

Marco Gross European Central Bank

Jérôme Henry European Central Bank

Willi Semmler The New School, New York and University of Bielefeld , Germany

Overleveraging of the Banking Sector:

Destabilising effects of bank overleveraging on real activity – An analysis based on a threshold-MCS-GVAR

(ECB working paper, no 2081, 2017)

Singapore, May, 22, 2018

Motivation: Issues

Macro-Finance linkage: ECB report "Stampe": 1. Macro Impacts on banks, 2. Excess leveraging of banking sector, 3. Connectedness and contagion. We focus on excess leveraging and macroeconomic stability

- US: Financial crisis 2007-09 caused by excessive leverage of US banking system
- Basel III (IV): Subsequent regulatory proposal 2009 and 2010, concentrating inter alia on excessive leveraging of banking sector
- EU Banking Union: Excessive leverage of EU banks appear as liability for stability of future Banking Union, ECB banking stress tests; see Stampe (2017), proposing deleveraging
- In theory: Excessive leverage makes banks more vulnerable to shocks, see Brunnermeier and Sannikov 2014, Stein 2012, Mittnik and Semmler 2013 (JEDC); Admati and Hellwig 2013; can lead to reduction in lending, and declining output, see
- In empirical work: nonlinear banking-macro linkage and effects of shocks are regime depending-- on high and low leveraging ratios of banks EUROPEAN CENTRAL BANK @ www.ecb.europa.eu

Motivation: Excess leveraging and macro stability

- Small scale stochastic model to explain sustainable (optimal) debt
- Derive from this the Markowitz mean-variance form of the optimal debt
- Measure excess debt as difference of actual and optimal and estimate both for 40 EU banks as well as aggregate excess debt for countries
- In a large scale model we employ excess debt as regime change variable in a T-GVAR model and study through IRS
- Study 3 policy scenarios to reduce leveraging, by means of the leverage ratio



A. Theoretical model; Small scale model

A.1 Model structure and solution

Based on Brunnermeier and Sannikov 2014, Stein 2012, Mittnik and Semmler 2013; optimal leveraging, in a finite horizon dynamic decision model.

Dynamic version, with net worth, and shocks :

$$V = \max_{c_{t}, f_{t}} E_{t} \sum_{t=0}^{N} \beta^{t} U(c_{t} x_{1,t})$$

s.t.

$$\begin{aligned} x_{1,t+1} &= x_{1,t} + hx_{1,t} [(1+f_t)(y+\nu_1 \ln x_{2,t}+r) - (i-\nu_2 \ln x_{2,t})f_t - a\varphi(x_{1,t}) - c_t] \\ x_{2,t+1} &= \exp(\rho \ln x_{2,t} + z_k) \end{aligned}$$

Hereby c and f are the two decision variables, with $c = C/x_1$, and $f = d/x_1$, with d, debt, h =step size, y =capital gains, driven by stochastic shocks, $\nu_1 log x_{2,t}, r$, the return on capital, i, the interest rate, also driven by stochastic shocks, $\nu_2 lng x_{2,t}$.

 $a\varphi(x_{1,t})$, convex adjustment cost, ρ , a persistence parameter, with $\rho = 0.9$, and z_k is an i.i.d. random variable with zero mean and a variance, $\sigma = 0.05$.

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A.1 Model structure and solution

Model solution: Nonlinear Model Predictive Control (NMPC), see Gruene and Pannek 2011, Gruene et al. 2013. NMPC (instead of DYNARE or DP).

NMPC: computes single (approximate) optimal trajectories for finite decision horizon. Advantage: no curse of dimensionality, limited information agents (Sims), regime changes, multi-phase dynamics.

maximize
$$\int_0^T e^{-\rho t} \ell(x(t), u(t)) dt$$
,

where x(t) satisfies $\dot{x}(t) = f(x(t), u(t))$, $x(0) = x_0$. Discretized as

maximize
$$\sum_{i=0}^{N} \beta^{i} \ell(x_{i}, u_{i}) dt$$
,

 x_i satisfies $x_{i+1} = \Phi(h, x_i, u_i)$. As iterative maximization of finite horizon problem

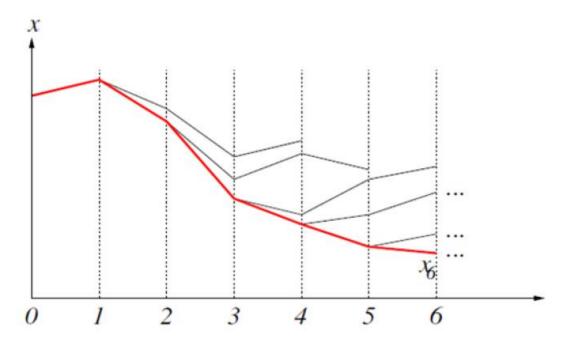
maximize
$$\sum_{k=0}^{N} \beta^{i} \ell(x_{k,i}, u_{k,i}) dt$$
,

 $N \in \mathbb{N}$ with $x_{k+1,i} = \Phi(h, x_{k,i}, u_{k,i})$

A.1 Model structure and solution

NMPC solves one optimal trajectory for finite T-horizon, without terminal constraints.

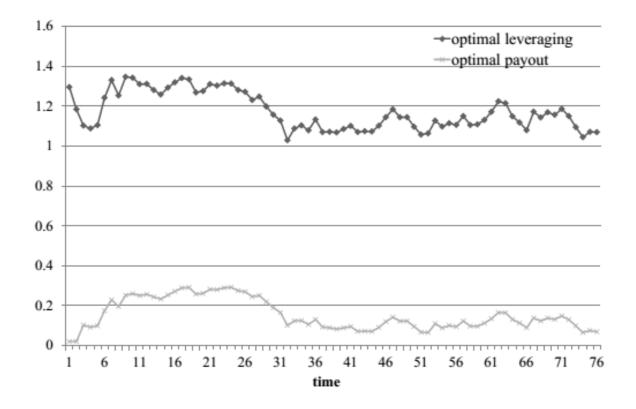
Black = predictions (open loop). Red = NMPC (closed loop), receding horizon



Question: Is the limiting behaviour of finite horizon similar to that of infinite horizon solution?

