65 years old. This typical demographic transition implies that $\mu = 1/80 = 0.0125$ and $d = 0.01$. Next, we translate the total fertility rate (TFR) into the labor entry rate ϕ as follows. Note that the TFR represents the number of children that are born to a woman over her childbearing years, and it was above 4 at the peak in 1948. Suppose that childbearing years correspond to the 20-year period that each worker spends as a young worker.[14] The TFR of 4 implies that each young worker reproduces 2 young workers over this 20-year period, corresponding to $\phi = 0.025(= 2/80)$. But given that our young worker's definition is somewhat narrower than the actual childbearing years, we choose to use $\phi = 0.02$. Note that the steady state of the economy is characterized by a constant population growth. Specifically, Equation (7) implies that the growth rate is given by $g = \phi - \mu = 0.02 - 0.0125 = 0.0075$, where $g = \frac{h_t}{h_{t-1}} - 1$.

4.2 Labor Market Transitions

Next, let us discuss the calibration of the parameter values related to labor market transitions. Note that transition rates are all expressed as quarterly values. First, to set separation rates, we refer to the several pieces of evidence presented by Lin and Miyamoto (2012) and Esteban-Pretel and Fujimoto (2012). Lin and Miyamoto (2012) show that the monthly aggregate job separation rate into unemployment averaged roughly around 0.4% over the period between 1980 and 2010. Based on this, we target the quarterly separation rate of

[14] This assumption is not exactly correct because childbearing years in the calculation of the TFR are between 15 and 49 years old. But we are considering only the middle 20-year period for simplicity.

1.2% per quarter. The paper by Esteban-Pretel and Fujimoto (2012) shows that the separation rate declines as workers get older. A visual inspection of their result suggests that the calibration where the young worker's separation rate is roughly twice as high that of the old worker is plausible. These considerations allow us to uniquely pin down s^i and s^e at 1.5% and 0.75% (per quarter). Next, we target the steady-state job finding rates $f^i(\theta^i)$ and $f^e(\theta^e)$ both at 35%. This value is roughly consistent with the monthly evidence presented by Lin and Miyamoto (2012). We also impose that, in the steady state $q^i(\theta^i) = f^i(\theta^i)$ and $q^i(\theta^i) = f^e(\theta^e)$. These assumptions allow us to determine the scale parameters of the matching functions (see (15) and (16)), $\overline{m}^i$ and $\overline{m}^e$ for a given value of α. By setting $\alpha = 0.5$, we can set both $\overline{m}^i$ and $\overline{m}^i$ to 0.35. The only remaining parameter within the steady-state stock-flow relationships is δ, which measures the risk of skill loss. We simply set this parameter at 0.8 in the benchmark calibration and show how the model's behavior changes as we lower its value.

4.3 Remaining Parameters

The discount factor is chosen to be 0.995, which implies the steady-state annual real interest at 2%. Our choice is based on the evidence in Figure 5 where the real interest rate fluctuated around 2% over the sample period. Note that, in our model, the steady-state real interest rate is invariant to the demographic structure. However, changes in the demographic structure induce fluctuations in the real interest rate over a long period of time, which is how we address its effects. Note also that our results are largely insensitive to the steady-state level of the real interest rate. In other words, whether we set the steady-state real interest rate at 4% or 1% instead of 2% has virtually no impact on the changes in the real interest rate along the transition dynamics of the model. The value of the CRRA is set to 2, a value that is within a plausible range in macro literature. It is also roughly consistent with the time-series evidence on consumption growth and the real interest rate presented in Figure 5, where one can see the volatility of the real interest rate is roughly twice as large as that of the growth rate of per-capita consumption.

We assume that the worker bargaining power η is 0.5, which is often used in the search/-matching literature given the lack of direct evidence. We set the experience premium γ at 60%, which is based on the observed wage profile in Japan. The observed profile shows that old workers (those who are between 40-64) on average make roughly 40% more than young workers (those who are between 20-39). Note that the slope of the observed profile does not measure $1 + \gamma$, because old workers in our model include those who have lost their firm specific skill and thus have become inexperienced. By setting $\gamma = 0.6$, we can match the observed wage premium of 40%. We set the level of unemployment insurance benefits by assuming that each worker receives 60% of their productivity while unemployed. This procedure implies that $b_y = 0.6$ and $b_o^e = (1 + \gamma) \times 0.6 = 0.96$ given that productivity of the young worker is normalized to 1 and old experienced worker enjoys 60% productivity premium. Regarding the old inexperienced worker, we assume that their benefit level is tied to their productivity level as an experienced worker, meaning that $b_o^i = b_o^e$.[15] Given the parameter values so far, we can compute all continuation values in the steady state of the economy. Using the steady-state values of J_y, J_o^e, and J_o^i in the free-entry conditions (24) and (23), we can back out the vacancy posting costs in the two matching markets. This procedure yields $\kappa_i = 0.251$ and $\kappa_e = 0.337$.

We set the remaining three parameters ψ_π, ϵ, and χ to the values standard in the new Keynesian literature. First, the elasticity of substitution ϵ is set to the conventional value of 6, which implies the steady-state markup at 20%. Given this steady-state markup, the Rotemberg (1982) price adjustment cost parameter χ is chosen to be 77. This value corresponds to the average price resetting frequency being once a year under the Calvo (1983) setup.[16] Finally, we set the policy reaction parameter in the Taylor rule ψ_π at 2.

