Habits and Leverage

Tano Santos
Columbia University Graduate School of Business

Pietro Veronesi
University of Chicago Booth School of Business
Motivation

• Much discussion in the academic literature and in policy circles about leverage and its impact on the real economy and on financial markets

• Various related themes, such as:
  
  – Excess credit supply may lead to financial crisis
  
  – The excessive growth of household debt and the causal relation between households’ deleveraging and their low future consumption growth
  
  – Leverage cycle: Leverage is high when prices are high and volatility is low
  
  – Active deleveraging of financial institutions generate “fire sales” of risky financial assets, which further crash asset prices
  
  – The leverage ratio of financial institutions is a risk factor
  
  – Balance sheet recessions
  
  – ....
What we do

• Study a frictionless dynamic general equilibrium model featuring heterogeneous agents with external habit preferences

  – Heterogeneous time varying risk-bearing capacity $\implies$ leverage dynamics
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  – Heterogeneous time varying risk-bearing capacity \(\implies\) leverage dynamics

• Our model predicts:
  1. Aggregate debt ↑ in good times when prices ↑ and volatility ↓
  2. Poorer agents borrow more than richer agents
  3. Leveraged agents enjoy a “consumption boom” in good times, followed by a consumption slump
  4. Crisis time \(\implies\) leveraged agents delever by “fire-selling” stocks, but their debt/wealth ratio ↑ due to strong discount effects.
  5. Intermediaries leverage is a priced risk factor.
  6. Wealth dispersion ↑ in good times
What we do

- Study a frictionless dynamic general equilibrium model featuring heterogeneous agents with external habit preferences
  
  - Heterogeneous time varying risk-bearing capacity \(\implies\) leverage dynamics

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  4. Crisis time \(\implies\) leveraged agents delever by “fire-selling” stocks, but their debt/wealth ratio \(\uparrow\) due to strong discount effects.
  5. Intermediaries leverage is a priced risk factor.
  6. Wealth dispersion \(\uparrow\) in good times

- Model aggregates to standard representative agent models with external habit
  
  \(\implies\) It can be calibrated to yield reasonable asset pricing quantities.
• Continuum of agents with external habit preferences:

\[ u(C_{i,t}, X_{i,t}, t) = e^{-pt} \log (C_{it} - X_{it}) \]
• Continuum of agents with external habit preferences:

$$u(C_{i,t}, X_{i,t}, t) = e^{-\rho t} \log (C_{it} - X_{it})$$

• Habit indices:

$$X_{it} = g_{it} \left( D_t - \int X_{jt} \, dj \right)$$
Preferences

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- External Habit in Utility: “Envy-the-Joneses”
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\[ g_{it} = a_i \ Y_t + b_i \]
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  \[ g_{it} = a_i \begin{pmatrix} Y_t \end{pmatrix} + b_i \]

  (i) heterogeneous: \( a_i > 0 \) with \( \int a_i d\bar{i} = 1 \)

  (ii) time varying: \( Y_t = \text{Recession Indicator} \) (next slide)
  \[ \Rightarrow \text{Habits matter more in bad times.} \]
Preferences

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    \[ \Rightarrow \] Habits matter more in bad times.

• Endowments \( w_i \) are also heterogeneous, with \( \int w_i \, di = 1 \)
• Aggregate output:

\[
\frac{dD_t}{D_t} = \mu_D dt + \sigma_D(Y_t) dZ_t
\]

\(-\sigma_D(Y_t) : Economic\ Uncertainty.\)
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• Recession indicator \(Y_t\):

\[
dY_t = k(\bar{Y} - Y_t) dt - \nu Y_t \left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right]
\]

\[\implies \text{Bad shocks:} \left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right] < 0 \implies Y_t \uparrow\]
• Aggregate output:

\[ \frac{dD_t}{D_t} = \mu_d dt + \sigma_D(Y_t) dZ_t \]

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• Recession indicator \( Y_t \):

\[ dY_t = k(\bar{Y} - Y_t)dt - v Y_t \left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right] \]

\( \implies \) Bad shocks: \( \left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right] < 0 \implies Y_t \uparrow \)

• Technical restrictions:

-\( Y_t > \lambda \geq 1 \) for all \( t \): \( \sigma_D(Y_t) \rightarrow 0 \) as \( Y_t \rightarrow \lambda \). Otherwise \( \sigma_D(Y_t) \) general.
Aggregate Output

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- Recession indicator \( Y_t \):
  \[
  dY_t = k(\overline{Y} - Y_t) dt - v Y_t \left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right]
  \]
  \( \Rightarrow \) Bad shocks: \( \left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right] < 0 \Rightarrow Y_t \uparrow \)

- Technical restrictions:
  - \( Y_t > \lambda \geq 1 \) for all \( t \): \( \sigma_D(Y_t) \to 0 \) as \( Y_t \to \lambda \). Otherwise \( \sigma_D(Y_t) \) general.
  - Endowments satisfy
  \[
  w_i > \frac{a_i(\overline{Y} - \lambda) + \lambda - 1}{\overline{Y}}
  \]
Optimal Risk Sharing

- No consumption externalities $\implies$ solve planner’s problem

- Consumption shares: $s_{it} = \frac{C_{it}}{D_t} = a_i + (w_i - a_i) \frac{\bar{Y}}{Y_t}$
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  - High endowment $w_i$ or low habit loading $a_i \implies s_{it} \uparrow$ when $Y_t \downarrow$ (good times)
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- Risk aversion (curvature):
  
  $$Curv_{it} = -\frac{C_{it}u_{cc}(C_{it}, X_{it}, t)}{u_c(C_{it}, X_{it}, t)} = 1 + \frac{a_i(Y_t - \lambda) + \lambda - 1}{w_i \bar{Y} - a_i(\bar{Y} - \lambda) - \lambda + 1}$$
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  - **Cross-section:** risk aversion $\downarrow$ if $w_i \uparrow$ or $a_i \downarrow$
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  - Cross-section: risk aversion $\downarrow$ if $w_i \uparrow$ or $a_i \downarrow$
  - Time-series: (1) all agents’ risk aversion $\uparrow$ if $Y_t \uparrow$
    - (2) risk aversion of $i \uparrow$ more if $w_i$ is low or $a_i$ is high
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- Less risk averse agents provide insurance to more risk averse agents
Competitive Equilibrium

- Given price processes \( \{P_t, r_t\} \), agents solve

\[
\max_{ \{C_{it}, N_{it}, N_{0it}\} } \mathbb{E}_0 \left[ \int_0^\infty e^{-\rho t} \log (C_{it} - X_{it}) \, dt \right] \quad \text{subject to}
\]

\[
dW_{it} = N_{it}(dP_t + D_t \, dt) + N_{0it} B_t r_t \, dt - C_{it} \, dt \quad \text{with} \quad W_{i,0} = w_i P_0
\]

- A competitive equilibrium is a set of stochastic processes for prices \( \{P_t, r_t\} \) and allocations \( \{C_{it}, N_{it}, N_{0it}\} \) such that agents maximize their utilities, and good and financial markets clear \( \int C_{it} \, di = D_t, \int N_{it} \, di = 1, \int N_{0it} = 0 \).
Our model aggregates to Menzly, Santos, and Veronesi (2004):

As in Campbell and Cochrane (1999), define

\[
Surplus\ consumption\ ratio = S_t = \frac{D_t - \int X_{it} di}{D_t} = \frac{1}{Y_t}
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(1)
Representative Agent and State Price Density

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(1)

- **Proposition.** The equilibrium state price density

\[
M_t = e^{-\rho t} D_t^{-1} S_t^{-1}
\]  

(2)

- which follows

\[
dM_t/M_t = -r_t dt - \sigma_{M,t} dZ_t \quad \text{with} \quad \sigma_{M,t} = (1 + v)\sigma_D(S_t)
\]

- We use \( S_t \) as state variable for notational convenience.
• **Proposition.** The competitive equilibrium has:

(Stock price) \[ P_t = \left( \frac{\rho + k\bar{Y}S_t}{\rho(\rho + k)} \right) D_t \]

(Risk-free rate) \[ r_t = \rho + \mu_D - (1 + v)\sigma_D(S_t)^2 + k\left(1 - \bar{Y}S_t\right) \]
Proposition. The competitive equilibrium has:

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(stock holdings) \[ N_{it} = a_i + (\rho + k) (1 + v) (w_i - a_i) H(S_t) \]

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where

\[
H(S_t) = \frac{\bar{Y}S_t}{\rho + k(1 + v)\bar{Y}S_t}
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• Stock and bond holdings depend on \( w_i - a_i \) and the function \( H(S_t) \).
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• Stock and bond holdings depend on \(w_i - a_i\) and the function \(H(S_t)\).