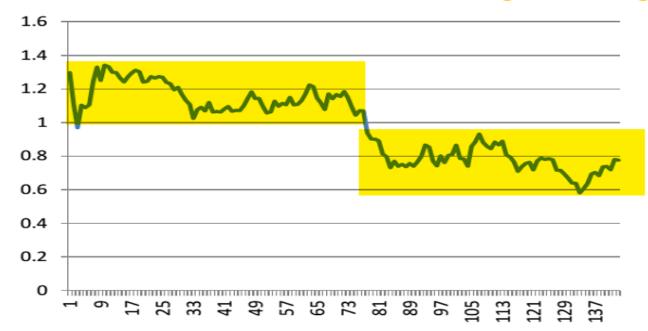
Indeed: with N \rightarrow very large \rightarrow HJB solution, for proof see Gruene and Pannek 2011

A.1 Model structure and solution – Results using NMPC



The fig. shows path of optimal payout (c) and optimal leveraging (x=1+f). x being the leveraging decision in BS and f the leveraging decision in the Stein model.

A.1 Model structure and solution – Results wrt regime change



- The fig. shows the path of leveraging. Upper regime with *i=0.02*, lower regime with *i=0.12*. Downward jump in optimal levering, implying lower optimal debt and higher excess debt.
- Interest rate mark-ups (Woodford, DSGE), or credit spreads, are also important, increases borrowing cost, and reduce credit flows (with asymmetric pass through), see QE exits and spill-over effects on emerging markets

A. Theoretical model on excess debt:

A.2 Deriving the mean-variance form; computation of optimal debt

- BS 2014 and Stein 2014 imply that optimal debt ratio can be derived in simplified case of log utility
- Stochastic differential equation for net worth:

 $dX(t) = X(t)[(1 + f(t))(dP(t)/P(t) + \beta(t)dt) - i(t)f(t) - cdt]$

- X(t) = net worth, f(t) = debt/net worth = L(t)/X(t), dP(t)/P(t) = capital gain or loss (stochastic), i(t) = interest rate (stochastic), (1+f(t)) = assets/net worth, β(t) = return on capital (dP(t)/P(t) could be model by jump-diffusion process).
- Optimal debt ratio f* maximises difference between mean and variance (risk/return trade-off)

$$f^* = \arg\max[M(f(t)) - R(f(t))] = [(a(t) + \beta(t) - i) - (\sigma_p^2 - \rho\sigma_i\sigma_b)]/\sigma^2$$

B. Empirics

B.1 Computing difference of actual and optimal debt: Excess debt

Optimal (sustainable) debt f*

- derived from theoretical model presented before
- solves the risk-return trade-off
- computed through the components of f* (capital gains, returns, interest rates, shocks...)

Actual debt: calculated as long-term and half short-term debt over TA

Actual-optimal debt: normalised gap measure, annual 1997-2014

B. Empirics

B.1 Computing optimal debt

Data: Thomson-Reuthers-Datastream, 40 European banks

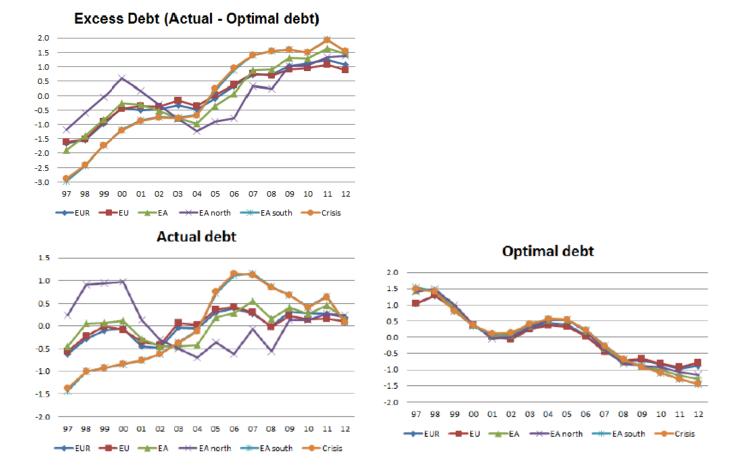
Returns of banks: net income over TA

Leverage: long plus short-term debt over TA

Country groups:

Group	Composition
Europe (EUR)	AT, BE, DK, FI, FR, DE, GR, IE, IT, NL, NO, PL, PT, ES, SE, CH, GB
EU	EUR less CH and NO
Euro area (EA)	EU less DK, PL, SE, GB
EA North	AT, BE, FI, FR, DE, NL
EA South	ES, GR, PT, IT
PIIGS	EA South and IE

B.1 Computing excess debt – Country vs group aggregates



Proc. 1: country aggregate, using country-specific mkt cap as weights. Proc. 2: group aggregate, each bank is weighted by group-specific mkt cap.

Credit/GDP: Positive correlation

	Europe	EU	EA	EA-N	EA-S	Crisis
corr	0.53	0.54	0.46	0.36	0.52	0.53

1998Q1–2013Q4, quarterly data

BEL	UK	FRA	GER	IRE	ITA	NLD	ESP	CH	PRT	GRE	AUT
0.47	0.40	0.38	0.03	0.23	0.25	0.45	0.70	-0.25	0.35	0.22	0.30
DEN	FIN	NOR	POL	SWE							
0.21	0.12	-0.04	0.25	0.37							

1998Q1–2013Q4, quarterly data; bold numbers indicate significance at 5% level

Positive correlation between credit and GDP

Negative correlation between excess debt, credit flow, and output

What constrains credit flows: "Banks' willingness to make new loans (and renew existing loans) would be affected by debt overhang...", Admati and Hellwig 2013.