4.4 Initial Steady State Equilibrium

Table 3 presents steady-state values of wages, employment levels, and unemployment rates at different levels of aggregation in the initial steady state. Note that employment levels are presented as a share of the total labor force (population) while unemployment rates are expressed as a share of the labor force of each type.

At the aggregate level, the unemployment rate is 5%, which is higher than the levels observed in the 1970s and 1980s. The main reason for this is that our model assumes that all young workers enter the labor force as unemployed, and in the initial steady state, the entry rate is assumed to be very high. One can see that the young worker's unemployment rate is 8%, while the old worker's unemployment rate is much lower at 2%. Within old workers, the experienced worker's unemployment rate is 1% while inexperienced worker's unemployment rate is 6%. The reason for the higher unemployment rate for the latter group is that 80% (δ) of the job loss flow from experienced employment goes into this I-matching pool in our calibration.[17]

The steady-state employment levels indicate that within all employed workers, roughly 43% are old workers and the rest are young workers. Roughly quarter of old workers are experienced (10% as a share of the labor force) and three quarters are inexperienced. The average wage is calculated as 0.98. Note that we normalize average productivity of the young worker and old inexperienced worker at 1 and the experienced worker enjoys the 60% skill premium. The value 0.98 is the weighted average of the wages of all three types. The wage of the experienced worker is the highest given the large skill premium. There are two reasons why w_o^i and w_y differ from each other, even though their productivities are equal. First, young worker's current wage incorporates the possibility that they become experienced in

[15] An alternative is to link it to the productivity level as an inexperienced worker. In light of the actual UI benefit scheme, the procedure we adopted appears more reasonable, given the replacement ratio is usually tied to the worker's wage prior to job loss. However, the alternative calibration does not change our results materially.

[16] As is well known, the aggregate supply equations under the Rotemberg (1982) and Calvo (1983) formulations are observationally equivalent up to the first-order approximation.

[17] A larger constant flow into the pool implies a larger stock in the steady state.

Table 3: Initial Steady-State Values

	Wages		Employment Levels		Unemployment Rates	
Aggregate	0.98	$\frac{n_y w_y + n_o^e w_o^e + n_o^i w_o^i}{n_y + n_o^e + n_o^i}$	0.95	$\frac{n_o^i + n_o^e + n_y}{h}$	0.05	$\frac{u_o^i + u_o^e + u_y}{h}$
Young	0.82	w_y	0.54	$\frac{n_y}{h}$	0.08	$\frac{u_y}{h_y}$
Old	1.19	$\frac{n_o^e w_o^e + n_o^i w_o^i}{n_o^e + n_o^i}$	0.41	$\frac{n_o^e + n_o^i}{h}$	0.02	$\frac{u_o^e + u_o^i}{h_o^e + h_o^i}$
Old Exp.	1.31	w_o^e	0.31	$\frac{n_o^e}{h}$	0.01	$\frac{u_o^e}{h_o^e}$
Old Inexp.	0.83	w_o^i	0.10	$\frac{n_o^i}{h}$	0.06	$\frac{u_o^i}{h_o^i}$

the future (see $J^e_{o,t+1}$ in Equation (38)), which raises w_y relative to w_o^i, whereas the old worker's continuation value drops to zero when hit by the d shock. However, old worker's wage is pushed up, because his flow outside option value b_o^i is higher (remember that we set b_o^i by linking it to their productivity as an experienced worker). The latter effect is larger than the first effect and thus w_o^i and higher than w_y in our calibration. The average wage of old workers (including both experienced and inexperienced) is roughly 40% higher than the young worker's wage. As mentioned above, this measured age premium is consistent with the observed wage profile in Japan.

5 Quantitative Exercises

In this section, we analyze the perfect foresight equilibrium paths of the economy, assuming that the labor force entry rate falls to a new steady-state value. The assumed path of the entry rate mimics the path of the series plotted in Figure 1. Demographic transitions take place only gradually and thus it takes a long time for the economy to converge to a new steady state, even though ϕ itself reaches a new value relatively quickly.

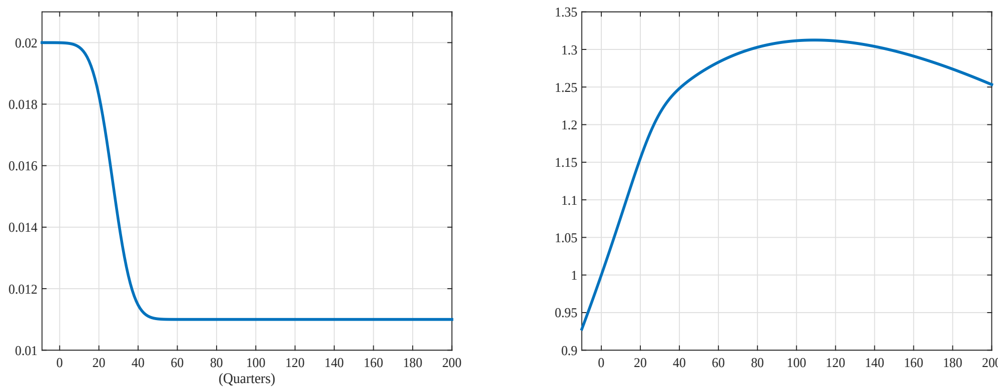
5.1 Labor Force Entry Path

Recall that the labor force entry rate ϕ was set to 2% per quarter in the initial steady state. We assume that it falls to 1.1% roughly over the following 10-year period. This assumed path is plotted in Panel (a) of Figure 6. The entry rate of 1.1% per quarter corresponds to the total fertility rate that is somewhat below 1.8, thus being roughly in line with the level of the birth rate after the baby boom. Remember that, in our model, ϕ represents the labor force entry rate, and one can see in Panel (b) of Figure 6 that labor force entry experienced a sharp drop throughout the 1970s. The assumed path of ϕ captures this decline that occurred over 10- to 12-year period, and one can associate the period 1 in our model experiment with the first quarter of 1970.

