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Motivation

Contrast these views:

“China is the one country that might be able to jump-start the global economic recovery; and yet its own economic growth is based on a foundation that is increasingly showing signs of strain”

Arvind Subramainian, Project Syndicate, 14/4/2016
Motivation

Contrast these views:

“The new normal for China is sub-8% growth, a level seen for most of the past decade as the government’s bottom line.

China will cast a long shadow from the ore mines of Brazil to the car factories of Germany. As the largest source of future economic growth globally, the world is relying on the Chinese”

Kate Allen & Simon Rabinovitch, “The China slowdown, in numbers”, FT, 15/7/2013
Motivation

Contrast these views:

“Invention abroad that gives China some of the comparative advantage that had belonged to the U.S. can induce for the U.S. permanent lost per capita income.”

Motivation

China’s rise in world GDP and trade, in pictures

These conflicting viewpoints then beg the following question:

“Just how much has China’s meteoric trade growth (picted) contributed to the growth and welfare of its trading partners over the past twenty years?”
Motivation

China’s rise in world GDP and trade, in pictures

Importantly, China’s trade has not grown evenly across all sectors...
Motivation

China’s rise in world GDP and trade, in pictures

(at least) 2 interesting facts to highlight about China’s trade growth:

1. A dramatic shift from non-manufactured exports towards manufacturing.
   - Plausibly may have made other manufacturing-exporters worse off by eroding their terms of trade.
     (Hicks, 1953; Samuelson, 2004)
Motivation

China’s rise in world GDP and trade, in pictures

(at least) 2 interesting facts to highlight about China’s trade growth:

1. A dramatic shift from non-manufactured exports towards manufacturing.

2. Within manufacturing, a pronounced shift towards increased trade in capital goods (e.g., machinery, equipment) in particular.

    ◊ Presents viable mechanism for trade-induced capital accumulation

    (Eaton and Kortum, 2001; Mutreja, Ravikumar, & Sposi, 2016)
Proposed Framework: *Model*

To deliver answers, I will build a dynamic, many-country trade model with the following main features:

- **“Capital accumulation”:** households making forward-looking investment decisions in each period
  - Provides main link between trade and growth

- Trade in (and use in production of) *Non-manufactured products* (e.g. Agriculture, Mining)
  - Upstream, capital-intensive, and important for developing countries

- China becomes a major producer and exporter of *traded capital goods* during the period - lowers the cost of investment in trading partners

- Input-output linkages between intermediate goods produced in China and more downstream goods produced abroad (and vice versa)
Proposed Framework: *Quantification*

- The model will be fitted to match trade, output, and capital accumulation for 72 developed and developing countries for the years 1993-2011.

- To quantify the model, I take inspiration from the “wedge accounting” methods of Eaton, Kortum, Neiman, & Romalis (2016) (“EKNR”)

  (previously: Chari, Kehoe, & McGrattan 2007; Kehoe, Ruhl, & Steinberg 2013)

My application

Recovering how China’s sectoral-level productivity growth and reductions in trade frictions contributed to *actual* real GDP growth observed in the data for other countries.
Proposed Framework: *Quantification*

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  (previously: Chari, Kehoe, & McGrattan 2007; Kehoe, Ruhl, & Steinberg 2013)

- However, the analysis performed in this paper adopts an overall larger-scale perspective than EKNR (72 countries, 6 sectors)

  This necessitates, in some places, introducing novel techniques:

  - A straightforward, scalable algorithm for solving dynamic trade models with input-output linkages
  - A fast, flexible “dummy variables only” method for estimating sectoral technology levels
  - A natural mapping between sectoral prices and the aggregate prices of consumption and investment
    *(main modeling innovation)*
Proposed Framework: *Limitations*

Before previewing the results, there are some **important limitations** left on the table that should be acknowledged:

1. I take from the trade literature the canonical assumptions of *constant returns to scale* and *perfect factor mobility* across industries
   - Latter assumption in particular is not innocuous in the case of China


3. All trade imbalances treated as exogenous. I explore endogenous imbalances in an extension.

4. No multinational activity or FDI.
Q1. “How much did increased trade with China contribute to growth in other countries?”

A. All told, China’s rapid trade expansion was responsible for 1.2% of the rest of the world’s real GDP growth between 1993 and 2007 and 8.8% for the period 2008-2011.
**Takeaways**

Q1. “*How much did increased trade with China contribute to growth in other countries?*”

A. All told, China’s rapid trade expansion was responsible for 1.2% of the rest of the world’s real GDP growth between 1993 and 2007 and 8.8% for the period 2008-2011.

Q2. Decomposition: “*How do we arrive at these numbers?*”

A. The model highlights 3 key ideas:

- **Geography** and **comparative advantage** w.r.t. China each play a key role: lower-income and Asia-Pacific countries enjoy the largest effects overall.

- **“Dynamic sectoral linkages”**: China’s change in comparative advantage from Non-Manufacturing to Manufacturing hurts some partners’ terms of trade in the short run, but generally promotes growth in the long run.

- **Capital adjusts slowly over time**: Model suggests that the majority of China’s effects on growth still have yet to be felt.
Takeaways

Q1. “How much did increased trade with China contribute to growth in other countries?”

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Q2. Decomposition: “How do we arrive at these numbers?”

A. The model highlights 3 key ideas:
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   - **“Dynamic sectoral linkages”**: China’s change in comparative advantage from Non-Manufacturing to Manufacturing hurts some partners’ terms of trade in the short run, but generally promotes growth in the long run.
   - **Capital adjusts slowly over time**: Model suggests that the majority of China’s effects on growth still have yet to be felt.

Q3. Looking ahead: “What can we say about the effects of slowdown in China?”

(to be continued)
Related Literature I

Quantifying the “China” Impact:
Samuelson (2004); Hsieh & Ossa (2011); Autor, Dorn, & Hanson (2013); Di Giovanni, Levchenko, & Zhang (2014)

Trade and Growth with Dynamics:
Anderson, Larch, & Yotov (2015); Eaton, Kortum, Neiman, & Romalis (2015); Ravikumar, Santacreu, & Sposi (2016)

Quantifying comparative advantage:
Shikher (2011, 2012); Costinot, Donaldson, & Komunjer (2012); Levchenko & Zhang (2016); Hanson, Lind, & Muendler (2015); Di Giovanni, Levchenko, & Zhang (2014)

Other related frameworks:
Caliendo & Parro (2015)
Related Literature II

Hecksher-Ohlin dynamic trade and growth models:

Evidence for the responsiveness of capital accumulation to trade:
Outline

1. A Dynamic Multi-sector Trade & Growth Model
2. Taking the Model to the Data
3. China vs. the World, 1993-2011
4. How much did China Contribute to World Growth?

Extension: endogenous trade imbalances

Discussion: A slowdown in China?
1. A Dynamic Multi-sector Trade & Growth Model
   
   **key message:**
   changes in sectoral composition of trade can have very different implications in a *static* (fixed capital) environment vs. a *dynamic* environment.

2. Taking the Model to the Data

3. China vs. the World, 1993-2011

4. How much did China Contribute to World Growth?

Extension: endogenous trade imbalances

Discussion: A slowdown in China?
Model: *Overview*

- **Production**: All goods are produced with a combination of labor, capital, and intermediate inputs produced by other industries.
  - Both factor intensities and intermediate input requirements differ by industries.
  - These requirements are taken directly from input-output tables.

- **Consumption & Utility**: Cobb-Douglas across industries and concave (log) across time

- **Investment**: Also Cobb-Douglas across industries, but with different share requirements than the utility function

- **Trade**: CES “Armington” (“love-of-varieties”) assumption: creates scope for *intra*-industry trade
  - Relative production cost differences across industries will also give rise to comparative advantage & *inter*-industry trade.
Model: *Overview*

An equilibrium in this model will be a (rational expectations) **Perfect Foresight Equilibrium**, where:

- Capital and investment satisfy an Euler condition in every period and satisfy a TVC as $t \to \infty$
- Trade, production, and prices within each period satisfy the competitive equilibrium conditions implied by the underlying trade model.

“Perfect foresight”

All agents can perfectly anticipate the future and are able to adjust their investment decisions accordingly.
Model: *Key Ideas*

4 main moving parts from the model:

- The investment choice \((I_{i,t})\)
- Factor rewards \((w_{i,t}, r_{i,t})\)
- Consumption and investment prices \((P_{i,c,t}, P_{i,IV,t})\)
- Trade, technology, and *sector* prices \((P_{i,k,t})\)

**Key idea: static vs dynamic gains from trade**

Changes in trade that lower the cost of production and/or consumption do not necessarily lower the price of investment or raise the return to capital.
Model: *Key Ideas*

1. **The investment choice** \((I_{i,t})\)

Real investment made by households in each period \((I_{i,t})\) obeys the following Euler equation:

\[
\frac{E_{i,C,t+1}}{E_{i,C,t}} \left( \frac{I_t}{K_t} \right)^{1-\kappa} = \rho \frac{\hat{\phi}_{i,t+1} X_{i,t}}{P_{i,IV,t}} \left\{ \kappa r_{i,t+1} + (1 - \kappa) \frac{E_{i,IV,t+1}}{K_{i,t+1}} + (1 - \delta) \frac{P_{i,IV,t+1}}{X_{i,t+1}} \left( \frac{I_{i,t+1}}{K_{i,t+1}} \right)^{1-\kappa} \right\}
\]

where:

- \( r_{i,t+1} \): future return to capital
- \( P_{i,IV,t} \): current price of investment
- \( \delta \): depreciation rate
- \( E_{i,C,t}, E_{i,IV,t} \): Consumption and investment expenditure

“Bells and whistles”

\( \kappa \): governs “capital adjustment costs”; \( \phi_{i,t} \) and \( \chi_{i,t} \): “structural residuals” needed to exactly match the data (more on these later).
Model: *Key Ideas*

2. **Factor rewards** \((w_{i,t}, r_{i,t})\)

Factor rewards in the model come from factor market clearing, respond to changes in sectoral output:

\[
\begin{align*}
w_{i,t}L_{i,t} &= \sum_k \beta_{i,k}^w \cdot Y_{i,k,t}; \quad r_{i,t}K_{i,t} &= \sum_k \beta_{i,k}^r \cdot Y_{i,k,t}
\end{align*}
\]

- \(\beta_{i,k}^w\): share of labor in production of sector \(k\)
- \(\beta_{i,k}^w\): share of capital in production of sector \(k\)

Trade raises the relative price of output in capital-intensive sectors \(\Rightarrow\) raises the relative return to capital

- creates link between neoclassical trade and neoclassical growth
Model: Key Ideas

3. Consumption and investment prices \((P_{i,C,t}, P_{i,IV,t})\)

Final goods prices also depend on the makeup of sectoral prices

\[
P_{i,C,t} = \prod_{k} P_{i,k,t}^{\gamma_{i,C,t}} \quad \quad \quad \quad P_{i,IV,t} = \prod_{k} P_{i,k,t}^{\gamma_{i,IV,t}}
\]

- \(\gamma_{i,C,t}\): usage share of sector \(k\) in consumption
- \(\gamma_{i,IV,t}\): usage share of sector \(k\) in investment

Lower relative prices in sectors used more intensively in investment \(\Rightarrow\) lower relative price of investment

- creates a second link between sectoral-level trade and capital accumulation
Model: *Trade, Prices, and Technology*

Trade between $i$ and $j$ in each sector $k$ takes the standard CES “gravity” form:

$$X_{ij,k} = \frac{A_{i,k} (c_{i,k} d_{ij,k})^{-\theta}}{P_{j,k}^{-\theta}} E_{j,k}$$

- $d_{ij,k}$ is an “iceberg” trade cost,
- $A_{i,k}$ is $i$’s “technology”-level / “TFP”,
- $c_{i,k}$ is the “input bundle cost”, and
- $P_{j,k}$ is the aggregate price index of sector $k$ in country $j$, given by:

$$P_{j,k}^{-\theta} = \sum_i A_{i,k} (c_{i,k} d_{ij,k})^{-\theta}$$

$\Rightarrow$ Note how $P_{j,k}$ very usefully aggregates the incidence of technology and geography!

