# Banking Dynamics and Capital Regulations

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### Preliminary. Please do not circulate or quote.

#### Abstract

This paper proposes a quantitative model of the banking sector to analyze potential aggregate impacts of the minimum capital requirements and counter-cyclical capital buffer in Basel III capital regulations. In the literature, an analysis on aggregate impacts of counter-cyclical capital buffer is limited. In order to fill this gap, the paper augments a standard banking model in two ways: (i) allows banks to default due to un-diversifiable risk based on the assumption of incomplete markets with respect to credit risk of bank loans and (ii) incorporates market-based funding with its equilibrium price reflecting individual-bank specific default premium. A numerical analysis of the model suggests that counter-cyclical capital buffer smoothes aggregate loan dynamics over time but its quantitative implication is limited during recessions. A larger quantitative impact can be obtained if the regulation allows the captail requirement to be lower also during recovery periods. Such state-contingent policies can raise bank default rates, posing a potential trade-off.

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## 1 Introduction

The recent financial crisis led to a creation of a new set of banking regulations, i.e., Basel III.<sup>1</sup> Although these regulations aim to improve global and domestic financial stability, their full and broad impacts on an economy are still being assessed. An increasing number of studies analyze bank capital requirements, one main part of Basel III, however multiple layers of capital requirements make the analysis complex. This paper contributes to the literature by adding an analysis of the effects of two such layers of the capital requirements: *minimum capital requirements* and *counter-cyclical capital buffer (CCyB)*. Minimum capital requirement defines the minimum level of bank capital relative to bank assets weighted by risk. Banks are required to hold capital above their minimum capital requirements under normal circumstances. In addition, counter-cyclical capital buffer requires banks to hold more capital during the boom and allows them to use/lower capital in recessions. It is designed so that banks build up capital buffers before a recession that stress banking activity and allows banks to drawn down their capital.<sup>2</sup> When banks violate these requirements, they are to rebuild it by reducing discretionary distribution of earnings, including dividend payments and staff bonus payments as well as potentially restricting the issuance of new loans.

An objective of these regulations is to reduce the likelihood of costly interventions by the government (including the use of deposit insurance) when negative shocks hit the banking sector. In addition, CCyB aims to limit socially valuable banking activities (e.g., bank loans) to decline during recessions when banks would need to reduce loans in the absence of CCyB that lowers the capital requirement in such periods. The paper measures the potential impacts and trade-offs of these regulations on the economy and ask how much bank lending, bank default and funding for banks change with and without these regulations.

To this end, we build a banking model that incorporates necessary features for the anal-

<sup>&</sup>lt;sup>1</sup>See Basel Committee on Banking Supervision (2011).

 $<sup>^{2}</sup>$ Besides counter-cyclical buffer, capital conservation buffer, (another layer of the capital requirements) also requires banks to maintain capital above the minimum threshold. Countercyclical buffer is designed to address a build-up of systemic risk in the economy when aggregate credit growth is "too high" whereas capital conservation buffer is designed for risk associated with individual banks.

ysis. Banks hold equity (i.e., bank capital) and deposits, and make risky long-term loans. In addition, banks raise risky funding with a premium reflecting their own default risk. In order to analyze default risk, banks in the model are heterogeneous. We assume that credit risk with respect to bank loans is idiosyncratic and markets are incomplete to diversify away this risk (e.g., no credit default swaps). Hence, individual bank's loan performance directly affects the evolution of its bank capital. In equilibrium, these assumptions lead to generate a distribution of banks with respect to their balance sheets. More specifically, banks in the model have technology to attract deposits and issue risky loans with cost. Banks pay dividends and accumulate retained earnings as bank equity. When in need for extra funding, banks can raise it through a competitive financial market. The price of this market-based funding incorporates individual bank's risk of default such that investors of this funding make zero profit. As a result, the price of market-based funding becomes specific to individual banks. Deposits are assumed to be insured by deposit insurance provided by the government and have a low interest rate.