Negative correlation: Banks' excess debt (actual minus optimal) and GDP and credit

	BEL	UK	FRA	GER	IRE	ITA	NLD	ESP	СН	PRT	GRE	AUT
GDP	-0.18	-0.25	-0.46	-0.11	-0.55	-0.62	-0.41	-0.66	-0.17	-0.83	-0.61	-0.17
Credit	-0.17	-0.36	-0.16	-0.26	-0.23	-0.25	-0.19	-0.33	0.65	-0.78	-0.70	-0.24
	DEN	FIN	NOR	POL	SWE							
GDP	-0.37	-0.43	-0.60	0.29	-0.44							
Credit	-0.09	-0.44	-0.05	0.53	-0.03							

1997–2012, annual data; the higher excess debt the higher banks' leverage ratio (actual minus optimal debt); bold numbers indicate significance at 5% level

Negative correlation between excess debt, credit flow, and output

Negative Correlation: Banks' excess debt (actual minus optimal) and real economy

	EUR	EU	EA	EA-N	EA-S	Crisis
GDP Growth	-0.57	-0.56	-0.59	-0.32	-0.73	-0.73
Credit Growth	-0.53	-0.53	-0.54	-0.35	-0.37	-0.38

1997–2012, annual data; the higher banks' excess debt the higher the difference between of actual minus optimal debt; bold numbers indicate significance at 5% level

- Negative correlation between credit growth and excess debt
- The higher excess debt, the lower GDP and credit growth
- Negative correlation with GDP growth by far highest in Southern EA countries

B.2 Small-scale model: Vector STAR (V-STAR) for Leveraging

- Vector STAR model: Switch between regime-specific dynamics corresponding to high and low leveraging regimes,
- Country-specific V-STAR model on quarterly data, 1998Q1-2013Q4
- Bivariate, including log diff of GDP and credit, exogenous transition variable as threshold variable (over/under-leveraging):

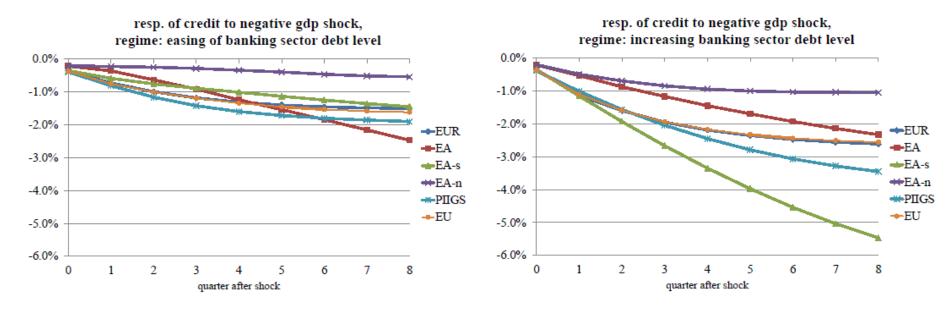
g(.): logistic transition functions, monotonically increasing in s_{ijt} , j = 1, ..., k, bounded between zero and one.

$$\mathbf{g}(s_{ijt}|\gamma_{ij}, c_{ij}) = [1 + \exp(-\gamma_{ij}(s_{ijt} - c_{ij}))]^{-1}, \ \gamma_{ij} > 0$$

 γ_{ij} : transition speed, c_{ij} : location parameter (threshold); s_{ijt} : transition variable.

B.2 Small scale model: Regime change in the VSTAR model

Figure: Cumulated response of credit after GDP shock



- After shock to output in low leverage regime: bank lending contracts; credit persistently negative
- After shock to output in high leverage regime: EA South and crisis countries respond more strongly with credit contraction

B. Empirics; Large Scale Model; T-GVAR and deleveraging B.3 Large-scale model: T-MCS-GVAR – The equation system

T-MCS-GVAR: Threshold mixed-cross-section dependence in GVAR

MCS: following Gross and Kok 2013. **Threshold**: make all model coefficients a function of leveraging regimes.

$$x_{it} = a_i + \sum_{p_{1=1}}^{P_1} \Phi_{ip_1} x_{i,t-p_1} + \sum_{p_{2=0}}^{P_2} \Lambda_{i,0,p_2} x_{i,t-p_2}^{*,C-C} + \sum_{p_{3=0}}^{P_3} \Lambda_{i,1,p_3} y_{i,t-p_3}^{*,C-B} + \sum_{p_{4=0}}^{P_4} \Lambda_{i,2,p_4} z_{i,t-p_4}^{*,C-CB} + \sum_{p_5=0}^{P_5} K_{i,p_5} v_{t-p_5} + \varepsilon_{it}$$

$$y_{jt} = b_i + \sum_{q_{1=1}}^{Q_1} \Pi_{jq_1} y_{j,t-q_1} + \sum_{q_{2=0}}^{Q_2} \Xi_{j,0,q_2} x_{j,t-q_2}^{*,B-C} + \sum_{q_{3=0}}^{Q_3} \Xi_{j,1,q_3} y_{j,t-q_3}^{*,B-B} + \sum_{q_{4=0}}^{Q_4} \Xi_{j,2,q_4} z_{j,t-q_4}^{*,B-CB} + \sum_{q_5=0}^{Q_5} E_{j,q_5} v_{t-q_5} + \omega_{jt}$$

$$z_{lt} = c_i + \sum_{r_{1=1}}^{R_1} \Gamma_{lr_1} z_{l,t-r_1} + \sum_{r_{2=0}}^{R_2} \Psi_{l,0,r_2} x_{l,t-r_2}^{*,CB-C} + \sum_{r_{3=0}}^{R_3} \Psi_{l,1,r_3} y_{l,t-r_3}^{*,CB-B} + \sum_{r_{4=0}}^{R_4} \Psi_{l,2,r_4} z_{l,t-r_4}^{*,CB-CB} + \sum_{r_{5=0}}^{R_5} T_{l,r_5} v_{t-r_5} + \tau_{lt}$$

Three sets of equations for countries, banking systems, and central banks.