$\theta \equiv 1 - \sigma$: reflects CES elasticity of substitution across varieties produced by different trade partners.
Model: *Trade, Prices, and Technology*

\[ X_{ij,k} = \frac{A_{i,k} \left( c_{i,k} d_{ij,k} \right)^{-\theta}}{P_{j,k}^{-\theta}} E_{j,k} \]  

(1)

The production technology for producing good \( k \) can be described via the “input bundle cost” \( c_{i,k} \):

\[ c_{i,k} = \left( \alpha_{w}^{\alpha_{w}^k} \cdot r_{i}^{\alpha_{r}^k} \right)^{\beta_{v}^k} \cdot \prod_{l} P_{i,l}^{\beta_{l}^i} \]  

(2)

- \( \alpha_{w}^k, \alpha_{r}^k \): factor intensities
- \( \beta_{v}^k \): value-added share
- \( \beta_{l}^i \): capture “Input-Output linkages” from input industry \( l \) to the using industry \( k \)

**Key Assumption**: Inputs to consumption, investment, and production all use the same aggregates from each industry

\( \Rightarrow \) “\( P \)” in (1) is the same as in (2)
Closing the Model

Transversality condition

\[ \lim_{t \to \infty} K_{i,t} = K_{i,SS} < \infty \]

Goods market clearing

\[ \sum_j X_{ij,k,t} = Y_{i,k,t} \implies Y_{i,k,t} = A_{i,k,t} c_{i,k,t} \cdot \sum_j \frac{d_{ij,k,t}^{-\theta}}{P_{j,k,t}^{-\theta}} E_{j,k,t} \]
Closing the Model II

Sectoral expenditure

\[ E_{i,k,t} = \left( x_{i,t} \cdot \gamma_{i,,IV,t}^{k} \cdot (GDP_{i,t} + D_{i,t}) \right) \]

\[ + (1 - x_{i,t}) \cdot \gamma_{i,C,t}^{k} \cdot (GDP_{i,t} + D_{i,t}) \]

\[ + \sum_{l} \beta_{i,l} Y_{i,l,t} \]

...in production

\[ \diamond \ x_{i,t} = E_{i,IV,t} / (GDP_{i,t} + D_{i,t}) \] is the national investment share

(must be solved for dynamically)
Model: *Key Ideas*

4 main moving parts from the model:

- The investment choice \((I_{i,t})\)
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- Consumption and investment prices \((P_{i,C,t}, P_{i,IV,t})\)
- Trade, technology, and *sector* prices \((P_{i,k,t})\)

**Key idea: static vs dynamic gains from trade**

Changes in trade that lower the cost of production and/or consumption do not necessarily lower the price of investment or raise the return to capital
Key Message: Static vs. Dynamic Gains from Trade

The change in steady state consumption can be approximated using the following formula:

\[ \hat{G}^{SS}_{i} \approx \prod_{k} \hat{\pi}_{ii,k} - \frac{\gamma_{i,C}}{\beta_{i,k} \theta} \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right] - \frac{\beta_{i,k} \gamma_{i,C}}{\beta_{i,k} \theta} \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right] - \gamma_{i,IV} \frac{\beta_{i,k} \gamma_{i,C}}{\beta_{i,k} \theta} \]

- \( \hat{\pi}_{ii} \): change in \( i \)'s internal trade share for sector \( k \)
- \( \theta \): trade elasticity parameter ("1 \( - \sigma \") governing intra-industry trade
- each sector must be weighted by its share in consumption, \( \gamma_{i,C} \)

This component: same “gains from trade” as a static model with no input-output structure.
Key Message: *Static vs. Dynamic Gains from Trade*

The role of input-output linkages is as in Caliendo & Parro (2015)

\[
\hat{G}_{i}^{SS} \approx \prod_{k} \frac{\gamma_{i,C}^{k}}{\beta_{i,k}^{W}} \frac{1}{\theta^{i,k}} \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\tilde{P}_{i,k}} \right]^{-\beta_{i,k}^{l} / \beta_{i,k}^{W}} \left[ \frac{\hat{P}_{i,l}}{\tilde{P}_{i,k}} \right]^{-\gamma_{i,C}^{l} / \beta_{i,k}^{W}} \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\tilde{P}_{i,k}} \right]^{-\gamma_{i,IV}^{l} / \beta_{i,k}^{W}}
\]

**Intuition:** real wage gains are higher if trade lowers the relative price of sectors that are used intensively as inputs to other sectors (high $\beta_{i,k}^{l}$)

- $\beta_{i,k}^{l}$: share requirement for use of $l$ needed for production of $k$ (from I-O table)
Key Message: *Static vs. Dynamic Gains from Trade*

In the **full model**, sectoral linkages contribute a second, strictly **dynamic** component:

\[ \hat{G}_{i}^{SS} \approx \prod_{k} \hat{\pi}_{i,k} \left( \frac{\gamma_{i,C}^{k}}{\beta_{i,k}^{W}} \right)^{\theta} \times \prod_{k} \prod_{l} \left( \frac{\hat{P}_{i,l}}{\hat{\mu}_{i,k}} \right)^{-\frac{\beta_{i,k}^{I} \gamma_{i,C}^{k}}{\beta_{i,k}^{W}}} \times \prod_{k} \prod_{l} \left( \frac{\hat{P}_{i,l}}{\hat{\mu}_{i,k}} \right)^{-\gamma_{i,IV}^{l} \frac{\beta_{i,k}^{r} \gamma_{i,C}^{k}}{\beta_{i,k}^{W}}} \]

When a given \( \hat{P}_{i,l} \) falls, there are additional dynamic benefits if its usage in investment \( \gamma_{i,IV}^{l} \) is high and/or its use of capital in production \( \beta_{i,k}^{r} \) is low.
Key Message: *Static vs. Dynamic Gains from Trade*

In the **full model**, sectoral linkages contribute a second, strictly **dynamic** component:

\[
\hat{G}_{i}^{SS} \approx \prod_{k} \hat{\pi}_{i,i,k} \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right]^{-\frac{\gamma_{i,C}^{k}}{\beta_{i,k}^{W}}} \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right]^{-\gamma_{i,IV}^{l} \frac{\beta_{i,k}^{r} \gamma_{i,C}^{k}}{\beta_{i,k}^{W}}} 
\]

**Upshot:**

The same change in sectoral-level trade can have very different effects for “static” vs. “dynamic” gains from trade.
1. A Dynamic Multi-sector Trade & Growth Model

2. Taking the Model to the Data
   “wedge accounting”

3. China vs. the World, 1993-2011

4. How much did China Contribute to World Growth?

Extension: endogenous trade imbalances

Discussion: A slowdown in China?
The full vector of “wedges” I need for the model to exactly match the data at time $t$ is

\[ \Psi_t = \{A_{i,k,t}, d_{ij,k,t}, \gamma^k_{i,C,t}, \gamma^k_{i,IIV,t}, \beta^v_{i,k,t}, D_{i,t}, L_{i,t}, \chi_{i,t}, \hat{\phi}_{i,t+1} \} \].

- $\Psi_t$ is allowed to vary in order to exactly match all observed data (e.g., from 1993-2007).
- It then remains unchanged thereafter (on the path to steady state).
- Counterfactuals will thus isolate the contribution of “China” to what actually occurred in other countries during this period.
Fitting the Model to Data: *Wedge Accounting*

The full vector of “wedges” I need for the model to exactly match the data at time $t$ is

$$\Psi_t = \{ A_{i,k,t}, d_{ij,k,t}, \gamma_{i,C,t}^k, \gamma_{i,I^V,t}^k, \beta_{i,k,t}^\nu, D_{i,t}, L_{i,t}, \chi_{i,t}, \hat{\phi}_{i,t+1} \}.$$  

**Example:** Observed investment choices identify the time-preference shock $\hat{\phi}_{i,t+1}$. 
Fitting the Model to Data:  *Wedge Accounting*

The full vector of “wedges” I need for the model to exactly match the data at time $t$ is

$$
\Psi_t = \{ A_{i,k,t}, d_{ij,k,t}, \gamma_i^k_{i,C,t}, \gamma_i^{lV,t}, \beta_i^{v,k,t}, D_i,t, L_i,t, \chi_i,t, \phi_i,t+1 \}.
$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Identified by</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{i,k,t}$</td>
<td>Sectoral technology levels</td>
<td>Estimated using “dummies only” gravity with <em>time-varying, symmetric</em> pair fixed effects†</td>
</tr>
<tr>
<td>$d_{ij,k,t}$</td>
<td>Bilateral trade frictions</td>
<td></td>
</tr>
<tr>
<td>$\chi_i,t$</td>
<td>Investment efficiency</td>
<td>Realization of next period capital $K_{t+1}$ given current period $l_t, K_t$</td>
</tr>
<tr>
<td>$\phi_i,t+1$</td>
<td>Inter-temporal preference</td>
<td>How much investment ($l_t$) is chosen at period $t$, given perfect foresight about the future.</td>
</tr>
</tbody>
</table>

Two different methodologies for recovering Technology Levels / TFPs...

1. “Closed economy” TFP accounting:

\[ Y = A^{1/\theta} \times L^{\beta_L} \times K^{\beta_K} \times \ldots \]

\diamond i.e., observe \( Y, K, L \ldots \); infer \( A^{1/\theta} \) as the residual.
Fitting the Model to Data: *Technology Levels & Trade Frictions*

Two different methodologies for recovering Technology Levels / TFPs...

1. “Closed economy” TFP accounting:

   \[ Y = A^{1/\theta} \times L^{\beta_L} \times K^{\beta_K} \times \ldots \]

   ◊ i.e., observe \( Y, K, L \); infer \( A^{1/\theta} \) as the residual.