Regarding the risk banks face, loans are risky due to their bank-specific idiosyncratic write-off shocks that reduce the quantity of loans. The impact of this risk is amplified by the maturity mismatch between long-term loans and short-term risky funding. Long-term loans are costly to liquidate and have a constant interest rate, whereas short-term risky funding is one period with changing interest rates over time as bank's individual default probability changes. This risk-taking by banks exhibits moral hazard as deposits are insured and banks operate under the limited liability such that the extent of the default risk that banks take is excessive from the perspective of the society. Minimum capital requirement alone could address this inefficiency. However, in the presence of aggregate shock, regulation that is contingent on the aggregate state (e.g., CCyB) could improve bank financial intermediation, especially, if loans are valued more during recessions. In order to analyze such regulation, we introduce an aggregate shock in the model. The aggregate shock changes the distribution of loan write-off shock such that in recessions loans are written-off relatively more than in booms.

A numerical example of the model is presented to help us understand the model dynamics and the impact of different regulations. The numerical analysis examines three types of policies regarding capital regulation: (i) the minimum capital requirement is non-state contingent and set at 10.5%; (ii) the minimum capital requirement is contingent on the aggregate state and declines during a recession (CCyB); (iii) the minimum capital requirement is contingent on the aggregate state and declines for extensive periods during and after a recession (CCyB with delayed tightening). Impulse-response analysis from a good aggregate state (i.e., boom) to a bad state (i.e., recession) and then again to the good state (i.e., recovery) shows that under state-contingent capital regulations the declines in loans and the use of market-based funding during the recession are smaller, implying smoother financial intermediation over the cycle in comparison to the non-state-contingent policy. Moreover, one margin of trade-off is observed in default probability. It is, on average, higher under state-contingent regulations than under non-state-contingent regulation. This is because lowering the capital requirement in the recession under state-contingent regulations allows banks to take more risk, leading more banks to default as a result.<sup>3</sup> However, our numerical results imply that the impcat of relaxed capital requirement on loans is limited during recessions. A more noticeable difference between the effects of non-state-contingent and state-contingent policies arises if the regulator maintains the lower capital requirement during recovery periods after the recession. This result has a policy implication that a state-contingent capital requirement should be tightened with a delay when an economy recovers from recessions.

Our paper builds on a growing body of the macro-banking literature (see, for example, Gertler and Kiyotaki (2010), Meh and Moran (2010), Repullo and Saurina (2011), Corbae and D'Erasmo (2012), He and Krishnamurthy (2012), Martinez-Miera and Suarez (2012), Martinez-Miera and Repullo (2017), Corbae and D'Erasmo (2013), Corbae, D'Erasmo, Galaasen,

 $<sup>^{3}</sup>$ Given a higher bank default rate, resulting potential liquidation cost, the use of deposit insurance and the time cost of rebuilding lost bank capital could amount to be higher than the benefits of CCyB. In this version of the paper, we do not address the question of net benefit of CCyB.

Irarrazabal, and Siemsen (2017), Brunnermeier and Sannikov (2014) and Bianchi and Bigio (2017)). One margin we contribute to the literature is the analysis of heterogenous banks. Analyzing banking regulations with bank default and their aggregate impacts would require banks that are endogenously different. For example, if regulations were to differ by bank size, banks would optimally take them into account in their decisions with implications for aggregate credit supply, banking sector risk, interest rates and so on.<sup>4</sup>

Along this line, there are two papers that are closely related to our study. De Nicolò, Gamba, and Lucchetta (2014) analyze how banks' decisions including lending and default change with capital requirement, liquidity requirement and Prompt Corrective Action (PCA) using a partial equilibrium model with heterogeneous banks. They find that PCA dominates the other regulations in terms of lending, bank's value and welfare through forcing banks to reaccumulate capital *ex post* to the required level when their capital ratios are lower than that. In addition, Mankart, Michaelides, and Pagratis (2016) study interactions between the leverage regulation and the capital requirement defined in terms of risk weighted assets in a partial equilibrium heterogeneous bank model. To analyze the substitution between safe assets and risky assets, their model allows banks to hold safe assets while borrowing in the wholesale market. They find that tighter capital requirement incentivizes banks to substitute risky assets with safe assets to reduce risk weighted assets rather than increase equity, whereas tighter leverage requirement increases lending and equity buffers. They do not analyze counter-cyclical capital buffer. In addition, our paper differ in that we incorporate risky market-based funding that reflects individual-bank risk premium in equilibrium.

The reminder of the paper is organized as follows. Section 2 briefly summarizes Basel III capital requirements. Section 3 describes the model. Section 4 discusses a numerical example of the model outcome. Section 5 concludes.

<sup>&</sup>lt;sup>4</sup>Corbae and D'Erasmo (2012, 2013) provide a model of heterogeneous banks, however they do not analyze counter-cyclical capital buffer with market-based funding.