• **Stock price** and **risk-free rate** are independent of distribution of \(w_i\) and \(a_i\).

\[\Rightarrow\] Prices and quantities have no causal relation with each other.
• **Results**: Agents with $w_i - a_i > 0$:
  
  (i) take on leverage ($N^0_{it} B_t < 0$);
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Implications: Leverage, Consumption, and Business Cycle

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  (iii) increase their debt in good times ($H'(S_t) > 0$)
    when $S_t \uparrow$, their risk aversion $\downarrow$, take on more aggregate risk
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  (iv) enjoy high consumption share $s_{it}$ when their debt is high
      
      * Leverage $\implies$ higher return $\implies$ higher consumption in good times
      
      * Lower risk aversion $\implies$ even more debt in good times
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  (v) suffer consumption decline after consumption boom
      
      * Spatial interpretation: e.g. counties with high $w_i$ or low $a_i$
      * Good times $\implies$ debt $\uparrow$ and consumption $\uparrow$ $\implies$ but lower future growth.
      * Crucial role of identification strategies to provide causal link between leverage and future consumption
• **Results (cntd.).** Agents with $w_i - a_i > 0$:
  
  (vi) increase stock holdings in good times (trend chasers)
**Implications: Active Trading**

- **Results (cntd.).** Agents with \( w_i - a_i > 0 \):
  
  (vi) increase stock holdings in good times (trend chasers)

(vii) drastically decrease stock holdings in bad times \((H(S)\text{ concave})\)
Implications for Intermediary Asset Pricing

- Much recent research on role of intermediaries’ leverage in asset prices
  - Households invest in risky assets through intermediaries, who issue debt
  - Empirically: leverage risk price is positive or negative depending on proxies
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• Let \( \ell_t = Q(S_t) \), and hence \( S_t = q(\ell_t) = Q^{-1}(\ell_t) \)

  \[ \Rightarrow SDF = M_t = e^{-\rho t} D_t^{-1} S_t^{-1} = e^{-\rho t} D_t^{-1} q(\ell_t)^{-1} \]
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\[ SDF = M_t = e^{-\rho t}D_t^{-1}S_t^{-1} = e^{-\rho t}D_t^{-1}q(\ell_t)^{-1} \]

• The risk premium for any asset with return $dR_{it} = (dP_{it} + D_{it})/P_{it}$ is

\[ E_t[dR_{it} - r_t dt] = Cov_t \left( \frac{dD_t}{D_t}, dR_{it} \right) + \frac{q'(\ell_t)}{q(\ell_t)} Cov_t (d\ell_t, dR_{it}) \]

  Consumption CAPM \hfill Leverage risk premium
Implications for Intermediary Asset Pricing

- Two potential measures of leverage:

  Debt/Output Ratio: \( \ell_t = Q_{it}^{D/O}(S_t) = -\frac{N_0^i B_t}{D_t} = v (w_i - a_i) H (S_t) \)

  Debt/Equity Ratio: \( \ell_t = Q_{it}^{D/W}(S_t) = -\frac{N_0^i B_t}{W_{it}} = \frac{\sigma_{W_i}(S_t)}{\sigma_P(S)} - 1 \)
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  Debt/Equity Ratio: \( \ell_t = Q^{D/W}_{it} (S_t) = -\frac{N^0_{it}B_t}{W_{it}} = \frac{\sigma_{Wi}(S_t)}{\sigma_P(S)} - 1 \)

- **Result:** The price of leverage risk is
  
  (a) \( \lambda^{D/O}_t = \frac{q^{D/O}'(\ell_t)}{q^{D/O}(\ell_t)} > 0 \) if \( \ell_t \) = Debt/Output Ratio ("book leverage").
  