2. “Trade-based” TFP accounting:

   \[ Y_{i,k,t} = A_{i,k,t} c_{i,k,t}^{-\theta} \cdot \sum_j \frac{d_{ij,k,t}^{-\theta}}{P_{j,k,t}^{-\theta}} E_{j,k,t} \]

   ◊ i.e., output = sum of shipments across all destinations

   ◊ trade frictions (\( d_{ij,k,t} \)’s) very important for correctly inferring technologies!

   e.g. Shikher (2012); Levchenko & Zhang (2016); EKNR...
Fitting the Model to Data: *Technology Levels & Trade Frictions*

Consider again the equation for trade flows:

\[ X_{ij,k,t} = \frac{A_{i,k,t} \left( c_{i,k,t} d_{ij,k,t} \right)^{-\theta}}{P_{j,k,t}^{-\theta}} E_{j,k,t} \]  

(3)

Note this expression has distinct *exporter*, *importer*, and *pair* components:

- \( A_{i,k,t} c_{i,k,t}^{-\theta} \): “absolute advantage” of the exporting country
- \( E_{j,k,t} / P_{j,k,t}^{-\theta} \): market size and price level of the importing country
- \( d_{ij,k,t}^{-\theta} \): bilateral (*pair*-specific) trade frictions

Motivates opportunity to *estimate* what I need from (3) using *fixed effects*...
Fitting the Model to Data: *Technology Levels & Trade Frictions*

The trade equation then takes the following (estimable) form:

\[
X_{ij,k,t} = \exp \left[ \ln \left( A_{i,k,t} c_{i,k,t}^{-\theta} \right) + \ln \left( \frac{E_{j,k,t}}{P_{j,k,t}} \right) + \ln d_{ij,k,t}^{-\theta} \ln \Gamma_{ikt} + \ln \Phi_{jkt} + \ln \eta_{ijkt} + \varepsilon_{ijkt} \right].
\]  

(3)

\( \Gamma_{ikt}, \Phi_{jkt}, \eta_{ijkt} \): fixed effects which are computed from a Poisson PML estimation of (3)

- “dummy variables only”: very flexible way of accounting for changes in trade costs.
- Two (standard) restrictions needed on trade costs are
  1. “symmetry”: \( \eta_{ijkt} = \eta_{jikt} \)
  2. internal trade is “frictionless”: all \( d_{ii,k,t} = 1 \)

- iterative methods can be used to quickly solve for any number of fixed effects
Fitting the Model to Data: *Technology Levels & Trade Frictions*

\[
X_{ij,k,t} = \exp \left\{ \ln \left( A_{i,k,t} c_{i,k,t}^{-\theta} \right) + \ln \left( \frac{E_{j,k,t}}{P_{j,k,t}^{-\theta}} \right) + \ln d_{ij,k,t}^{-\theta} + \ln \eta_{ijkt} \right\} + \varepsilon_{ijkt}. \quad (3)
\]

Prices, \( \{ P_{j,k,t} \} \), then follow directly from \( \Phi_{jkt} \), data on \( E_{j,kt} \).
Fitting the Model to Data: *Technology Levels & Trade Frictions*

\[ X_{ij,k,t} = \exp \left[ \ln \left( A_{i,k,t} c_{i,k,t}^{-\theta} \right) + \ln \left( \frac{E_{j,k,t}}{P_{j,k,t}^{-\theta}} \right) + \ln \left( d_{ij,k,t}^{-\theta} \right) + \ln \left( \frac{\Gamma_{ikt}}{\Phi_{jkt}} \right) + \ln \left( \frac{\kappa_{ijkt}}{\eta_{ijkt}} \right) \right] + \epsilon_{ijkt}. \] (3)

Prices, \( \{ P_{j,k,t} \} \), then follow directly from \( \Phi_{jkt} \), data on \( E_{j,k,t} \)

\( c_{i,k,t} = c(w, r, P) \) can be computed using \( \{ P_{j,k,t} \} \), data on \( \{ w \} \), \( \{ r \} \)
Fitting the Model to Data: *Technology Levels & Trade Frictions*

\[
X_{ij,k,t} = \exp \left[ \ln \left( A_{i,k,t} c_{i,k,t} \right) - \ln \Gamma_{ikt} \right] + \ln \left( \frac{E_{j,k,t}}{P_{j,k,t}} \right) + \ln \left( d_{ij,k,t} \right) + \ln \eta_{ijkt} + \varepsilon_{ijkt}. \tag{3}
\]

Prices, \( \{P_{j,k,t}\} \), then follow directly from \( \Phi_{jkt} \), data on \( E_{j,kt} \)

\( c_{i,k,t} = c(w, r, P) \) can be computed using \( \{P_{j,k,t}\} \), data on \( \{w\}, \{r\} \)

Technologies \( \{A_{i,k,t}\} \) then follow from the estimated \( \Gamma \)'s.
Data Sources & Construction I

Countries/Regions included (72)

- OECD (32) plus 39 non-OECD countries plus 1 “Rest of World” aggregate
- “Rest of World” based on available data for excluded countries, absorbs residual trade imbalances and contributes residual world GDP (roughly ~7% of world GDP).

Industry groupings (6):

1. “Non-Manufacturing”: Agriculture, Fishing, Forestry, & Mining
2. “Capital-intensive Manufacturing”: Food & Beverages, Refined Fuels, Chemicals, Metal Products
4. “Capital goods”: Electrical Machinery, Office computing equipment, Medical/Optical Equipment, Telecommunications Equipment, Motor vehicles, Machinery & Equipment n.e.c., Manufacturing n.e.c.
5. “Construction”
6. “Other Services”: all other services besides construction.

(based on ISIC rev 3 industry codes)
Data Sources & Construction II

Bilateral Trade
UN COMTRADE

Production
OECD STAN, UNIDO INDSTAT, and UN National Accounts

Production Technologies
OECD Input-Output Tables (incl. data for 23 non-OECD countries)

GDP, Investment, & Trade Balances
OECD STAN and UN National Accounts

Investment and Consumption Prices, Factor Endowments
Penn World Tables v8.1

All prices are deflated to 1993 USD equivalents, which serves as a numeraire
## Production Linkages

### Input Output Table (Median Coefficients)

<table>
<thead>
<tr>
<th>Input industry</th>
<th>NM</th>
<th>MK</th>
<th>ML</th>
<th>K</th>
<th>F</th>
<th>O</th>
<th>Final Use</th>
<th>C</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Manufacturing (NM)</td>
<td>0.096</td>
<td>0.263</td>
<td>0.072</td>
<td>0.006</td>
<td>0.018</td>
<td>0.016</td>
<td>0.038</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>Capital-Intensive Manufacturing (MK)</td>
<td>0.074</td>
<td>0.167</td>
<td>0.099</td>
<td>0.084</td>
<td>0.086</td>
<td>0.031</td>
<td>0.121</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>Labor-Intensive Manufacturing (ML)</td>
<td>0.012</td>
<td>0.034</td>
<td>0.185</td>
<td>0.091</td>
<td>0.162</td>
<td>0.022</td>
<td>0.042</td>
<td>0.020</td>
<td></td>
</tr>
<tr>
<td>Capital Goods (K)</td>
<td>0.012</td>
<td>0.008</td>
<td>0.016</td>
<td>0.255</td>
<td>0.050</td>
<td>0.244</td>
<td>0.042</td>
<td>0.283</td>
<td></td>
</tr>
<tr>
<td>Construction (F)</td>
<td>0.007</td>
<td>0.003</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
<td>0.017</td>
<td>0.000</td>
<td>0.446</td>
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</tr>
<tr>
<td>Other Services (O)</td>
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<td>0.200</td>
<td>0.255</td>
<td>0.226</td>
<td>0.196</td>
<td>0.277</td>
<td>0.672</td>
<td>0.177</td>
<td></td>
</tr>
</tbody>
</table>

### Value Added

- Value added share ($\beta_v$): 0.623 0.286 0.305 0.286 0.358 0.596
- Labor share ($\alpha^w$): 0.260 0.440 0.570 0.570 0.560 0.520
- Capital share ($\alpha^c$): 0.740 0.560 0.430 0.430 0.440 0.480
## Parameters

<table>
<thead>
<tr>
<th>Industry</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade elasticity ($\theta$)</td>
<td>4.00</td>
</tr>
<tr>
<td>Investment adjustment ($\kappa$)</td>
<td>0.55</td>
</tr>
<tr>
<td>Depreciation ($\delta$)</td>
<td>0.05</td>
</tr>
<tr>
<td>Time preference ($\rho$)</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Outline

1. A Dynamic Multi-sector Trade & Growth Model

2. Fitting the Model to Data

3. China vs. the World, 1993-2011
   - Accounting results

4. How much did China Contribute to World Growth?

Extension: endogenous trade imbalances

Discussion: A slowdown in China?
China vs. the World (by sector)

Non-manufacturing

Figure: (Log) changes in sectoral productivity and trade barriers
China vs. the World (*by sector*)

**Labor-Intensive Intermediates**

![Graph](image)

**Figure:** (Log) changes in sectoral productivity and trade barriers
China vs. the World (by sector)

Capital-Intensive Intermediates

Figure: (Log) changes in sectoral productivity and trade barriers
China vs. the World (*by sector*)

**Capital Goods**

*Figure:* (Log) changes in sectoral productivity and trade barriers
China vs. the World: 1993-2007

China’s productivity growth and globalization vs. the Rest of the World, 1993-2007

<table>
<thead>
<tr>
<th>Industry</th>
<th>$\hat{A}_{\text{nonCHN}}^{1/\theta}$</th>
<th>$\hat{A}_{\text{CHN}}^{1/\theta}$</th>
<th>$\hat{A}_{\text{CHN+}}^{1/\theta}$</th>
<th>$\hat{d}_{\text{nonCHN}}$</th>
<th>$\hat{d}_{\text{CHN}}$</th>
<th>$\hat{d}_{\text{CHN+}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Manufacturing</td>
<td>-.008</td>
<td>-.003</td>
<td>.004</td>
<td>-.007</td>
<td>-.012</td>
<td>-.005</td>
</tr>
<tr>
<td>Capital-intensive Manuf.</td>
<td>-.008</td>
<td>.023</td>
<td>.032</td>
<td>-.006</td>
<td>-.011</td>
<td>-.005</td>
</tr>
<tr>
<td>Labor-intensive Manuf.</td>
<td>.008</td>
<td>.029</td>
<td>.021</td>
<td>-.002</td>
<td>-.004</td>
<td>-.002</td>
</tr>
<tr>
<td>Capital Goods</td>
<td>.012</td>
<td>.042</td>
<td>.030</td>
<td>-.005</td>
<td>-.026</td>
<td>-.022</td>
</tr>
<tr>
<td>Construction</td>
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<td>-.01</td>
<td>-.001</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Other services</td>
<td>.005</td>
<td>-.002</td>
<td>-.007</td>
<td>-.001</td>
<td>-.049</td>
<td>-.048</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>.002</td>
<td>.032</td>
<td>.030</td>
<td>-.004</td>
<td>-.016</td>
<td>-.012</td>
</tr>
<tr>
<td>Total</td>
<td>.002</td>
<td>.024</td>
<td>.022</td>
<td>-.003</td>
<td>-.015</td>
<td>-.013</td>
</tr>
</tbody>
</table>

Notes: Annualized percentage changes over time. Shocks highlighted in bold are those are “subtracted” in the counterfactuals.

