Investing Like China

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Abstract

There are several puzzling facts that we document about the Chinese economy in this paper: i) the private lending rate is very high, while the average return to capital is much lower; ii) the aggregate investment rate in China is almost 50%; iii) the wage premium between skilled and unskilled workers are declining during recent years. This paper attempts to explain the above unique features of the capital and labor markets in the Chinese economy using a two-sector model. Our model shows that these puzzling facts that exist in Chinese capital and labor market are likely to be caused by the distortion in the government investment, which is modeled as government's subsidizing the infrastructure sector by offering a lower loan rate, and related structural change. Given that the infrastructure sector can absorb capital with low rate, large amount of investment flows into this sector, which also drives up the demand for unskilled labor. The rising demand for unskilled labor leads to an increase of unskilled wage. On the other hand, since infrastructure sector can borrow at a low rate, it has the incentive to borrow a lot, thus pushing up the market interest rate for the general good sector. The resulting reduction of investment in the general good sector reduces the demand for skilled labor and hence the growth of skilled labor wage, leading to a fall in the wage premium. The model is calibrated to match the Chinese data. Our results suggest that the government has strong incentive to invest in the infrastructure sector if they are only interested in the short-run output level. This also helps to explain the high local government debt during recent years, as the local government has strong incentive to invest in the infrastructure sector, they borrow more. We also find that the more government deviates from the non-distortionary rate, the larger the welfare loss. More interestingly, when there exists abundant unskilled labor due to frictions to migration from the rural area, distortionary government investment could in fact increase the marginal product of labor and help overcome the frictions, and hence increase welfare. However, once rural to urban migration is completed, there is only welfare loss associated with distortionary government investment.

JEL Codes: E25, O16, O41, O53, P23.

Keywords: Credit Market Imperfections; Economic Growth; Transition; Migration; Abundant Unskilled Labor Supply; Wage Premium.

1 Introduction

There are several puzzling facts we document for Chinese economy in this paper: i) the private lending rate is very high, while the average return to capital is much lower; ii) the aggregate investment rate in China is almost as high as 50%, even though the private lending rate is high and marginal return to capital is lower; iii) it seems that demand for skilled labor (e.g., college students) is falling over these year, while demand for unskilled worker is rising. As the result, the wage premium between skilled and unskilled workers are declining in recent years, driven by a faster rise in the wage of unskilled worker and a slower rise in the wage of skilled worker, such as the college graduates. This paper attempt to explain the above unique features of capital and labor markets in the Chinese economy using a two-sector model.

From 2000 to 2012, the investment rate has increased from 35% to 48%. This happened when the rate of return to capital (net of taxes on production and enterprise income) dropped from 10.5% to 6.6%. In 2012, the lending rates are higher than the rate of return. The quarterly average bank lending rates for one year loan were 7.97%, 7.55%, 7.18%, and 7.07%. Because it is very difficult for enterprises to get loans from the bank, they are often forced to bear other costs such as service fees hence the effective lending rates are higher than the reported rates. Furthermore, many enterprises are shut out of bank loans. They have to borrow from the shadow banking sector, paying much higher rates.

On the labor market, there is a significant shrinking of the skill premium in wages. In 2012, the average wage for migrant workers, who are generally unskilled, increased by 14%, but that for urban residents, who are generally better skilled, only increased by 12.5%; the respectively growth rates for the first three quarters of 2013 were 12.6% and 8.8%. Between 2008 and 2012, the average wage in the construction sector, where the average school years of the labor force is about 9, went up by 72%, while the average wage in the IT sector, where the average school years of the labor force is about 13, grew by only 47%. The relative wage of the IT sector with respect to the construction sector has been steadily declining since 2005. More generally, if we look across one-digit sectors, there is significant negative relationship between the growth of average wage and the average school years of the labor force.

(...discuss relevant literature here...)

Our model shows that these puzzling facts that exist in Chinese capital and labor market are caused by the distortion in the government investment and related structural change. In particular, we assume that the economy has two production sectors, an infrastructure sector and a general good sector. Goods from the two sectors are combined into final good for consumption and investment. The infrastructure sector uses unskilled labor and capital for production while the general good sector uses skilled labor, unskilled labor and capital. There is a banking sector which takes deposits from the household, lends part of it to the infrastructure sector with government subsidized lower rate and lends the rest to the general goods sector with the market rate. Bank makes zero profit.

Given that the infrastructure sector can absorb capital with a lower rate, they invest and at the same time hire. The rising demand for the unskilled labor drives up the unskilled wage. On the other hand, the expanding infrastructure sector crowds out the demand from the other sector, hence demand for skilled labor also declines and the skilled labor wage falls, leading to a fall in the wage premium. Since infrastructure sector can borrow at a low rate, it has the incentive to borrow a lot, thus driving up the market rate for the general good sector.

The model is calibrated to match the Chinese data and simulation results show that if the government starts with a non-distortionary interest rate and gradually increases the interest rate subsidies, the model predicts that the wage premium falls, market interest rate rises, infrastructure sector expands and general good sector shrinks, which are all in line with data observations. More interestingly, aggregate output increases first but falls soon after the boom while consumption falls from the beginning, which suggests that if they are only interested in the short-run output level, the government has strong incentive to invest in the infrastructure sector. We also find that the more the government deviates from the non-distortionary rate, the larger the welfare loss.

We also consider an extension of the model with rural migration. When there are abundant unskilled labor, for example immigration from the rural area, distortionary government investment could in fact increase the marginal product of labor, and hence increase the welfare. However, once immigration is finished, the previous analysis holds and welfare loss is incurred.

2 Data Evidence

China has maintained close to 10% annual growth rate over the last thirty years, which still remains a miracle to many researchers as well as policy makers. In this section, we present a set of empirical facts during China's rapid growth period. We document the dynamics of multi-sector growth pattern, the wage premium, private loan rates and marginal return to capital, aggregate investment, and migration of unskilled worker to urban area. The aim is to provide a set of stylized facts that capture the unique features of chinese economy. These facts will be the basis of the theoretical discussion in the subsequent sections of the paper.

2.1 Unskilled-labor intensity and growth pattern across sectors

One striking feature of the Chinese economy is that infrastructure investment has been growing very rapidly. Since construction sector is closely associated with infrastructure investment, Figure 1 illustrates the changes of the share of construction sector in the economy. We can see that the share has been increasing, especially in recent years. Note that the construction sector is unskilled labor intensive with a average school years of the labor force to be 8.95 years which is the lowest among all one-digit sectors except for agriculture.

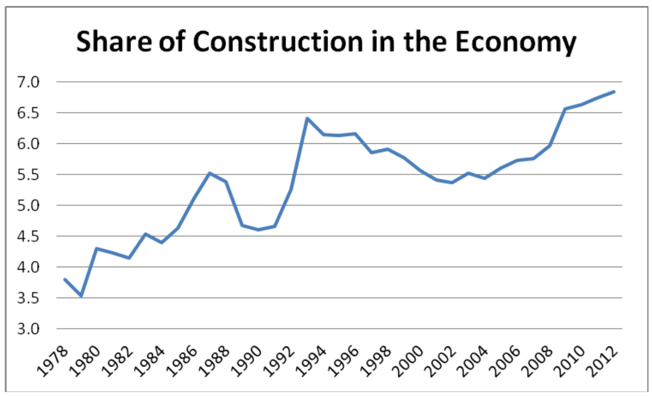


Figure 1; Source: China Statistical Yearbook 2013

On the other hand, the share of skilled labor intensive sector, such as IT sector has been declining over years as shown in Figure 12.

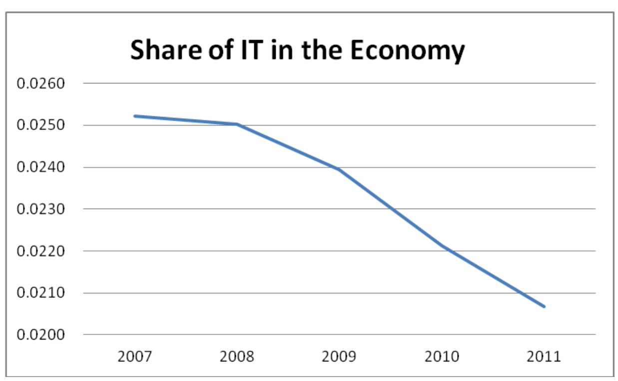
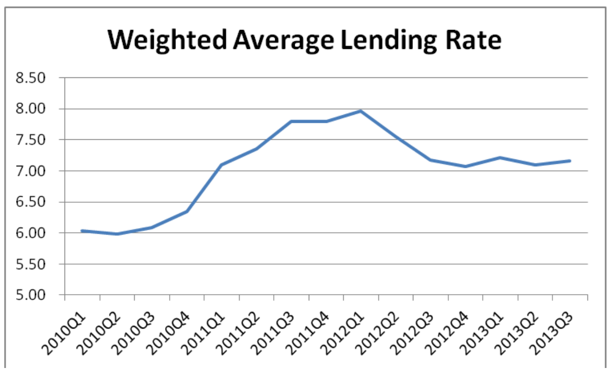


Figure 12; Source: Authors' calculation from 2013 China Statistical Yearbook.

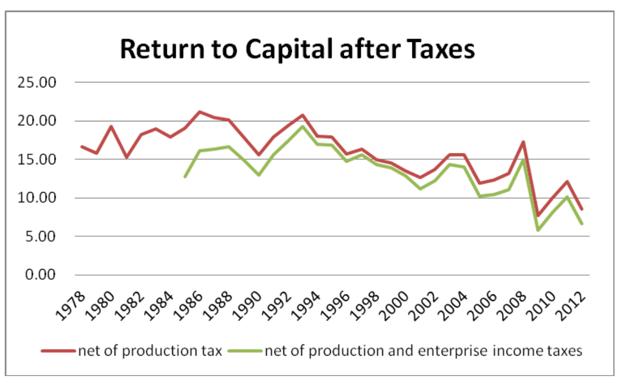
2.2 Private loan rates, deposit rates, and marginal return to capital

In China, the deposit rates are under strict regulation. Although the loan rates are market-determined, most of the major state-owned firms can obtain the loan at a rate that is much lower than the market rates. That is, the loan rates to the state-owned firms are subsidized by the government. Figures 2 presents the average one-year bank loan interest rate published by the People's Bank of China. Since it's hard for the private firms to get loans, especially small and medium firms, they face a higher effective loan rates. The premium always take the form of other costs such as service fees. Moreover, many of these enterprises are forced to get financing from the shadow banking sector, where the loan rates are substantially higher than those in the formal banking sector.

A puzzling phenomenon is that although the private lending rate is high, the average return to capital is low. Figure 3 illustrates the rate of return to capital estimated from the national accounts data (Bai, Hsieh and Qian, 2006 and its update). For the most recent years, the aggregate rates of return to capital are lower than the market loan rates many enterprises have to pay.



Figures 2; Source: China Monetary Policy Report, PBOC, various issues.



Figures 3; Source: Bai, Hsieh, and Qian (2006) and its update.

2.3 High and rising aggregate investment rate

Even though the marginal (average) return to capital is very low, the aggregate investment rate in China keeps increasing during recent years which is incompatible with the standard Neo-classical growth theory. We observe from Figure 4 that the investment rate now reaches a level as high as over 50%.

