Urban Structure, Land Prices and Volatility

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Motivating Observations

- Cities have very different house price/rent volatilities

- Avg office rent volatility between 1988-2014 (std. of log):
  - 5.71% in LA
  - 6.95% in Phoenix
  - 21.98% in New York
  - 20.52% in Dallas

- Commercial real estate tends to be more volatile than residential (Kwong and Leung 2000)

- City configurations also differ tremendously:
  - Houston (2016): population=2.3 million; area=1,553 $km^2$
  - NYC (2016): population=8.6 million; area=784 $km^2$
Questions

- How do city configuration and city land price dynamics depend on city characteristics?

- We consider a rich set of city characteristics:
  - transportation infrastructure
  - land/housing supply constraints
  - strength of production externality
  - relative share of capital, land, and labor in production
Approach

1. Construct a general equilibrium model and characterize the equilibria
   ▶ perfect mobility of capital and labor across cities
   ▶ monocentric circular cities
   ▶ multiple equilibria may exist

2. Study comparative statics about land rent, wage and population
   ▶ analytical results about land rent elasticities with respect to productivity

3. Simulate a dynamic model to study
   ▶ land rent volatility
   ▶ land rent serial correlation
   ▶ rent to value ratio of land
Literature

- Theoretical
  - Glaeser et al. (2006): a simple model that assume land supply constraint $\iff$ supply elasticity
  - Saiz (2010) shows how supply constraint leads to low supply elasticity
  - We extend the simple model in Saiz (2010) in major ways:
    - allow for feedback from population growth to TFP
    - go beyond supply constraints to study a rich set of city characteristics
  - We show land supply constraint doesn’t necessarily lead to more volatile prices

- Empirical
  - Focus on one city characteristic: land/house supply constraints
  - Glaeser et al. (2006), Saiz (2010), Hilber and Vermeulen (2016)
Model

- A monocentric circular city is occupied by firms and workers.

- Competitive firms operate in the CBD, produces tradable goods.

- Workers receive reservation utility and choose
  - consumption of tradable goods and land
  - location of residence

- Absentee landlords take all the economic surplus

- Transportation cost ($j=$distance; $N=$population):
  \[ f(j, N) = \beta_0 + \beta_1 j + \beta_2 jN \]
Workers

\[
\max_{c,h} = u(c, h)
\]

\[
s.t.
\]

\[
c + p_r(j)h = w \times e^{-f(j,N)}
\]

where

- \( c = \) non-tradable goods
- \( h = \) land
- \( w = \) wage
- \( p_r(j) = \) land rent in location \( j \)
- \( u(c, h) = c^{1-\theta}h^\theta \)
Residential Bid-rent

- perfect labor mobility \( \Rightarrow \) reservation utility \( u \)

- In each location, the landlord charge a rental rate such that workers achieve the reservation utility

\[
p_r(j) = \left[ \frac{(1 - \theta)^{1-\theta} \theta^\theta}{u} \right]^{1/\theta} \exp^{-f(j;N)}
\]

- the rent \( p_r(j) \)
  - increases with wage
  - decreases with transportation cost
  - decreases with reservation utility
Firms

\[
\max_{\ell,n} F(\ell, k, n) - wn - rk - q_c \ell \\
\text{s.t.} \\
F(\ell, k, n) = A^{\ell_\sigma} k^\xi n^{1-\sigma-\xi}
\]

- \(k=\text{capital}, \ell=\text{land}, n=\text{labor}\)
- \(r=\text{capital rent}, \text{exogenous given}\)
- \(A=\text{TFP that firms take as given}\)
- Firms take TFP as given, FOCs are

\[
\begin{align*}
\ell/n &= \frac{\sigma w}{1 - \sigma - \xi p_c} \\
k/n &= \frac{\xi w}{1 - \sigma - \xi r} \\
l/k &= \frac{\sigma r}{\xi p_c}
\end{align*}
\]
Commercial Bid-rent

- perfect capital mobility + constant return to scale production function $\Rightarrow$ zero profit

- In CBD, the landlord change a rental rate of commercial land such that firms’ profit is zero

$$p_c = \left[ \frac{A \sigma^\sigma \xi^\xi (1 - \sigma - \xi)^{1-\sigma-\xi}}{r^\xi w^{1-\sigma-\xi}} \right]^{\frac{1}{\sigma}}$$