Basis for counterfactuals: How would the world economy have evolved differently if China had only grown and opened its borders at the same rate as the rest of the world?
China vs. the World: 2008-2011

China’s productivity growth and globalization vs. the Rest of the World, 2008-2011

<table>
<thead>
<tr>
<th>Industry</th>
<th>$\hat{A}_{\text{nonCHN}}^{1/\theta}$</th>
<th>$\hat{A}_{\text{CHN}}^{1/\theta}$</th>
<th>$\hat{A}_{\text{CHN}+}^{1/\theta}$</th>
<th>$\hat{d}_{\text{nonCHN}}$</th>
<th>$\hat{d}_{\text{CHN}}$</th>
<th>$\hat{d}_{\text{CHN}+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Manufacturing</td>
<td>.031</td>
<td>.076</td>
<td><strong>.046</strong></td>
<td>.006</td>
<td>-.01</td>
<td><strong>-.016</strong></td>
</tr>
<tr>
<td>Capital-intensive Manuf.</td>
<td>-.029</td>
<td>.014</td>
<td><strong>.044</strong></td>
<td>-.006</td>
<td>.01</td>
<td><strong>.016</strong></td>
</tr>
<tr>
<td>Labor-intensive Manuf.</td>
<td>-.008</td>
<td>.053</td>
<td><strong>.061</strong></td>
<td>-.001</td>
<td>-.008</td>
<td><strong>-.006</strong></td>
</tr>
<tr>
<td>Capital Goods</td>
<td>.007</td>
<td>.067</td>
<td><strong>.060</strong></td>
<td>-.002</td>
<td>.004</td>
<td><strong>.006</strong></td>
</tr>
<tr>
<td>Construction</td>
<td>-.018</td>
<td>-.029</td>
<td>-.011</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Other services</td>
<td>.002</td>
<td>.003</td>
<td>.001</td>
<td>-.002</td>
<td>-.051</td>
<td>-.049</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>-.016</td>
<td>.039</td>
<td>.055</td>
<td>-.004</td>
<td>.001</td>
<td>.005</td>
</tr>
<tr>
<td>Total</td>
<td>.000</td>
<td>.038</td>
<td>.038</td>
<td>.000</td>
<td>-.002</td>
<td>-.002</td>
</tr>
</tbody>
</table>

**Notes:** Annualized percentage changes over time. Shocks highlighted in bold are those are “subtracted” in the counterfactuals.

**Basis for counterfactuals:** How would the world economy have evolved differently if China had only grown and opened its borders at the same rate as the rest of the world?
Outline

1. A Dynamic Multi-sector Trade & Growth Model
2. Taking the Model to the Data
3. China vs. the World, 1993-2011
4. How much did China Contribute to World Growth?

Extension: endogenous trade imbalances

Discussion: A slowdown in China? A tariff war between the U.S. in China?
Model Results (1993-2007)

Examples of Model Output: Sweden vs. Indonesia

China’s productivity growth and trade liberalization between 1993 and 2007 raised Sweden’s 2007 real GDP by 0.1%, Indonesia’s by 0.6%.
Model Results (1993-2007)

Figure: China’s effects on 2007 real GDPs vs. Income per capita (all countries)

slope: -.370 (s.e.: .082)
How China’s productivity growth and globalization contributed to growth (1993-2007):

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real GDP</td>
<td>$\hat{r}/\hat{w}$</td>
</tr>
<tr>
<td>Australia</td>
<td>0.0043</td>
<td>0.0088</td>
</tr>
<tr>
<td>China</td>
<td>0.6386</td>
<td>0.0442</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>0.0066</td>
<td>0.0009</td>
</tr>
<tr>
<td>Germany</td>
<td>0.0001</td>
<td>0.0061</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.0004</td>
<td>0.0031</td>
</tr>
<tr>
<td>Japan</td>
<td>0.0009</td>
<td>0.0026</td>
</tr>
<tr>
<td>Malaysia</td>
<td>0.0127</td>
<td>0.0020</td>
</tr>
<tr>
<td>Peru</td>
<td>0.0052</td>
<td>0.0083</td>
</tr>
<tr>
<td>USA</td>
<td>0.0018</td>
<td>0.0013</td>
</tr>
<tr>
<td>Vietnam</td>
<td>0.0242</td>
<td>-0.0117</td>
</tr>
<tr>
<td>World</td>
<td>0.0272</td>
<td>0.0097</td>
</tr>
<tr>
<td>Non-China</td>
<td>0.0028</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

**Left:** How much do China’s changing sectoral productivities and trade liberalization contribute to 2007 real GDP (and other outcomes) in a “static” (fixed capital) setting?

**Right:** Results from the full dynamic model with *capital accumulation* factored in.
Model Results (1993-2007)

How China’s productivity growth and globalization contributed to growth (1993-2007):

<table>
<thead>
<tr>
<th>Model Outcomes for Selected Countries</th>
<th>Static Model (2007 values)</th>
<th>Dynamic Model (2007 values)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real GDP ( \hat{r}/\hat{w} )</td>
<td>( \hat{P}_{IV}/\hat{P}_C )</td>
</tr>
<tr>
<td>selected countries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>0.0043</td>
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<td>0.0097</td>
</tr>
<tr>
<td>Non-China</td>
<td>0.0028</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

**Take-away #1:** China’s productivity growth and globalization increased non-China 2007 real GDP by 0.48% (1.21% of GDP growth since 1993).

About 42% of the rest of the world’s real GDP gains as of 2007 are due to capital accumulation.
**Model Results (1993-2007)**

How China’s productivity growth and globalization contributed to growth (1993-2007):

<table>
<thead>
<tr>
<th>Model Outcomes for Selected Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static Model (2007 values)</strong></td>
</tr>
<tr>
<td>(selected countries)</td>
</tr>
<tr>
<td>Real GDP</td>
</tr>
<tr>
<td>Australia</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td>Ethiopia</td>
</tr>
<tr>
<td>Germany</td>
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<tr>
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<td>Peru</td>
</tr>
<tr>
<td>USA</td>
</tr>
<tr>
<td>Vietnam</td>
</tr>
<tr>
<td>World</td>
</tr>
<tr>
<td>Non-China</td>
</tr>
</tbody>
</table>

**Take-away #2:** Developing, resource-oriented, and Asian economies tend to gain more across the board. Highlights the roles of geography and comparative advantage.

Compare, e.g., results for Germany and Italy with those for Malaysia and Peru.
Model Results (1993-2007)

How China’s productivity growth and globalization contributed to growth (1993-2007):

<table>
<thead>
<tr>
<th>Model Outcomes for Selected Countries</th>
<th>Static Model (2007 values)</th>
<th>Dynamic Model (2007 values)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real GDP</td>
<td>( \hat{r}/\hat{w} )</td>
</tr>
<tr>
<td>Australia</td>
<td>0.0043</td>
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<td>Peru</td>
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<td>Vietnam</td>
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<td>-0.0117</td>
</tr>
<tr>
<td>World</td>
<td>0.0272</td>
<td>0.0097</td>
</tr>
<tr>
<td>Non-China</td>
<td>0.0028</td>
<td><strong>0.0029</strong></td>
</tr>
</tbody>
</table>

**Take-away #3:** China’s trade growth has generally raised the return to capital and lowered the price of investment in the rest of the world.

Notice how most of the effect on 2007 real GDP effects for Germany and Italy are only apparent in the dynamic model.
Model Results (1993-2007)

How China’s productivity growth and globalization contributed to growth (1993-2007):

<table>
<thead>
<tr>
<th>Model Outcomes for Selected Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Model (2007 values)</td>
</tr>
<tr>
<td>Real GDP</td>
</tr>
<tr>
<td>---------------------------------------</td>
</tr>
<tr>
<td>(selected countries)</td>
</tr>
<tr>
<td>Australia</td>
</tr>
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<tr>
<td>Ethiopia</td>
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<td>Italy</td>
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<td>Japan</td>
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<tr>
<td>USA</td>
</tr>
<tr>
<td>Vietnam</td>
</tr>
<tr>
<td>World</td>
</tr>
<tr>
<td>Non-China</td>
</tr>
</tbody>
</table>

**Take-away #4:** Long-run (steady state) effects are an order of magnitude larger than 2007 effects.

⇒ majority of China’s effects on growth actually yet to be felt.
Other Results

- Decomposing the effects of “technological change” vs. “globalization”
- Results for the 2008-2011 period
- Isolating the contribution of “dynamic sectoral linkages”
- Varying key parameters:
  - trade elasticity ($\theta$)
  - capital adjustment ($\kappa$)
1. A Dynamic Multi-sector Trade & Growth Model

2. Fitting the Model to Data

3. China vs. the World, 1993-2011

4. How much did China Contribute to World Growth?

Extension: endogenous trade imbalances

Discussion: A slowdown in China?
Takeaways

Q1. “How much did increased trade with China contribute to growth in other countries?”

A. All told, China’s rapid trade expansion was responsible for 1.2% of the rest of the world’s real GDP growth between 1993 and 2007 and 8.8% for the period 2008-2011.

Q2. Decomposition: “How do we arrive at these numbers?”

A. The model highlights 3 key ideas:

- **Geography** and **comparative advantage** w.r.t. China each play a key role: lower-income and Asia-Pacific countries enjoy the largest effects overall

- **“Dynamic sectoral linkages”**: China’s change in comparative advantage from Non-Manufacturing to Manufacturing hurts some partners’ terms of trade in the short run, but generally promotes growth in the long run.

- **Capital adjusts slowly over time**: Model suggests that the majority of China’s effects on growth still have yet to be felt.

Q3. Looking ahead: “What can we say about the effects of slowdown in China?”
Rich framework for teasing out the effects of changes in the sectoral composition of trade:

- Comparative advantage, geography, I-O linkages, trade in capital goods all play a role

Highlights the role of “dynamic sectoral linkages” in shaping the gains from trade

- Explain three-fourth’s of China’s effects on capital accumulation in other countries
- These can take a long time to truly manifest, however.

Main result:
China’s “exceptional” trade liberalization and productivity growth between 1993-2007 in tradeables added about half a point to the rest of the world’s 2007 real GDP. I also find a similar result for the (much shorter) period 2008-2011.
Future work: Optimal trade policy; A U.S.-China tariff war

- How much does the U.S.’s trade deficit (especially with respect to China) matter for its incentives to use trade policy?
## Model Results (2008-2011)

### Using shocks to both technologies and trade frictions

<table>
<thead>
<tr>
<th>Static Model (2011 values)</th>
<th>Dynamic Model (2011 values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>$\hat{r}/\hat{w}$</td>
</tr>
<tr>
<td>(selected countries)</td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>0.0041</td>
</tr>
<tr>
<td>China</td>
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<td>0.0259</td>
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<tr>
<td>Non-China</td>
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</tr>
</tbody>
</table>

The noteworthy result here is that China’s percentage contribution to non-China world GDP over this 4 year period (0.59%) is actually larger than it was for the entire 14 year period 1993-2007

*(Take-away #5)*
Model Results (1993-2007)

*Decomposition: using changes in China’s productivity changes only*

<table>
<thead>
<tr>
<th>Model Outcomes for Selected Countries</th>
<th>Static Model (2007 values)</th>
<th>Dynamic Model (2007 values)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real GDP</td>
<td>$\hat{r}/\hat{w}$</td>
</tr>
<tr>
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When we consider productivity changes *only*, the number of countries who suffer negative consequences in the static setting.