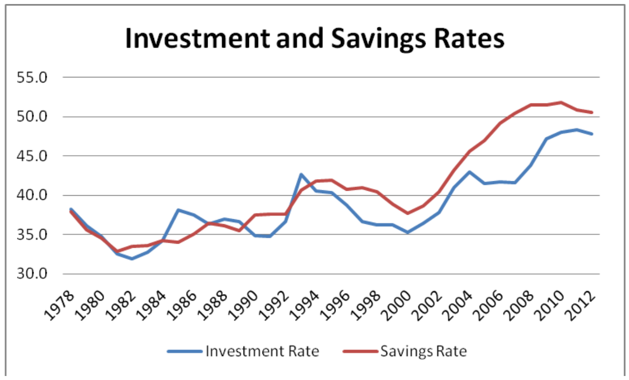


Figure 4; Source: China Statistical Yearbook, 2013.

2.4 Large migration of unskilled labor from rural area to urban industry

During the chinese transitional period, we observe a large amount of reallocation of labor force across sectors. A remarkable fact is the large scale migration of unskilled labor from rural area to urban industry. The migration, among which are unskilled labor, occurred at 1978 with the amount of 2 million, and has surged up to 145.33 million in 2009. By the third quarter of 2013, migrant workers numbered 173.92 million. The mobilization of labor from rural area provides excess supply of unskilled labor to urban industry, which contribute to the persistent growth of Chinese economy.

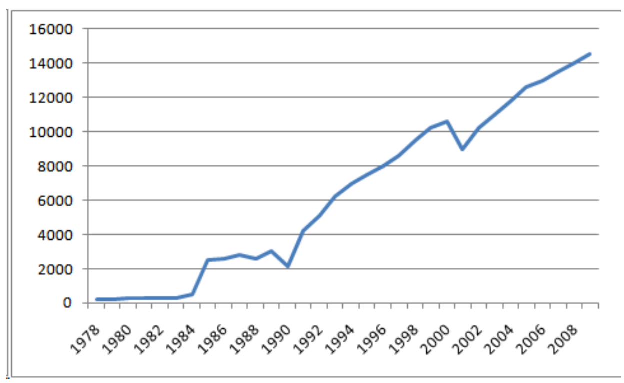


Figure 5; Source: China Statistical Yearbook, 2013.

2.5 Skill premium in the wage

The economic transition in China has been accompanied by increasing income inequality, especially the wage inequality between skilled labor and unskilled labor. Using the Chinese Urban Household Survey Data (1988-2009), we compute the average real wage for skilled workers and unskilled workers, as illustrated in Figure 6. We can observe a clear difference in the real wage between skilled workers and unskilled workers, and the average real wages of the two groups even diverge in recent years. Figure 7 presents the relationship between the average wage of a sector and the skill intensity of the sector. The skill intensity is defined as the average school years of its labor force. From Figure 7, we can also see significant skill premium in the wages.

Interestingly, the wage premium seems to be declining in the most recent years. In Figure 8, we show that wages for migrant workers grow faster than that of urban residents. Figure 9 shows that between 2008 and 2012, the average wages tend to grow faster in sectors with lower skill intensity. Figure 10 illustrates the relative wage of the IT sector with high skill intensity to the construction sector with low skill intensity. We observe a declining of wage premium between these two sectors since 2003. All these charts suggest that wage premium has been declining in recent years.

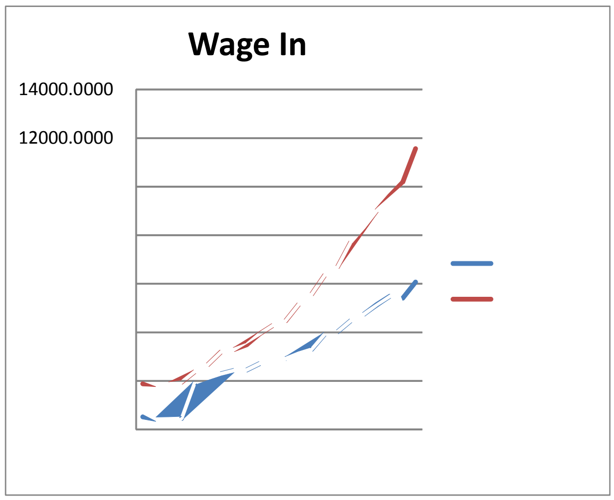


Figure 6; Source: China Statistical Yearbook, 2013.

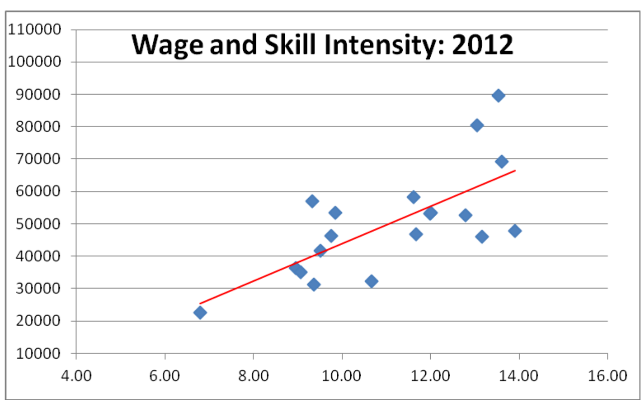


Figure 7; Source: Authors' calculation from 2013 China Statistical Yearbook and 2005 population survey.

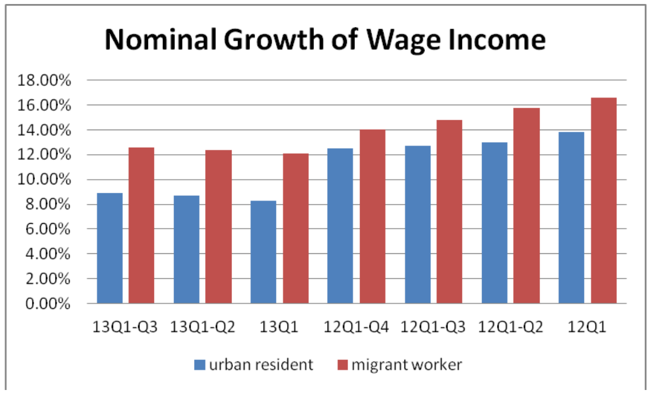


Figure 8; Source:Authors' calculation using data from the quarterly economic analysis by the National Statistical Bureau.

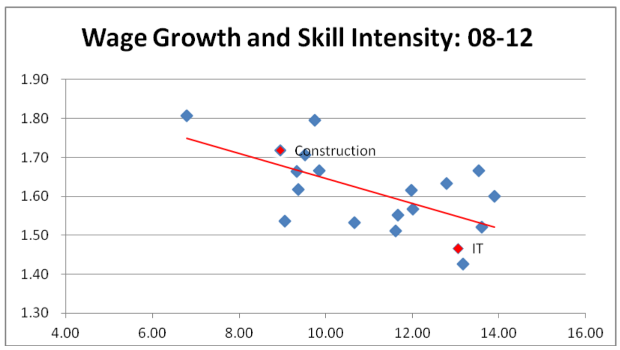


Figure 9; Source: Authors' calculation from 2013 China Statistical Yearbook and 2005 population survey.

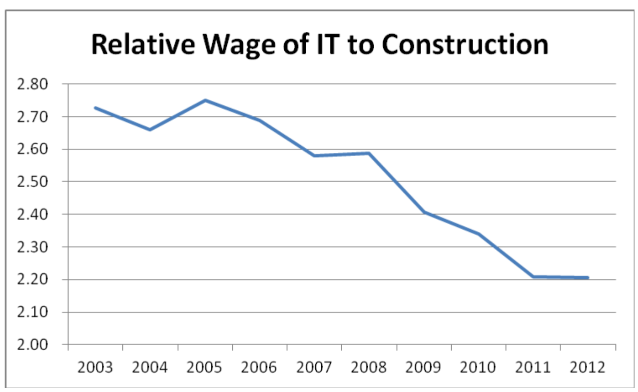


Figure 10; Source: China Statistical Yearbook, 2013.

2.6 Skill Intensity and Return to Capital

There is a positive relation between the skill intensity of a sector and its averge return to capital. As shown in Figure 11, the lower the skill intensity, the lower return to asset. The intuition behind this fact is that given most of the infrastructure projects are backed by the government, they receive government funding at very low cost and also get cheap bank loans guaranteed by the government. Because they face low cost of capital, they will invest so much that the marginal return to capital is low. We assume that the average return to capital is positively correlated with the marginal return to capital, then infrastructure projects should generally yield low return to assets. If we look at sector specific return, the construction sector yields only about one third of the average ROA in the IT sector.

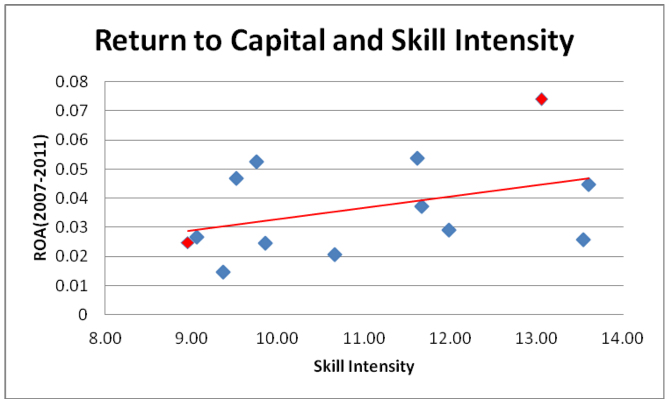


Figure 11; Source: Authors' calculation.

3 Benchmark Model

3.1 Model Setup

In this section, we outline a two-sector neo-classical growth model. The economy consists of four sectors: a household sector, a production sector, a banking sector and a government sector. There are three types of firms in the production sector: two intermediate firms which produce infrastructure good and general good, and a final good producer. Infrastructure good producer uses capital and unskilled labor in the production while general good producer uses capital, skilled labor and unskilled labor. The household sector is populated with one infinitely lived representative agent who saves and makes deposit to the bank. The bank takes household's deposit and lends it to the infrastructure good producer and the general good sector. However, there is distortion in the banking sector: the two firms face different loan rates. The general good producer borrows at the market rate while the infrastructure firm borrows at a government regulated rate, which is much lower than the market rate. The following subsections detail the choice faced by the agents, the structure of production and banking, and the relevant market clearing conditions.

3.1.1 Household Sector

There is one infinitely lived representative household in this economy who has a standard CRRA preference

$$U\left(c_t\right)=\frac{c_t^{1-\gamma}}{1-\gamma}+\eta.$$

The household chooses consumption c_t, provides skilled labor s_t and unskilled labor l_t, makes a saving decision a_{t+1} to maximize his life-time utility

$$\max_{c_t, l_t, s_t, a_{t+1}} \sum_{t=0}^{\infty} \beta^t U(c_t)$$

$$\text{s.t. } c_t + a_{t+1} = w_{Lt} l_t + w_{St} s_t + (1 + r_{dt}) a_t$$

In each period, the agent receives wage income by working at both infrastructure good sector and general good sector. He also receives interest r_{dt} from his bank deposit. He uses his after-tax income to consume c_t and to save a_{t+1}. Given prices $\{w_{Lt}, w_{St}, r_{dt}\}$, the following first-order conditions describe household's optimal choice:

$$\frac{U'(c_t)}{U'(c_{t+1})} = \beta(1 + r_{dt+1}); \tag{3.1}$$

$$l_t = L$$

$$s_t = S$$

3.1.2 Production Sector

There are three types of firms in this economy: an infrastructure good producer, a general good producer and a final good producer. Two intermediate goods are produced by the infrastructure sector and the general sector produce separately. The intermediate goods are then used as the inputs for final good production.

Intermediate Good Producer Since all the intermediate good production exhibits constant return to scale, we assume that there is one competitive firm operating in each sector. The firms live for one period.