- the rent $p_c(j)$
  - decreases with wage
  - increases with TFP
City Level Variables

- **TFP:** \( A = \tilde{A}N^\lambda \), where
  - \( \tilde{A} \) = exogenous productivity
  - \( N \) = total number of workers (population)
  - \( \lambda \) = agglomeration parameter

- **\( S \) = total area of CBD (pre-specified)**

- **\( K \) = total amount of capital (MPK=\( r \))**

- **\( J \) = distance from CBD to city boundary \( (p_r(J) = \underline{p}) \)**
  - \( \underline{p} \) = agricultural land rent (exogenous)

- **\( \Lambda \) = share of undevelopable residential land**
General Equilibrium

- Three endogenous prices:
  - wage (w)
  - commercial land rent \( p_c \)
  - residential land rent \( p_r \)

- Four endogenous quantities: \( \{N, K, J, A\} \)

- Seven equations for seven endogenous variables

- General equilibrium can be summarized by two equations:
  - aggregate labor supply equation
  - aggregate labor demand equation
Aggregate Labor Supply

- A positive relationship between population and wage
- Derived from residential land market equilibrium
  - higher wage $\Rightarrow$ higher residential and rent (bid-rent)
  - higher rent $\Rightarrow$ more land in the periphery is developed
  - more land $\Rightarrow$ more workers are housed in the city
Aggregate Labor Demand

- The relationship between population and wage

- Derived from residential land market equilibrium
  - larger population $\Rightarrow$ higher TFP (agglomeration)
  - higher TFP $\Rightarrow$ firms can afford higher wage and land rent
  - Since land is immobile, land rent rises more quickly than wage, therefore higher TFP $\Rightarrow$ larger $\frac{N}{S}$

- The relationship can be positive if the agglomeration effect is strong enough.
Ignoring congestion effect, the aggregate labor supply curve is a straight line, and equilibrium is always unique, e.g. Lucas and Rossi-Hansberg (2002).

Whenever multiple equilibria exist, we focus on the good equilibrium.
Elasticities

▶ The economy starts from a steady state
▶ It receives an exogenous shock to productivity $\tilde{A}$
▶ It reaches a new steady state
▶ Changes between the two steady states are:
  ▶ $\zeta_w = \frac{dw/w}{d\tilde{A}/\tilde{A}} = \text{wage elasticity}$
  ▶ $\zeta_N = \frac{dN/N}{d\tilde{A}/\tilde{A}} = \text{population elasticity}$
  ▶ $\zeta_{p_c} = \frac{dp_c/p_c}{d\tilde{A}/\tilde{A}} = \text{commercial rent elasticity}$
  ▶ $\zeta_{pr(j)} = \frac{dp_r(j)/p_r(j)}{d\tilde{A}/\tilde{A}} = \text{residential rent elasticity in location } j$
▶ elasticity $\approx$ volatility in the dynamic model
wage and Population Elasticities

\[ \zeta_N = 1 - \lambda + \sigma + (1 - \xi)F \]

\[ \frac{\zeta_w}{\zeta_N} = F \]

- \( F = \) cost of travelling from CBD to periphery

- larger \( F \) implies:
  - more increase in wage
  - less increase in population

- consistent with Glaeser et al. (2006)
Residential Rent

\[
\zeta_{pr} = \frac{1}{\theta} \times \frac{F - \beta_2jN}{-\lambda + \sigma + (1 - \xi)F}
\]

where \(\beta_2jN\) is the congestion effect in transportation cost function.

- We can show that \(\zeta_{pr} > 0\) (unless the city can grow explosively), since
  - \(F - \beta_2jN > 0\)
  - \(-\lambda + \sigma + (1 - \xi)F > 0\)

- \(\zeta_{pr}\) decreases with distance to CBD, i.e., rent of close-in land is more volatile.
Residential Rent Elasticity and Production Function

\[ \zeta_{pr} = \frac{1}{\theta} \times \frac{F - \beta_2 jN}{-\lambda + \sigma + (1 - \xi)F} \]

which is:

- increasing in \( \lambda \) and \( \xi \) but decreasing in \( \sigma \) in each location.
  - \( \lambda \) = agglomeration parameter
  - \( \xi \) = capital share in production
  - \( \sigma \) = land share in production

- decreasing in \( F \) if \( \lambda - \sigma > (1 - \xi)\beta_2 jN \); and increasing otherwise.
Proposition