When capital is endogenous, however, everyone realizes higher real GDP.
Model Results (1993-2007)

*Decomposition: Using China’s reductions in trade frictions only*

<table>
<thead>
<tr>
<th>Static Model (1993 values)</th>
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<td>$\hat{P}_{IV}/\hat{P}_C$</td>
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(Selected countries)

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<th>$\hat{r}/\hat{w}$</th>
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All countries benefit from trade liberalization, however. Thus, trade liberalization contributes a relatively larger share of the “static” gains from trade here.
Other Results (1993-2007): varying the trade elasticity

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<tr>
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<tbody>
<tr>
<td></td>
<td>Real GDP</td>
<td>$\hat{r}/\hat{w}$</td>
<td>$\hat{P}_{IV}/\hat{P}_C$</td>
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<tr>
<td>A. Lower trade elasticity ($\theta = 2.00; \kappa = 0.55$)</td>
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<td>B. Higher trade elasticity ($\theta = 6.00; \kappa = 0.55$)</td>
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Notes: Table shows how much changes in China’s sectoral TFPs and trade barriers during the period 1993-2007 contributed to actual outcomes for a small selection of countries, versus a counterfactual where China’s sectoral TFP changes and trade barrier reductions matched those of its trade partners. Each panel experiments with varying a key parameter from the model.
Other Results: *varying capital adjustment costs*

### Model Outcomes for Selected Countries

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<tr>
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<tbody>
<tr>
<td></td>
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<td>$\hat{r}/\hat{w}$</td>
<td>$\hat{P}_{IV}/\hat{P}_C$</td>
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<td><strong>C. Lower capital adjustment costs</strong> ($\theta = 4.00; \kappa = 0.75$)</td>
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<td>-0.0051</td>
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<td>-0.0080</td>
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<td>USA</td>
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<td>0.0013</td>
<td>-0.0051</td>
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<tr>
<td>All Non-China</td>
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<td>0.0029</td>
<td>-0.0058</td>
</tr>
</tbody>
</table>

| **D. Higher capital adjustment costs** ($\theta = 4.00; \kappa = 0.35$) |
| CHN                   | 0.6386   | 0.0442           | -0.2005                      | 0.7397   | 0.1414   | 0.0621   | 2.4603   | 3.2415   | 0.6963   |
| DEU                   | 0.0001   | 0.0061           | -0.0051                      | 0.0009   | 0.0017   | 0.0061   | 0.0460   | 0.0977   | 0.0002   |
| KOR                   | 0.0037   | 0.0009           | -0.0080                      | 0.0049   | 0.0017   | 0.0025   | 0.0521   | 0.0868   | 0.0035   |
| PER                   | 0.0052   | 0.0083           | -0.0080                      | 0.0068   | 0.0030   | 0.0103   | 0.1899   | 0.2881   | 0.0087   |
| USA                   | 0.0018   | 0.0013           | -0.0051                      | 0.0023   | 0.0009   | 0.0028   | 0.0411   | 0.0726   | 0.0021   |
| VNM                   | 0.0242   | -0.0117          | -0.0100                      | 0.0256   | 0.0020   | -0.0023  | 0.0971   | 0.1171   | 0.0231   |
| All Non-China         | 0.0028   | 0.0029           | -0.0058                      | 0.0043   | 0.0017   | 0.0035   | 0.0891   | 0.1208   | 0.0045   |

**Notes:** Table shows how much changes in China’s sectoral TFPs and trade barriers during the period 1993-2007 contributed to actual outcomes for a small selection of countries, versus a counterfactual where China’s sectoral TFP changes and trade barrier reductions matched those of its trade partners. Each panel experiments with varying a key parameter from the model.
Model Results (1993-2007, *big*)

Using shocks to both technologies and trade frictions

<table>
<thead>
<tr>
<th>Model Outcomes for Selected Countries</th>
<th>Static Model (1993 values)</th>
<th>Dynamic Model (2007 values)</th>
<th>Dynamic Model (Steady State)</th>
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</table>
Dynamic sectoral linkages (*revisited*)

In the **full model**, sectoral linkages contribute a second, strictly dynamic component:

\[
\hat{G}_{i}^{SS} \approx \prod_{k} \hat{\pi}_{i,k} \beta_{i,k} \theta \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right]^{-\frac{\beta_{i,k} \gamma_{i,c}}{\beta_{i,k} \gamma_{i,c}}} \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right]^{-\gamma_{i,IV} \frac{\beta_{i,k} \gamma_{i,c}}{\beta_{i,k} \gamma_{i,c}}}
\]

This third term **drops out completely** if:

- All sectors are used in the same proportions in final demand
  
  (i.e., \( \gamma_{i,c}^k = \gamma_{i,IV}^k = \gamma_i^k \))

- Relative capital intensities are the same across sectors
  
  (i.e., \( \beta_{i,k}^r / \beta_{i,k}^w = \beta_i^r / \beta_i^w \))
Model Results (1993-2007)

Appraising “dynamic sectoral linkages”

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</table>

When “dynamic sectoral linkages” are removed, the dynamic portion of China’s contribution to growth in other countries falls by 1/2.
## Appraising “dynamic sectoral linkages”

### Model Outcomes for Selected Countries

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<tbody>
<tr>
<td></td>
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<td>( \tilde{P}_{IV}/\tilde{P}_C )</td>
</tr>
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<td>C. Remove differences in final demand shares only ((\gamma^k_{i,C} = \gamma^k_{i,IV} = \gamma^k_i))</td>
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</table>
References I


Related Literature

EKNR in more detail

- **Huge contribution** bridging trade and macro, establishing “dynamic trade accounting” methodology

- Influences several modeling choices to be presented here

- My setting differs from EKNR’s in the following key respects:
  - More active sectors (necessitates different accounting techniques)
  - My model matches (in levels) national statistics on capital stocks, investment spending, and investment prices
  - Aside from construction, all non-manufacturing activity in ENKR is “hidden”
Related Literature

Differences from EKNR (cont’d)

- Focus here is more on quantifying and decomposing gains from trade and globalization. In particular:

  “How do changes in the sectoral *structure* of international trade lead to dynamic vs. static gains from trade?”

  (old question, but has proven difficult to answer)

- These additions come via the following innovations and data sources

  ◦ A straightforward, scalable algorithm for solving dynamic trade models with complex sectoral production linkages

  ◦ A fast, flexible “dummy variables only” method for estimating changes in technology levels over time

  ◦ A method for mapping sectoral price changes to changes in the national “investment price”
Related Literature

Differences from EKNR (cont’d)

- Only one capital series per country: invested by households, used by firms.
- Annual perspective, rather than monthly.
- Trade frictions are assumed to be symmetric, recovered via estimation
- Economic activity in all sectors is endogenously determined
  - Only construction is non-traded
  - “Services” are traded subject to trade frictions recovered from the data.
  - (but trade balances are taken as exogenous)
Dynamic Gains from Trade

The complete formula for the steady state real consumption change is:

\[
\hat{G}_{i}^{SS} = \left( \frac{1 - x_i}{\hat{\theta}_w} \right) \times \prod_{k} \left\{ \frac{-\gamma_i C_{i,k}}{\beta_{i,k} \theta} \times \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right] \right\} \times \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right]^{-\gamma_{i,IV} \frac{\beta_{i,k} \gamma_{i,C}}{\beta_{i,k}}}
\]

where:
- The numerator \( \left( \frac{1 - x_i}{\hat{\theta}_w} \right) \) captures the standard intertemporal tradeoff.
- The product \( \prod_{k} \left\{ \frac{-\gamma_i C_{i,k}}{\beta_{i,k} \theta} \times \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right] \right\} \) represents the static real wage gains.
- The product \( \prod_{k} \prod_{l} \left[ \frac{\hat{P}_{i,l}}{\hat{P}_{i,k}} \right]^{-\gamma_{i,IV} \frac{\beta_{i,k} \gamma_{i,C}}{\beta_{i,k}}} \) accounts for dynamic sectoral linkages.
Inter-temporal preference “wedge” (from Euler equation):

\[
\hat{\phi}_{i,t+1} = \frac{\tilde{\chi}_{i,t} \cdot \frac{E_{C,t+1}}{E_{C,t}} \cdot P_i^{\kappa} \cdot \frac{E_{i,IV,t}^{-\kappa}}{K_{i,t}^{1-\kappa}}}{\kappa \cdot r_{i,t+1} + (1 - \kappa) \frac{E_{i,IV,t+1}}{K_{i,t+1}} + (1 - \delta) \frac{P_i^{\kappa} \cdot E_{i,IV,t+1}^{1-\kappa}}{\tilde{\chi}_{i,t+1} \cdot K_{i,t+1}^{1-\kappa}}}
\]

(4)

Investment efficiency:

\[
\chi_{i,t} = \frac{K_{i,t+1} - (1 - \delta) \cdot K_{i,t}}{I_i^{\kappa} K_{i,t}^{1-\kappa}}
\]
Why PPML?

\[ X_{ij,k,t} = \exp \left[ \ln \left( A_{i,k,t c_{i,k,t}^{-\theta}} \right) + \ln \left( \frac{E_{j,k,t}}{P_{j,k,t}^{-\theta}} \right) + \ln d_{ij,k,t}^{-\theta} \right] + \varepsilon_{ijkt} \] (5)

Versus the typical alternative (log-OLS), PPML...

- ...assigns more weight to larger trade flows; allows for zeros (Santos Silva & Tenreyro 2006)
- ...ensures \( i \) and \( j \) fixed effects \( \Gamma_{ikt} \) and \( \Phi_{jkt} \) are consistent with market clearing from the model and can be interpreted structurally (Fally, 2014)

Another useful consideration is that a PPML regression on just dummy variables can be computed numerically and efficiently for an arbitrary number of dummy variables (Guimarães & Portugal, 2010)
Fitting the Model to Data

**Construction and Services Sectors**

Finally, how to model sectors for which bilateral trade flows are not available?
Fitting the Model to Data

Construction and Services Sectors

Finally, how to model sectors for which bilateral trade flows are not available?

