A Shadow Rate New Keynesian Model

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Issues caused by ZLB: UMP

Before ZLB
- Federal funds rate is the primary instrument of monetary policy
- Economists rely on it to study monetary policy
  - monetary VAR
  - New Keynesian model

At ZLB
- Unconventional policy tools
  - large-scale asset purchases
  - lending facilities
  - forward guidance

How do we accommodate the ZLB and unconventional monetary policy?
Issues caused by ZLB: counterfactual implications of standard NK models

Anomalies at the ZLB without unconventional policy

- **Negative supply shock**
  - before: decreases output
  - at zlb: increases output

- **Government spending shock**
  - before: $< 1$
  - at zlb: $> 1$
Issues caused by ZLB: computational challenges

The ZLB imposes one of the biggest challenges for solving and estimating these models:

- nonlinearity
- multiple equilibria

Existing methods

- Shortcut
  - greatly simplify the solution, but
  - have undesirable economic implications
  - cannot match data
  - hide nonlinear interactions

- Global projection method
  - seriously solve the model, but
  - computationally demanding → estimation impossible
Contributions

- presents new empirical evidence relating the shadow rate with
  - private interest rates
  - Fed’s balance sheet
  - Taylor rule
- proposes a New Keynesian model with the shadow rate
  - accommodates both conventional and unconventional policies
- maps unconventional policy tools into the shadow rate framework
  - QE
  - lending facilities
- makes two anomalies disappear
  - a negative supply shock decreases output
  - government-spending multiplier is back to normal
- restores traditional solution and estimation methods
Outline

1. Shadow rate New Keynesian model (SRNKM)
2. Microfoundation I: Mapping QE into SRNKM
3. Microfoundation II: Mapping lending facilities into SRNKM
4. Quantitative analyses
Standard NK model

Definition

A standard New Keynesian model consists of the IS curve

\[ y_t = -\frac{1}{\sigma} (r_t - \mathbb{E}_t \pi_{t+1} - s) + \mathbb{E}_t y_{t+1}, \]

New Keynesian Phillips curve

\[ \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa (y_t - y^*_t), \]

and the Taylor rule with zero lower bound

\[ s_t = \phi_s s_{t-1} + (1 - \phi_s) [\phi_y (y_t - y^*_t) + \phi_\pi \pi_t + s], \]
\[ r_t = \max(0, s_t). \]
Long-term interest rate interpretation

\[ y_t = -\frac{1}{\sigma} \sum_{i=1}^{n} E_t(r_{t+i-1} - \pi_{t+i} - r) + E_t y_{t+n} \]

\[ = -\frac{1}{\sigma} n r_{t,t+n} - \frac{1}{\sigma} \sum_{i=1}^{n} E_t(-\pi_{t+i} - r) + E_t y_{t+n}. \]

- long-term rate matters for decision making instead of short rate
- UMP works through long term rates to affect the economy
- this link is missing in standard NK models

Two ways to fill the gap:
- model UMP separately – structural break
- use the shadow rate to capture UMP – no structural break
Shadow rate NK model

Definition

The shadow rate New Keynesian model consists of the shadow rate IS curve

\[
y_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t\pi_{t+1} - s) + \mathbb{E}_ty_{t+1},
\]

New Keynesian Phillips curve

\[
\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa(y_t - y^n_t),
\]

and shadow rate Taylor rule

\[
s_t = \phi s_{t-1} + (1 - \phi_s)[\phi y(y_t - y^n_t) + \phi \pi \pi_t + s].
\]
Shadow rate

- Black (1995): \( r_t = \max(s_t, r) \)

Wu-Xia Shadow Federal Funds Rate

Sources: Board of Governors of the Federal Reserve System and Wu and Xia (2015)
Empirical 1: shadow rate and private rates

- the fed funds rate is at the ZLB
- shadow rate moves in response to unconventional monetary policy
- private rates move with the shadow rate $r_t^B = s_t + rp$
- private rates are the relevant rates for agents
Empirical 2: shadow rate and Fed’s balance sheet

Correlation
- full sample: -0.74
- QE1 - QE3: -0.94
Empirical 3: shadow rate Taylor rule

\[ s_t = \beta_0 + \beta_1 s_{t-1} + \beta_2 (y_t - y^*_t) + \beta_3 \pi_t + \varepsilon_t \]
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4. Quantitative analyses
Large-scale asset purchases (QE)