Importantly, with this assumed path of the entry rate, the labor force begins to fall at roughly the same timing as in the actual data. The earlier figure (Panel (b) of Figure 2) showed that the Japanese labor force started shrinking in the late 1990s, while one can see

Figure 6: The Entry Rate and Labor Force

(a) Assumed Entry Rate Path (b) Implied Path of the Labor Force



Notes: The labor force is normalized to 1 in period 0.

in Panel (b) of Figure 6 that the labor force in the model peaks around 110th quarter, which corresponds to the late 1990s in our interpretation of the first period being the start of 1970.

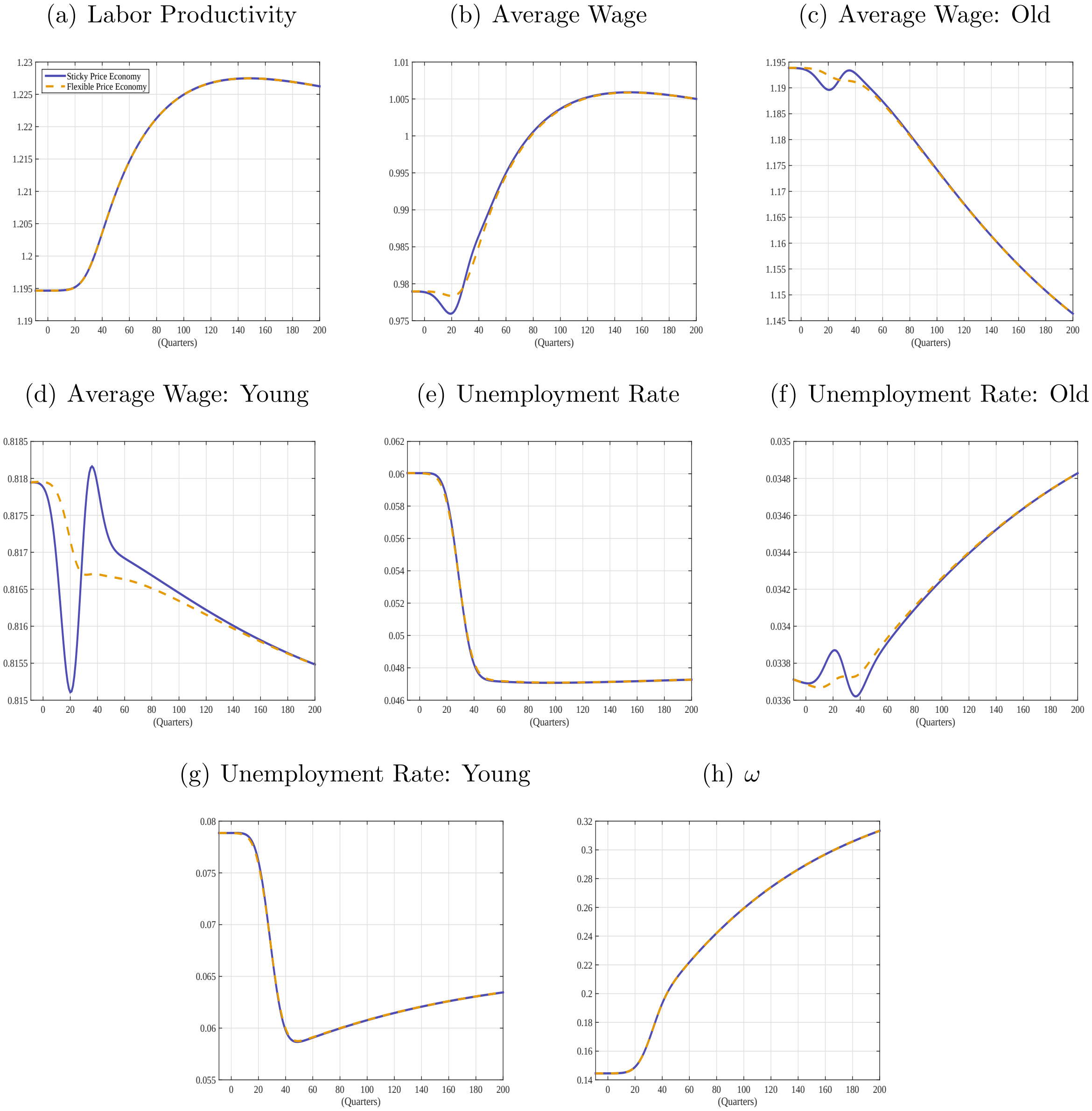
5.2 Perfect Foresight Equilibrium

We now present our main results. While the model presented above features sticky prices, we also solve for the equilibrium paths of the flexible price economy. Each figure below presents the paths of each variable under the two economies.