The price levels for these sectors can be backed out from data on investment and consumption price levels.

\[ P_{i,F}^{\gamma_{i,IV}} = \frac{P_{i,IV}}{\prod_{k \neq F} P_{i,k}^{\gamma_{i,IV}}} \quad P_{i,O}^{\gamma_{i,IV}} = \frac{P_{i,C}}{\prod_{k \neq O} P_{i,k}^{\gamma_{i,C}}} \]
Fitting the Model to Data

Construction and Services Sectors

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Construction is non-traded \( \implies A_{i,F} = P_{i,F}^{-\theta} / c_{i,F}^{-\theta} \)

For Other Services, \( A_{i,O} \) follows from \( \pi_{ii,O} = A_{i,O} c_{i,O}^{-\theta} / P_{i,O,t}^{-\theta} \).
Fitting the Model to Data

Construction and Services Sectors

To exactly match services trade, I can also compute (aggregated) “export-side” and “import-side” trade costs for services, using only data on a country’s total services exports and imports (from UN National Accounts)

These can be solved for from the following system:

\[ d_{m, O, t}^{ex-\theta} = \frac{EX_{m, O, t}}{A_{m, O, t} c_{m, O, t} \sum_{j \neq m} \frac{E_{j, O, t}}{P_{j, O, t}} d_{j, O, t}^{im-\theta}}; \quad d_{m, O, t}^{im-\theta} = \frac{IM_{m, O, t}}{E_{m, O, t} P_{m, O, t} \sum_{j \neq m} A_{j, O, t} c_{j, O, t} d_{j, O, t}^{ex-\theta}}. \]

This will exactly match services trade balances in the data and allow services to be endogenously traded in counterfactuals.
## Included countries

### Table: Included Countries

**OECD (32 countries/regions):** Australia, Austria, Belgium-Luxembourg, Canada, Switzerland, Chile, Czech Republic, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, South Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Sweden, Turkey, United States

**Non-OECD (40 countries/regions):** Argentina, Bangladesh, Bulgaria, Bolivia, Brazil, China, Colombia, Costa Rica, Ecuador, Egypt, Ethiopia, Fiji, Ghana, Guatemala, Honduras, Hungary, Indonesia, India, Iran, Jordan, Kenya, Sri Lanka, Mauritius, Nigeria, Nepal, New Zealand, Panama, Pakistan, Peru, Russia, Senegal, Thailand, Trinidad & Tobago, Tanzania, Ukraine, Uruguay, Venezuela, Vietnam, South Africa, “Rest of World”
China vs. the World

Services

Figure: (Log) changes in sectoral productivity and trade barriers
Definitions

“Real GDP” in each period

Expenditure-side (“welfare relevant”) measure:

\[
realGDP = \frac{\sum_k \beta_k, Y_k}{P_c^{1-x} \cdot P_{iV}^x}
\]

“Consumption equivalent units” (“\(\lambda\)“):

Permanent increase in consumption that produces same change in welfare:

\[
\sum_{t=0}^{\infty} \rho^t \phi_{i,t} \ln (1 + \lambda_i) = \sum_{t=0}^{\infty} \rho^t \phi_{i,t} \ln C_{i,t} - \sum_{t=0}^{\infty} \rho^t \phi_{i,t} \ln C'_{i,t},
\]
Extension: *Endogenous Trade Balances*

To endogenize the trade balance, replace the household budget constraint with:

\[ w_{i,t}L_{i,t} + r_{i,t}K_{i,t} + B_{i,t} - \varphi_{i,t}R_t B_{i,t-1} + Z_{i,t} = P_{i,C,t}C_{i,t} + P_{i,IV,t}I_{i,t}, \]

which elaborates on each country’s trade balance as the difference between new borrowing, \( B_{i,t} \), and interest payments on the previous period’s borrowing, \( R_t B_{i,t-1} \).

\( \varphi_{i,t} \) is a “capital tax” wedge which would now be needed to match each country’s trade balance.

\( Z_{i,t} \) is an implicit transfer which rebates capital taxes to households.
Extension: *Endogenous Trade Balances*

Households now equalize consumption growth to match the local real interest rate:

\[
\frac{C_{i,t+1}}{C_{i,t}} = \rho \hat{\phi}_{i,t+1} \varphi_{i,t+1} R_{t+1} \times \frac{P_{i,t}}{P_{i,t+1}},
\]

where the world *nominal* interest rate, \( R_t \), must always adjust so that trade stays globally balanced:

\[
\sum_i D_{i,t} = \sum_i B_{i,t} - R_t B_{i,t-1} = 0, \forall t.
\]

(draws on Reyes-Heroles 2015; Ravikumar, Santacreu, & Sposi 2016)
Extension: *Endogenous Trade Balances*

Results with vs. without endogenous trade balances

China’s effects on 2007 real GDPs

China’s effects on 2007 real GDP are generally twice as large with endogenous balances than without.
Extension: *Endogenous Trade Balances*

**Results with vs. without endogenous trade balances**

![Graph showing China's effects on 2007 real GDPs (log-log)]
Model Results (1993-2007)

Figure: China’s effects on 2007 real GDPs vs. Distance from China (all countries)

slope: -.298 (s.e.: .171)
Model: *Households*

**Household Consumption, Investment, and Utility**

The (aggregated) inter-temporal problem is to maximize

\[ U_i = \sum_{t=0}^{\infty} \rho^t \phi_{i,t} \cdot \log C_{i,t} \tag{6} \]

such that

\[ w_{i,t} L_{i,t} + r_{i,t} K_{i,t} + D_{i,t} = P_{i,C,t} \cdot C_{i,t} + P_{i,IV,t} \cdot I_{i,t} \tag{7} \]

\[ K_{i,t+1} = K(K_t, I_t, \chi_{i,t}) \tag{8} \]

\[ P_{i,C,t} = \prod_k P_{i,k,t}^{\gamma_{i,C}^k} \quad P_{i,IV,t} = \prod_k P_{i,k,t}^{\gamma_{i,IV}^k} \]

\( \phi_{i,t} \): “time preference” shock. \( \chi_{i,t} \): “investment efficiency” shock.

\( \gamma_{i,C}^k \) and \( \gamma_{i,IV}^k \): (Cobb-Douglas) consumption and investment share parameters.

\( D_{i,t} \): trade deficit (treated as exogenous)
Model: Households

Household Consumption, Investment, and Utility

The (aggregated) inter-temporal problem is to maximize

$$U_i = \sum_{t=0}^{\infty} \rho^t \cdot \phi_{i,t} \cdot \log C_{i,t}$$  \hspace{1cm} (6)

such that

$$w_{i,t}L_{i,t} + r_{i,t}K_{i,t} + D_{i,t} = P_{i,C,t} \cdot C_{i,t} + P_{i,IV,t} \cdot I_{i,t}$$  \hspace{1cm} (7)

$$K_{i,t+1} = K(K_t, I_t, \chi_{i,t})$$  \hspace{1cm} (8)

Eq (6)-(8) describe a standard inter-temporal problem:

Households trade-off some consumption today in the form of investment, which enhances future income via capital accumulation.
Model: Households

Household Consumption, Investment, and Utility

The (aggregated) inter-temporal problem is to maximize

$$U_i = \sum_{t=0}^{\infty} \rho^t \cdot \phi_{i,t} \cdot \log C_{i,t}$$  \hspace{1cm} (6)

such that

$$w_{i,t} L_{i,t} + r_{i,t} K_{i,t} + D_{i,t} = P_{i,C,t} \cdot C_{i,t} + P_{i,I,t} \cdot I_{i,t}$$  \hspace{1cm} (7)

$$K_{i,t+1} = \chi_{i,t} K_{i,t}^{1-\kappa} I_{i,t}^\kappa + (1 - \delta) K_{i,t}$$  \hspace{1cm} (8)

The specific law of motion for $K$ follows EKNR and Lucas and Prescott (1971):

- $\delta$: depreciation of last-period capital
- $\kappa$: governs “adjustment costs” for investments made on top of a small existing level of capital
- $\chi_{i,t}$: efficiency/yield of investment
Model: *Households*

**Household Consumption, Investment, and Utility**

The (aggregated) inter-temporal problem is to maximize

\[ U_i = \sum_{t=0}^{\infty} \rho^t \cdot \phi_{i,t} \cdot \log C_{i,t} \]  \hspace{1cm} (6)

such that

\[ w_{i,t} L_{i,t} + r_{i,t} K_{i,t} + D_{i,t} = P_{i,C,t} \cdot C_{i,t} + P_{i,IV,t} \cdot I_{i,t} \]  \hspace{1cm} (7)

\[ K_{i,t+1} = \chi_{i,t} K_{i,t}^{1-\kappa} I_{i,t}^\kappa + (1 - \delta) K_{i,t} \]  \hspace{1cm} (8)

The Euler equation associated with this problem is:

\[ \frac{P_{IV,t}}{E_{C,t}} \left( \frac{I_t}{K_t} \right)^{1-\kappa} = \rho \frac{\hat{\phi}_{i,t+1} \chi_{i,t}}{E_{C,t+1}} \left\{ \kappa r_{t+1} + (1 - \kappa) \frac{E_{IV,t+1}}{K_{t+1}} + (1 - \delta) \frac{P_{IV,t+1}}{\chi_{t+1}} \left( \frac{I_{t+1}}{K_{t+1}} \right)^{1-\kappa} \right\} \]

(*i* subscript is suppressed)
Trade, Prices, and Productivities

Trade between \( i \) and \( j \) in each sector \( k \) takes the following standard “gravity” form:

\[
X_{ij,k} = \frac{A_{i,k} \left( c_{i,k} d_{ij,k} \right)^{1-\sigma}}{P_{j,k}^{1-\sigma}} E_{j,k}
\]  

(9)

where \( d_{ij,k} \) is an iceberg trade cost, \( A_{i,k} \) is \( i \)'s “technology”-level, \( c_{i,k} \) is the production cost and

\[
P_{j,k}^{1-\sigma} = \sum_i A_{i,k} \left( c_{i,k} d_{ij,k} \right)^{1-\sigma}
\]

captures the aggregate price index for industry \( k \) in market \( j \), by the structure of the CES Armington trade model

(as well as other such models)
Model: *Trade, Prices, and Productivities*

Trade, Production, and Prices

\[ X_{ij,k} = \frac{A_{i,k} (c_{i,k} d_{ij,k})^{1-\sigma}}{P_{j,k}^{1-\sigma}} E_{j,k} \] (9)

The combined “trade elasticity” parameter \( \sigma - 1 \) can be treated as a single parameter, “\( \theta \)”

- Emphasizes generality
- Illustrates connection with original Eaton & Kortum (2002) model (and, by extension, that of EKNR)
Model: *Trade, Prices, and Productivities*

**Trade, Production, and Prices**

\[
X_{ij,k} = \frac{A_{i,k} \left( c_{i,k} d_{ij,k} \right)^{-\theta}}{P_{j,k}^{-\theta}} E_{j,k}
\]

The combined “trade elasticity” parameter \( \sigma - 1 \) can be treated as a single parameter, “\( \theta \)”

- Emphasizes generality
- Illustrates connection with original Eaton & Kortum (2002) model (and, by extension, that of EKNR)
Model: *Trade, Prices, and Productivities*

**Trade, Production, and Prices**

\[
X_{ij,k} = \frac{A_{i,k} \left( c_{i,k} d_{ij,k} \right)^{-\theta}}{P_{j,k}^{-\theta}} E_{j,k}
\] (9)

The production technology for producing good \( k \) can be described via the “input bundle cost” \( c_{i,k} \):

\[
c_{i,k} = \left( w_i^{\alpha^w_k} \cdot r_i^{\alpha^r_k} \right)^{\beta^v_{i,k}} \cdot \prod_l P^{\beta^l_{i,l}}
\] (10)

- \( \alpha^w_k, \alpha^r_k \): factor intensities
- \( \beta^v_{i,k} \): value-added share
- \( \beta^l_{i,k} \): capture “Input-Output linkages” from input industry \( l \) to the using industry \( k \)