Table 1:	Various	Capital	Requirements	
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	Requirement (% of risk-weighted assets)
Minimum Total Capital	8.0
Minimum CET 1 Capital	4.5
Capital Conservation Buffer	2.5
Counter Cyclical Capital Buffer	0-2.5
Additional Capital for Systemically Important Banks	1-3.5

## 2 Basel III Capital Requirements

The Basel Committee on Banking Supervision issued a document containing a framework of new post-crisis regulations, Basel III.<sup>5</sup> This section briefly discusses an overview of the capital requirements in Basel III.<sup>6</sup> Table 1 summarizes the four layers of capital requirements in Basel III.

The definition of "capital" used in the first row of the Table 1 is *Total Capital* which, for example, include common shares, retained earnings, preferred stocks, subordinated debt and loan loss provisions. The rest of the rows use *Common Equity Tier 1 (CET 1) Capital* which mainly consists of common shares and retained earnings.<sup>7</sup> Banks will need to maintain the minimum capital requirements at all times both in terms of total and CET 1 capital. Other requirements are added on top of the minimum requirements.

More specifically, capital conservation buffer of 2.5% is added on top of the minimum requirements of 8% or 4.5% depending on the definition of capital, *Total* or *CET 1*, respectively. As the name suggests, it is a buffer below which capital can fall in a period of stress while banks should maintain capital above it during normal times. When buffers have been

<sup>&</sup>lt;sup>5</sup>See Basel Committee on Banking Supervision (2011) for the details of the information presented in this section.

<sup>&</sup>lt;sup>6</sup>Basel III consists of two broad categories of regulations: one on capital framework and another on liquidity standard.

<sup>&</sup>lt;sup>7</sup>Numbers in the table are the required percentage of capital with respect to risk-weighted assets. A risk weight is assigned to each asset class by the Basel Committee and represents a degree of riskiness of the underlying asset. Hence, the higher it is the risk, the more capital is required. In our model, banks hold only one type of risky assets, hence, assuming the risk weight of one.

drawn down, banks should rebuild them by reducing discretionary distributions of earnings, such as dividend payouts and staff bonus payments.

The forth row of Table 1 lists the counter-cyclical capital buffer (CCyB) of 0 to 2.5%. This buffer aims to address the risks of system-wide stress that varies with the macro-financial environment. The requirement is turned on by national jurisdictions when aggregate credit growth is deemed excessive and turned off when financial risk materializes. When CCyB is on, banks need to increase up to 2.5% of additional capital.

Finally, additional requirements are placed as extra loss absorbency for banks that are systemically important.<sup>8</sup> Banks are deemed systemically important based on indicators under several categories: cross-jurisdictional activity, size, interconnectedness, substitutability/financial institution infrastructure and complexity. Systemically important banks are charged with additional 1 to 3.5% of capital, depending on the values of the indicators.

In the next section, we present a model that aims to incorporate features addressing regulatory concerns for banks. In this version of the paper, the analysis of the model focuses on the minimum capital requirements and the counter-cyclical capital buffer (CCyB), however, the model provides a framework in which other layers of capital regulations can also be analyzed.

## 3 Model

We build a dynamic heterogenous-bank model with bank default. Banks have technology to attract insured deposits at zero interest rates, can raise additional market-based funding with potential risk premium, and issue long-term risky loans at interest rates r with increasing loan-issuance costs. The risk premium for market-based funding reflects the default probability of the bank in equilibrium. Moral hazard problem with risk-taking arises due to the combination of the limited liability for banks and the existence of insured deposit whose

<sup>&</sup>lt;sup>8</sup>These are banks that are deemed to potentially cause system-wide adverse impacts in case of their default. There are both global and domestic systemically important banks. See Basel Committee on Banking Supervision (2013) for discussions of global systemically important banks and Office of Superintendant of Financial Institutions (2013) for domestic systemically important banks in Canada.

interest rate does not adjust to the default probability of banks. In addition to the moral hazard problem, regulation to make banks safer will be socially beneficial if liquidation and bankruptcy costs of a failed bank is large and/or if banks grow and become more efficient over time. Below, we describe the environment and problem for banks in detail.