Global exogenous (v_t) or local exogenous variables can be included.

B.3 Large-scale model: T-MCS-GVAR – The equation system

$$\begin{aligned} x_{it} &= a_{i} + \sum_{p_{1=1}}^{P_{1}} \Phi_{ip_{1}} x_{i,t-p_{1}} + \sum_{p_{2}=0}^{P_{2}} \Lambda_{i,0,p_{2}} x_{i,t-p_{2}}^{*,C-C} + \sum_{p_{3}=0}^{P_{3}} \Lambda_{i,1,p_{3}} y_{i,t-p_{3}}^{*,C-B} + \sum_{p_{4}=0}^{P_{4}} \Lambda_{i,2,p_{4}} z_{i,t-p_{4}}^{*,C-CB} + \sum_{p_{5}=0}^{P_{5}} K_{i,p_{5}} v_{t-p_{5}} + \varepsilon_{it} \\ y_{jt} &= b_{i} + \sum_{q_{1=1}}^{Q_{1}} \Pi_{jq_{1}} y_{j,t-q_{1}} + \sum_{q_{2}=0}^{Q_{2}} \Xi_{j,0,q_{2}} x_{j,t-q_{2}}^{*,B-C} + \sum_{q_{3}=0}^{Q_{3}} \Xi_{j,1,q_{3}} y_{j,t-q_{3}}^{*,B-B} + \sum_{q_{4}=0}^{Q_{4}} \Xi_{j,2,q_{4}} z_{j,t-q_{4}}^{*,B-CB} + \sum_{q_{5}=0}^{Q_{5}} P_{j,q_{5}} v_{t-q_{5}} + \omega_{jt} \\ z_{lt} &= c_{i} + \sum_{r_{1=1}}^{R_{1}} \Gamma_{lr_{1}} z_{l,t-r_{1}} + \sum_{r_{2}=0}^{R_{2}} \Psi_{l,0,r_{2}} x_{l,t-r_{2}}^{*,CB-C} + \sum_{r_{3}=0}^{R_{3}} \Psi_{l,1,r_{3}} y_{l,t-r_{3}}^{*,CB-B} + \sum_{r_{4}=0}^{R_{4}} \Psi_{l,2,r_{4}} z_{l,t-r_{4}}^{*,CB-CB} + \sum_{r_{5}=0}^{R_{5}} T_{l,r_{5}} v_{t-r_{5}} + \tau_{lt} \end{aligned}$$

Fully endogenous (though constrained) cross-cross-section dependence via weighted variable vectors ("star-variables").

To establish link b/w 3 cross-sections, up to 9 weight matrices needed. Some weight sets not needed due to exclusion restrictions.

B.3 T-MCS-GVAR – Model variables / Sample / Weights

Countries (EU28): Nominal GDP, GDP deflator inflation

Banking systems (14): Nominal loan growth, capital ratios, loan interest rates, Merton-model bank PDs (asset-weighted country aggregates), ECB BSI data

Central banks (ECB + 10 other non-EA): Short-term policy rates

Regime-determining variable: Overleveraging, as used for VSTAR

Sample: 1995Q1-2013Q4

Weights: --- Countries (trade shares)

- ---- Banking system (cross country exposure)
- --- Central banks (weighted average in Taylor Rule)

B.3 T-MCS-GVAR – Simulation types / Sign constraints

#	Scenario	Shock	Sign constraints
1	Contractionary deleveraging shock	LEV down	L down, I up
2	Expansionary deleveraging shock	LEV down	Lup, I down
3	Deleveraging shock No sign constraints	LEV down	-

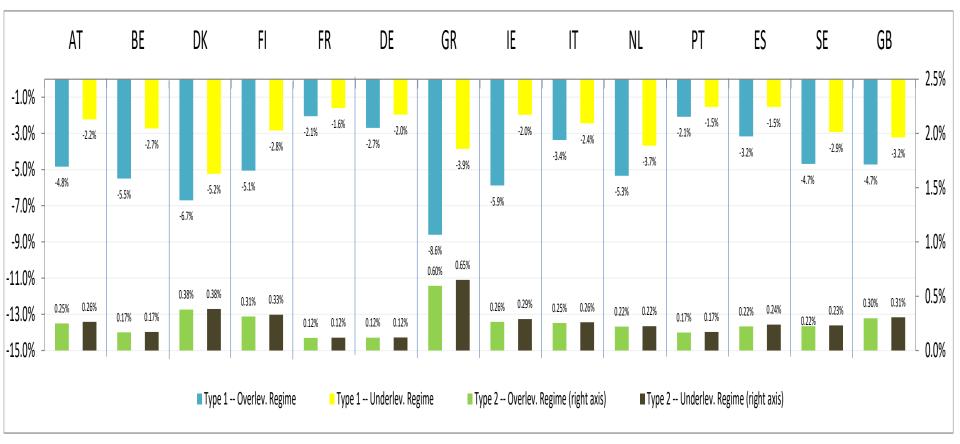
Scenario Type 1: Banks get to lower leverage / higher capital ratio by letting business mature and not renewing it or selling assets \rightarrow negative credit supply shock: $A_{\downarrow} E \rightarrow D_{\downarrow}$

Scenario Type 2: Banks raise capital and invest it \rightarrow positive credit supply shock: At Et D \rightarrow

Scenario Type 3: Unconstrained deleveraging shock, to see how banks went over deleveraging process historically: $A \rightarrow E \uparrow D \downarrow$

B.3 T-MCS-GVAR: Implied Type 1 and 2 loan supply shocks

<u>Figure 4</u>: Type 1 and Type 2 loan supply shocks (implied by otherwise identical capital ratio shocks) under over- and under-leveraging regime