Infrastructure Good Producer The infrastructure sector operates under the Cobb-Douglas production technology. The firm uses unskilled labor L_{It} and capital K_{It} to produce infrastructure good Y_{It}. Unskilled labor is rented from the household while capital is rented from the bank at a government-regulated rate r_{St}. The production is subject to a sectoral technology shock z_{It}.

$$Y_{It} = e^{z_{It}} A_I (K_{It})^{1-\alpha_I} (L_{It})^{\alpha_I};$$

Given factor prices $\{w_{Lt}, r_{St}, p_{It}\}$, the infrastructure firm solves the following problem

$$\max \{p_{It}Y_{It} - w_{Lt}L_{It} - r_{St}K_{It}\}; \tag{3.2}$$
$$w_{Lt} = \alpha_I p_{It} e^{z_{It}} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{1-\alpha_I};$$
$$r_{St} = (1-\alpha_I)\, p_{It} e^{z_{It}} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{-\alpha_I};$$

Note that all the factor prices and good prices are measured in terms of final good.

General Good Producer General good producer also operates under the Cobb-Douglas production function subjected to sectorial technology shock. However, besides capital K_{Ct}, he uses both skilled labor S_{Ct} and unskilled labor L_{Ct} to produce general consumption good Y_{Ct}. General good producer rents capital from the bank at the market rate r_{Lt} and rents both skilled and unskilled labor from the household at rate w_{Lt} and w_{St}. General good producer is less unskilled labor intensive than the infrastructure sector, hence $\alpha_C < \alpha_I$.

$$Y_{Ct} = e^{z_{Ct}} A_C (K_{Ct})^{1-\alpha_C-\beta_C} (S_{Ct})^{\beta_C} (L_{Ct})^{\alpha_C}$$

Given factor prices $\{w_{Lt}, w_{St}, r_{Lt}, p_{Ct}\}$, the general good firm solves the following problem

$$\max \{p_{Ct}Y_{Ct} - w_L L_C - w_S S_C - r_l K_C\}; \tag{3.3}$$
$$w_{Lt} = \alpha_C p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}}\right)^{1-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C};$$
$$w_{St} = \beta_C p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}}\right)^{1-\beta_C};$$
$$r_{Lt} = (1-\alpha_C-\beta_C)\, p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C};$$

Final Good Producer The final good producer uses infrastructure good and general consumption good to produce the final good under a CES aggregator

$$Y_t = \left(\varphi (Y_{It})^{\frac{\sigma-1}{\sigma}} + (Y_{Ct})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}};$$

The final good firm solves the following problem in period t

$$\max \{Y_t - p_{It}Y_{It} - p_{Ct}Y_{Ct}\}$$

$$\text{s.t. } Y_t = \left(\varphi \left(Y_{It}\right)^{\frac{\sigma-1}{\sigma}} + \left(Y_{Ct}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

which leads to the first-order condition of

$$\frac{Y_I}{Y_C} = \left(\varphi \frac{p_C}{p_I}\right)^{\sigma}$$

and the standard price aggregation:

$$\left[\varphi^{\sigma} \left(p_I\right)^{1-\sigma} + \left(p_C\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = 1;$$

3.1.3 Banking Sector

We assume there is one representative bank in the economy. This bank can convert household's savings into capital goods. For simplicity, we assume one-for-one capital formation. In each period, the bank takes household's savings, converts them into capital, and rents them to both the infrastructure firm and the general good firm. There is government distortion in the banking sector: the bank has to rent to the infrastructure firm at a lower-than-market-rate r_{St} while renting to the general good sector at market rate r_{Lt}. Since the bank makes zero profit, household's deposit rate r_{dt} is pinned down by the following condition

$$\left(1 + r_{dt}\right) a_t = \left(1 - \delta + r_s\right) K_{It} + \left(1 - \delta + r_l\right) K_{Ct}$$

where

$$r_{Lt} \geqslant r_{dt} \geqslant r_{St}.$$

3.1.4 Market Clearing Conditions

There are three sets of market clearing conditions: the capital market clearing, the labor market clearing and the final good market clearing. The sum of capital used in infrastructure sector and general good sector is the total capital stock in the economy.

$$K_{It} + K_{Ct} = K_t$$

The total of unskilled labor in infrastructure sector and general good sector consists of the total unskilled labor supply in the economy.

$$L_{It} + L_{Ct} = L_t$$

Finally, we have the standard final good market clearing

$$C_t + I_t = Y_t.$$

The law of motion for capital is the following

$$K_{t+1} = I_t + (1 - \delta) K_t.$$

3.2 Competitive Equilibrium

Given initial labor and capital endowment, L_{t_0}, S_{t_0}, and K_{t_0}, a set of exogenous rental rate, and sectorial TFP $\{r_{st}, A_{It}, A_{Ct}\}_{t \geq t_0}$. A *competitive equilibrium* consists of:

- Sequences of good prices and factor prices, $\{p_{It}, p_{Ct}, w_{Lt}, w_{St}, r_{dt}, r_{lt}\}_{t \geq t_0}$;
- Firms allocations, $\{K_{It}, K_{Ct}, L_{It}, L_{Ct}\}_{t \geq t_0}$;
- Household allocations, $\{c_t, a_{t+1}\}_{t \geq t_0}$;

such that:

1. Given the sequence of prices, the firm allocation solves (FP);
2. Given the sequence of prices, the household allocation solves (HP);
3. Market clearing condition:
   - Capital allocation across sectors: $K_{It} + K_{Ct} = K_t$;
   - Unskilled-labor allocation across sectors: $L_{It} + L_{Ct} = L_t$;
   - Goods market: $C_t + I_t = Y_t$
   - Asset market: $K_{t+1} = I_t + (1 - \delta) K_t$
4. Competitive banking:

$$(1 + r_{dt}) a_t = (1 - \delta + r_{St}) K_{It} + (1 - \delta + r_{lt}) K_{Ct}.$$

3.3 Equilibrium Condition

In this section, we summarize the equilibrium conditions for this model. In an equilibrium, the following conditions must hold:

- From household side:
  - Euler equation:
$$\frac{U'(c_t)}{U'(c_{t+1})} = \beta\left(1 + r_{dt+1}\right); \tag{3.4}$$
  - household budget constraint
$$c_t + a_{t+1} = w_{Lt}l_t + w_{St}s_t + \left(1 + r_{dt}\right)a_t; \tag{3.5}$$
- From firm side:
  - FOC for firms
$$w_{Lt} = \alpha_I p_{It} A_{It}\left(\frac{K_{It}}{L_{It}}\right)^{1-\alpha_I} = \alpha_C p_{Ct} A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{1-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}; \tag{3.6}$$
$$w_{St} = \beta_C p_{Ct} A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{1-\beta_C}; \tag{3.7}$$
$$r_{lt} = \left(1 - \alpha_C - \beta_C\right) p_{Ct} A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}; \tag{3.8}$$
$$r_{st} = \left(1 - \alpha_I\right) p_{It} A_{It}\left(\frac{K_{It}}{L_{It}}\right)^{-\alpha_I}; \tag{3.9}$$
  - CES aggregation for final good production:
$$Y_t = \left(\varphi\left(Y_{It}\right)^{\frac{\sigma-1}{\sigma}} + \left(Y_{Ct}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}; \tag{3.10}$$
$$\frac{Y_{It}}{Y_{Ct}} = \left(\varphi\frac{p_{Ct}}{p_{It}}\right)^{\sigma}; \tag{3.11}$$
$$\left[\varphi^{\sigma}\left(p_{It}\right)^{1-\sigma} + \left(p_{Ct}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = 1; \tag{3.12}$$
  - Technology:
$$Y_{It} = A_{It}\left(K_{It}\right)^{1-\alpha_I}\left(L_{It}\right)^{\alpha_I}; \tag{3.13}$$
$$Y_{Ct} = A_{Ct}\left(K_{Ct}\right)^{1-\alpha_C-\beta_C}\left(S_{Ct}\right)^{\beta_C}\left(L_{Ct}\right)^{\alpha_C}; \tag{3.14}$$
- From banking sector:
  - Competitive banking:
$$\left(1 + r_{dt}\right)a_t = \left(1 - \delta + r_{st}\right)K_{It} + \left(1 - \delta + r_{lt}\right)K_{Ct}. \tag{3.15}$$

– Capital accumulation

$$K_{t+1} = I_t + (1-\delta) K_t \tag{3.16}$$

- Market clearing conditions:

  – Goods market:

  $$Y_t = I_t + C_t; \tag{3.17}$$

  government spending doesn't show up here because it goes into investment.

  – Asset market:

  $$A_t = K_t \tag{3.18}$$

  – Capital allocation across sectors:

  $$K_{It} + K_{Ct} = K_t; \tag{3.19}$$

  – Unskilled-labor allocation across sectors:

  $$L_{It} + L_{Ct} = L_t; \tag{3.20}$$

Therefore, we have 17 variables: $\{p_{It}, p_{Ct}, w_{Lt}, w_{St}, r_{dt}, r_{lt}\}$; $\{K_{It}, K_{Ct}, L_{It}, L_{Ct}, K_t, Y_{It}, Y_{Ct}, Y_t\}$; $\{C_t, I_t, A_{t+1}\}$. The equilibrium is characterized by above 17 equilibrium conditions (household's $B.C.$ and banking's capital accumulation is for dynamics).[1]

3.4 Quantitative Analysis

In this section, we first calibrate this model to match the Chinese data. With the calibrated parameters, we first solve for steady state. Then we solve for transitional dynamics: we let the economy to start from an initial state and compute the entire path until it converges to steady state.

3.4.1 Calibration

We now proceed to choose parameter values, setting some numbers on the basis of a prior information and setting others according to the steady-state conditions. A period in the model corresponds to one year. The sample period in the data is from XX to XX. The table below summarizes cali-

[1] Note that the non-arbitrage condition, i.e., equation (3.15), is redundant, given the resourse constraint.

bration. [*insert detailed calibration here later on ...*]

Parameter	Description	Value
β	discount factor	0.96
γ	risk aversion	2
σ	elasticity of substitution between infrastructure and general good	2
φ	infrastructure good share	0.5
α_I	unskilled labor share for infrastructure good production	0.6
α_C	unskilled labor share for general good production	0.3
β_C	skilled labor share for general good production	0.3
δ	depreciation	0.1
A_I	technology of infrastructure production	1
A_C	technology of general consumption good production	1
L	total unskilled labor supply	20
S	total skilled labor supply	1

3.4.2 Steady State

In the steady state, we have 17 variables $\{p_I, p_C, w_L, w_S, r_d, r_L, K_I, K_C, L_I, L_C, K, Y_I, Y_C, Y, C, I, A\}$ and 17 equilibrium conditions. In the derivation below, we express all the other 16 variables in terms of p_I and use the household budget constraint to pin down p_I.