#### **3.1** Environment

Bank technology is a set of exogenous parameters,  $\xi = \{\xi_d, \xi_{n,1}, \xi_{n,2}\}$ , which follow a Markov process with a transition function,  $\xi' \sim \Gamma(\xi)$ .  $\xi_d$  is the value of deposits and  $(\xi_{n,1}, \xi_{n,2})$ are the parameters of new loan issuance costs. In the beginning of a period, a bank has its liquid asset position, a, and its long term loans  $\ell$  that mature at rate  $\lambda$  per period. Banks issue new loans n, distribute dividends c and raise market-based funding b' at price  $q(\ell, n, b', \xi, z)$ . Banks face idiosyncratic write-off shocks on their existing loans at rate  $\delta$ . A bank default occurs when  $\delta$  is high enough that its equity diminishes, and either the bank is unable to pay back their liabilities or finds its outside option more attractive. As existing banks default, the same number of new banks enter and start their operations with small a and  $\ell$ . There is a persistent aggregate shock, z, following a Markov process,  $z' \sim \Gamma(z)$ . It changes the probability density function of  $\delta$ ,  $\pi(\delta|z)$ , capturing the changing overall riskiness of loans in the banking sector over time.

Banks's decisions to issue new loans and raise market-based funding are subject to bank capital regulation in the form of a minimum capital requirement. That is, the ratio of bank equity to risk-weighted assets needs to be above a certain level,  $\theta(z)$ , where we interpret the dependence of  $\theta$  on the aggregate shock z to capture the nature of dynamic capital requirements such as the counter-cyclical capital buffer. Banks do not necessarily default or enter the liquidation stage when violating the capital requirement. Instead, they become subject to further restrictions by the regulator such as lower dividend payout and/or no issuance of new loans.<sup>9</sup> When these additional restrictions do not prevent a bank to continue

<sup>&</sup>lt;sup>9</sup>See, for example, the Prompt Correction Action in the Federal Deposit Insurance Act in the USA at https://www.fdic.gov/regulations/laws/rules/1000-4000.html.

Table 2: Bank Balance Sheet

ASSET		LIABILITY & EQUITY	
Liquid Assets		Insured Deposit	$\xi_d$
Long-Term Loan $n + \ell$		Market-Based Funding	qb'
		Equity	e

losing its capital, the bank will be resolved and liquidated. This happens at some threshold  $\underline{\theta}$  such that  $\underline{\theta} < \theta(z)$ .

## **3.2** Bank problem

The timing of events involving bank's decisions are as follows:

- In the beginning of a period, a bank observes the realizations of shocks, ξ, z and δ, as well as its existing long-term loans (ℓ) and liquid asset position (a) as a result of the shocks.
- 2. The bank decides whether to continue their operation or default.
- 3. If no default, it chooses c, n and b' subject to the capital regulation,  $\theta(z)$ .
- 4. If the regulation is successfully met, the bank operates according to its choices. If not, the regulator forces the bank to set c = 0 and n = 0 in the spirit of PCA.
- 5. At the end of the period,  $\xi'$ , z' and  $\delta'$  realize.

Accordingly, the balance sheet position of the bank after its choices on c, n and b' is given by Table 2. Note that q is a discount price of b' and a function of the bank decisions and state as it will be discussed below. Furthermore, s is bank's holding liquid assets at the risk-free rate of return.<sup>10</sup> Then, the risk weighted capital ratio of the bank is given by  $\frac{e}{\omega_s s + \omega_r (n+\ell)}$ , where  $\omega_s$  and  $\omega_r$  are the relative risk weight for safe (s) and risky assets  $(n+\ell)$ . The balance sheet gives the identity that  $e \equiv s + n + \ell - \xi_d - qb'$ .

<sup>&</sup>lt;sup>10</sup>In equilibrium, s = -qb with b < 0 as liquid assets are held only by banks that do not need market-based funding given its low rate of return.