The risk premium channel

- government purchases outstanding loans
- decrease interest rates through reducing risk premium
  - Gagnon et al. (2011) and Hamilton and Wu (2012)
- The same mechanism works for government bonds or corporate bonds
Households’ problem

Households’ utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{t}^{1-\sigma}}{1 - \sigma} - \frac{L_t^{1+\eta}}{1 + \eta} \right)$$

budget constraint

$$C_t + \frac{B_{t}^H}{P_t} = \frac{R_{t-1}^B B_{t-1}^H}{P_t} + W_t L_t + T_t$$

Euler equation

$$C_t^{-\sigma} = \beta R_t^B E_t \left[ \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \right]$$

The linear Euler equation

$$y_t = -\frac{1}{\sigma} \left( r_t^B - E_t \pi_{t+1} - r^B \right) + E_t y_{t+1}$$
Bond return and policy rate

Define

\[ rp_t \equiv r^B_t - r_t \]

Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2012), and Hamilton and Wu (2012) suggest

\[ rp'_t(b^G_t) < 0 \Rightarrow rp_t(b^G_t) = rp - \varsigma(b^G_t - b^G) \]

- During normal times, \( b^G_t = b^G, r_t = s_t \)
  \[ r^B_t = r_t - rp_t(b^G_t) = r_t + rp = s_t + rp \]

- At the ZLB, \( r_t = 0 \)
  \[ r^B_t = r_t - rp_t(b^G_t) = rp_t = rp - \varsigma(b^G_t - b^G) = s_t + rp \]

if \( s_t = -\varsigma(b^G_t - b^G) \)
Shadow rate equivalence for QE

Proposition

The shadow rate New Keynesian model represented by the shadow rate IS curve

\[ y_t = -\frac{1}{\sigma} (s_t - \mathbb{E}_t \pi_{t+1} - s) + \mathbb{E}_t y_{t+1} \]

New Keynesian Phillips Curve, shadow rate Taylor rule

\[ s_t = \phi_s s_{t-1} + (1 - \phi_s) \left[ \phi_y (y_t - y^n_t) + \phi_\pi \pi_t + s \right]. \]

nests both conventional Taylor interest rate rule and QE operation that changes risk premium if

\[ \begin{cases} r_t = s_t, & b^G_t = b^G, \quad & \text{for } s_t \geq 0 \\ r_t = 0, & b^G_t = b^G - \frac{s_t}{\varsigma}, \quad & \text{for } s_t < 0. \end{cases} \]
Quantifying assumption in proposition

\[ s_t = -\zeta (b_t^G - b^G) \]

- Linear assumption: correlation = 0.92
- \( \zeta = 1.83 \)
  - Fed increases its bond holdings by 1%, the shadow rate decreases by 0.0183%
  - QE1: 490 billion to 2 trillion \( \Rightarrow \) 2.5% decrease in the shadow rate
  - QE3: 2.6 trillion to 4.2 trillion \( \Rightarrow \) 0.9% decrease in the shadow rate
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3. Microfoundation II: Mapping lending facilities into SRNKM

4. Quantitative analyses
Lending facilities

Government injects liquidity to the economy

- Term Asset-Backed Securities Loan Facility in the US
- valuation haircuts in Eurosystem
- credit controls in UK

Combine this with a tax on interest rates
Model features

Entrepreneurs

- produce intermediate goods with labor and capital
- maximize utility
- discount factor $\gamma < \beta$
- borrow from households with a loan-to-value ratio $M$
- accumulate capital
- use capital as collateral

Government policy at the ZLB

- lending facilities
  - lend directly to entrepreneurs
  - change the loan-to-value ratio from $M$ to $M_t$
- tax (subsidy) on the interest rate income (payment)
Entrepreneurs’ problem

Utility function

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \gamma^t \log C_t^E$$

production function

$$Y_t^E = AK_{t-1}^\alpha (L_t)^{1-\alpha}$$

capital accumulation

$$K_t = I_t + (1 - \delta)K_{t-1}$$

budget constraint

$$\frac{Y_t^E}{X_t} + \tilde{B}_t = \frac{R_{t-1}^B \tilde{B}_{t-1}}{\mathcal{T}_{t-1} \Pi_t} + W_tL_t + I_t + C_t^E$$

borrowing constraint

$$\tilde{B}_t \leq M_t \mathbb{E}_t \left( \frac{K_t \Pi_{t+1}}{R_t^B} \right)$$
Entrepreneurs’ FOCs