- From Euler equation,

$$r_d = \frac{1}{\beta} - 1; \tag{3.21}$$

- Firm's FOC, $r_{st} = (1-\alpha_I)\, p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{-\alpha_I}$,

$$\frac{K_{It}}{L_{It}} = \left[\frac{(1-\alpha_I)\, p_{It} A_{It}}{r_{st}}\right]^{\frac{1}{\alpha_I}}; \tag{3.22}$$

- Firm's FOC, $w_{Lt} = \alpha_I p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{1-\alpha_I}$,

$$w_{Lt} = \alpha_I p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{1-\alpha_I}; \tag{3.23}$$

- Price aggregate, $\left[\varphi^\sigma (p_I)^{1-\sigma} + (p_C)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = 1$,

$$p_C = \left[1 - \varphi^\sigma (p_I)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}; \tag{3.24}$$

- Firm's technology, $Y_I = A_I (K_I)^{1-\alpha_I} (L_I)^{\alpha_I}$,

$$\frac{Y_I}{L_I} = A_I \left(\frac{K_I}{L_I}\right)^{1-\alpha_I}; \tag{3.25}$$

- Firm's technology, $\frac{Y_C}{L_C}=A_C\left(\frac{K_C}{L_C}\right)^{1-\alpha_C}\left(\frac{K_C}{S_C}\right)^{-\beta_C}$, and FOC, $w_{Lt}=\alpha_C p_{Ct}A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{1-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}$,

$$\frac{Y_C}{L_C}=\frac{w_{Lt}}{\alpha_C p_{Ct}}; \tag{3.26}$$

- Optimal allocation across sectors, $\frac{Y_I}{Y_C}=\left(\varphi\frac{p_C}{p_I}\right)^{\sigma}$,

$$\frac{L_I}{L_C}=\frac{\frac{Y_C}{L_C}}{\frac{Y_I}{L_I}}\frac{Y_I}{Y_C}=\frac{\frac{Y_C}{L_C}\left(\varphi\frac{p_C}{p_I}\right)^{\sigma}}{\frac{Y_I}{L_I}}; \tag{3.27}$$

- Labor allocation, $L_I+L_C=L$;

$$L_C=\frac{L}{1+\frac{L_I}{L_C}}; \tag{3.28}$$

$$L_I=L-L_C; \tag{3.29}$$

- Capital, output allocation:

$$K_I=L_I\left(\frac{K_I}{L_I}\right); \tag{3.30}$$

$$Y_I=L_I\left(\frac{Y_I}{L_I}\right); \tag{3.31}$$

$$Y_C=L_C\left(\frac{Y_C}{L_C}\right); \tag{3.32}$$

- Goods market clear condition:

$$C=\left(\varphi\left(Y_{It}\right)^{\frac{\sigma-1}{\sigma}}+\left(Y_{Ct}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}-\delta K; \tag{3.33}$$

- Firm's technology, $\frac{Y_C}{L_C}=A_C\left(\frac{K_C}{L_C}\right)^{1-\alpha_C}\left(\frac{K_C}{S_C}\right)^{-\beta_C}$,

$$K_C=\left[\frac{\frac{Y_C}{L_C}\left(L_C\right)^{1-\alpha_C}\left(S_C\right)^{-\beta_C}}{A_C}\right]^{\frac{1}{1-\alpha_C-\beta_C}}; \tag{3.34}$$

- Firm's FOC, $w_{St}=\beta_C p_{Ct}A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{1-\beta_C}$, and $r_{lt}=\left(1-\alpha_C-\beta_C\right)p_{Ct}A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}$

$$w_{St}=\beta_C p_{Ct}A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{1-\beta_C} \tag{3.35}$$

$$r_{lt}=\left(1-\alpha_C-\beta_C\right)p_{Ct}A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}; \tag{3.36}$$

- Then, we can use B.C. to pin down p_I :

$$C+\overline{\tau}=w_L L+w_S S+r_d A$$

$$C=w_L L+w_S S+r_d K-(1+r_d)\,\overline{\tau}; \tag{3.37}$$

- No-arbitrage condition in banking sector, $(1+r_d)\,K=(1-\delta+r_S)\,K_I+(1-\delta+r_L)\,K_C$ are automatically satisfied.

Given the calibration and our algorithm described just now, we can solve for steady state. We first solve the case when the government imposed a state interest rate for the infrastructure sector $r_S=0.11$ and zero lump-sum tax $\tau=0$. In steady state, we have $r_d=0.0417$, $r_L=0.1879$. The corresponding effective state interest rate $(r_S-\delta)$ is 0.01 and effective market rate $(r_L-\delta)$ is 0.0897. The market rate $r_L-\delta$ is much higher than the state rate $r_S-\delta$ while the deposit rate r_d lies between the market rate and the deposit rate. The wage premium is positive with $w_L=0.51$ and $w_S=3.78$. The prices of the intermediate goods in terms of final goods are $p_I=0.54$ and $p_C=1.86$. Steady state capital level is 65.94.

We also computed the case where there is no government distortion. In this case, all the interest rates are equal: $r_S-\delta=r_L-\delta=r_d=0.0417$. The unskilled labor wage is $w_L=0.51$, the skilled labor wage is $w_S=4.22$. The prices of the intermediate goods in terms of final goods are $p_I=0.60$ and $p_C=1.71$. Steady state capital level is 68.07 which is higher than the case with distortion.

3.4.3 Algorithm for Computing Transition Path

Shooting method is used to solve the trasitional dynamics. We compute a path where the economy starts from a given state and eventually goes back to steady state. We assume it takes less than $T=100$ periods for the economy to go back to steady state. The shooting algorithm is described as follow:

1. The economy starts from an initial capital stock level K_1. We guess a range $\left[\underline{K},\overline{K}\right]$ for second period capital level K_2.

2. Let $K_2=\left(\underline{K}+\overline{K}\right)/2$. Given K_1 and K_2, we can solve the system for T periods.

   (a) Given K_t, we could solve for static variables $\{p_{Ct}$, p_{It}, L_{Ct}, L_{It}, K_{Ct}, K_{It}, Y_{Ct}, Y_{It}, w_{Lt}, w_{St}, r_{Lt}, $r_{dt}\}$ in period t.

(b) Similarly given K_{t+1}, we solve the the static variables in period $t+1$: $\{p_{Ct+1}$, p_{It+1}, L_{Ct+1}, L_{It+1}, K_{Ct+1}, K_{It+1}, Y_{Ct+1}, Y_{It+1}, w_{Lt+1}, w_{St+1}, r_{Lt+1}, $r_{dt+1}\}$.

(c) Since we know $\{w_{Lt}, w_{St}, r_{dt}, K_t, K_{t+1}\}$, c_t is solved from household's budget constraint.

(d) From Euler equation

$$\frac{U'(c_t)}{U'(c_{t+1})} = \beta(1 + r_{dt+1})$$

we know c_{t+1}.

(e) Given $\{c_{t+1}, w_{Lt+1}, w_{St+1}, r_{dt+1}, K_{t+1}\}$, from household budget constraint we can solve for K_{t+2}.

(f) Since we solved for K_{t+2}; repeat the procedure (a)-(e), we can solve for $K_{t+3}, K_{t+4}, \ldots K_T$.

3. If the value of K_2 we guessed above is higher than its true value, the economy will accumulate more and more capital, and thus eventually diverge with either $c_t \leq 0$ or $r_{dt} \leq 0$ at some point in the future. Similarly, if the guess of K_2 is too low, the economy will consume too much and thus accumulate less and less capital. Eventually, the economy will diverge with If $K_t \leq 0$. Therefore, in any period t,

   (a) If $c_t \leq 0$ or $r_{dt} \leq 0$, then $\overline{K} = K_2$ and go back to step 2.

   (b) If $K_t \leq 0$, then $\underline{K} = K_2$ and go back to step 2.

4. If $|\overline{K} - \underline{K}| \leq 10^{-15}$ then stop. Otherwise, go back to step 2.

We describe the details in 2.(a) and (b) here. Given any K_t using equation (3.38) - (3.51), we can rewrite $\{p_{Ct}, L_C, L_I, K_C, K_I, Y_C, Y_I, w_L, w_s, r_l\}$ as functions of p_{It}. Then we could use equation (3.52) to pin down p_{It}. Immediately, we can solve for r_{dt} by non-arbitrage condition (3.53).

- Firm's FOC, $r_{st} = (1-\alpha_I)\, p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{-\alpha_I}$,

$$\frac{K_{It}}{L_{It}} = \left[\frac{(1-\alpha_I)\, p_{It} A_{It}}{r_{st}}\right]^{\frac{1}{\alpha_I}}; \tag{3.38}$$

- Firm's FOC, $w_{Lt} = \alpha_I p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{1-\alpha_I}$,

$$w_{Lt} = \alpha_I p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{1-\alpha_I}; \tag{3.39}$$

- Price aggregate, $\left[\varphi^{\sigma}\left(p_I\right)^{1-\sigma}+\left(p_C\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}=1$,

$$p_C=\left[1-\varphi^{\sigma}\left(p_I\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}; \tag{3.40}$$

- Firm's technology, $Y_I=A_I\left(K_I\right)^{1-\alpha_I}\left(L_I\right)^{\alpha_I}$,

$$\frac{Y_I}{L_I}=A_I\left(\frac{K_I}{L_I}\right)^{1-\alpha_I}; \tag{3.41}$$

- Firm's technology, $\frac{Y_C}{L_C}=A_C\left(\frac{K_C}{L_C}\right)^{1-\alpha_C}\left(\frac{K_C}{S_C}\right)^{-\beta_C}$, and FOC, $w_{Lt}=\alpha_C p_{Ct} A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{1-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}$,

$$\frac{Y_C}{L_C}=\frac{w_{Lt}}{\alpha_C p_{Ct}}; \tag{3.42}$$

- Optimal allocation across sectors, $\frac{Y_I}{Y_C}=\left(\varphi\frac{p_C}{p_I}\right)^{\sigma}$,

$$\frac{L_I}{L_C}=\frac{\frac{Y_C}{L_C}}{\frac{Y_I}{L_I}}\frac{Y_I}{Y_C}=\frac{\frac{Y_C}{L_C}\left(\varphi\frac{p_C}{p_I}\right)^{\sigma}}{\frac{Y_I}{L_I}}; \tag{3.43}$$

- Labor allocation, $L_I+L_C=L$;

$$L_C=\frac{L}{1+\frac{L_I}{L_C}}; \tag{3.44}$$

$$L_I=L-L_C; \tag{3.45}$$

- Capital, output allocation:

$$K_I=L_I\left(\frac{K_I}{L_I}\right); \tag{3.46}$$

$$Y_I=L_I\left(\frac{Y_I}{L_I}\right); \tag{3.47}$$

$$Y_C=L_C\left(\frac{Y_C}{L_C}\right); \tag{3.48}$$

- Firm's technology, $\frac{Y_C}{L_C}=A_C\left(\frac{K_C}{L_C}\right)^{1-\alpha_C}\left(\frac{K_C}{S_C}\right)^{-\beta_C}$,

$$K_C=\left[\frac{\frac{Y_C}{L_C}\left(L_C\right)^{1-\alpha_C}\left(S_C\right)^{-\beta_C}}{A_C}\right]^{\frac{1}{1-\alpha_C-\beta_C}}; \tag{3.49}$$

- Firm's FOC, $w_{St}=\beta_C p_{Ct} A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{1-\beta_C}$, and $r_{lt}=\left(1-\alpha_C-\beta_C\right) p_{Ct} A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}$

$$w_{St}=\beta_C p_{Ct} A_{Ct}\left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C}\left(\frac{K_{Ct}}{S_{Ct}}\right)^{1-\beta_C} \tag{3.50}$$

$$r_{lt} = (1 - \alpha_C - \beta_C)\, p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}} \right)^{-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}} \right)^{-\beta_C}; \tag{3.51}$$

Then, we can use $K_{It} + K_{Ct} = K_t$ to pin down p_{It}.

$$K_{It} + K_{Ct} = K_t \tag{3.52}$$

and solve for r_{dt}

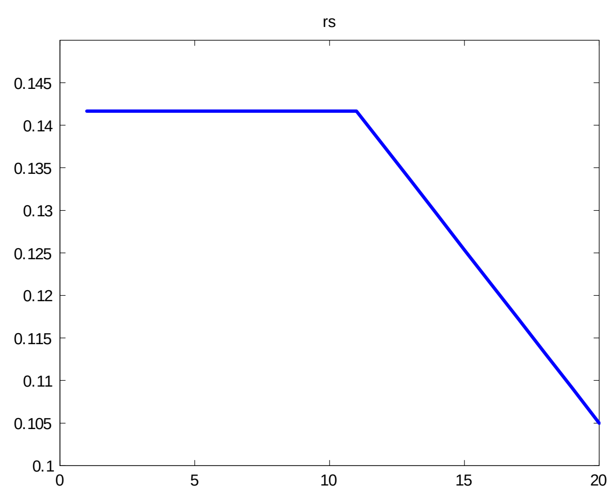
- Competitive banking:

$$(1 + r_{dt})\, A_t = (1 - \delta + r_{st})\, K_{It} + (1 - \delta + r_{lt})\, K_{Ct}. \tag{3.53}$$

3.5 Results

We would like to study the effect of a gradually introduced interest rate distortion on the economy. First, the economy stays on the balanced growth path where there are no distortion. The three interest rates are all equal, i.e., $r_d = r_s - \delta = r_l - \delta$. Then, the government starts to subsidize the infrastructure sector by providing loan with lower than market interest rate, $r_s < r_l$.