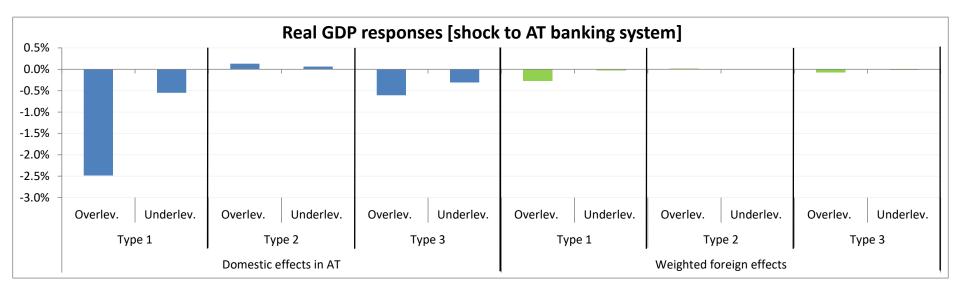


Pronounced asymmetry in Type 1 loan supply shocks under over- vs underleveraging (blue and yellow), and positive effect under Type 2.

B. Empirics; T-GVAR and deleveraging B.3 T-MCS-GVAR: Results

Example: Shock to AT banking system capital ratio (+0.24pp)

Figure: Cumulative effect for real GDP after 3 years in PP deviation from baseline growth.

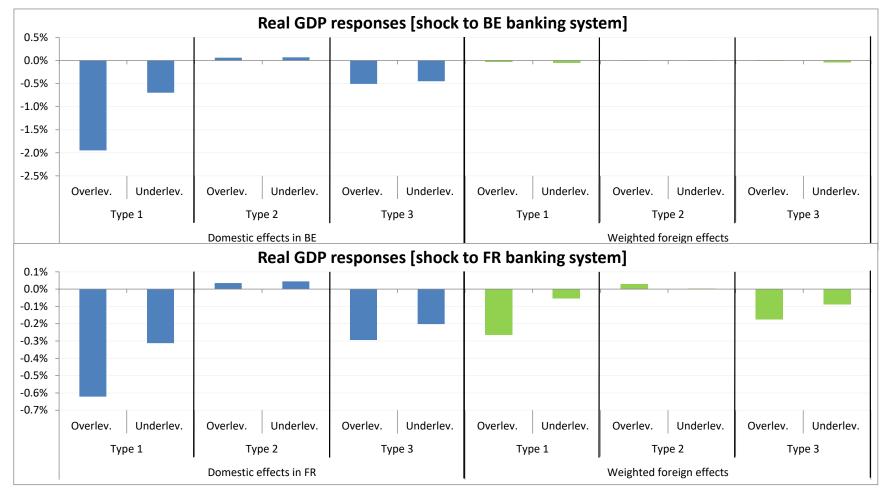


•GDP response stronger under Type 1 than under Type 2

•GDP response stronger when starting conditional on overleveraging regime

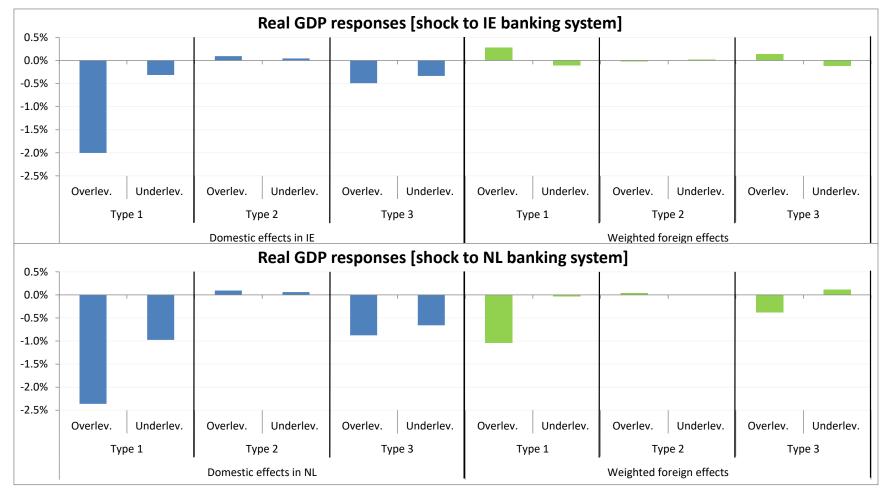
B. Empirics; T-GVAR and deleveraging B.3 T-MCS-GVAR: Results

Shocks to <u>BE</u> (+0.16pp CAPR) and <u>FR</u> (+0.11pp CAPR) banking system



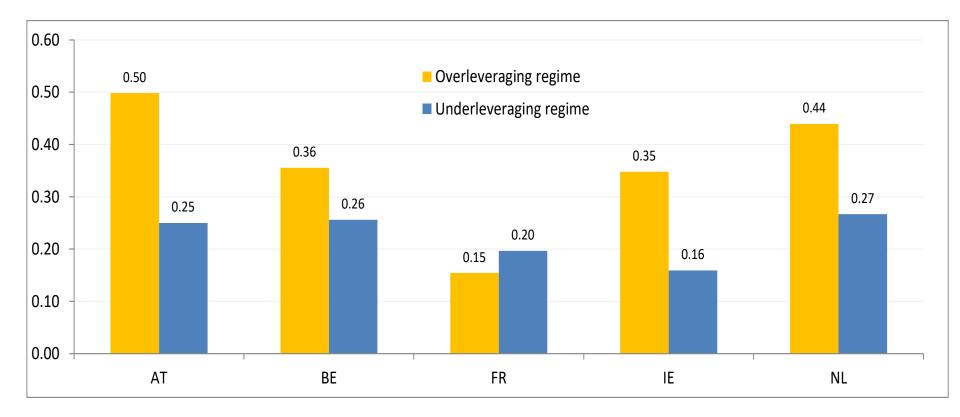
B. Empirics; T-GVAR and deleveraging B.3 T-MCS-GVAR: Results