Labor demand

\[ W_t = \frac{(1 - \alpha) AK_{t-1}^\alpha L_t^{-\alpha}}{X_t} \]

Euler equation

\[ \frac{1}{C_t^E} \left( 1 - \frac{M_t E_t \Pi_{t+1}}{R_t^B} \right) = \gamma E_t \left[ \frac{1}{C_{t+1}^E} \left( \frac{\alpha Y_{t+1}^E}{X_{t+1} K_t} - \frac{M_t}{T_t} + 1 - \delta \right) \right] \]
Households’ problem

Households’ utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{L_t^{1+\eta}}{1 + \eta} \right)$$

budget constraint

$$C_t + \tilde{B}_t^H = \frac{R_t^B \tilde{B}_t-1^H}{\prod_{t-1}^t} + W_t L_t + T_t$$

Euler equation

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left( R_t^B \frac{C_{t+1}^{-\sigma}}{\prod_{t+1}^{t+1} T_t} \right)$$

labor supply

$$W_t = C_t^\sigma L_t^\eta$$
Sources of funding

Entrepreneurs’ borrowing constraint

\[ \tilde{B}_t \leq M_t \mathbb{E}_t \left( \frac{K_t \Pi_{t+1}}{R_t^B} \right) \]

Households lend

\[ \tilde{B}_t^H \leq M \mathbb{E}_t \left( \frac{K_t \Pi_{t+1}}{R_t^B} \right) \]

- During normal times \( \tilde{B}_t = \tilde{B}_t^H \), and \( M_t = M \)
- At the ZLB \( M_t > M \)

Government lends the rest

\[ \tilde{B}_t^G = (M_t - M) \mathbb{E}_t \left( \frac{K_t \Pi_{t+1}}{R_t^B} \right) \]
Conventional and unconventional policy

Suppose $R_t^B = R_t RP$

Conventional and unconventional policy tools appear in the model in pairs:

- $R_t/T_t$ – HH Euler equation, HH&E budget constraints
- $R_t/M_t$ – E borrowing constraint, E Euler equation
- $M_t/T_t$ – E Euler equation

Decreasing $R_t$ is equivalent to increasing $T_t$ and $M_t$. 
Shadow rate equivalence for lending facilities

Proposition

If

\[
\begin{align*}
R_t &= S_t, T_t = 1, M_t = M \quad \text{for } S_t \geq 1 \\
T_t &= M_t/M = 1/S_t \quad \text{for } S_t < 1,
\end{align*}
\]

then \( R_t/T_t = S_t, R_t/M_t = S_t/M, M_t/T_t = M \ \forall S_t. \)

- \( S_t \) summarizes both conventional and unconventional policies
- Equivalence in the non-linear model
Shadow rate equivalence for lending facilities

**Proposition**

The shadow rate New Keynesian model represented by the Euler equation

\[ c_t = -\frac{1}{\sigma} (s_t - \mathbb{E}_t \pi_{t+1} - s) + \mathbb{E}_t c_{t+1} \]

the shadow rate Taylor rule, Phillips curve, ..., nests both conventional Taylor interest rate rule and lending facility – tax policy if

\[
\begin{cases}
    r_t = s_t, \tau_t = 0, m_t = m & \text{for } s_t \geq 0 \\
    \tau_t = m_t - m = -s_t & \text{for } s_t < 0.
\end{cases}
\]
Quantitative model

Iacoviello (2005, AER) with
- unconventional policy
- technology shock to investigate the impact of negative supply shocks at the ZLB
- government spending to investigate fiscal multiplier at the ZLB
- preference shocks to create ZLB
Methodology

Notations

- **standard model**: w/o unconventional policy \( r_t = 0 \)
- **shadow rate model**: w/ unconventional policy \( s_t < 0 \)

Methodology for **standard model**:

- piecewise linear – Guerrieri and Iacoviello (2005, JME): toolkit for models with occasionally binding constraints

Methodology for **shadow rate model**:

- solve linear model with shadow rate
- then use propositions mapping shadow rate into various UMP
Preference shock and the ZLB

Preference shock

% dev. from S.S.