Among cities with more undevelopable land (i.e. larger $\Lambda$)

1. have lower residential land rent elasticities if and only if
   \[ \lambda - \sigma > (1 - \xi)\beta_2 jN, \text{ given the same population.} \]

2. have a larger geographical size if and only if
   \[ \lambda - \sigma < (1 - \xi)\beta_2 jN. \]
Commercial Rent Elasticity

$$\zeta_{pc} = \frac{1 + F}{-\lambda + \sigma + (1 - \xi)F}$$

which is:

- increasing in \(\lambda\) and \(\xi\) but decreasing in \(\sigma\),
- decreasing in transportation cost \(F\).
Elasticity: Commercial Land vs Residential Land

\[ \zeta_{pc} > \zeta_{pr}(j=0) \iff F < \frac{\theta}{1-\theta} \]

Low transportation cost \( F \) (relative to \( \frac{\theta}{1-\theta} \) which measure the importance of land consumption)

\[ \rightarrow \text{easy to develop new residential land in periphery} \]

\[ \rightarrow \text{residential land supply is elastic} \]
Supply Constraint and Rent Elasticity

- Old Demand
- New Demand (Constrained City)
- New Demand ($\lambda - \sigma$ large)
- Land Supply (Unconstrained City)
- Land Supply (Constrained City)
- Land Rent
- Land Quantity

Diagram showing the relationship between land rent and land supply with different demand and supply conditions.
Model Extensions (Proposition 6 in the paper)

Proposition

Relative to the benchmark model, the following is true:

1. fixing the city boundary,
   1.1 residential land rent elasticity is lower if \( \lambda - \sigma > \beta_2 jN(1 - \xi) \) for all \( j \),
   1.2 commercial land rent elasticity is lower if \( \lambda - \sigma > -(1 - \xi) \).

2. allowing the CBD to expand and contract, land rent elasticity is higher than the benchmark model if and only if \( F < \frac{\theta}{1-\theta} \).

3. assuming immobile capital (i.e. fixing the city-level capital stock), both commercial land and residential land have lower rent elasticities.
Dynamic Model

\[ A_t = \tilde{A}_t N_t^\lambda \]
\[ \log \tilde{A}_t = \log \tilde{A}_{t-1} + \epsilon_t, \]
\[ \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2) \]

- The agglomeration effect on productivity depends on lagged city population.
- Rise and fall of cities are persistent due to the lagged feedback.
- With the dynamic model, we study
  - serial correlation of land rent
  - land rent-to-value ratios
  - land rent volatilities
Calibration

Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\epsilon$</td>
<td>stdev. of productivity shocks</td>
<td>0.003</td>
</tr>
<tr>
<td>$\theta$</td>
<td>land share in preference</td>
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</tr>
<tr>
<td>$\xi$</td>
<td>capital share in production</td>
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</tr>
<tr>
<td>$u$</td>
<td>reservation utility</td>
<td>0.118</td>
</tr>
<tr>
<td>$p$</td>
<td>agricultural rent (per 100 km$^2$)</td>
<td>0.447</td>
</tr>
<tr>
<td>$\bar{A}$</td>
<td>initial productivity</td>
<td>2.735</td>
</tr>
</tbody>
</table>

Commuting Cost

\[
f(j, N, \tau = \text{car}) = 0.0073 + 0.00008 \times j + 2.2 \times 10^{-9} \times j \times N
\]

\[
f(j, N, \tau = \text{rail}) = 0.0201 + 0.0005 \times j + 8.0 \times 10^{-9} \times j \times N
\]
Initial City Configuration

<table>
<thead>
<tr>
<th></th>
<th>Pop (million)</th>
<th>CBD (km²)</th>
<th>Radius (km)</th>
<th>Wage</th>
<th>$p_c$ (100m²)</th>
<th>$p_r$ (100m²)</th>
<th>Density (pop/100m²)</th>
</tr>
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<tbody>
<tr>
<td>$\lambda=0.08$</td>
<td>$\sigma=0.05$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda = 0.0$</td>
<td>5.00</td>
<td>30</td>
<td>16.00</td>
<td>3.14</td>
<td>3.49</td>
<td>0.73</td>
<td>62.20</td>
</tr>
<tr>
<td>$\Lambda = 0.4$</td>
<td>3.88</td>
<td>30</td>
<td>18.28</td>
<td>3.11</td>
<td>2.68</td>
<td>0.71</td>
<td>61.66</td>
</tr>
<tr>
<td>$\lambda=0.076$</td>
<td>$\sigma=0.15$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda = 0.0$</td>
<td>1.72</td>
<td>30</td>
<td>9.79</td>
<td>2.82</td>
<td>3.73</td>
<td>0.52</td>
<td>57.06</td>
</tr>
<tr>
<td>$\Lambda = 0.4$</td>
<td>1.58</td>
<td>30</td>
<td>12.09</td>
<td>2.84</td>
<td>3.46</td>
<td>0.53</td>
<td>57.41</td>
</tr>
</tbody>
</table>
## Serial Correlation