  (b) \( \lambda^{D/W}_t = \frac{q^{D/W}'(\ell_t)}{q^{D/W}(\ell_t)} < 0 \) if \( \ell_t \) = Debt/Equity Ratio ("market leverage").

- In bad times:
  
  - agents deleverage \( \Rightarrow \) debt/output ↓ \( \Rightarrow \) book leverage risk price > 0.
  
  - high discounts \( \Rightarrow \) debt/equity ↑ \( \Rightarrow \) market leverage risk price < 0.
Quantitative Predictions

• Previous results independent of the functional form of $\sigma_D(Y_t)$.

• Assume now a specific functional form to make model comparable to MSV and obtain reasonable asset pricing implications:

$$\sigma_D(Y_t) = \sigma_{\text{max}} (1 - \lambda Y_t^{-1})$$

• $\Rightarrow$ Economic uncertainty increases in bad times, but bounded between $[0, \sigma_{\text{max}}]$

• $\Rightarrow$ Obtain same process for $Y_t$ as in MSV $\Rightarrow$ Use their same parameters.
  - Additional parameter $\sigma_{\text{max}}$ chosen to fit average consumption volatility

• All asset pricing results are similar (or stronger) than MSV.
### Table 1. Parameters and Moments

**Panel A. Parameters (MSV)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.0416</td>
</tr>
<tr>
<td>$k$</td>
<td>0.1567</td>
</tr>
<tr>
<td>$\bar{Y}$</td>
<td>34</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>20</td>
</tr>
<tr>
<td>$\bar{v}$</td>
<td>1.1194</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.0218</td>
</tr>
<tr>
<td>$\sigma_{max}$</td>
<td>0.0641</td>
</tr>
</tbody>
</table>

**Panel B. Moments (1952 – 2014)**

<table>
<thead>
<tr>
<th></th>
<th>$E[R]$</th>
<th>Std($R$)</th>
<th>$E[r_f]$</th>
<th>Std($r_f$)</th>
<th>$E[P/D]$</th>
<th>Std($P/D$)</th>
<th>$SR$</th>
<th>$E[\sigma_t]$</th>
<th>Std($\sigma_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>7.13%</td>
<td>16.55%</td>
<td>1.00%</td>
<td>1.00%</td>
<td>38</td>
<td>15</td>
<td>43%</td>
<td>1.41%</td>
<td>0.52%</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td>8.19%</td>
<td>25.08%</td>
<td>0.54%</td>
<td>3.77%</td>
<td>30.30</td>
<td>5.80</td>
<td>32.64%</td>
<td>1.43%</td>
<td>1.18%</td>
</tr>
</tbody>
</table>

**Panel C. P/D Predictability $R^2$**

<table>
<thead>
<tr>
<th></th>
<th>1 year</th>
<th>2 year</th>
<th>3 year</th>
<th>4 year</th>
<th>5 year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>5.12%</td>
<td>8.25%</td>
<td>9.22%</td>
<td>9.59%</td>
<td>12.45%</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td>14.18%</td>
<td>23.67%</td>
<td>30.01%</td>
<td>33.81%</td>
<td>35.92%</td>
</tr>
</tbody>
</table>

- Model matches asset pricing moments well.
Conditional Moments

A. Stationary Distribution

B. Price-Consumption Ratio

C. Risk Premium, Volatility, and Risk Free Rate

D. Sharpe Ratio
The Cross-Section of Agents’ Behavior: Who Levers?
The Cross-Section of Agents’ Behavior: Who Levers?

Uniform distribution of habit $a_i$

A. Distribution of Habit Loadings $a_i$

B. Distribution of Endowments $w_i$

C. Relation between $w_i$ and $a_i$

D. Leveraged Agents

LEVERAGED AGENTS

UNLEVERAGED AGENTS
The Cross-Section of Agents’ Behavior: Who Levers?

A. Distribution of Habit Loadings $a_i$

B. Distribution of Endowments $w_i$

C. Relation between $w_i$ and $a_i$

D. Leveraged Agents

Positively skewed distribution of $w_i$
The Cross-Section of Agents’ Behavior: Who Levers?