In our numerical exercise, the initial loan rate for the infrastructure sector, r_s, is set at the non-distortionary level, $r_s = r_d^{ss} + \delta = 0.1417$. After 10 periods, the government starts to subsidize the infrastructure sector by gradually pushing down the loan rate to $r_S = 0.105$ (We assume it takes 10 periods to reach r_S). The path of r_S is shown in the figure below.

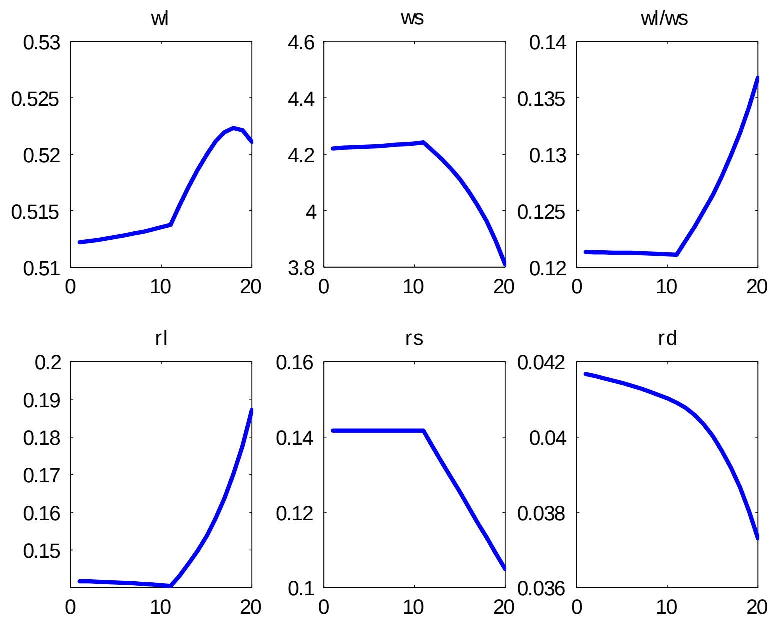


3.5.1 Factor Prices and Allocations

Figure X shows the changes in wages, interest rates and capital labor allocations along the transitional path. As government introduces low interest rate for the infrastructure sector from period

10 to period 20, we observe sustaintial changes in the factor prices and allocations. First of all, we observe that unskilled labor wage increases and skilled labor wage decreases. With these changes, the relative wage for unskilled labor increases substantially. Second, the market loan rate increases sharply and the deposit rate falls. Third, capital and labor all shift to the infrastrucuture sector. All these predications are consistent with our data observations discussed in the previous section. The intuition behind the changes is the following. As the lower interest rate is introduced to the infrastructure sector, investment flows into the infrastructure sector. When the firms in infrastructure sector increase their capital stock, they also hire more unskilled workers. Threrefore we observe that capital and unskilled labor are moving from the consumption sector to the infrastructure sector. In other words, as the interest rate falls, the infrastructure sector crowds out the consumption sector. The increasing demand for unskilled labor from the expanding infrastructure sector drives up unskilled wage. On the other hand, the shrinking consumption sector demands less skilled labor which leads to a decrease of skilled labor wage.

As infrastructure sector takes out most of the available capital, the falling supply of capital drives up the market interest rate. Given that the bank makes zeros profit, the fall in the deposit rate represents a fall in the average rate of return to capital in the economy. Hence in this model, we see that although investment increases a lot, the average return is low. The intuition is straightforward: infrastructure sector can rent the capital at a lower cost, they tend to invest more and the marginal return to capital is low; the other sectors has to rent the capital at market rate, which is distortionarily higher. The aggregate rate of return to capital is lower.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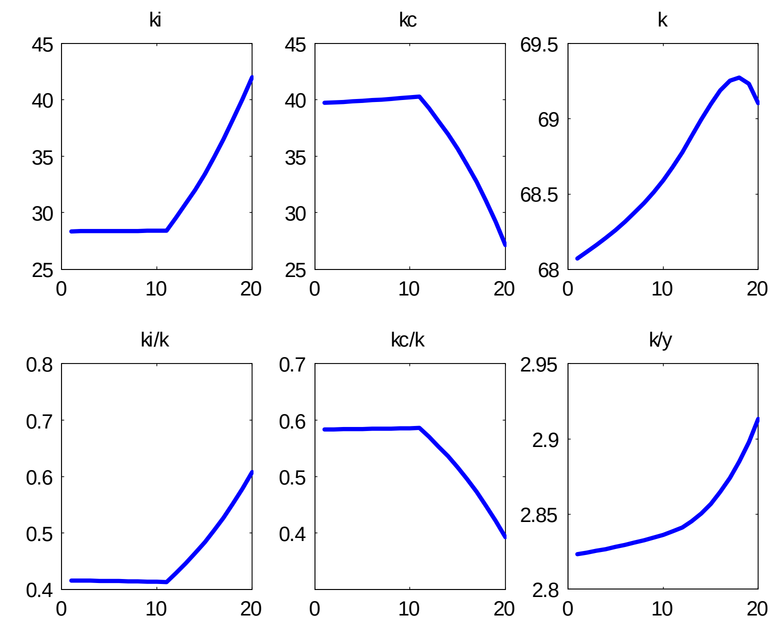
0.525
0.52
0.515
0.51
0 10 20
ws
4.6
4.4
4.2
4
3.8
0 10 20
wl/ws
0.14
0.135
0.13
0.125
0.12
0 10 20
rl
0.2
0.19
0.18
0.17
0.16
0.15
0 10 20
rs
0.16
0.14
0.12
0.1
0 10 20
rd
0.042
0.04
0.038
0.036
0 10 20


ki
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40
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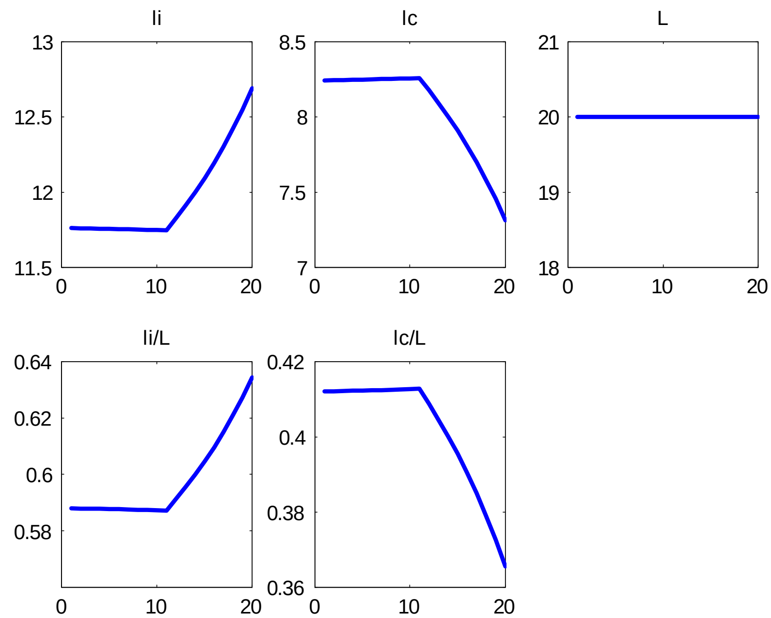
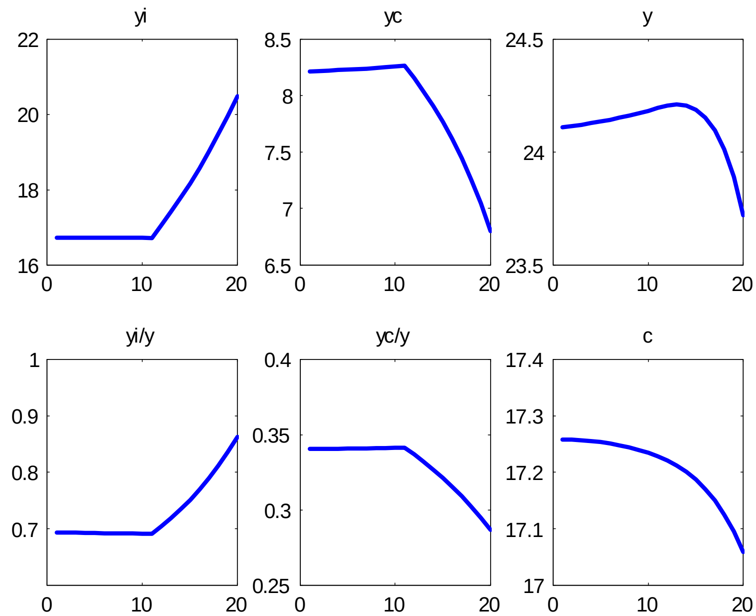
30
25
0 10 20
kc
45
40
35
30
25
0 10 20
k
69.5
69
68.5
68
0 10 20
ki/k
0.8
0.7
0.6
0.5
0.4
0 10 20
kc/k
0.7
0.6
0.5
0.4
0 10 20
k/y
2.95
2.9
2.85
2.8
0 10 20



3.5.2 Aggregate Output and Consumption

Figure X shows the changes in the output and consumption level. We observe that the output of infrastructure rises while the output of consumption sector falls. This is consistent with the rising capital stock and labor employed in the infrastructure sector and falling capital and labor in the consumption sector. More interestingly, the aggregate output increase initially, however, it falls soom after a boom. On the other hand, the consumption falls during the entire transitional period. An interesting implication of our results is that for a government who is interested in the short-run output level, it will have a strong incentive to apply such distortionary policy to promote the output, even though such a policy does not generate more consumption and eventually the output level indeed falls because of the distortions. This finding may also explain the high debt level of the local government across China, when local governments have strong incentives to investment in the subsidized sector, they borrow more.



4 An Extended Model with Rural/Urban Migration

In this section, we relax the assumption that the total population is fixed and study a model where there is excess labor supply. The economy has a rural sector and an urban sector. In the rural sector, unskilled workers are engaged in subsistence agriculture whose output is consumed only in the rural area. In the urban sector, similar to the previous set up, two types of goods are produced using inputs of unskilled labor, skilled labor, and capital. The economy's supply of skilled labor and unskilled labor is exogenously given, although the allocation of unskilled labor between the rural and urban sectors is driven by household migration decisions. The supply of capital to the urban sector is assumed to be endogenously determined by the savings decisions of urban households.

4.1 The Rural Area

In this section, we describe the production in the rural area and derive the conditions determining the allocation of unskilled labor between the rural and urban areas. The total population of the unskilled labor is denoted by $\bar{N}$ which consists of a group of size N residing in the rural area and a group of size L located in the urban area.