Let us denote  $V(a, \ell, \xi, z)$  and  $W(a, \ell, \xi, z)$  to be the value function of the bank before the default decision and that of after it, respectively. We then have

$$V(a,\ell,\xi,z) = \max\left\{\varphi, W(a,\ell,\xi,z)\right\},\tag{1}$$

where  $\varphi$  is the outside option of the bank if defaulted. The continuation value of the bank that does not default further break down into three sub-values depending on the feasible choice set to satisfy different levels of the capital ratio:

$$W(a,\ell,\xi,z) = \begin{cases} W^N(a,\ell,\xi,z) & \text{if } \frac{e}{\omega_s s + \omega_r(n+\ell)} \ge \theta \text{ is feasible,} \\ \\ W^P(a,\ell,\xi,z) & \text{if } \frac{e}{\omega_s s + \omega_r(n+\ell)} \ge \theta \text{ is not feasible and} \\ \\ \\ \frac{e}{\omega_s s + \omega_r(n+\ell)} \in [\underline{\theta},\theta) \text{ is feasible, or} \\ \\ -\infty & \text{otherwise.} \end{cases}$$

 $W^N$  represents the value of normally-operating banks with feasible choices that satisfy the capital requirement and  $W^P$  the value of banks with positive equity but its capital ratio is in the penalty zone with no feasible choices that satisfy the minimum capital requirement. We define  $W^N$  as follows.<sup>11</sup>

$$W^{N}(a,\ell,\xi,z) = \max_{n \ge 0, c \ge 0, b'} u(c) + \beta \sum_{z',\xi',\delta'} \Gamma_{z,z'} \Gamma_{\xi,\xi'} \pi(\delta'|z') V\left(a'(\delta'), \ell'(\delta'), \xi', z'\right)$$
(2)

subject to

$$\ell' = (1 - \lambda) (1 - \delta') \ell + (1 - \overline{\delta})n \tag{3}$$

$$a' = (\lambda + r)(1 - \delta')\ell + r (1 - \overline{\delta})n - \xi_d - b'$$
(4)

$$c + n + \chi(n, \xi_{n,1}, \xi_{n,2}) \le a + q(\ell, n, b', \xi, z)b' + \xi_d$$
(5)

$$\frac{e}{\omega_s s + \omega_r (n+\ell)} \ge \theta(z) \tag{6}$$

<sup>&</sup>lt;sup>11</sup>Given that s = -qb when b < 0 in equilibrium, we pre-impose this condition and abstract from the decision on s.

Equations 3 and 4 are the law of motion for the long-term loans and the liquid asset position, respectively. Equation 5 is the budget constraint and 6 the regulatory capital requirement. We assume that the preference for dividend displays diminishing marginal returns. The Markov processes,  $\Gamma_{z,z'}$  and  $\Gamma_{\xi,\xi'}$ , are written in discrete forms. New loans come with a small constant write-off probability,  $\overline{\delta}$ , to allow for the distinction that the performance of new loans are better monitored by banks than of older loans.  $\chi(n, \xi_{n,1}, \xi_{n,2})$  is a convex issuance cost function for new loans.

The discount price of market-based funding,  $q(\ell, n, b', \xi, z)$ , is an equilibrium object that incorporates the bank-specific default risk premium. Under the assumption that the marketbased funding is competitively priced, investors providing this funding to the bank break even in equilibrium such that

$$q(\ell, n, b', \xi, z) = \frac{1 - \Pr(\delta > \underline{\delta}(\ell, n, b', \xi', z'), \xi', z'|\xi, z)}{1 + r_f},$$

where  $r_f$  is the risk-free rate and  $\underline{\delta}$  is such that, when  $\delta = \underline{\delta}(\ell, n, b', \xi', z')$ , the two terms inside the max operator in Problem 1 are equal to each other. Note that  $q(\ell, n, b', \xi, z)$  becomes the discount price based on the risk-free rate,  $r_f$ , when the default probability of the bank is zero.

Moreover, the value function of the bank in penalty,  $W^P$ , is given as follows.

$$W^{P}(a,\ell,\xi,z) = \max_{b'} u(0) + \beta \sum_{z',\xi',\delta'} \Gamma_{z,z'} \Gamma_{\xi,\xi'} \pi(\delta'|z') V\left(a'(\delta'),\ell'(\delta'),\xi',z'\right)$$
(7)

subject to

$$\ell' = (1 - \lambda) (1 - \delta') \ell \tag{8}$$

$$a' = (\lambda + r)(1 - \delta')\ell - \xi_d - b'$$
(9)

$$0 \le a + q(\ell, 0, b', \xi; z)b' + \xi_d \tag{10}$$

$$\frac{e}{\omega_s s + \omega_r (n+\ell)} \ge \underline{\theta} \tag{11}$$

Note that banks are restricted to set c = 0 and n = 0 in Problem 7 as penalty in violation of the minimum capital requirement.