5.2.1 Labor Market Responses to Aging

Figure 7 presents the paths of the key labor market variables. One can immediately see that the paths under the sticky price and flexible price economies are very similar to each other. This is interesting but not surprising, considering that our experiment considers economy's responses that spread over a long-period of time. Panel (a) presents the path of labor productivity. Labor productivity increases in the initial phase of the aging process, because this phase corresponds to the situation where more and more young workers are gaining experience, thus becoming more productive. However, after this initial phase is over, labor productivity growth slows down, and eventually turns negative. The slowdown occurs simply because there are fewer young workers (whose future productivity is higher). The decline in the level of productivity in the late phase of aging is due to the presence of firm-specific human capital in the model. Remember that old (experienced) workers are subject to the risk of job destruction that is accompanied by the loss of the firm-specific skill. As the workforce gets older, there are more and more workers that are subject to this risk. When the experienced worker is hit by the δ shock, the worker is employable only at the entry-level job, and therefore, a higher share of old (experienced) workers eventually leads to a decline in labor productivity. The quantitative importance of this latter effect depends on the value

Figure 7: Perfect Foresight Equilibrium Paths (I)



(a) Labor Productivity (b) Average Wage (c) Average Wage: Old

(d) Average Wage: Young (e) Unemployment Rate (f) Unemployment Rate: Old

(g) Unemployment Rate: Young (h) ω

of δ. We will discuss this below.

Panel (b) plots average wage in the economy. The behavior of this series is similar to that of labor productivity. Overall, the hump-shaped pattern comes from the same mechanism behind the hump-shaped pattern in labor productivity. In Panel (c), one can see average wage of old workers steadily fall over time. There are two reasons for this result. The main reason is the changing composition within these old workers. As the labor-force aging progresses, the share of inexperienced workers (within old workers) increases, thereby putting a downward pressure on average wage of old workers. Although wages of two types workers (experienced and inexperienced) within the old are affected by changing market tightness in each matching market, these effects turn out to be quantitatively small. Average wage of young workers falls over time, as indicated by Panel (d), because of the increase in ω, the share of old inexperienced workers in the I-matching market. A higher ω lowers job creation in this market, because wage of old and inexperienced workers is higher than that of young workers, and hence, a higher ω translates into a lower return to posting vacancies in that market.

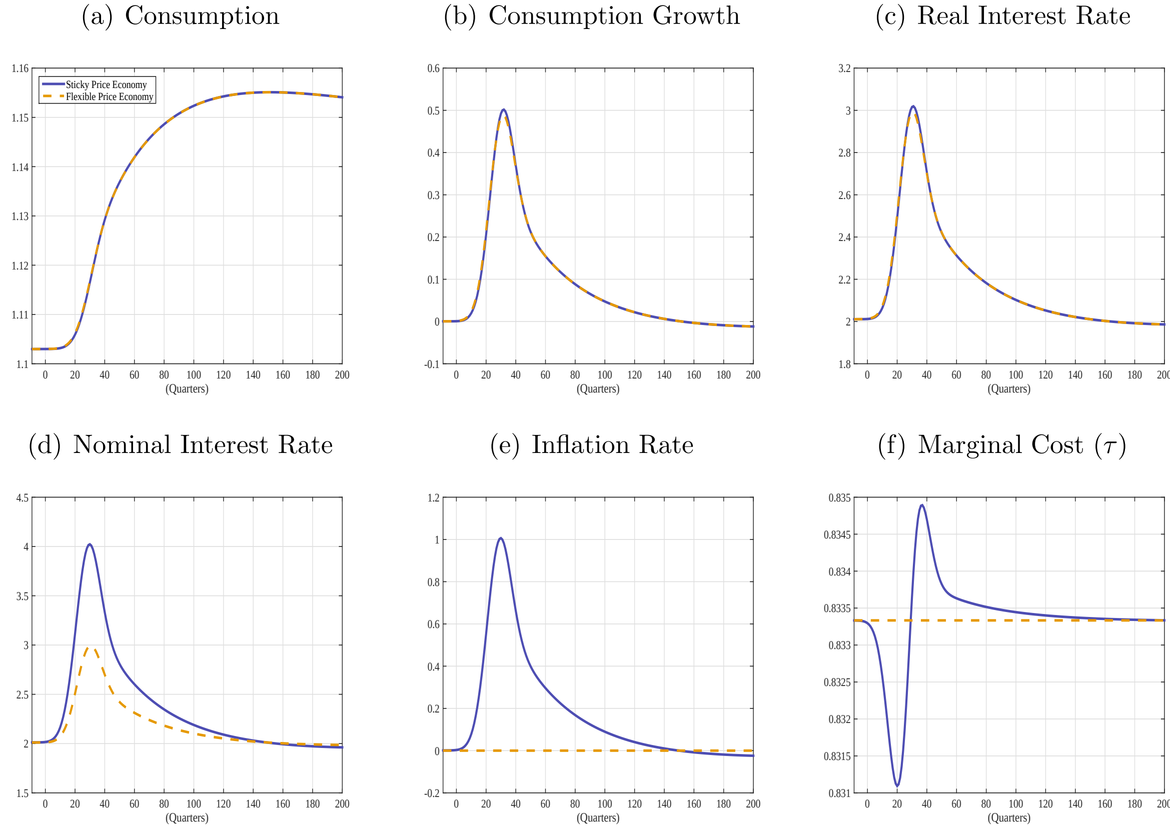
The aggregate unemployment rate falls rather sharply initially and then stays roughly flat thereafter. The initial fall in the aggregate unemployment comes directly from the decline in labor-force entry. Remember that young workers enter the labor force as unemployed and thus a smaller volume of this flow directly reduces the unemployment rate. The effect can be seen more clearly in Panel (g), where the unemployment rate of the young workers drops almost 2 percentage points. However, the young worker's unemployment increases after the initial drop. The reason for the upturn is the increase in ω. As explained in the previous paragraph, the increase in the share of old workers in the I-matching market discourages job creation in that market, thus resulting in a higher unemployment rate of the young workers. The old worker's unemployment (Panel (f)) steadily increases over time and the main reason is again a higher share of inexperienced workers within old workers. The level of the inexperienced worker's unemployment rate is higher than that of experienced workers (mainly because $s_e < s_i$ in our calibration) and thus a higher share of inexperienced workers results in a higher unemployment among old workers.