0 0.05 0.1 0.15 0.2 0.25

5 10 15 20 25 30 35 40

r_t policy rate

percentage

0 0.5 1 1.5 2 2.5

5 10 15 20 25 30 35 40
Negative technology shock

1. $s$ shadow rate
2. $r_p$ policy rate
3. $r_t^{\beta_t}$ private rate
4. $\pi_t$ inflation
5. $r_t$ real interest rate
6. $r_p^p$ risk premium - QE
7. $\tau_t$ tax - Facilities
8. $M_t$ loan-to-value ratio - Facilities
9. $Y_t$ output
10. $C_t$ consumption
11. $I_t$ investment
12. $A_t$ TFP

Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP
Economic implication 1: negative supply shock

Technology shock

\[ a_t \downarrow = \rho_a a_{t-1} + e_{a,t} \downarrow \]

Phillips Curve

\[ \pi_t \uparrow = \beta \mathbb{E}_t \pi_{t+1} + \kappa (y_t - y) - \frac{\kappa (1 + \eta)}{\sigma + \eta} a_t \downarrow \]
Economic implication 1: negative supply shock

Standard model

Monetary policy

\[ r_t = \max \{ \phi_s s_{t-1} + (1 - \phi_s) [\phi_y (y_t - y_t^n) + \phi_\pi \pi_t + s], 0 \} \]

Real interest rate

\[ rr_t = r_t - \mathbb{E}_t [\pi_{t+1}] \]

IS curve

\[ y_t = -\frac{1}{\sigma} (rr_t - r) + \mathbb{E}_t y_{t+1} \]

normal times: \( \pi \uparrow \rightarrow r \uparrow \uparrow \rightarrow rr \uparrow \rightarrow y \downarrow \)

ZLB without UMP: \( \pi \uparrow \rightarrow r = 0 \rightarrow rr \downarrow \rightarrow y \uparrow \) Counterfactual
Economic implication 1: negative supply shock

**Shadow rate model**

Shadow rate Taylor rule

\[ s_t = \phi_s s_{t-1} + (1 - \phi_s) [\phi_y (y_t - y_t^n) + \phi_\pi \pi_t + s] \]

Real interest rate

\[ rr_t = s_t - \mathbb{E}_t[\pi_{t+1}] \]

Shadow rate IS curve

\[ y_t = \frac{1}{\sigma} (s_t - \mathbb{E}_t \pi_{t+1} - r) + \mathbb{E}_t y_{t+1} \]

normal times: \[ \pi \uparrow \rightarrow r \uparrow \uparrow \rightarrow rr \uparrow \rightarrow y \downarrow \]

ZLB without UMP: \[ \pi \uparrow \rightarrow r = 0 \rightarrow rr \downarrow \rightarrow y \uparrow \text{Counterfactual} \]

ZLB with UMP: \[ \pi \uparrow \rightarrow s \uparrow \uparrow \rightarrow rr \uparrow \rightarrow y \downarrow \text{Data consistent} \]
Government-spending shock

Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP
Economic implication 2: government spending multiplier

Government spending shock

\[ g_t \uparrow = (1 - \rho_g)g + \rho_g g_{t-1} + e_{g,t} \uparrow \]

Market-clearing condition

\[ y_t \uparrow = c_y c_t + g_y g_t \uparrow \]

Phillips Curve

\[ \pi_t \uparrow = \beta \mathbb{E}_t \pi_{t+1} + \frac{\kappa}{\delta + \eta} (\sigma (c_t - c) + \eta (y_t \uparrow - y)) \]
Economic implication 2: government spending multiplier

Standard model

Monetary policy

\[ r_t = \max \{ \phi_s s_{t-1} + (1 - \phi_s) [\phi_y (y_t - y^n_t) + \phi_\pi \pi_t + s], 0 \} \]

Real interest rate

\[ rr_t = r_t - \mathbb{E}_t [\pi_{t+1}] \]

IS curve

\[ c_t = -\frac{1}{\sigma} (rr_t - r) + \mathbb{E}_t c_{t+1} \]

normal times: \[ \pi \uparrow y \uparrow \rightarrow r \uparrow \uparrow \rightarrow rr \uparrow \rightarrow c \downarrow \rightarrow \Delta y < \Delta g \]

ZLB without UMP: \[ \pi \uparrow y \uparrow \rightarrow r = 0 \rightarrow rr \downarrow \rightarrow c \uparrow \rightarrow \Delta y > \Delta g \]
Shadow rate NK model and Anomaly 2