Agents missing due to endowment constraint
The Cross-Section of Agents’ Behavior: Who Levers?

A. Distribution of Habit Loadings $a_i$

B. Distribution of Endowments $w_i$

C. Relation between $w_i$ and $a_i$

D. Leveraged Agents

Poor agents Borrow

Wealthy agents borrow
Leverage in Good and Bad Times

Panel A. Agents’ Debt/Asset: Model.
Leverage in Good and Bad Times

Panel A. Agents' Debt/Asset: Model.

Panel B. Agents' Debt / Assets: Data.
“Fire Sales” in a Simulation Run

A. Surplus Consumption Ratio

B. Economic Uncertainty

C. Price / Dividend Ratio

D. Return Volatility

E. Leverage and Stock Holdings

F. Aggregate Debt/Wealth
• Consumption boom of levered agents during good times
• But expected negative consumption growth going forward
Wealth and Wealth Dispersion

- **Proposition.** Agent $i$’s wealth/output ratio:

\[
\frac{W_{it}}{D_t} = \frac{1}{\rho} \left[ \frac{\rho}{\rho + k} a_i (1 - \overline{Y}S_t) + w_i \overline{Y}S_t \right]
\]

- and wealth share:

\[
\frac{W_{it}}{\int W_{jt}dj} = a_i + (w_i - a_i) \frac{(\rho + k)\overline{Y}S_t}{\rho + k\overline{Y}S_t}
\]

- Higher $w_i$ or lower $a_i \implies$ higher wealth in good times
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- Higher $w_i$ or lower $a_i \implies$ higher wealth in good times

**Proposition.** Let $w_i$ and $a_i$ be independent. Then:

$$Var^{CS} \left( \frac{W_{it}}{\int W_{jt} dj} \right) = Var^{CS} (a_i) \left( 1 - \frac{(\rho + k) \bar{Y} S_t}{\rho + k \bar{Y} S_t} \right)^2 + Var^{CS} (w_i) \left( \frac{(\rho + k) \bar{Y} S_t}{\rho + k \bar{Y} S_t} \right)^2$$

- Endowment dispersion $\implies$ higher wealth dispersion in good times
- Preference heterogeneity $\implies$ U-shaped wealth dispersion
  * Less risk averse richer in good times but poorer in bad times
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and wealth share:

$$
\frac{W_{it}}{\int W_{jt} d j} = a_i + (w_i - a_i) \frac{(\rho + k) \bar{Y} S_t}{\rho + k \bar{Y} S_t}
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- Endowment dispersion \implies higher wealth dispersion in good times
- **Preference heterogeneity** \implies U-shaped wealth dispersion
  - Less risk averse richer in good times but poorer in bad times
- Level effect: Wealth/output dispersion increases in good times
- Relative effect: Wealth-share dispersion decreases on some range
  - Poor but very leveraged agents become better off as times get better
• Relative wealth dispersion now increases in good times
  – Only agents with high endowment (i.e. $w_i > a$) borrow $\implies$ they become even wealthier in good times
Conclusions

- A frictionless dynamic general equilibrium model with heterogeneous agents and external habits seem consistent with many stylized facts.
- Risk sharing motives generate endogenous leverage dynamics.
- Our model predicts:
  1. Aggregate debt $\uparrow$ in good times when prices $\uparrow$ and volatility $\downarrow$
  2. Poorer agents borrow more than richer agents.
  3. Leveraged agents enjoy a “consumption boom” in good times, followed by a consumption slump.
  4. Crisis time $\implies$ leveraged agents delever by “fire-selling” stocks, but their debt/wealth ratio $\uparrow$ due to strong discount effects.
  5. Intermediaries leverage is a priced risk factor.
  6. Wealth dispersion $\uparrow$ in good times.

- Leverage dynamics is due to the differential impact of aggregate shocks on agents’ risk aversion.
The Cross-Section of Consumption and Wealth

A. Average Consumption Diffusion

B. Average Wealth Diffusion

C. Average Expected Consumption Growth

D. Average Expected Excess Return on Wealth