4.1.1 Subsistence Farming

The production technology in the rural is described as follows:[2]

$$y = \begin{cases} A_r E^{1-\alpha_R} N^{\alpha_R}; & N < N_R \\ \bar{C}; & N \geq N_R \end{cases};$$

where E is land, N is unskilled labor input and $N_R = \left(\frac{\bar{C}}{A_r E^{1-\alpha_R}} \right)^{\frac{1}{\alpha_R}}$. This production technology represents the idea that when population is above certain shreshold, increasing labor input will not increase the total output. Therefore, the MPL for unskilled labor in rural area follows such a step function:

$$MPL = \begin{cases} \alpha_R A_r E^{1-\alpha_R} N^{\alpha_R - 1}; & N < N_R \\ 0; & N \geq N_R \end{cases}.$$

Note that if the population above the threshold N_R moves into urban industry, then their MPL will change from zero to a positive number.

4.1.2 Migration Cost

The unskilled labor can choose to move to urban area to work, and thus enjoy the final urban goods. However, they need to pay a migration cost $D(L, l)$, which is described by

$$D(L, l) = \phi_0 \left(\frac{L}{\bar{N} - N_R} \right)^{-\xi} \frac{l^{1+\psi}}{(1+\psi)};$$

where $\bar{N}$ is the total number of unskilled labor in the economy, l is the number of migrants who chooses to move to urban area, and L is the total number of unskilled labor located in the urban area. Note that l is a choice variable for the rural household, and L is an aggregate variable which is taken as given when making the migration decision. The above migration cost function shows externality of migration: the more unskilled labor migrates, the smaller the cost of migration. ξ here captures the effect of such an externality.

We can conjecture that if the government subsidizes the infrastructure sector which increases employment of unskilled worker, then the migration cost is lower and migration is hence encouraged. Given that the unskilled worker moves into the urban industry, the productivity is increased and the government distortionary policy is welfare enhancing. However, once the migration is complete, the government distortionary policy will reduce the welfare as discussion in the previous set up.

[2] As the first step, we can assume that $\alpha_R = 1$, which means that we don't consider the contribution of land for now.

4.1.3 The Rural/Urban Migration Decision

By migrating to the urban area, an unskilled worker earns the unskilled wage w and consumes instead the aggregate good Y. For the representative worker who migrates into the urban area, we have

$$w_L = \phi_0 \left(\bar{N} - N_R\right)^{\xi} L^{-\xi} l^{\psi}.$$

If the total population of the unskilled worker already moved into the urban area, then $L = \bar{N} - N_R$. This gives us the threshold migration wage $\bar{w}$

$$\bar{w} = \phi_0 \left(\bar{N} - N_R\right)^{\psi}$$

For wages lower than the threshold, only part of the unskilled population moves into the urban area. Therefore the migration condition is

$$L = \begin{cases} \left(\frac{w_L}{\phi_0\left(\bar{N}-N_R\right)^{\xi}}\right)^{\frac{1}{\psi-\xi}}; & w_L < \bar{w} \\ \bar{N} - N_R; & w_L \geq \bar{w} \end{cases}.$$

4.2 The Urban Area

The urban area is modelled the same way as the benchmark model. The only difference here is that the total supply of unskilled labor may depend on the stage of the economy. In particular, there are two stages of the economy:

1. Stage 1: there exists abundant unskilled labor supply

   At this stage, only a part of the unskilled labor from the rural area moves to the urban area. The total labor supply is given by $L = \left(\frac{w}{\phi_0\left(\bar{N}-N_R\right)^{\xi}}\right)^{\frac{1}{\psi-\xi}} < \left(\bar{N} - N_R\right)$.

2. Stage 2: all unproductive unskilled labors have already moved to urban area

   At this stage, the total labor supply in urban sector is $\left(\bar{N} - N_R\right)$. The model works the same way as our benchmark model.

4.3 Equilibrium Conditions

We assume that the economy starts from a state where there is potential future migration. During the migration phase, we can write $\{p_{Ct}, L_C, L_I, K_C, K_I, Y_C, Y_I, w_L, w_s, r_l\}$ as a function of p_{It} and L. Then using equation (4.15) and (4.16) to pin down p_{It} and L. Once the factor allocations and prices are solved, we solve for r_{dt} by non-arbitrage condition and then c_t by budget constraint. For every period, we check if $L = \left(\bar{N} - N_R\right)$ holds. Once $L = \left(\bar{N} - N_R\right)$, then for all the periods that follow we solve the equilibrium as in section 6.3.

- Firm's FOC, $r_{st} = (1-\alpha_I)\, p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{-\alpha_I}$,

$$\frac{K_{It}}{L_{It}} = \left[\frac{(1-\alpha_I)\, p_{It} A_{It}}{r_{st}}\right]^{\frac{1}{\alpha_I}}; \tag{4.1}$$

- Firm's FOC, $w_{Lt} = \alpha_I p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{1-\alpha_I}$,

$$w_{Lt} = \alpha_I p_{It} A_{It} \left(\frac{K_{It}}{L_{It}}\right)^{1-\alpha_I}; \tag{4.2}$$

- Price aggregate, $\left[\varphi^{\sigma} (p_I)^{1-\sigma} + (p_C)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = 1$,

$$p_C = \left[1 - \varphi^{\sigma} (p_I)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}; \tag{4.3}$$

- Firm's technology, $Y_I = A_I (K_I)^{1-\alpha_I} (L_I)^{\alpha_I}$,

$$\frac{Y_I}{L_I} = A_I \left(\frac{K_I}{L_I}\right)^{1-\alpha_I}; \tag{4.4}$$

- Firm's technology, $\frac{Y_C}{L_C} = A_C \left(\frac{K_C}{L_C}\right)^{1-\alpha_C} \left(\frac{K_C}{S_C}\right)^{-\beta_C}$, and FOC, $w_{Lt} = \alpha_C p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}}\right)^{1-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}$,

$$\frac{Y_C}{L_C} = \frac{w_{Lt}}{\alpha_C p_{Ct}}; \tag{4.5}$$

- Optimal allocation across sectors, $\frac{Y_I}{Y_C} = \left(\varphi \frac{p_C}{p_I}\right)^{\sigma}$,

$$\frac{L_I}{L_C} = \frac{\frac{Y_C}{L_C}}{\frac{Y_I}{L_I}} \frac{Y_I}{Y_C} = \frac{\frac{Y_C}{L_C}\left(\varphi \frac{p_C}{p_I}\right)^{\sigma}}{\frac{Y_I}{L_I}}; \tag{4.6}$$

- Labor allocation, $L_I + L_C = L$;

$$L_C = \frac{L}{1 + \frac{L_I}{L_C}}; \tag{4.7}$$

$$L_I = L - L_C; \tag{4.8}$$

- Capital, output allocation:

$$K_I = L_I \left(\frac{K_I}{L_I}\right); \tag{4.9}$$

$$Y_I = L_I \left(\frac{Y_I}{L_I}\right); \tag{4.10}$$

$$Y_C = L_C \left(\frac{Y_C}{L_C}\right); \tag{4.11}$$

- Firm's technology, $\frac{Y_C}{L_C} = A_C \left(\frac{K_C}{L_C}\right)^{1-\alpha_C} \left(\frac{K_C}{S_C}\right)^{-\beta_C}$,

$$K_C = \left[\frac{\frac{Y_C}{L_C} (L_C)^{1-\alpha_C} (S_C)^{-\beta_C}}{A_C}\right]^{\frac{1}{1-\alpha_C-\beta_C}}; \tag{4.12}$$

- Firm's FOC, $w_{St} = \beta_C p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}}\right)^{1-\beta_C}$, and $r_{It} = (1-\alpha_C-\beta_C)\, p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}$

$$w_{St} = \beta_C p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}}\right)^{1-\beta_C} \tag{4.13}$$

$$r_{It} = (1-\alpha_C-\beta_C)\, p_{Ct} A_{Ct} \left(\frac{K_{Ct}}{L_{Ct}}\right)^{-\alpha_C} \left(\frac{K_{Ct}}{S_{Ct}}\right)^{-\beta_C}; \tag{4.14}$$

Then, we can use the following equations to pin down p_{It} and L

$$K_{It} + K_{Ct} = K_t \tag{4.15}$$

$$L = \left(\frac{w_L}{\phi_0 \left(\bar{N} - N_R\right)^{\xi}}\right)^{\frac{1}{\psi-\xi}} \tag{4.16}$$

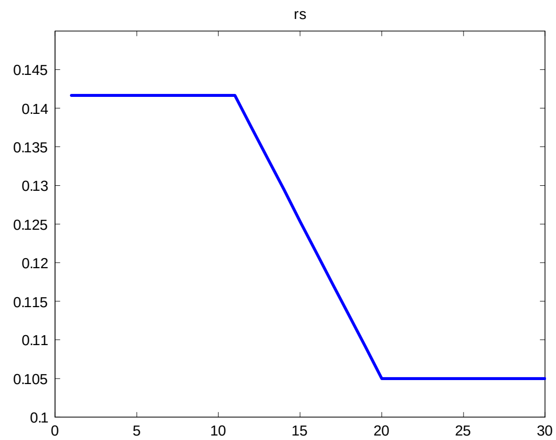
4.4 Calibration: Rural Area

The parameter values for the urban economy are set the same level as before. The new parameters for the rural area and migration cost are set as follows:

$\overline{N}$	$\overline{C}$	ϕ_0	ξ	ψ	α_R	E	A_R	$\overline{U}$
25	5	0.0011	1	2	1	1	1	−2

4.5 Results

We follow the same government distortionary policy as in the previous section. First, the economy stays on the balanced growth path where there are no distortions for 10 periods, then government starts to subsidize the infrastructure sector by gradually pushing down the loan rate for another 10 periods. The path of r_S is shown in the figure below.



In the following section, we summarize the effect of the policy on the key macroeconomic variables for two cases: the economy with migration and the economy without migration.[3]

4.5.1 Factor Allocations and Prices

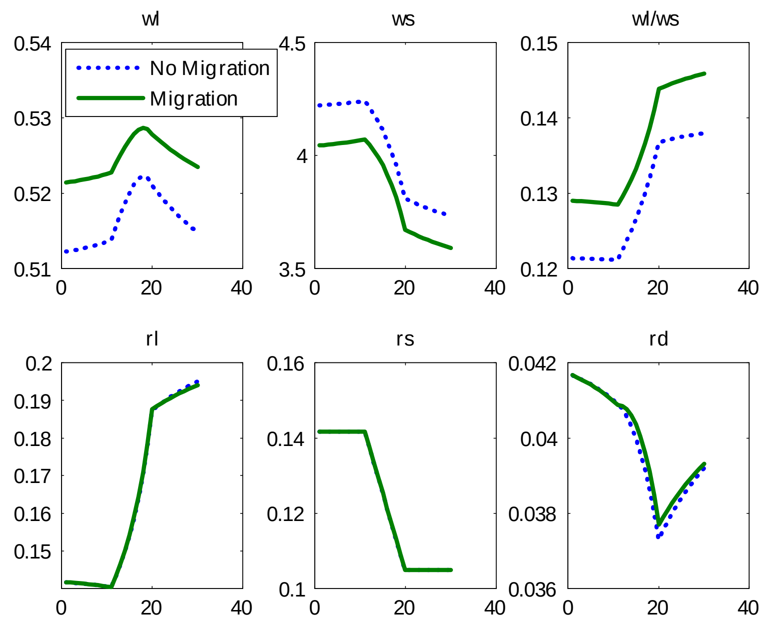
The figures below show the changes in the factor allocations and prices. The solid line shows the paths where where the country has a rural area and the dash line shows that case that the urban area is absent. Given government's distortionary policy, we obtain similar results as in Section 3.