Shocks to IE (+0.25pp CAPR) and NL (+0.21pp CAPR) banking system



B.3 T-MCS-GVAR: Results – Credit to GDP long-run effect ratios

Figure: Real GDP to nominal credit long-run shock response ratios under two different regimes



Markedly stronger GDP to credit effect ratios when starting under overleveraging regime. 26 EUROPEAN CENTRAL BANK ©

C. Conclusions -- Regime dependence

- Nonlinearities in banking-macro linkages; amplifying macro-feedback loops between banks and macro in overleveraged regime (differences across countries, EA South more affected)
- As to policies we distinguish explicitly between asset side deleveraging (Type 1), equity raising and investing (Type 2) and unconstrained capital ratio shocks (Type 3), there is a difference whether deleveraging is accomplished via Type 1 or 2 or 3
- Deleveraging policy is stronger when starting from overleveraging regime: Decline of GDP growth to credit volume stronger; capital ratio shock translates into much stronger asset side reaction (see Type 1 simulation) than under the low-leverage;
- Macro responses are distinctively stronger under overleveraging; i.e. cumulative growth decline is stronger under overleveraging (see type 1 simulation) than under the low-leverage.

C. Conclusions-- Extensions

- (1) In "Stampe" (ECB report on banking stress test analytics), there are detailed studies of bank net income, credit growth, risk assessments of banks, default probabilities, connectedness, contagion effects, liquidity tests, households and firms leveraging.
- (2) Extension: For credit flows depends on risk prima/credit spread (interest rate markups, DSGE), with asymmetric pass through, more specifically in Gross et al. (2018).
- (3) Extension: Effect of normalization? Policy rate up? Will there be a downward jump in net worth of banks, rising of excess debt and further macro instability?
- (4) Extension: dP(t)/P(t) can be modelled by jump-diffusion process (oil price jump and banking system of oil exporting countries, see Isser/Semmler, 2018).
- (5) Extension: Cross-border spillover effects appear more pronounced when using financial stress measures, defining financial stress regimes, see also Chen/Semmler (2018), JEDC.

THANKS FOR YOUR ATTENTION!

Background Slides

A. Theoretical model on excess debt

A.2 Computation of path of optimal debt

Model 1: Optimal debt ratio; Stein considers $\beta(t)$ as deterministic.

$$dP(t)/P(t) = (r + \alpha(0 - y))dt + \sigma_p dw_p$$

$$f^*(t) = \left[(r-i) + \beta - \alpha y(t) - (\frac{1}{2})\sigma_p^2 + \rho \sigma_i \sigma_p\right] / \sigma^2$$

Model 2: Price equation is the first of the following three. Drift is $a(t)dt=\pi dt$. With diffusion term $\sigma_p dw_p$.

$$dP(t)/P(t) = \pi dt + \sigma_p dw_p$$

$$f^*(t) = [(\pi - i + \beta(t)) - (\sigma_p^2 - \rho\sigma_i\sigma_p)]/\sigma^2$$

$$\sigma^2 = \sigma_i^2 + \sigma_p^2 - 2\rho\sigma_i\sigma_p$$

Appendix: weights

From	То	Weights
	- Countries	Bilateral trade (sum of nominal inports and exports)
Countries -	- Banking system	Transpose of Banking system (banks) - Countries matrices
	- Central Banks	Unit weights for countries to their respective central bank; e.g. for EA countries set unit weight to ECB
	-Countries	BSI domestic and cross-border exposure data
Banking system -	- Banking system	BSI cross-banking system exposure to financial institutions
System -	- Central Banks	Unit weights for banking systems to respective central bank; e.g. for EA systems unit weight on ECB
	-Countries	HICP official weights for the Taylor rule, for both GDP and inflation
Central banks -	- Banking system	not needed given the current model structure
	- Central Banks	not needed given the current model structure

Motivation (ctd)

- Does banks' degree of leveraging effect credit flows and economic activity,
- Is there a difference whether deleveraging is accomplished by reducing assets (credit supply) or raising capital (eventually allowing for investing in new assets)?
- How significant are cross-bank and cross-border effects of bank deleveraging shocks, in terms of loan supply and economic activity?
- Does the strength of the cross-bank and cross-border effects change depending on the leveraging regimes?
- Contribute to discussion around macroeconomic effects of tighter capital requirements (leverage ratio)

B. Empirics

B.3 The equation system

- See the difference between MCS structure and standard GVAR with variable-specific weights!
- Example: loan growth equation

→ Standard GVAR w/ v-specific weights: e.g. BIS weights on credit but trade weights on GDP (!)

→ MCS-GVAR: exposure of bank or banking system vis-à-vis countries, i.e. reflecting its activity there and hence susceptibility to macro

- For a bank it doesn't matter how much the country in which it's located trades with other countries; its x-border exposure matters
- Predictive performance tests confirm MCS makes more sense

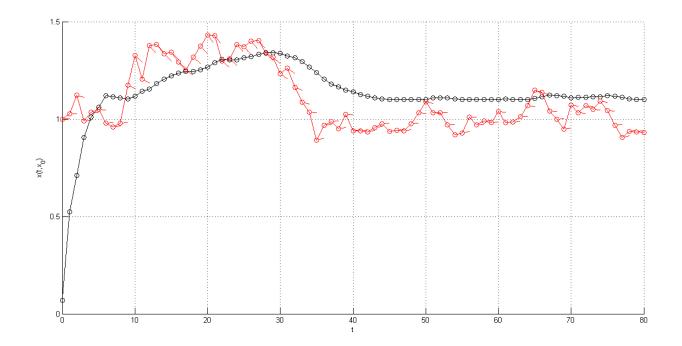
B. Empirics

B.3 T-MCS-GVAR – Solving the global model

- Solve model based on 2013Q4 weights
- Define regime constellation → all banking systems either simultaneously in overleveraging or underleveraging regime
- Other regime constellations possible