## 3.3 Equilibrium price

The only relevant equilibrium price is the zero profit condition among investors of marketbased funding for banks. This condition leads to  $\underline{\delta}^*(\ell, n, b', \xi', z')$  and  $q^*(\ell, n, b', \xi, z)$  such that the discount price of the funding just reflects the default probability of banks in the equilibrium discounted by the risk-free rate:

$$q^*(\ell, n, b', \xi, z) = \frac{1 - \Pr(\delta > \underline{\delta}^*(\ell, n, b', \xi', z'), \xi', z'|\xi, z)}{1 + r_f}$$

## 4 Numerical Example

This section presents quantitative results as a numerical example. Specifically, we derive the impulse-response functions with respect to an adverse aggregate shock, which increases both the mean and the variance of loan write-offs and bank default as a result, under three different capital requirements. For simplicity, we assume that the aggregate shock has two states: "G" and "B", corresponding to low and high average loan write-off states, respectively. Table 3 presents the three policies for the capital requirements. The first policy in the table is non-contingent on the aggregate state, the second is the counter-cyclical capital buffer that lowers the capital requirement in the bad state, and the third one represents a policy that maintains the lower capital requirement even afer the aggregate state returns to the good state.

Table 4 summarizes the functional forms and parameter values used for the analysis. We discretize the values of z into two and assume that  $\xi_{n,1}$ ,  $\xi_{n,2}$  and  $\xi_d$  are constant. In addition, we assume that  $\delta'$  follows the Beta distribution whose parameters in each aggregate state imply the average and the variance of loan write-offs given in Table 5. Given these

Always 10.5%	$\theta(z) = 0.105 \; \forall \; z$
CCyB	$\theta(z=G) = 0.105$ and $\theta(z=B) = 0.08$
CCyB + 8% during recovery	same as CCyB except $\theta(z) = 0.08$ during recovery

Table 3: Three Poli	cies for the	e Capital Re	quirement
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Parameter	Value	Description
$(\xi_{n,1},\xi_{n,2})$	(0.075, 0.15)	$\chi(n,\xi_{n,1},\xi_{n,2}) = \xi_n^1 n + 0.5 \xi_n^2 n^2$
$\xi_d$	5	Deposits
$\beta$	0.95	Subjective discount factor
$\lambda$	0.2	Maturity rate of long-term loans
r	0.1	Bank lending rate
$r_f$	0.005	Risk-free rate
σ	0.9	$u(c) = c^{\sigma}$
$\omega_r$	1	Risk weight on risky loans
$\omega_s$	0	Risk weight on safe assets
$\Gamma_{z=G,z'=G}$	0.99	$\Pr(z' = G   z = G)$
$\Gamma_{z=B,z'=B}$	0.8	$\Pr(z' = B   z = B)$
$(\alpha_{\delta'}, \beta_{\delta'})_{z=G}$	(0.38,  15)	Loan write-off process in $z=G$
$(\alpha_{\delta'}, \beta_{\delta'})_{z=B}$	(0.34, 8.2)	Loan write-off process in $z=B$
arphi	0	Outside option
$\underline{\theta}$	0.02	Default threshold under PCA

 Table 4: Model Parameters

parameter values, we numerically solve the model and simulate the economy under each of the three policies for the capital requirements. For each policy, the common starting point of its impulse-response analysis is given by an equilibrium after z = G for a long period of time under the "Always 10.5%" regulation such that the distribution of banks converges. Starting from such an initial state, the simulation of the aggregate shock follows the path such that  $z_{t=1} = G$  (initial state),  $z_{t=2} = z_{t=3} = B$  (recession) and  $z_{t>3} = G$  (recovery). We formally define the common initial state as follows:

**Definition 1.** The common initial state is given by the capital requirement,  $\theta(z)$ , the realization of aggregate shocks,  $z_t = G \quad \forall t$ , a measure of banks,  $\Omega^*(a, \ell, \xi)$ , a price of market-based funding,  $q^*(\ell, n, b', \xi, z)$ , and decisions,  $\{c^*(a, \ell, \xi, z), n^*(a, \ell, \xi, z), b'^*(a, \ell, \xi, z)\}$ , such that

Table 5: Loan write-off processes

State	Average	Variance
z = G	0.025	0.0015
z = B	0.04	0.004

- $\{c^*(a, \ell, \xi, z), n^*(a, \ell, \xi, z), b'^*(a, \ell, \xi, z)\}$  solve Problems 1, 2 and 7;
- investors get the market return,  $q^*(\ell, n, b', \xi, z)$ ; and
- the measure,  $\Omega^*(a, \ell, \xi)$ , converges to a limit.