<table>
<thead>
<tr>
<th></th>
<th>( \lambda=0.08, \sigma=0.05 )</th>
<th>( p_c )</th>
<th>( p_{r (j=0)} )</th>
<th>( p_{r (j=5)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td></td>
<td>0.366</td>
<td>0.356</td>
<td>0.357</td>
</tr>
<tr>
<td>Rail</td>
<td></td>
<td>0.455</td>
<td>0.432</td>
<td>0.433</td>
</tr>
<tr>
<td>( \Lambda=0.4 )</td>
<td></td>
<td>0.391</td>
<td>0.380</td>
<td>0.381</td>
</tr>
<tr>
<td>Fix capital</td>
<td></td>
<td>0.139</td>
<td>0.139</td>
<td>0.139</td>
</tr>
<tr>
<td>Fix boundary</td>
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<td>0.131</td>
<td>0.131</td>
<td>0.132</td>
</tr>
<tr>
<td>( \lambda=0.076, \sigma=0.15 )</td>
<td></td>
<td>0.357</td>
<td>0.351</td>
<td>0.351</td>
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<tr>
<td>Baseline</td>
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<td>0.386</td>
<td>0.380</td>
<td>0.379</td>
</tr>
<tr>
<td>Rail</td>
<td></td>
<td>0.345</td>
<td>0.342</td>
<td>0.342</td>
</tr>
<tr>
<td>( \Lambda=0.4 )</td>
<td></td>
<td>0.126</td>
<td>0.125</td>
<td>0.125</td>
</tr>
<tr>
<td>Fix capital</td>
<td></td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>Fix boundary</td>
<td></td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
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</table>
## Volatility (std. of log)

<table>
<thead>
<tr>
<th>$\lambda=0.08$, $\sigma=0.05$</th>
<th>$\bar{A}$</th>
<th>Wage</th>
<th>Pop</th>
<th>$p_c$</th>
<th>$p_r_{j=0}$</th>
<th>$p_r_{j=5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline (Car)</td>
<td>1.849</td>
<td>1.715</td>
<td>9.564</td>
<td>11.276</td>
<td>5.715</td>
<td>4.780</td>
</tr>
<tr>
<td>Rail</td>
<td>2.161</td>
<td>1.847</td>
<td>13.712</td>
<td>15.552</td>
<td>6.158</td>
<td>5.180</td>
</tr>
<tr>
<td>$\Lambda = 0.4$ (car)</td>
<td>1.869</td>
<td>1.694</td>
<td>10.286</td>
<td>11.977</td>
<td>5.647</td>
<td>4.861</td>
</tr>
<tr>
<td>Fix capital (car)</td>
<td>1.354</td>
<td>0.577</td>
<td>3.110</td>
<td>3.686</td>
<td>1.922</td>
<td>1.610</td>
</tr>
<tr>
<td>Fix boundary (car)</td>
<td>1.341</td>
<td>1.493</td>
<td>2.944</td>
<td>4.437</td>
<td>4.976</td>
<td>4.679</td>
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</table>