[see background slides for derivation of global solution]

A.1 Model structure and solution – Results wrt regime change

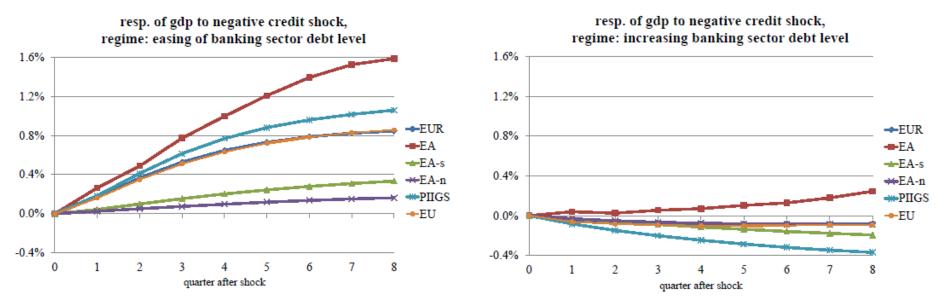


The fig. shows the path of net worth. Upper regime with i=0.02, lower regime with i=0.12. Downward jump in net worth.

B. Empirics

B.2 Regime change in the VSTAR model

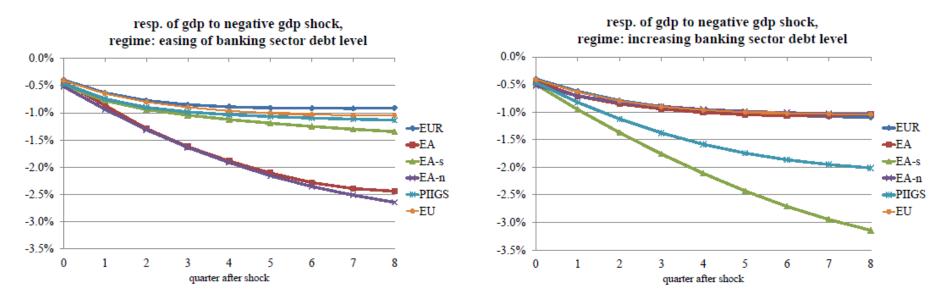
Figure: Cumulated response of GDP to credit shock



- After negative credit shock starting from low leverage regime: GDP moves up
- After negative credit shock starting from high leverage regime: GDP down, in particular in EA South
- In EU and EA, response of GDP after credit shock less in high leverage regime

B. Empirics B.2 Regime change in the VSTAR model

Figure: Cumulated response of GDP after GDP shock



- After shock to GDP: banks constrain their lending
- Indicating presence of adverse feedback loop
- South and crisis countries suffer more than North

Motivation – The deleveraging debate

- Credit cycles a common feature of financial systems that tend to positively correlate with the business cycle, reflecting fluctuations in borrowers' demand for external financing [Borio et al. 2001, Brunnermeier and Shin 2009]
- Cycles in credit developments and thereby implicitly in financial sector leverage (i.e. asset-to-equity ratios) are exacerbated by the inherent procyclical behaviour of financial intermediaries [Allen-Gale, 2004; Fostel-Geanakoplos, 2008; Brunnermeier-Petersen, 2009; Adrian-Shin, 2010]
- Deleveraging is not all bad. A necessary correction towards a more sustainable equilibrium, creating scope for new lending to finance more profitable business, supporting the recovery of economic activity [e.g. Scandinavian banking crises, see Laeven-Valencia, 2010]
- But deleveraging processes can be long and painful, especially in cases where they occur simultaneously with shocks to the financial sector [e.g. Japan in the 1990s and early 2000s and US+Europe since 2007, see Caballero-Hoshi-Kashyap, 2008; Greenlaw-Hatzius-Kashyap-Shin, 2008]

A. The semi-structural MCS-GVAR model A.6 Solving the global model – STEP 1/4

Step 1: Generate A-matrices. One starts by stacking the within-cross-section vectors along with the cross-cross-section weighted variable vectors in (here) three vectors m_{it}^x , m_{jt}^y , and m_{lt}^z .

$$\begin{split} m_{it}^{x} &= \left(\begin{array}{ccc} x_{it\prime} & x_{it}^{*,C-C\prime} & y_{it}^{*,C-B\prime} & z_{it}^{*,C-CB\prime} \end{array} \right)' \\ m_{jt}^{y} &= \left(\begin{array}{ccc} y_{jt\prime} & x_{jt}^{*,B-C\prime} & y_{jt}^{*,B-B\prime} & z_{jt}^{*,B-CB\prime} \end{array} \right)' \\ m_{lt}^{z} &= \left(\begin{array}{ccc} z_{lt\prime} & x_{lt}^{*,CB-C\prime} & y_{lt}^{*,CB-B\prime} & z_{lt}^{*,CB-CB\prime} \end{array} \right)' \end{split}$$

The equation system can be re-written with these m vectors as follows.

$$\underbrace{\begin{pmatrix} I_{k_{i}^{x}} & -\Lambda_{i,0,0} & -\Lambda_{i,1,0} & -\Lambda_{i,2,0} \end{pmatrix}}_{=A_{i0}^{x}} m_{it}^{x} = a_{i} + \underbrace{\begin{pmatrix} \Phi_{i1} & \Lambda_{i,1,1} & \Lambda_{i,2,1} \end{pmatrix}}_{=A_{i1}^{x}} m_{i,t-1}^{x} + \dots + \epsilon_{it}$$

$$\underbrace{(I_{g_{j}^{y}} & -\Xi_{j,0,0} & -\Xi_{j,1,0} & -\Xi_{j,2,0} \end{pmatrix}}_{=A_{j0}^{y}} m_{jt}^{y} = b_{j} + \underbrace{\begin{pmatrix} \Pi_{j1} & \Xi_{j,1,1} & \Xi_{j,2,1} \end{pmatrix}}_{=A_{j1}^{y}} m_{j,t-1}^{y} + \dots + \omega_{jt}$$

$$\underbrace{(I_{k_{l}^{z}} & -\Psi_{l,0,0} & -\Psi_{l,1,0} & -\Psi_{l,2,0} \end{pmatrix}}_{=A_{l0}^{z}} m_{lt}^{z} = c_{l} + \underbrace{\begin{pmatrix} \Gamma_{l1} & \Psi_{l,1,1} & \Psi_{l,2,1} \end{pmatrix}}_{=A_{l1}^{z}} m_{l,t-1}^{z} + \dots + \tau_{lt}$$