Figure 1 shows the value function and decision rules associated with the common initial state. In the defaulting region, the value is equal to the outside option. New loans and dividend are non-decreasing functions of cash in hand and existing loans, and both are constrained to be zero in the penalty region. New loans are valuable for all banks since the loan failure rates are lower than those of existing loans. However, the issuance of new loans requires convex cost. Small banks smooth out these costs over time by investing a part of fundings raised through deposits in safe assets rather than distributing them as dividends. This appears as negative market-based funding in Panel 1c. On the other hand, larger banks need to raise fundings from the market to maintain or expand loan volumes above the level financed by deposits. For these reasons, the market-based funding increases in existing loans and decreases in cash in hand.

Table 6 provides average observations of bank variables in the common initial state. All numbers, except for those shown with %, are normalized by the value of equity. Table 7 compares some key statistics in the Canadian data and in the common initial state. The table shows that our model captures these important features of the Canadian banking industry reasonably well.

Figure 2 displays the distribution in the common initial state (blue) and that after the adverse aggregate shock (orange). As the realization of write-off shocks worsens in the sim-

Default Probability	0.26%
New Loans	1.25
Existing Loans	5.63
Market-Based Funding	1.50
Dividend	0.30
Equity	1.00
Average Capital Ratio	14.40%
Fraction of Banks in Penalty	1.063%

Table 6: Banking Sector in the Common Initial State

Table 7: Model and Data Moments

	Canadian Data%	Model (Always 10.5%)
Default Probability	0.22%	0.26%
Market-Based Funding/Total Assets	27%	22%
Average Capital Ratio	14.40%	14.38%

ulation, banks holdings of cash-in-hand and loans both decline. As a result, the measure of banks with fewer cash-in-hand and loans increases.

Figure 3 displays a set of resulting impulse-response functions. Panels 3a and 3b show that the decline in new loan issuance and the associated decline in total loans outstanding are smaller if the regulation lowers the capital requirement during the crisis. This is due to the fact that the counter-cyclical policies mitigate the decline in market-based funding by allowing banks to operate with lower capital ratios. In addition, counter-cyclical policies allow some banks that would otherwise be penalized by PCA to operate as normal banks, which in turn leads to more provision of loans. However, the quantitative implication on loans of a 2.5 percentage point decline of capital requirement during the crisis is limited. A noticeable difference between the non-contingent and contingent policies arises only when the capital requirement continues to remain lower than 10.5% after the crisis. Such an extension of lower capital requirement allows banks to attract more market-based funding while the outlook of the economy improves after the crisis. Finally, bank default probability in Panel 3f gives additional insight into bank behaviour under different policies. Compared to "Always 10.5% policy, CCyB and "CCyB + 8% during recovery", on average, lead to higher default probabilities. This occurs as the counter-cyclical policies allow more risk-taking by banks through lower capital requirement. Our model, thus, generates a trade-off associated with relaxing the capital requirement in response to the crisis.

## 5 Conclusions

This paper proposes a structural model framework for banking sector dynamics that can analyze Basel III capital regulations, including minimum requirements and counter-cyclical capital buffer. The model incorporate bank default based on the credit risk on the loan book. The risk is amplified due to maturity mismatch with respect to long-term loans and short-term funding where the short-term market-based funding is priced for individual-bank default risk. There is moral hazard in banks' risk-taking and default due to the use of insured deposit as well as the limited liability framework.

Results from a numerical example of the model under different policies for the capital requirement are presented and discussed. The model captures reasonably well the average balance sheet of banks and bank default rates in Canada. The impulse-response analysis in this paper provides a rationale for delaying the tightening of capital requirement when the recession ends. It is also shown that, under CCyB, there exist a trade-off between loan provisions over the cycle and bank default probability. Further analysis is necessary to measure the net effects of this trade-off. In addition, future work would benefit from making loan and deposit rates endogenous such that general equilibrium impacts from these policies can be assessed.



Figure 1: Value Function and Decision Rules of Banks in the Common Initial State



Figure 2: Comparison of Distributions Before and After the Shock



Figure 3: Impulse-Response Function of the Banking Industry under Three Policies

(a) New Loans

(b) Total Loans

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