<table>
<thead>
<tr>
<th>$\lambda=0.076$, $\sigma=0.05$</th>
<th>$\bar{A}$</th>
<th>Wage</th>
<th>Pop</th>
<th>$p_c$</th>
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<th>$p_r_{j=5}$</th>
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<th>$p_r_{j=0}$</th>
<th>$p_r_{j=5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline (car)</td>
<td>1.840</td>
<td>0.469</td>
<td>9.765</td>
<td>10.233</td>
<td>1.564</td>
<td>1.219</td>
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<tr>
<td>Rail</td>
<td>1.935</td>
<td>0.349</td>
<td>11.044</td>
<td>11.392</td>
<td>1.163</td>
<td>0.908</td>
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<tr>
<td>$\Lambda = 0.4$ (car)</td>
<td>1.805</td>
<td>0.512</td>
<td>9.307</td>
<td>9.818</td>
<td>1.708</td>
<td>1.406</td>
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<tr>
<td>Fix capital (car)</td>
<td>1.333</td>
<td>0.300</td>
<td>2.952</td>
<td>3.252</td>
<td>0.999</td>
<td>0.813</td>
</tr>
<tr>
<td>Fix boundary (car)</td>
<td>1.293</td>
<td>1.165</td>
<td>2.405</td>
<td>3.570</td>
<td>3.882</td>
<td>3.712</td>
</tr>
</tbody>
</table>
Dispersion of Rent-to-value Ratios

(c) model

(d) data
## Rent-to-value Ratios

<table>
<thead>
<tr>
<th></th>
<th>10\textsuperscript{th} percentile</th>
<th>90\textsuperscript{th} percentile</th>
<th>Dispersion 100 × (90\textsuperscript{th} − 10\textsuperscript{th}) / mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda=0.08 )</td>
<td>( \sigma=0.05 )</td>
<td>( L_c ) ( L_r ) ( L_r )</td>
<td>( L_c ) ( L_r ) ( L_r )</td>
</tr>
<tr>
<td>Baseline</td>
<td>4.01 4.25 4.60</td>
<td>5.27 4.87 4.83</td>
<td>27.32 13.54 4.70</td>
</tr>
<tr>
<td>Rail</td>
<td>3.85 4.24 4.60</td>
<td>5.72 4.92 4.86</td>
<td>39.18 14.90 5.41</td>
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<tr>
<td>( \Lambda=0.4 )</td>
<td>3.91 4.24 4.61</td>
<td>5.40 4.88 4.84</td>
<td>31.98 13.98 4.89</td>
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<tr>
<td>Fix K</td>
<td>4.36 4.46 4.58</td>
<td>4.74 4.65 4.66</td>
<td>8.24 4.28 1.76</td>
</tr>
<tr>
<td>Fix J</td>
<td>4.32 4.29 4.59</td>
<td>4.77 4.79 4.84</td>
<td>9.89 11.09 5.16</td>
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<tr>
<td></td>
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<tr>
<td>( \lambda=0.076 )</td>
<td>( \sigma=0.15 )</td>
<td>( L_c ) ( L_r ) ( L_r )</td>
<td>( L_c ) ( L_r ) ( L_r )</td>
</tr>
<tr>
<td>Baseline</td>
<td>4.01 4.48 4.58</td>
<td>5.11 4.64 4.63</td>
<td>24.14 3.49 1.14</td>
</tr>
<tr>
<td>Rail</td>
<td>3.95 4.50 4.57</td>
<td>5.17 4.62 4.62</td>
<td>26.75 2.60 0.89</td>
</tr>
<tr>
<td>( \Lambda=0.4 )</td>
<td>4.04 4.47 4.58</td>
<td>5.09 4.65 4.64</td>
<td>23.08 3.85 1.25</td>
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<tr>
<td>Fix K</td>
<td>4.39 4.51 4.57</td>
<td>4.72 4.61 4.61</td>
<td>7.25 2.22 0.89</td>
</tr>
<tr>
<td>Fix J</td>
<td>4.36 4.35 4.59</td>
<td>4.74 4.74 4.78</td>
<td>8.31 8.54 4.19</td>
</tr>
</tbody>
</table>
Conclusion

- We develop a framework for thinking about how design of a city and the firms that inhabits it affect its
  - configuration
  - land values
  - risk of real estate.

- large $\lambda$ (agglomeration) + small $\sigma$ (land share in production) $\Rightarrow$
  - high density, high wage, large population (e.g. NYC)
  - high volatility and large serial correlation in rent
  - more dispersion in rent-to-value ratio

- Land supply constraints do not necessarily lead to more land rent volatility, because constraints
  - suppress agglomeration effect
  - cause land demand curve to be shifted less.
Future Work

- Add buildings and adjustment cost to the model:
  - Study the endogenous response of real estate development to house price volatility
  - House price volatility is a fixed point

- Allow multiple CBDs to arise endogenously (lot of implications on Chinese cities)

- Consider migration costs of labor
  - Workers in rising cities receive higher utility than workers in falling cities.
  - Implications on labor misallocation (Hseih and Moretti 2017)