Modelling yields at the lower bound through regime shifts\(^1\)

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\(^1\)The views expressed do not necessarily reflect those of the BIS or the ECB.
Motivation

- Lessons from recent effective lower bound (ELB) experiences
  - ELB is not zero, due to cash storage costs; its exact level is uncertain
  - ELB spells can be extremely persistent; and expected to be persistent
  - US experience suggests very slow pace of normalisation after exiting ELB

- Shadow rate models (e.g. Bauer&Rudebusch 2015; Wu&Xia 2016) empirically successful (and parsimonious), but:
  - they impose a hard constraint, arguably too strong an assumption
  - the state vector dynamics are the same at the ELB as in normal times
  - this suggests fast pace of normalisation after exiting ELB
We study an alternative model of yields at the ELB. Two regimes: Normal (N) and Lower bound (L), with stochastic switches
- allows for different dynamics conditional on regime

Regime-switching probabilities are state dependent: the probability of switching to L is high when the policy rate is close to 0; the prob. of switching to N increases as the short rate rises

Benefit: explicit account of state nonlinearity at the ELB; allows ELB episodes to be very persistent; bond prices reflect these features — also after exiting

Cost: more parameters → use solely observable state variables
Results (so far)

- Application to US term structure using yield factors
- Good fit; clear identification of regimes
- The RS model rules out a deeply negative policy rate (but allows it to dip below the estimated LB)
- The model implies a slow pace of policy rate normalisation in coming years
- Compared to an affine model: higher term premia in recent years / lower average expected policy rates
- Regime shift risk is priced by investors, but magnitude is small
Regime-switching model

• State vector

\[ x_{t+1} = \mu^j + \Phi^j x_t + \Sigma^j \varepsilon_{t+1} \]

with \( j = N, L \) and \( x_t = \begin{bmatrix} c_t & s_t & r_t \end{bmatrix}' \); i.e. curvature, slope, \( r \).

• By assumption, under \( L \) the policy rate:
  
  • is expected to remain constant;
  
  • does not affect the other factors

\[ r_{t+1} = \mu_r^L + \sigma^L \varepsilon_{r,t+1}, \]

\[ \Phi^L = \begin{bmatrix} \phi_{cc}^L & \phi_{cs}^L & 0 \\ \phi_{sc}^L & \phi_{ss}^L & 0 \\ 0 & 0 & 0 \end{bmatrix}. \]
Regime-switching model

- **RS probabilities are state-dependent:**
  - general intuition: the lower $r$, the more likely a switch to $L$; the higher $r$, the more likely a switch to $N$.

- **Specifically:**
  \[ \pi_{t}^{\text{P},NL} = \int_{\theta}^{\theta_r} \frac{1}{\sigma_r^N \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{r - \mu^N_{t+1}}{\sigma_r^N} \right)^2 \right) dr. \]

  and \[ \pi_{t}^{\text{P},LN} = 1 - \pi_{t}^{\text{P},NL}. \]

- Assume constant Q-RS probabilities, $\pi^{Q,NL}$ and $\pi^{Q,LN}$. 
Adding yields

- Pricing according to Dai, Singleton and Yang (2007) and Bansal and Zhou (2002)
- Recall state vector

\[ x_{t+1} = \mu^j + \Phi^j x_t + \Sigma^j \varepsilon_{t+1} \]

- Risk-aversion induces

\[ x_{t+1} = \mu^{Qj} + \Phi^{Qj} x_t + \Sigma^j \varepsilon_{t+1} \]

and, using a conditionally log-normal approximation

\[ y_{t,n} = \frac{A^j_n}{n} + \frac{B^j_n}{n} x_t \]

where

\[ A^j_n = \sum_{k=1}^{S} \pi^{Qj} (\delta^j_0 + A^k_{n-1} + B^k_{n-1} \mu^{Qj} - \frac{1}{2} B^k_{n-1} \Sigma^j \Sigma^j (B^k_{n-1})' ) \]

\[ B^j_n = \sum_{k=1}^{S} \pi^{Qj} (\delta^j_x + B^k_{n-1} \Phi^{Qj} ) \]
Pricing consistency