Note that the unemployment rate in Japan increased substantially in the 1990s and the model does not replicate the increase. This is not surprising, because our model does not allow the TFP trend to change. The decline in trend TFP growth is known to be an important feature accounting for the Japan's so-called "Lost Decade." Relatedly, in our experiment, job separation rates are assumed to be constant, and as we will see below, the increases in the unemployment rate in the 1990s are associated with higher separation rates. In a model with endogenous job separation, a decline in trend TFP growth is likely to cause the job separation rate and thus the unemployment rate to increase as in the data.

5.2.2 Real Interest Rate and Inflation Responses

Figure 8 displays the effects of workforce aging on the variables related to the real interest rate and inflation. The first three panels of the figure indicates that the allocation of the economy is hardly affected by the presence of price-setting frictions. The hump-shaped pattern in the level of per-capita consumption comes from the same mechanism discussed with respect to the hump-shaped pattern in labor productivity. Panel (b) expresses consumption in the annualized growth rate in percent. Consumption growth increases roughly 0.5 percentage point at its peak and then gradually comes down. Given the value of CRRA parameter of 2, the real interest rate rises 1 percentage point and follows a similar gradual declining path. Note that the peak of these two variables occurs roughly 7 to 8 years after the entry rate ϕ begins to fall. This timing corresponds to the late 1970s in the data. In Table 1 earlier, we showed that consumption growth and the real interest rate fell 1.3 percentage points and

Figure 8: Perfect Foresight Equilibrium Paths (II)



Notes: Consumption growth, real interest rate, nominal interest rate, and inflation are expressed as annualized percent.

2.3 percentage points, respectively, between the 1980s and 2000s. Our experiment implies that aging of the labor force accounts for roughly 40% of these declines. Below we will consider how these numbers change under the alternative calibrations. Another subtle but important observation in Panels (b) and (c) is that, in the late phase of aging (around 150th quarter), consumption growth turns negative (before coming back its steady-state level again later from below).[18] This negative growth occurs only in the economy with high δ as in our benchmark calibration. As discussed with respect to the path of labor productivity, there are more and more old workers who are (re-)employed only in the entry-level job.

The second row of Figure 8 presents the equilibrium paths of the nominal interest rate, inflation, and the marginal cost. As discussed in the model section, the nominal interest rate is set by a standard Taylor rule which does not internalize the effect of the changing natural rate of interest. The assumption here is that the monetary authority fails to recognize the low-frequency movements in the natural rate, and simply follows the standard Taylor rule. Under this assumption, the inflation rate, after initially increasing roughly 1 percentage

[18] The negative consumption growth path corresponds to the path of the real interest rate that falls below its steady-state value of 2%, as can be seen in Panel (c).

point, follows a gradual declining trend over the following decades. In particular, the inflation rate turns negative (deflation) around 150th quarter, which roughly corresponds to mid 2000s. Obviously, the inflation path would change, when a different value of the Taylor rule parameter ψ_π (which is currently set to 2) is used. Moreover, there is a monetary policy rule that completely neutralizes the effect of the demographic change in this economy, namely, the one where the intercept term in the policy rule equals consumption growth. But again, the presumption here is that the monetary authority cannot do so, because of the various uncertainties surrounding the measurement of the natural rate of interest. The lesson here is that avoiding deflation would require a more aggressive monetary policy reaction.

5.3 Results Under Alternative Calibrations

To better understand the benchmark results so far, we consider the following three alternative calibrations.

1. Lower δ: The parameter δ is set to a lower value 0.4 instead of 0.8 in the benchmark calibration. This case reduces the effect of specificity of worker's skill and thus is useful in highlighting the importance of human capital specificity in generating our results.[19]

2. Higher separation rates s_o and s_y: Both separation rates, s_o and s_y, are doubled and set to 0.015 and 0.03, respectively. In our model, the separation decision is taken to be exogenous, but the data show that the job separation rate has increased significantly in the 1990s and 2000s in Japan, raising the unemployment rate in those years. The higher values for these two parameters are chosen by referring to the evidence by Lin and Miyamoto (2012) who show that the employment-to-unemployment transition rate doubled in the 1990s and 2000s. The separation rate (especially s_o) has an important implication for the model dynamics since it means a higher risk of the skill loss for the experienced workers.

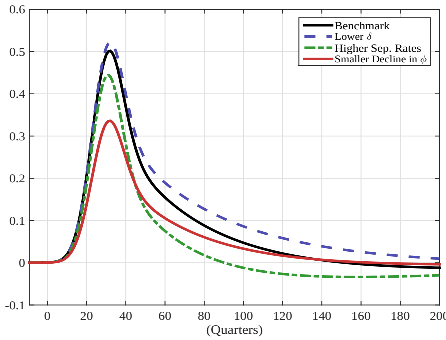
3. Smaller decline in ϕ: ϕ is assumed to drop to 0.014 (instead of 0.011 in the benchmark calibration) from 0.02. Under this calibration, population keeps growing even in the new steady state (albeit at a slower pace). With 0.014, the path of the labor force roughly mimics the observed path of the U.S. labor force. The intention with this exercise is to demonstrate that labor force growth itself influences dynamics of per-capita real variables and inflation.