**Key Assumption:** Inputs to consumption, investment, and production all use the same aggregates from each industry

\( \Rightarrow \) “\( P \)” in (9) is the same as in (10)
Closing the Model I

Goods market clearing

\[
\sum_{j} X_{ij,k,t} = Y_{i,k,t} \implies Y_{i,k,t} = A_{i,k,t} c_{i,k,t}^{-\theta} \cdot \sum_{j} \frac{d_{ij,k,t}^{-\theta}}{P_{j,k,t}} E_{j,k,t}
\]

Factor market clearing

\[
\begin{align*}
    w_{i,t} L_{i,t} &= \sum_{k} \alpha_{k}^{w} \cdot \beta_{i,k}^{\nu} \cdot Y_{i,k,t}; \\
    r_{i,t} K_{i,t} &= \sum_{k} \alpha_{k}^{w} \cdot \beta_{i,k}^{\nu} \cdot Y_{i,k,t}
\end{align*}
\]

Transversality condition

\[
\lim_{t \to \infty} K_{i,t} = K_{i,SS} < \infty
\]
Sectoral expenditure

\[ E_{j,k,t} = \gamma_{i,t}^k \cdot (GDP'_{i,t} + D_{i,t}) + \sum_l \beta_{i,l}^k Y'_{i,l,t} \]

Diamond \( \gamma_{i,t}^k = (1 - x_{i,t}) \cdot \gamma_{i,C,t}^k + x_{i,t} \cdot \gamma_{i,IV,t}^k \) is a sectoral absorption share

Diamond \( x_{i,t} = E_{i,IV,t} / (GDP_{i,t} + D_{i,t}) \) is the national investment share
Equilibrium: *Overview*

An equilibrium in this model will be a (rational expectations) **Perfect Foresight Equilibrium**, where:

- Capital and investment satisfy the Euler condition in every period and satisfy the TVC at $t \to \infty$
- Trade, production, and prices within each period satisfy the competitive equilibrium conditions implied by the trade model.

The initial equilibrium will be constructed to perfectly match GDP growth, factor endowment changes, and industry-level trade flows for, e.g., 1993-2007 (and beyond, until steady state).

Counterfactuals will thus isolate the contribution of different “shocks” to what actually occurred during this period.
Equilibrium: *Solving the Static Model*

**Static Trade Equilibrium**

\[ c_{i,k} = \left( w_k^{\alpha_w} \cdot r_k^{\alpha_r} \right)^{\beta_{i,k}} \cdot \prod_l P_{i,l}^{\beta_{i,k}} \]  \hspace{1cm} (11)

\[ P_{j,k}^{-\theta} = \sum_i A_{i,k} \cdot (c_{i,k} d_{ij,k})^{-\theta} \]  \hspace{1cm} (12)

\[ Y_{i,k} = \sum_j \frac{A_{i,k} \cdot (c_{i,k} d_{ij,k})^{-\theta}}{P_{j,k}^{-\theta}} E_{j,k} \]  \hspace{1cm} (13)

\[ GDP_i = \sum_k \beta_{i,k}^{\nu} \cdot Y_{i,k} \]  \hspace{1cm} (14)

\[ E_{i,k} = \gamma_i^{k} \cdot (GDP_i + D_i) + \sum_l \beta_{i,l}^{k} Y_{i,l} \]  \hspace{1cm} (15)

\[ w_i = \frac{\sum_k \alpha_k w \cdot \beta_{i,k}^{\nu} \cdot Y_{i,k}}{L_i}; \]  \hspace{1cm} (16a)

\[ r_i = \frac{\sum_k \alpha_k r \cdot \beta_{i,k}^{\nu} \cdot Y_{i,k}}{K_i} \]  \hspace{1cm} (16b)

These 6 equations describe a general equilibrium given endowments, technologies, and trade frictions.
Equilibrium: Solving the Static Model

Static Trade Equilibrium

\[ c_{i,k} = \left( w_i^\alpha \cdot r_i^\alpha \right)^{\beta^v_{i,k}} \cdot \prod_l P_{i,l}^{\beta^l_{i,k}} \]  
(9)

\[ P_{j,k}^{-\theta} = \sum_i A_{i,k} \cdot (c_{i,k}d_{ij,k})^{-\theta} \]  
(10)

\[ Y_{i,k} = \sum_j \frac{A_{i,k} \cdot (c_{i,k}d_{ij,k})^{-\theta}}{P_{j,k}^{-\theta}} E_{j,k} \]  
(11)

\[ GDP_i = \sum_k \beta^v_{i,k} \cdot Y_{i,k} \]  
(12)

\[ E_{i,k} = \gamma_i^k \cdot (GDP_i + D_i) + \sum_l \beta^k_{i,l} Y_{i,l} \]  
(13)

\[ w_i = \frac{\sum_k \alpha^w_k \cdot \beta^v_{i,k} \cdot Y_{i,k}}{L_i}; \quad \text{(14a)} \]

\[ r_i = \frac{\sum_k \alpha^r_k \cdot \beta^v_{i,k} \cdot Y_{i,k}}{K_i}; \quad \text{(14b)} \]

**Note:** the absorption share \( \gamma_i^k \equiv x_i \cdot \gamma_{i,l,v}^k + (1 - x_i) \cdot \gamma_{i,c}^k \) and capital stock \( K_i \) come from the dynamic component of the model.
Equilibrium: **Solving the Static Model**

Static Trade Equilibrium

\[ c_{i,k} = \left( w_i^{\alpha'_w} \cdot r_i^{\alpha'_r} \right)^{\beta'_{i,k}} \cdot \prod_l P_{i,l}^{\beta_{i,l}} \]  

(9)

\[ P_{j,k}^{-\theta} = \sum_i A_{i,k} \cdot \left( c_{i,k} d_{ij,k} \right)^{-\theta} \]  

(10)

\[ Y_{i,k} = \sum_j \frac{A_{i,k} \cdot \left( c_{i,k} d_{ij,k} \right)^{-\theta}}{P_{j,k}^{-\theta}} E_{j,k} \]  

(11)

\[ GDP_i = \sum_k \beta'_{i,k} \cdot Y_{i,k} \]  

(12)

\[ E_{i,k} = \gamma_i^k \cdot \left( GDP_i + D_i \right) + \sum_l \beta_{i,l} Y_{i,l} \]  

(13)

\[ w_i = \frac{\sum_k \alpha'_w \cdot \beta'_v \cdot Y_{i,k}}{L_i} ; \]  

(14a)

\[ r_i = \frac{\sum_k \alpha'_r \cdot \beta'_v \cdot Y_{i,k}}{K_i} \]  

(14b)

The linkages between trade, factor rewards, and output/expenditure are best illustrated by examining the static *equilibrium in changes*

(e.g., as in Dekle, Eaton, & Kortum, 2007)
Equilibrium: \textit{Solving the Static Model}

\section*{Static Trade Equilibrium (in changes)}

\begin{align*}
c_{i,k} &= \left( w^{\alpha^w_k} \cdot r^{\alpha^r_k} \right)^{\beta^y_{i,k}} \cdot \prod_l P^{\beta^l_{i,k}} \tag{9} \\
P^{-\theta}_{j,k} &= \sum_i A_{i,k} \cdot \left( c_{i,k} d_{ij,k} \right)^{-\theta} \tag{10} \\
Y_{i,k} &= \sum_j \frac{A_{i,k} \cdot \left( c_{i,k} d_{ij,k} \right)^{-\theta}}{P^{-\theta}_{j,k}} E_{j,k} \tag{11} \\
\text{GDP}_i &= \sum_k \beta^y_{i,k} \cdot Y_{i,k} \tag{12} \\
E_{i,k} &= \gamma^k_i \cdot (\text{GDP}_i + D_i) \\
&\quad + \sum_l \beta^l_{i,l} Y_{i,l} \tag{13} \end{align*}

Let’s consider:

A set of trade cost shocks \( \hat{d}_{ij,k} = d^{'}_{ij,k} / d_{ij,k} \) and/or “technology” shocks \( \hat{A}_{i,k} = A^{'}_{i,k} / A_{i,k} \)

These will enter directly only through eq. (12’) and (13’).
Equilibrium: Solving the Static Model

Static Trade Equilibrium (in changes)

\[ \hat{c}_{i,k} = \left( \hat{w}^{\alpha_k^w} \cdot \hat{r}^{\alpha_k^r} \right)^{\beta_{i,k}} \prod_k \hat{P}_{i,k}^{\beta_{i,k}} \]  \hspace{1cm} (9')

\[ \hat{P}_{j,k}^{-\theta} = \sum_i \pi_{ij,k} \cdot \hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta} \]  \hspace{1cm} (10')

\[ Y'_{i,k} = \sum_j \pi_{ij,k} \cdot \frac{\hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta}}{\hat{P}_{j,k}^{-\theta}} E'_{j,k} \]  \hspace{1cm} (11')

\[ GDP'_{i} = \sum_k \beta_{i,k}^v \cdot Y'_{i,k} \]  \hspace{1cm} (12')

\[ E'_{i,k} = \gamma_{i}^k \cdot \left( GDP'_{i} + D_i \right) + \sum_l \beta_{i,l}^k Y'_{i,l} \]  \hspace{1cm} (13')

\[ \hat{w}_i = \frac{L_i \sum_k \alpha_{k}^w \cdot \beta_{i,k}^v \cdot Y'_{i,k}}{L_i' \sum_k \alpha_{k}^w \cdot \beta_{i,k}^v \cdot Y_{i,k}}; \]  \hspace{1cm} (14'a)

\[ \hat{r}_i = \frac{K_i \sum_k \alpha_{k}^r \cdot \beta_{i,k}^v \cdot Y'_{i,k}}{K_i' \sum_k \alpha_{k}^r \cdot \beta_{i,k}^v \cdot Y_{i,k}} \]  \hspace{1cm} (14'b)

Let’s consider:

A set of trade cost shocks \( \hat{d}_{ij,k} = d'_{ij,k} / d_{ij,k} \) and/or “technology” shocks \( \hat{A}_{i,k} = A'_{i,k} / A_{i,k} \)

These will enter directly only through eq. (12’) and (13’).
Equilibrium: *Solving the Static Model*

Static Trade Equilibrium *(in changes)*

\[ \hat{c}_{i,k} = \left( \hat{w}_i^{\alpha_k^w} \cdot \hat{r}_i^{\alpha_k^r} \right)^{\beta_{i,k}} \cdot \prod_{k} \hat{P}_{i,l}^{\beta_{i,k}} \quad (9') \]

\[ \hat{P}_{j,k}^{-\theta} = \sum_i \pi_{ij,k} \cdot \hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta} \quad (10') \]

\[ Y_{i,k}' = \sum_j \pi_{ij,k} \cdot \frac{\hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta} \hat{P}_{j,k}^{-\theta}}{E_{j,k}} \quad (11') \]

\[ GDP_i' = \sum_k \beta_{i,k}^v \cdot Y_{i,k}' \quad (12') \]

\[ E_{i,k}' = \gamma_i^k \cdot (GDP_i' + D_i) + \sum_l \beta_{i,l}^k Y_{i,l}' \quad (13') \]

\[ \hat{w}_i = \frac{L_i \sum_k \alpha_k^w \cdot \beta_{i,k}^v \cdot Y_{i,k}'}{L_i' \sum_k \alpha_k^w \cdot \beta_{i,k}^v \cdot Y_{i,k}}; \quad (14'a) \]

\[ \hat{r}_i = \frac{K_i \sum_k \alpha_k^r \cdot \beta_{i,k}^v \cdot Y_{i,k}'}{K_i' \sum_k \alpha_k^r \cdot \beta_{i,k}^v \cdot Y_{i,k}} \quad (14'b) \]