- Recall $x_t = \begin{bmatrix} c_t & s_t & r_t \end{bmatrix}'$. We set $c_t = r_t + y_{t,120} - 2y_{t,36}$ and $s_t = y_{t,120} - r_t$.
- Hence $y_{t,120} = s_t + r_t$ and $y_{t,36} = \frac{1}{2} (s_t - c_t) + r_t$.
- Need to ensure consistency with $y_{t,120} = \frac{1}{120} \left( A_{120}^j + B_{120}^j x_t \right)$, so that
  - $A_{120}^j = 0$ and $B_{120}^j = \begin{bmatrix} 0 & 120 & 120 \end{bmatrix}$
- and $y_{t,36} = \frac{1}{36} \left( A_{36}^j + B_{36}^j x_t \right)$ so that
  - $A_{36}^j = 0$ and $B_{36}^j = \begin{bmatrix} -18 & 18 & 36 \end{bmatrix}$
- This induces nonlinear constraints on two rows of $\mu^Q$ and $\Phi^Q$.
- We impose these constraints in ML estimation.
Data

- Zero-coupon yields from Fed Board (Gürkaynak, Sack, Wright, 2006)
- 1m, 3y, 10y yields used for factors; not included among yields
We have a large number of parameters, so we need to impose some restrictions.

We estimate the $N$ parameters on a sub-sample when the economy clearly was in the $N$ regime (1987 — 2007).

We estimate the VAR $L$ parameters under $\mathbb{P}$ on the Dec. 2008 — Oct. 2015 sample.

We set the short rate threshold $\theta_r$ used in $\pi_t^{\mathbb{P},NL}$ to the 10th percentile of the distribution of $r$.

Remaining parameters are estimated using maximum likelihood.
Results
Parameter estimates

- Mean levels of state variables

<table>
<thead>
<tr>
<th>Conditional means</th>
<th>$N$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.473</td>
<td>0.962</td>
</tr>
<tr>
<td>$s$</td>
<td>1.581</td>
<td>2.361</td>
</tr>
<tr>
<td>$r$</td>
<td>3.286</td>
<td>0.136</td>
</tr>
</tbody>
</table>

- The lower bound is estimated at 13.6 basis points.
- The steady state short rate in the normal regime is 3.29%.
- Standard deviation of yield measurement errors is $\sigma^m = 0.124$. 
The RS model effectively combines the $N$ and $L$ dynamics to ensure consistency with actual data.
Filtered probabilities of N/L regimes

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Risk premia
RS model implies higher premia than 1-regime affine model recently.

ACM premium source: Adrian, Crump, Moench (2013)
Expected short rate and term premia

- Interest rate expectations matter more for yields during normal times; the term premium dominates at the LB.
Is regime shift risk priced?

- Regime shift premia essentially zero during $N$-regime; small during $L$-regime.

Hördahl and Tristani (ABFER 2017)
Forecasts
Yield forecasts at end-2011

- Wide confidence bands, but rate distributions do not include deeply negative values.
Gradual increase in interest rates and yields.
Short rate forecasts at end-April 2017

- Forecast from regime switching model more in line with SPF than shadow rate model (WX) forecasts.
Highly skewed interest rate forecasts increases the future $L$-regime probability, influencing longer term bond pricing today.
We propose a dynamic term structure model with regime switches to account for lower bound spells.

Application to US term structure using yield factors: good fit; clear identification of regimes.

The RS model rules out a deeply negative policy rate (but allows it to dip below the estimated LB).

Compared to an affine model: higher term premia in recent years; lower average expected policy rates.

Regime shift risk is priced by investors, but magnitude is small.

The model implies a slow pace of policy rate normalisation in coming years, in line with SPF forecasts.