Note that, in the first and second cases, we recalibrate the initial steady state, following the same procedure as in the benchmark calibration. We then simulate the perfect foresight equilibrium path of the economy, assuming that the entry rate follows the same exogenous path as in the benchmark case. In the third case, the economy is assumed to be in the same

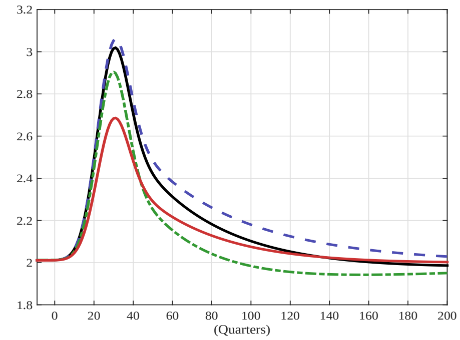
[19]Recall that the skill premium $\gamma = 0.6$ in the benchmark calibration is chosen such that the old worker makes 40% more than the young worker. This choice is conditional on the level of skill loss probability. With the lower value of δ, we adjust γ to a lower value (0.53) in this alternative calibration to achieve the observed premium of 40%.

Figure 9: Perfect Foresight Equilibrium Paths (Alternative Calibrations)

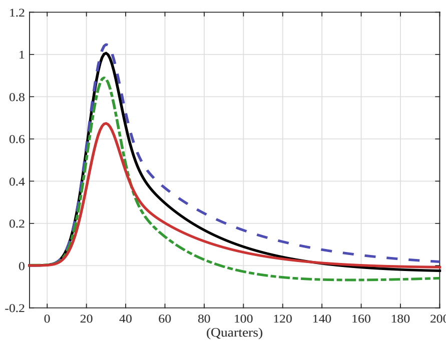
(a) Consumption Growth



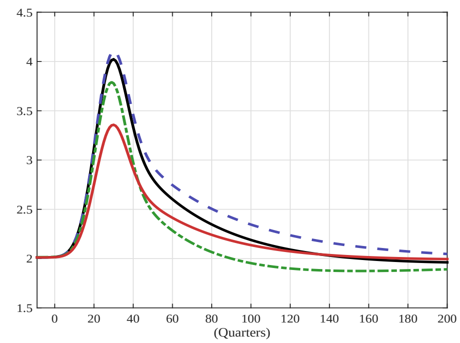
(b) Real Interest Rate



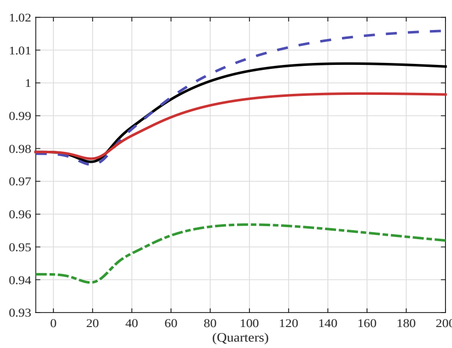
(c) Inflation Rate



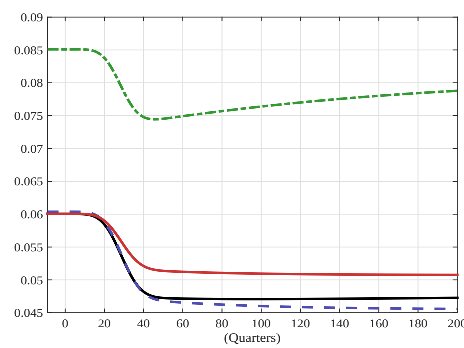
(d) Nominal Interest Rate



(e) Aggregate Wage



(f) Unemployment Rate



steady state initially, but ϕ falls to a higher new steady state value (0.014). All results are put together in Figure 9.

5.3.1 Low Human Capital Specificity

The results under this case are plotted by blue dashed lines in Figure 9. Under this specification, consumption grows somewhat more in the initial stage of aging than under the benchmark calibration, but more important, the deceleration of consumption growth occurs more gradually. The latter fact is a direct consequence of the smaller value of δ. As we discussed above the source of the decline in the consumption level in the benchmark case is a high value of the specificity parameter. With the smaller value of this parameter, consumption growth never turns negative as shown in the first panel. The real interest rate, the nominal interest rate, and inflation all follow similar paths. One notable result is that inflation always stays above zero in this case. The results here suggest that the labor market institution (represented by a high degree of skill specificity in this paper) carries important implications for low frequency movements in interest rates and inflation.

The path of aggregate wage is consistent with that of consumption growth. It grows somewhat more rapidly initially than in the benchmark case, and keeps rising albeit at slower pace. Regarding the unemployment rate, it drops rapidly initially as in the benchmark case. But further gradual declines follow, in contrast to the flat path in the benchmark case. Remember that a high value of δ in the benchmark calibration puts an upward pressure on the aggregate unemployment rate, because it implies a large number of experienced workers becoming inexperienced as the aging process progresses. Conversely, a smaller value of δ reduces this effect.