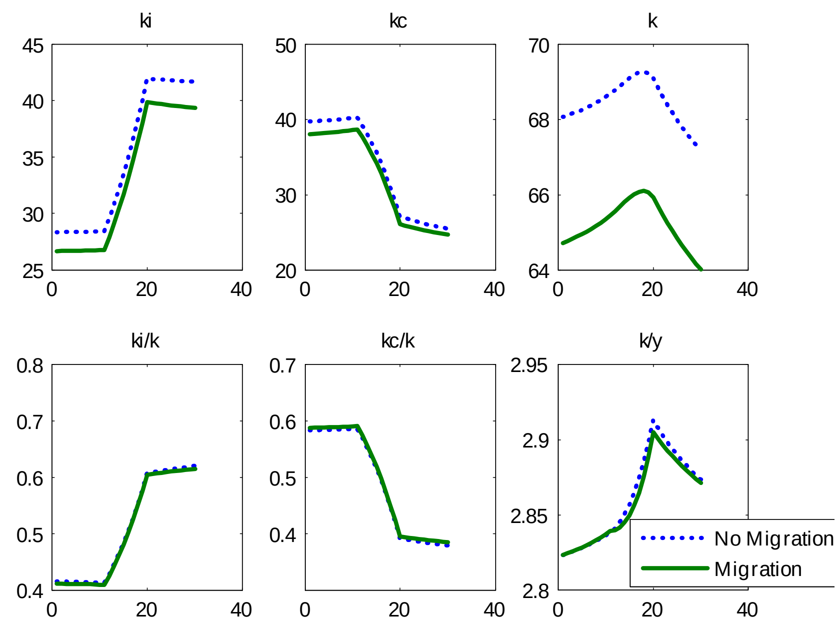
We observe that the rise in unskilled labor wage and the fall in the skilled labor wage leads to a declining of wage premium. The market loan rate increases sharply and deposit interest rate does not change that much. It falls at first and then returns back to its original level gradually. Capital and labor move from the consumption sector to the infrastructure sector. Moreover, part of the unskilled population moves from the rural area into the urban area.

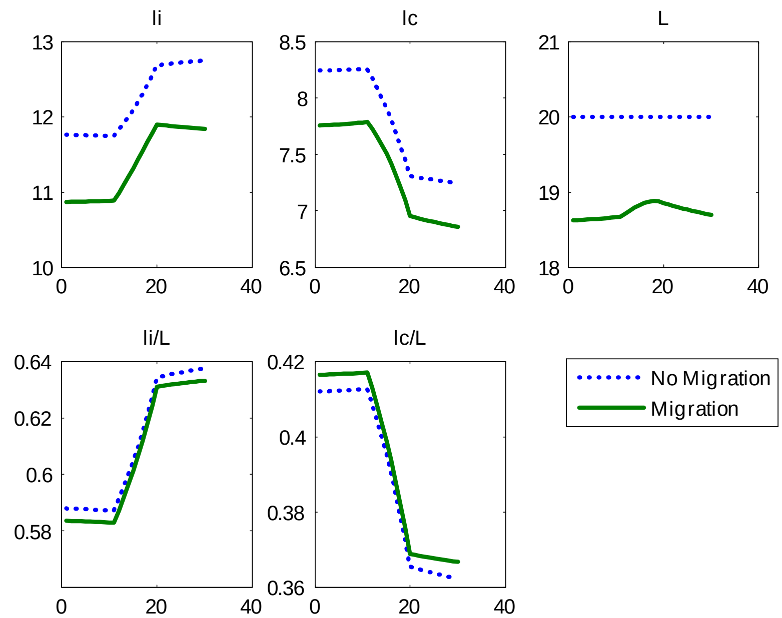
The logic is the following. As the lower interest rate is introduced to the infrastructure sector, investment flows into the infrastructure sector. When the firms in infrastructure sector increase their capital stock, they also hire more unskilled workers. Threrefore we observe that capital and unskilled labor are moving from the consumption sector to the infrastructure sector. In other words, as the interest rate falls, the infrastructure sector crowds out the consumption sector. The increasing demand for unskilled labor from the expanding infrastructure sector drives up unskilled wage. Therefore the unskilled worker moves from rural area into the urban industry. On the other hand, the shrinking consumption sector demands less skilled labor which leads to a decrease of skilled labor wage. Note that not all the population in the rural area moves into the urban

[3] To illustrate the effect of the distortionary policy on the economy, we only show the first 30 period of the transition path. We summarize the full path of the transition dynamics in the Appendix A.

industry, this is because that the unskilled wage is not high enough at this stage. To sum up, the movements in the factor allocation and prices are similar for the two cases except that part of the rural population moves into the urban industry in the extended model.



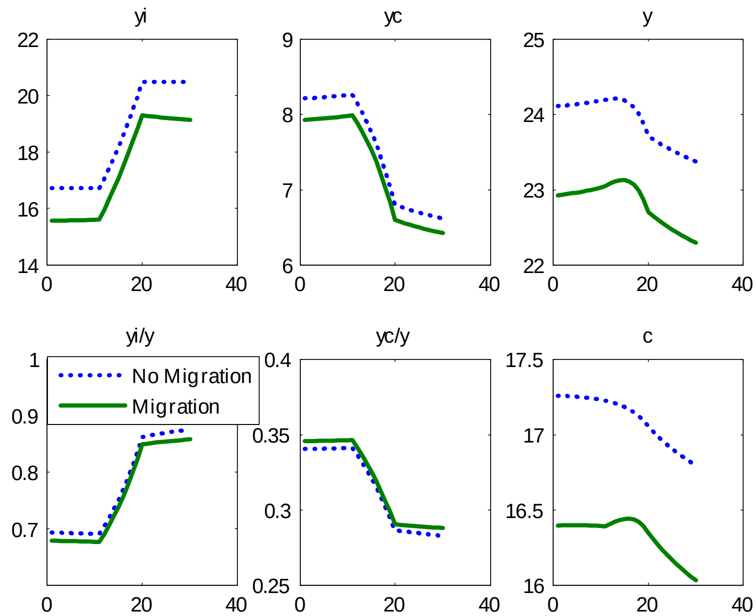




4.5.2 Aggregate Output and Consumption

The implications on output and consumption level are more interesting. Figure X shows the trasitional path of ouput and consumption level. The pattern of consumption is clearly different. For the case with migration, consumption increases and then falls. However, for the case without migration, the consumption level falls right from the beginning. The movements of the rest of the macro variables are the same for the two cases. The infrastructure sector expands due to the distortionary policy and the consumption good sector shrinks. Output experiences a short-lived boom and falls back to distortionary steady state.

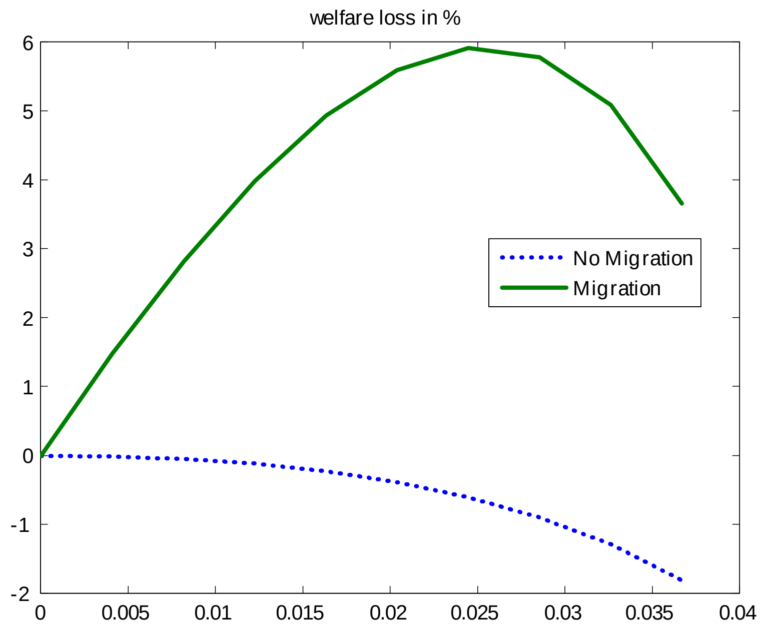
The difference in the consumption pattern is because when unskilled worker moves from the rural area into the urban area, their marginal product of labor increases. Hence their consumption level rises from the subsistence level to the urban household level. The interesting implication of this finding is that there is a trade-off between reallocating the unskilled labor to more productive sector and government biased investment policy. If there is excess labor supply, biased investment policy would potentially increase the aggregate consumption level in short-run. Hence the government can increase output and consumption at the same time.



5 Discussion

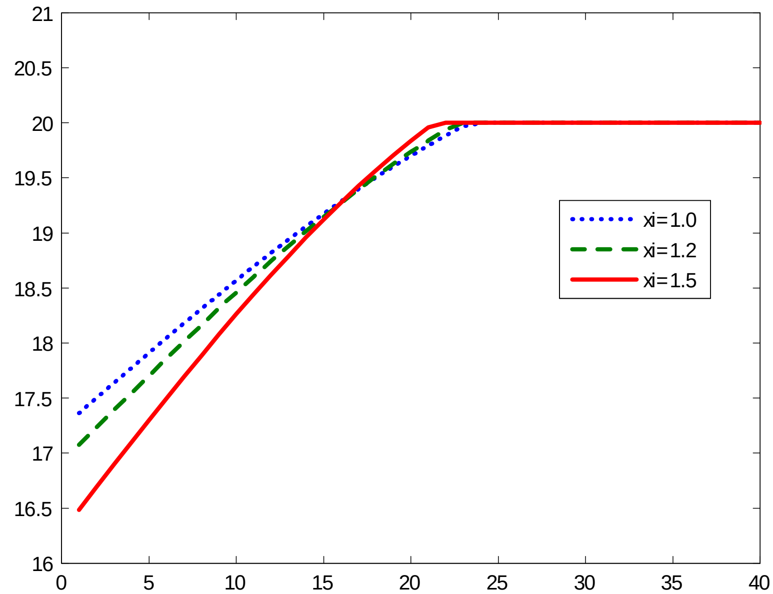
5.1 Welfare

Note that the gap between subsidized rate, r_s, and market rate, $r_{l.}$ actually captures the degree of distortion in our economy. We compute the welfare loss associated with the distortion, by comparing welfare under economy with and without distortion. Our results clearly show that when the economy has a fixed labor supply, the welfare loss increases with the degree of the distortion. However, when there is excess labor supply, distortionary government policy could actually increase welfare. When the distortion is relatively small, as it increases, the welfare is improving. After certain degree of distortion, the economy still benefits from the policy by having a welfare gain, however, the size of welfare gain is falling. The inverse U-shaped welfare gain reflects trade-off between two forces: when unskilled labor moves into more productive sector, welfare increases. When government distortionary interest rate misallocates the resources within the economy, the welfare falls. Therefore, there exists an optimal extent of intervetion of government distortionary policy.



5.2 Migration Externality

We explores the relationship between migration cost and the migration speed here. The figure below shows the population size in the urban area for different migration costs. We assume that the economy faces a permanent level of interest rate distortion. These three cases all start from the same level of capital stock. A smaller ξ represents a lower degree of externality and hence a higher migration cost. The figure shows that when the migration cost is high, the migration speed is low as demonstrated by the slope of the population line.



5.3 Transition Dynamics from an off-balanced-path Initial State

In previous section, we assume that the economy initially stays at steady state, and thus examine how it reacts when applying the distortionary policy. Therefore, the results illustrated above indeed capture the short-run effect of policies. However, it is also interesting to examine the long run effect of such policies on the economy during its transitional stage. In this section, we assume the economy still locates on its transition path to the S.S.. In particular, the initial capital stock of the economy is only half of its steady state level. To show the long-run effect of distortionary policy, we compare the transition path of the economy with distortion to that without distortion. Similarly, we consider two scenarios: i) economy with no migration; ii) economy with migration from rural area Interestingly, the transition paths under long run distortionary policies presents quite different patterns than previous sections. We summarize our results in Appendix B.