**Shadow rate model**

Shadow rate Taylor rule

\[ s_t = \phi_s s_{t-1} + (1 - \phi_s) [\phi_y (y_t - y^n_t) + \phi_\pi \pi_t + s] \]

Real interest rate

\[ rr_t = s_t - \mathbb{E}_t[\pi_{t+1}] \]

Shadow rate IS curve

\[ c_t = -\frac{1}{\sigma} (s_t - \mathbb{E}_t\pi_{t+1} - r) + \mathbb{E}_t c_{t+1} \]

normal times: \( \pi \uparrow y \uparrow \rightarrow r \uparrow \uparrow \rightarrow rr \uparrow \rightarrow c \downarrow \rightarrow \Delta y < \Delta g \)

ZLB without UMP: \( \pi \uparrow y \uparrow \rightarrow r = 0 \rightarrow rr \downarrow \rightarrow c \uparrow \rightarrow \Delta y > \Delta g \)

ZLB with UMP: \( \pi \uparrow y \uparrow \rightarrow s \uparrow \uparrow \rightarrow rr \uparrow \rightarrow c \downarrow \rightarrow \Delta y < \Delta g \)
Conclusion

We build a shadow rate NK model, capturing
▶ the conventional interest rate rule at normal times
▶ unconventional monetary policy at the ZLB

The shadow rate policy can be implemented by
▶ QE
▶ lending facilities

Economic implications
▶ a negative supply shock is not stimulative
▶ government-spending multiplier is as usual

Model solution
▶ the ZLB is not associated with a structural break
Lending facilities

\[ c_t = -\frac{1}{\sigma}(r_t^B - \tau_t - \mathbb{E}_t \pi_{t+1} - r - rp) + \mathbb{E}_t c_{t+1} \]

\[ \Rightarrow c_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t \pi_{t+1} - r) + \mathbb{E}_t c_{t+1} \]

\[ C^E c_t^E = \alpha \frac{Y}{X} (y_t - x_t) + Bb_t - R^B B(r_{t-1}^B + b_{t-1} - \tau_{t-1} - \pi_{t-1}) - li_t + \Lambda_1 \]

\[ \Rightarrow C^E c_t^E = \alpha \frac{Y}{X} (y_t - x_t) + Bb_t - R^B B(s_{t-1} + rp + b_{t-1} - \pi_{t-1}) - li_t + \Lambda_1 \]

\[ b_t = \mathbb{E}_t (k_t + \pi_{t+1} + m_t - r_t^B) \]

\[ \Rightarrow b_t = \mathbb{E}_t (k_t + \pi_{t+1} + m - s_t - rp) \]
Lending facilities

\[ 0 = \left(1 - \frac{M}{R^B}\right)(c_t^E - \mathbb{E}_t c_{t+1}^E) + \frac{\gamma \alpha Y}{XK} \mathbb{E}_t(y_{t+1} - x_{t+1} - k_t) \]
\[ + \frac{M}{R^B} \mathbb{E}_t(\pi_{t+1} - r_t^B + m_t) + \gamma M(\tau_t - m_t) + \Lambda_2 \]

\[ \Rightarrow 0 = \left(1 - \frac{M}{R^B}\right)(c_t^E - \mathbb{E}_t c_{t+1}^E) + \frac{\gamma \alpha Y}{XK} \mathbb{E}_t(y_{t+1} - x_{t+1} - k_t) \]
\[ + \frac{M}{R^B} \mathbb{E}_t(\pi_{t+1} - s_t - rp + m) - \gamma Mm + \Lambda_2 \]
## Calibration

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<td>1</td>
</tr>
<tr>
<td>$rp$</td>
<td>steady-state risk premium</td>
<td>3.6% risk premium annually</td>
<td>1.009</td>
</tr>
</tbody>
</table>
Preference shock and the ZLB

1. $s_t$: shadow rate
2. $r_t$: policy rate
3. $r_{t-t_0}$: private rate
4. $\pi_t$: inflation
5. $r_t$: real interest rate
6. $r_p$: risk premium - QE
7. $\tau_t$: tax - Facilities
8. $M_t$: loan-to-value ratio - Facilities
9. $Y_t$: output
10. $C_t$: consumption
11. $I_t$: investment
12. Preference shock

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