5.3.2 Higher Separation Rates

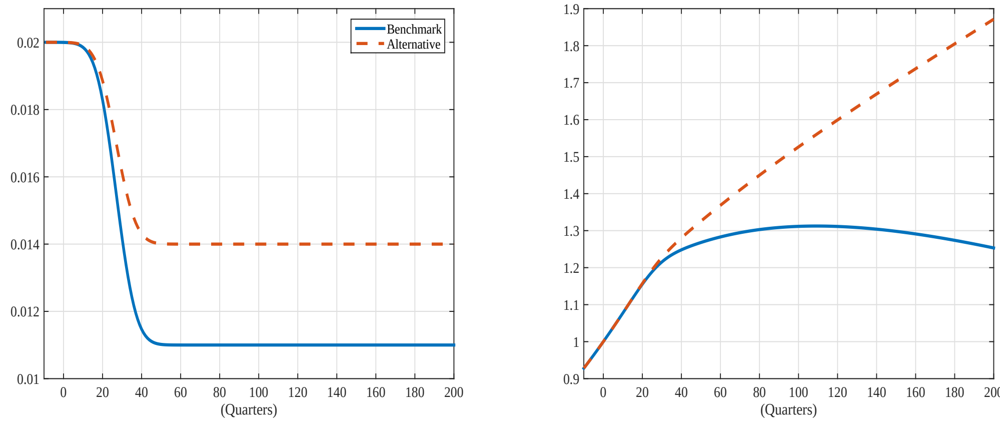
With the higher separation rates, consumption growth in the first 5-10 years of the process is less rapid, and it is followed by steeper declines. Again, the paths of interest rates and inflation are similar. Note that the higher separation rate for the experienced worker s^e implies a higher risk of skill loss, and thus puts a downward pressure on labor productivity, aggregate wage, and consumption, when aging of the labor force reaches its later stage. With the higher value of s^e, the timing of negative consumption growth comes much earlier, and the size of the decline is larger than in the benchmark calibration. This pattern in consumption growth translates into different inflation dynamics. That is, the economy experiences larger deflation, and deflation itself occurs much earlier.

The unemployment rate in the initial steady state is higher (given the higher values of separation rates). In contrast to the benchmark case, the unemployment rate rises following the initial decline. Again, the initial decline is due to a lower entry rate. The unemployment rate starts increasing, once the adverse effect of aging (due to the larger and lager number of inexperienced old workers in the I-matching market) becomes more evident. The path of real wage is also intuitive, given the discussions so far. It increases less and its decline that follows is more pronounced.

In this exercise, we assume that the separation rate was higher from the beginning and thus not entirely realistic. However, this exercise shows that, when the separation rate increases (which indeed occurred throughout the 1990s), it puts a downward pressure on real wages, interest rates, and thus inflation (again, assuming that the monetary authority follows the standard Taylor rule). In this sense, the results under the benchmark calibration may be viewed as an underestimate of the impacts of the labor force aging.

Figure 10: The Entry Rate and Labor Force

(a) Alternative Entry Rate Path (b) Implied Path of the Labor Force



Notes: The labor force is normalized to 1 in the initial period.

5.3.3 Smaller Decline in the Labor Force Entry Rate

In this hypothetical scenario, the labor force entry rate drops in a manner similar to the one assumed in the benchmark calibration, but less to a higher level (0.014). Note that steady-state labor force growth in the model is given by $\phi - \mu$. Given that μ is set to 0.0125, the benchmark calibration implies that labor force growth in the new steady state is negative ($g = 0.011 - 0.0125 = -0.0015$) whereas, in the alternative scenario, labor force continues to be positive even after the economy reaches the new steady state (at a rate of 0.0015 per quarter). Figure 10 compares the paths of the entry rate and the labor force under the benchmark calibration (blue solid line) and this alternative calibration (red dashed line).

Figure 9 indicates that this seemingly innocuous change makes significant differences in the subsequent behavior of the economy. The difference results from the fact that different labor force growth implies a different age compositions along the economy's transition path. A smaller decline in ϕ implies a smaller initial increase in consumption growth and, accordingly, the deceleration in consumption growth is less pronounced and occurs more smoothly. In general, the economy experiences a smaller low-frequency swing because the workforce composition does not change as much as in the benchmark calibration. This result highlights the fact that the changing demographics composition is an important element in understanding the low-frequency movement of the economy

5.3.4 Summary

In Table 4, we present how much of the observed declines in per-capita consumption growth and the real interest rate can be accounted for by the model under the three alternative calibrations as well as under the benchmark calibration. Specifically, we compute the difference between the peak level and the level 40 years (160 quarters) after the start of the experiment

Table 4: Changes in Real Interest Rate and Consumption Growth

	Real Interest Rate (%)	Consumption Growth (%)
Data	-2.27	-1.26
Benchmark	-1.02	-0.51
Lower δ	-1.00	-0.50
Higher separation rate	-0.96	-0.48
Smaller decline in ϕ	-0.67	-0.33

Notes: Data refers to the difference between the average levels in 1980s and 2000s. See Table 1. The decline in the two variables in the model is computed as the difference between the peak level and the level at 160th quarter (40 years later) after the start of the decline in the entry rate.

(period 1 in the figures). As mentioned above, under the benchmark calibration, the model accounts for roughly 40% of the declines in consumption growth and the real interest rate.

First note that the smaller decline in ϕ implies different contributions, not surprisingly. However, an interesting result is that the contributions of labor force aging remain roughly the same at around 40% under the other two alternative calibrations. As we saw above, these two calibrations imply a fairly different timing at which consumption growth and the real interest rate fall below their respective steady-state levels. However, in terms of the difference from their peak levels, all three calibrations (benchmark and two alternative calibrations) yield essentially the same implication on the contributions of aging on the declines in consumption growth and the real interest rate over the past several decades.