A. The semi-structural MCS-GVAR model A.6 Solving the global model – STEP 2/4

Step 2: Generate L-matrices ("link" matrices). With a global, stacked variable vector $s_t = (x'_{1t}, ..., x'_{Nt}, y'_{1t}, ..., y'_{Mt}, z'_{1t}, ..., z'_{Bt})$ at hand, the cross-section-specific variable vectors m_{it}^x , m_{jt}^y , and m_{lt}^z to s_t can be linked. The link matrices L_i^x , L_j^y , and L_l^z are used to map the local cross-section variables into the global vector, which involve the weights from the weight matrices W.

$$\begin{split} m_{it}^{x} &= L_{i}^{x} s_{t} & \to & A_{i0}^{x} L_{i}^{x} s_{t} = a_{i} + A_{i1}^{x} L_{i}^{x} s_{t-1} + ... + \epsilon_{it} \\ m_{jt}^{y} &= L_{j}^{y} s_{t} & \to & A_{j0}^{y} L_{j}^{y} s_{t} = b_{j} + A_{j1}^{y} L_{j}^{y} s_{t-1} + ... + \omega_{jt} \\ m_{lt}^{z} &= L_{l}^{z} s_{t} & \to & A_{l0}^{z} L_{l}^{z} s_{t} = c_{l} + A_{l1}^{z} L_{l}^{z} s_{t-1} + ... + \tau_{lt} \end{split}$$

A. The semi-structural MCS-GVAR model A.6 Solving the global model – STEP 3/4

Step 3: Generate G-matrices. The equation-by-equation system can now be stacked into a global system.

$$\begin{array}{l} G_{0}^{x} = \begin{pmatrix} A_{10}^{x} L_{1}^{x} \\ \dots \\ A_{N0}^{x} L_{N}^{x} \end{pmatrix}, G_{1}^{x} = \begin{pmatrix} A_{11}^{x} L_{1}^{x} \\ \dots \\ A_{N1}^{x} L_{N}^{x} \end{pmatrix}, \dots, a = \begin{pmatrix} a_{1} \\ \dots \\ a_{N} \end{pmatrix} \\ G_{0}^{y} = \begin{pmatrix} A_{10}^{y} L_{1}^{y} \\ \dots \\ A_{M0}^{y} L_{M}^{y} \end{pmatrix}, G_{1}^{y} = \begin{pmatrix} A_{11}^{y} L_{1}^{y} \\ \dots \\ A_{M1}^{y} L_{M}^{y} \end{pmatrix}, \dots, b = \begin{pmatrix} b_{1} \\ \dots \\ b_{M} \end{pmatrix} \\ G_{0}^{z} = \begin{pmatrix} A_{10}^{z} L_{1}^{z} \\ \dots \\ A_{B0}^{z} L_{B}^{z} \end{pmatrix}, G_{1}^{z} = \begin{pmatrix} A_{11}^{z} L_{1}^{z} \\ \dots \\ A_{B1}^{z} L_{B}^{z} \end{pmatrix}, \dots, c = \begin{pmatrix} c_{1} \\ \dots \\ c_{B} \end{pmatrix}$$

These cross-section-specific G matrices can be further combined to a set of global G matrices. The intercept vectors a, b, and c will be combined in a vector d. That is,

A. The semi-structural MCS-GVAR model A.6 Solving the global model – STEP 4/4

Step 4: Generate H-matrices. The global system can now be pre-multiplied by the inverse of G_0 . The system is now ready to be used for shock simulation and forecast purposes.

$$s_{t} = \underbrace{G_{0}^{-1}d}_{\equiv H_{0}} + \underbrace{G_{0}^{-1}G_{1}}_{\equiv H_{1}} \mathbf{s}_{t-1} + \dots + G_{0}^{-1}\varphi_{t}$$

Outline

Α	Theoretical model					
В	Empirics					
С	Conclusions / The way forward					

Outline

Α	Theo	retical model				
	A.1 Model structure and solution					
	A.2	Computation of optimal debt				
В	Empirics					
С	Conclusions / The way forward					

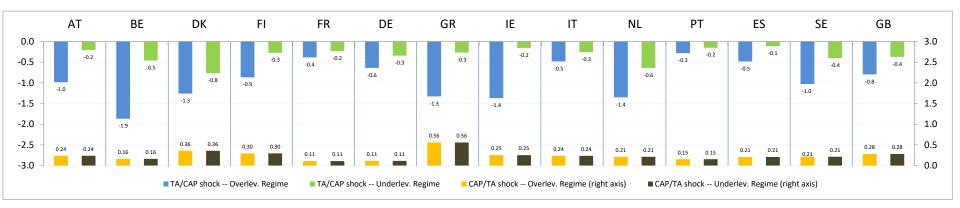
Outline

Α	Theo	Theoretical model						
В	Empi	Empirics						
	B.1	Computing optimal debt						
	B.2	Small-scale model: Vector STAR						
	B.3	Large-scale model: T-MCS-GVAR						
С	Conc	lusions / The way forward						

B. Empirics

B.3 T-MCS-GVAR – Shock size calibration

<u>Figure</u>: Capital ratio (CAP/TA) and leverage (TA/CAP) shocks under over- and under-leveraging regime



Capital ratio shocks are the same by assumption under both regimes (see bars at bottom of figure).