Intuitively, shocks in/with other countries are transmitted via the “trade share”, \( \pi_{ij,k} \)

By consistently aggregating these shocks to the country level, (12’) and (13’) dramatically reduce the dimensionality of the problem.
Equilibrium: *Solving the Static Model*

**Static Trade Equilibrium (in changes)**

$$\hat{c}_{i,k} = \left( \hat{w}_i^w \cdot \hat{r}_i^r \right)^{\beta_{i,k}} \prod_k \hat{P}_{i,l}^{\beta_{i,k}} \quad (9')$$

$$\hat{P}_{j,k}^{-\theta} = \sum_i \pi_{ij,k} \cdot \hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta} \quad (10')$$

$$y'_{i,k} = \sum_j \pi_{ij,k} \cdot \frac{\hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta}}{E_{j,k}} \quad (11')$$

$$GDP_{i}' = \sum_k \beta_{i,k}^v \cdot y'_{i,k} \quad (12')$$

**Step I**

Note first that, given \{\hat{w}, \hat{r}, E'\} one can solve for output, producer costs, and intermediate prices using (11')-(13')
Equilibrium: **Solving the Static Model**

Static Trade Equilibrium (*in changes*)

\[ \hat{c}_{i,k} = \left( \hat{w}_i^{\alpha^w_k} \cdot \hat{r}_i^{\alpha^r_k} \right)^{\beta_{i,k}} \cdot \prod_k \hat{P}_{i,l}^{\beta_{i,k}} \]  \hspace{1cm} (9')

\[ \hat{P}_{j,k}^{-\theta} = \sum_i \pi_{ij,k} \cdot \hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta} \]  \hspace{1cm} (10')

\[ Y'_{i,k} = \sum_j \pi_{ij,k} \cdot \frac{\hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta}}{\hat{P}_{j,k}^{-\theta}} E'_{j,k} \]  \hspace{1cm} (11')

\[ GDP'_i = \sum_k \beta^v_{i,k} \cdot Y'_{i,k} \]  \hspace{1cm} (12')

\[ E'_{i,k} = \gamma_i^k \cdot (GDP'_i + D_i) + \sum_l \beta_{i,l}^k Y'_{i,l} \]  \hspace{1cm} (13')

\[ \hat{w}_i = \frac{L_i \sum_k \alpha^w_k \cdot \beta^v_{i,k} \cdot Y'_{i,k}}{L_i' \sum_k \alpha^w_k \cdot \beta^v_{i,k} \cdot Y_{i,k}} \]  \hspace{1cm} (14'a)

\[ \hat{r}_i = \frac{K_i \sum_k \alpha^r_k \cdot \beta^v_{i,k} \cdot Y'_{i,k}}{K_i' \sum_k \alpha^r_k \cdot \beta^v_{i,k} \cdot Y_{i,k}} \]  \hspace{1cm} (14'b)

**Step II**

Changes in factor rewards, GDP, and expenditure follow immediately after obtaining \{Y'^k_i\}
Equilibrium: Solving the Static Model

Static Trade Equilibrium (in changes)

\[
\hat{c}_{i,k} = \left( \hat{w}_i^{\alpha_k^w} \cdot \hat{r}_i^{\alpha_k^r} \right)^{\beta_{i,k}} \cdot \prod_k \hat{P}_{i,k}^{\beta_{i,k}} \tag{9'}
\]

\[
\hat{P}_{j,k}^{-\theta} = \sum_i \pi_{ij,k} \cdot \hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta} \tag{10'}
\]

\[
Y_{i,k}' = \sum_j \pi_{ij,k} \cdot \frac{\hat{A}_{i,k} \left( \hat{c}_{i,k} \hat{d}_{ij,k} \right)^{-\theta}}{\hat{P}_{j,k}^{-\theta}} \cdot E_{j,k}' \tag{11'}
\]

\[
GDP_{i}' = \sum_k \beta_{i,k}^{\gamma_k} \cdot Y_{i,k}' \tag{12'}
\]

\[
E_{i,k}' = \gamma_i^k \cdot (GDP_{i}' + D_i) + \sum_l \beta_{i,l}^{\gamma_l} Y_{i,l}' \tag{13'}
\]

\[
\hat{w}_i = \frac{L_i}{L_i'} \sum_k \alpha_k^w \cdot \beta_{i,k}^{\gamma_k} \cdot Y_{i,k}' \tag{14'a}
\]

\[
\hat{r}_i = \frac{K_i}{K_i'} \sum_k \alpha_k^r \cdot \beta_{i,k}^{\gamma_k} \cdot Y_{i,k}' \tag{14'b}
\]

Steps III, IV, V...

Plugging \( \{ \hat{w}, \hat{r}, E' \} \) back into (11')-(13'), and continuously iterating, converges very quickly to a set of \( Y_i^{k} \)'s that solves the above system.
Equilibrium: *Solving the dynamic model*

To account for *dynamic* linkages (via capital accumulation) what needs to be added to the above iteration system is:
To account for dynamic linkages (via capital accumulation) what needs to be added to the above iteration system is:

1. **Update investment at time** $t$ (*via the Euler equation*):

$$\frac{x'_{i,t}}{1 - x'_{i,t}} = \frac{\rho \cdot \phi_{i,t+1} \chi_{i,t}}{\hat{P}_{i,t+1} \hat{r}_{i,t+1} + (1 - \kappa) \frac{E'_{i,IV,t+1}}{K_{i,t+1}} + (1 - \delta) \frac{\hat{P}^\kappa_{i,IV,t+1}}{\chi_{i,t+1}} \frac{E'_{i,IV,t}^{1-\kappa}}{K_{i,t}^{1-\kappa}}},$$

where:

- $x' = \frac{E'_{i,IV}}{GDP'} + D$ is the updated investment rate
- $\hat{P}_{IV} = \prod_k \hat{P}_{IV}^{k\gamma_{i,IV}^k}$ is the change in the price of investment
- $E'_C$ and $E'_{IV}$ are updated consumption and investment spending
- initial equilibrium $r_{t+1}$ can be computed from data.
Equilibrium: *Solving the dynamic model*

To account for **dynamic** linkages (via capital accumulation) what needs to be added to the above iteration system is:

1. **Update investment at time** \( t \) (*via the Euler equation*):

\[
\frac{x'_{i,t}}{1 - x'_{i,t}} = \frac{\rho \hat{\phi}_{i,t+1} x_{i,t}}{E'_{i,C,t+1}} \cdot \frac{\kappa \cdot r_{i,t+1} \hat{r}_{i,t+1} + (1 - \kappa) \frac{E'_{i,IV,t+1}}{K_{i,t+1}} + (1 - \delta) \frac{\hat{P}^\kappa_{i,IV,t+1}}{\chi_{i,t+1}} \frac{E'_{i,IV,t}}{K_{i,t}^{1-\kappa}}}{\hat{P}^\kappa_{i,IV,t} \cdot \frac{E'_{i,IV,t}}{K_{i,t}^{1-\kappa}}},
\]

2. **Update capital at time** \( t + 1 \) (*via the Law of Motion*):

\[
K'_{i,t+1} = x_{i,t} K_{i,t}^{1-\kappa} \left[ \frac{x'_{it} \cdot (GDP'_{i,t} + D_{i,t})}{\hat{P}_{i,IV,t}} \right]^{\kappa} + (1 - \delta) K_{i,t}
\]
Equilibrium: *Solving the dynamic model*

To account for dynamic linkages (via capital accumulation) what needs to be added to the above iteration system is:

1. **Update investment at time** $t$ (*via the Euler equation*):

   $$
   \frac{x_{i,t}'}{1 - x_{i,t}'} = \rho \frac{\phi_{i,t+1} x_{i,t}}{E_{i,C,t+1}} \cdot \kappa \cdot r_{i,t+1} \hat{r}_{i,t+1} + (1 - \kappa) \frac{E_{i,IV,t+1}'}{K_{i,t+1}} + (1 - \delta) \frac{\hat{P}_{i,IV,t+1}'}{\chi_{i,t+1}} \cdot \frac{E_{i,C,t}'}{1 - \kappa} \cdot \left( \frac{\hat{P}_{i,IV,t+1}'}{K_{i,t}^{1-\kappa}} \right)
   $$

2. **Update capital at time** $t + 1$ (*via the Law of Motion*):

   $$
   K_{i,t+1}' = x_{i,t} K_{i,t}^{1-\kappa} \left[ \frac{x_{i,t}' \cdot (GDP_{i,t}' + D_{i,t})}{\hat{P}_{i,IV,t}} \right]^{\kappa} + (1 - \delta) K_{i,t}
   $$

3. **Update new** $\{ \hat{r}, \hat{P}_{IV}, E_C', E_{IV}' \}$ **from the static model at time** $t + 1$
Equilibrium: *Solving the dynamic model*

To account for dynamic linkages (via capital accumulation) what needs to be added to the above iteration system is:

1. **Update investment at time** $t$ *(via the Euler equation)*:

   \[
   \frac{x_{i,t}'}{1 - x_{i,t}'} = \rho \frac{\hat{\phi}_{i,t+1} \chi_{i,t}}{E_{i,C,t+1}'} \cdot \frac{\kappa \cdot r_{i,t+1} \hat{r}_{i,t+1} + (1 - \kappa) \frac{E_{i,IV,t+1}'}{K_{i,t+1}} + (1 - \delta) \frac{\hat{P}_{i,IV,t+1}'}{\chi_{i,t+1}} \frac{E_{i,IV,t}'}{K_{i,t}^{1-\kappa}}}{\hat{P}_{i,IV,t}^\kappa \cdot \frac{E_{i,IV,t}'}{K_{i,t}^{1-\kappa}}}
   \]

2. **Update capital at time** $t + 1$ *(via the Law of Motion)*:

   \[
   K_{i,t+1}' = \chi_{i,t} K_{i,t}^{1-\kappa} \left[ \frac{x_{i,t}' \cdot (GDP_{i,t}' + D_{i,t})}{\hat{P}_{i,IV,t}} \right]^{\kappa} + (1 - \delta) K_{i,t}
   \]

3. **Update new** $\{\hat{r}, \hat{P}_{IV}, E_C', E_{IV}'\}$ *from the static model at time* $t + 1$

4. **Iterate repeatedly on** $\{K_{i,t}\}_{1}^{T_{SS}}$ *from** $\{K_{i,1}\}$ *to** $\{K_{i,T_{SS}}\}$ *until capital paths converge for all countries.*

   ◦ Competitive equilibrium conditions necessarily satisfied in every period
   ◦ Need to iterate *twice*, first time for initial capital path