6 Conclusion

This paper has studied the implications of aging on the low-frequency behavior of the Japanese economy. Specifically, we find that the collapse in labor force entry in the 1970s (associated with the end of the baby boom roughly 20 years before) could be an important driver of low frequency movements in consumption growth and the real interest rate as well as important labor market variables such as labor productivity and real wage. Furthermore, if the monetary authority follows a simple standard Taylor rule that fails to internalizes the time-varying nature of the natural rate of interest, the economy experiences a sizable low-frequency movement in the inflation rate results. Specifically, under our benchmark calibration, the decline in the labor force entry in the 1970s leads to deflation 35 years later.

Our quantitative experiments suggest that aging of the labor force accounts for roughly 40% of the declines in per-capita consumption growth and the real interest rate in the last three decades in Japan. The declining trend in the real interest rate in the U.S. has also received much attention (see, for example, Greenspan (2005)). One of the leading explanations for the phenomenon is the global saving glut by Bernanke (2005). In this paper, we took an alternative view motivated by the empirical study by Feyrer (2007) that the changing workforce composition itself leads to low frequency movements in aggregate labor productivity growth. How this productivity channel affects real interests in a open-economy setup would be an important future research topic, especially given rapid labor-force aging in all major developed economies.

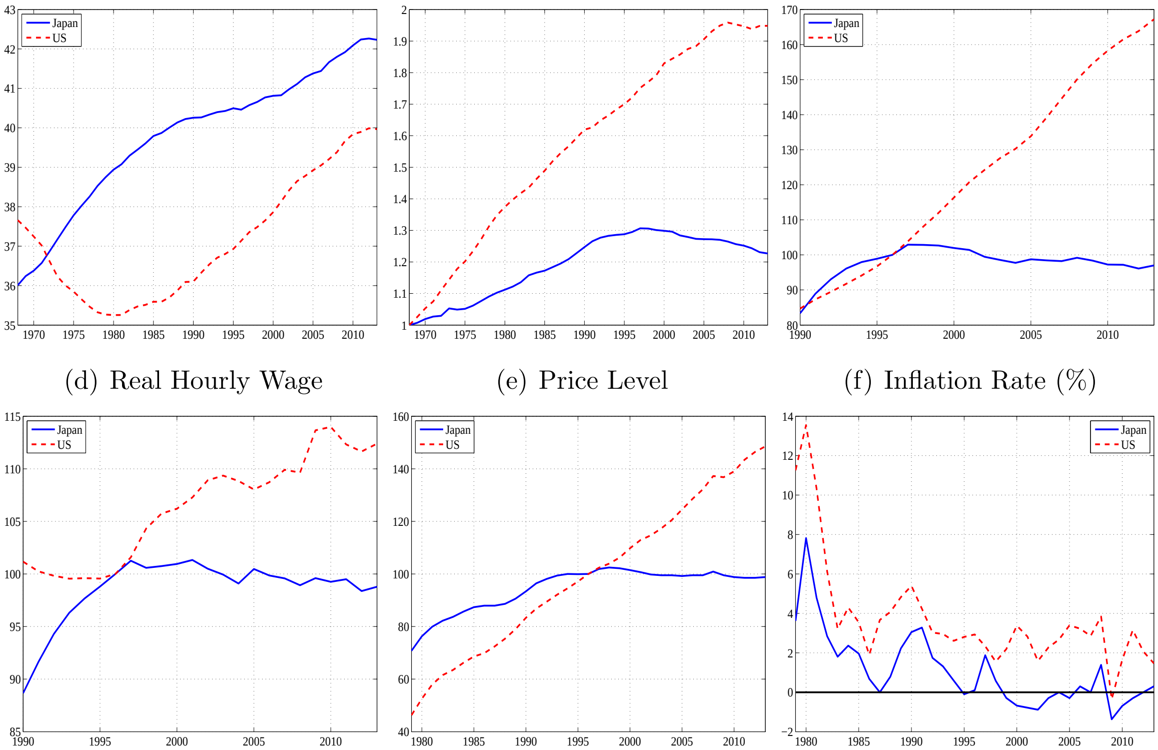
A Comparison Between Japan and the US

Our discussion in Section 2 focused on the Japanese data. Here we compare each series with the corresponding series for the U.S. economy. Panel (a) of Figure 11 compares the average age of the labor force in Japan and the U.S. One can see that the US labor force has also been getting older since 1980. However, the US labor force is still younger by more than 2 years than the Japanese labor force as of 2013. A more striking difference can be observed in Panel (b) which shows that the U.S. labor force expanded at a much more rapid pace over the last 45 years. In particular, the U.S. labor force grew 40% between 1996 and 2013, while the Japanese labor force has been shrinking. Wages in the two countries have behaved dramatically different in the last 25 years (Panels (c)-(d)). Nominal wage in the U.S. increased 70% between 1996 and 2013 while that in Japan fell more than 5%. A Similar pattern can be observed in the comparison of the price level and inflation (Panel (e)-(f)).

Figure 11: Comparison Between Japan and the U.S.

(a) Average Age of Labor Force (b) Labor Force Size (c) Nominal Hourly Wage

(d) Real Hourly Wage (e) Price Level (f) Inflation Rate (%)



Notes: Labor force size is normalized to 1 in 1968. Wage and price series are normalized to 100 in 1996.

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