5.4 Implications on Education Policy

Our current framework also has important implications on the education policy. China started to expand its college enrollment from 1999 and this policy has always been critisized as the reason for falling college wage premium. If declining wage premium is due to expansion of college enrollment, then we should probably enroll fewer college students. However, our model implies that the decline of wage premium may not be due to college expansion. The evidence is that, if supply of skilled labor increases, we should see the expansion of skilled labor intensive sectors, but we observe the opposite in the data. This rules out the supply side story and leads to our demand side story: demand of skilled labor is low due to capital market distortions, so wage premium declines and skilled labor intensive sectors grow slower.

5.5 Robustness

We experimented with different sets of parameter values, and the transition patterns under both short-run policy and long-run policy are pretty much the same.

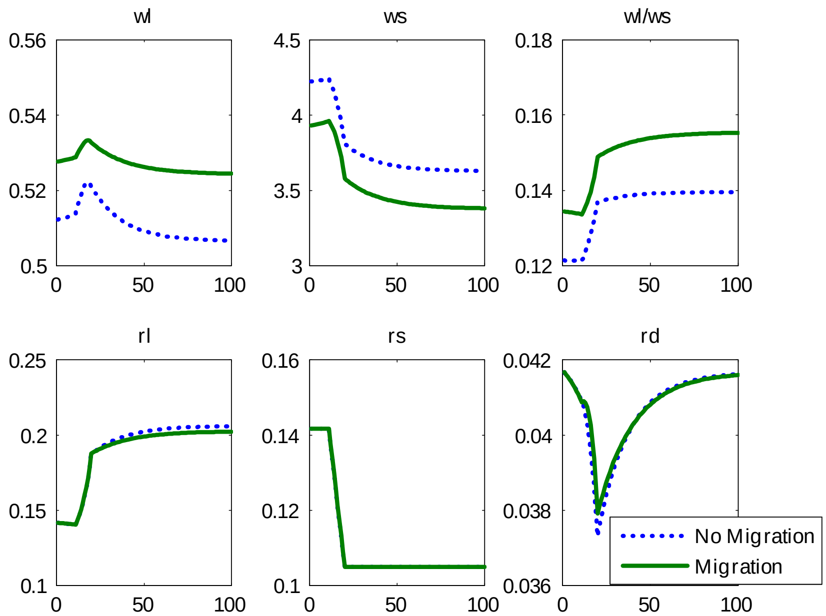
6 Conclusions

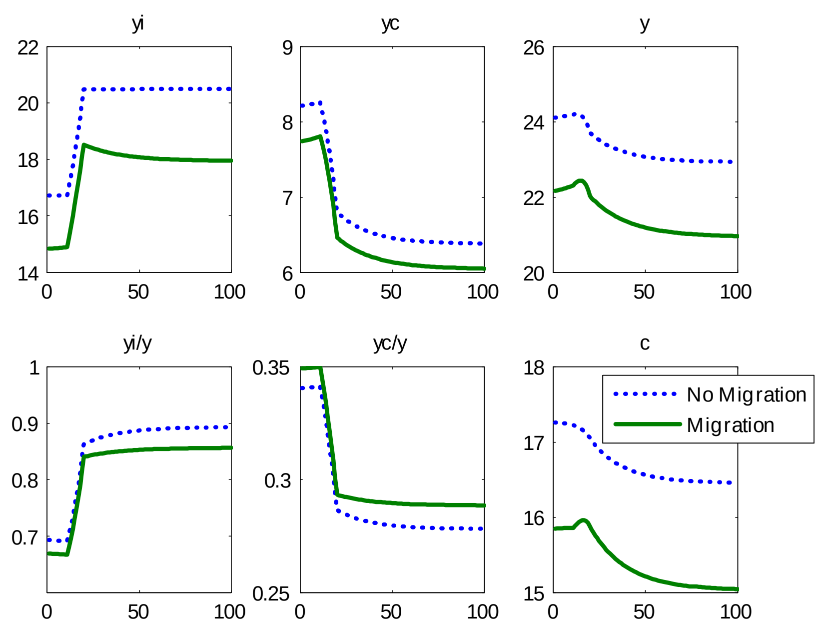
In this paper, we build a two-sector growth model with government policy biased towards the infrastructure sector. The model is consistent with salient features of the Chinese economy. Our model can explain i) high private loan rate with low average return to capital; ii) high investment rate with low marginal return to capital; iii) falling wage premium between skilled and unskilled worker. A calibrated version of the model is shown to be consistent with the above facts.

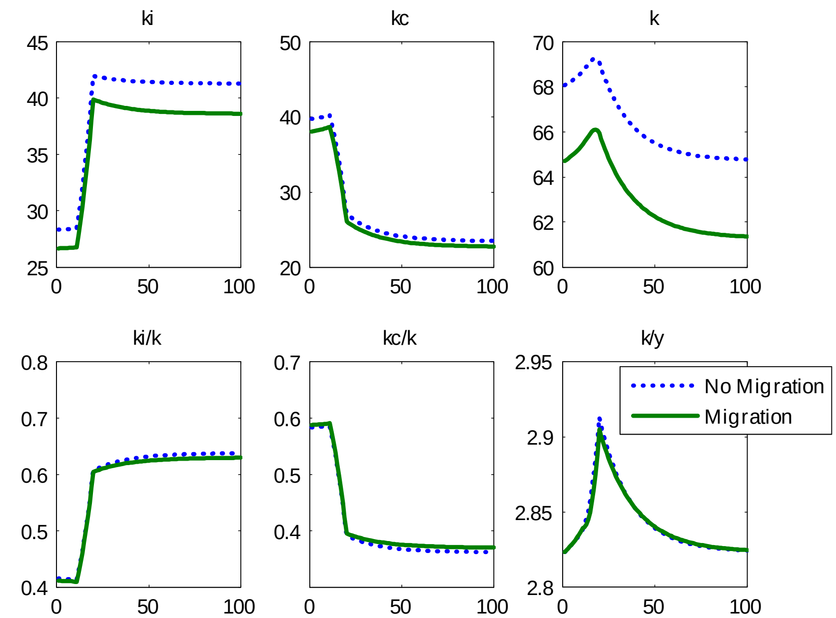
We believe that the model has many important policy implications. First of all, there is an optimal extent of intervention on subsidizing investment. When the rural area has excess supply of labor, subsidizing infrastructure sector could increase total productivity and at the same time increases welfare. However, when the migration is finished, the intervention could only increase output while reduces welfare. Second, when the government is only interested in the short-run output level, then it has strong incentive to subsidize the infrastructure sector. This helps to explain the high local government debt in China during recent years. Finally, this model also has implication on the enducation policy. We show that the expansion of college enrollment is not the main driving force for the declining wage premium.

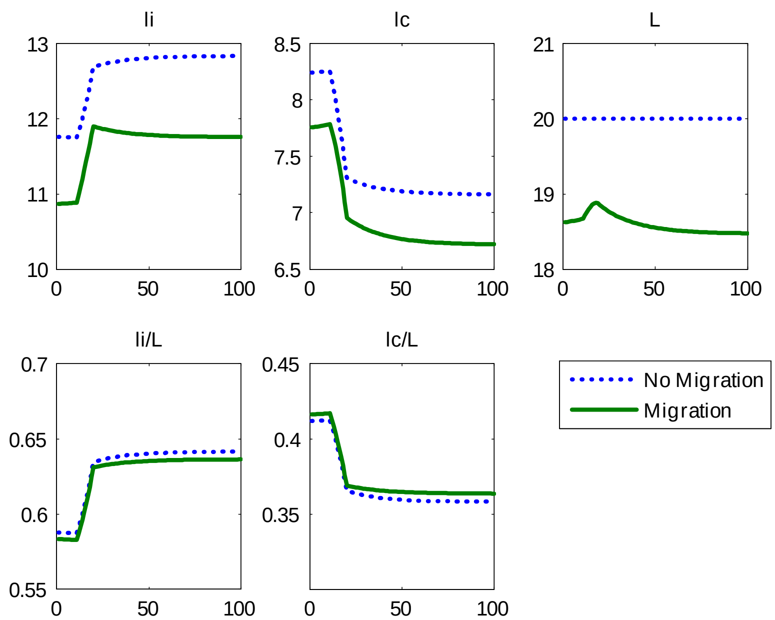
Appendix A

Here we show the full transition paths of the economy when they choose to apply the distortionary policy at period 10.






ki
kc
k
ki/k
kc/k
k/y
No Migration
Migration

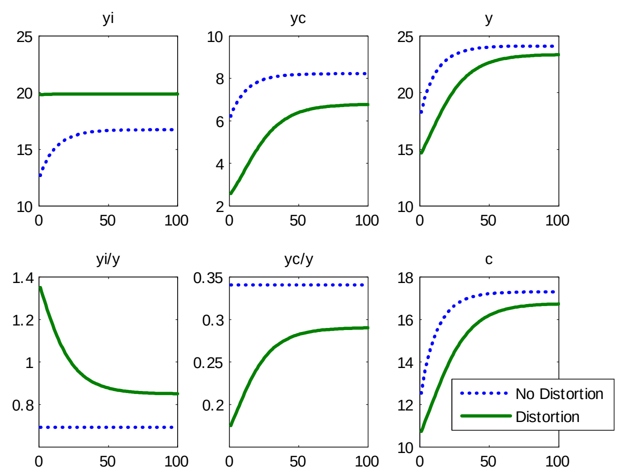

li
lc
L
li/L
lc/L
No Migration
Migration

Appendix B

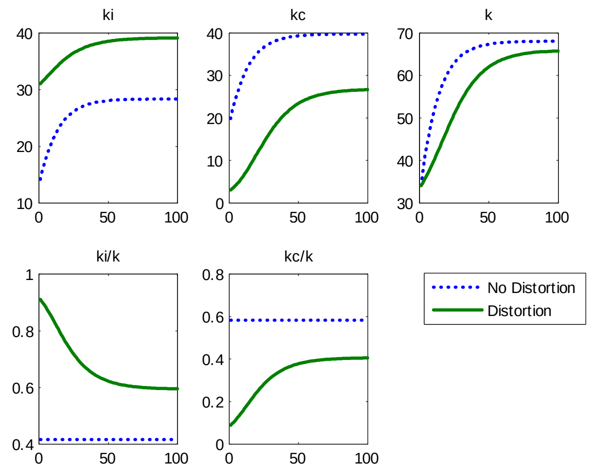
We examine the long run impact of the distortionary government policy here.

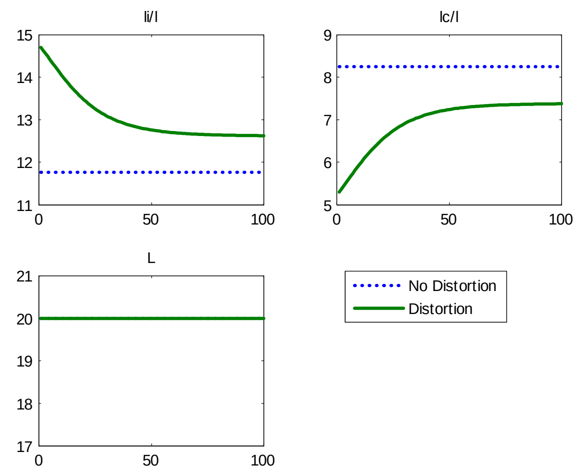
.1 Transition Dynamics without Migration

We start from an initial state where capital is only half of the steady state capital level. The graphs below shows the transitional dynamics for economies with and without distortions. With distortion in the capital market, the social welfare is lower. Welfare is -1.59 without distortion and -2.01 with distortion. In general, economy without distortion reaches a high level of consumption and output.

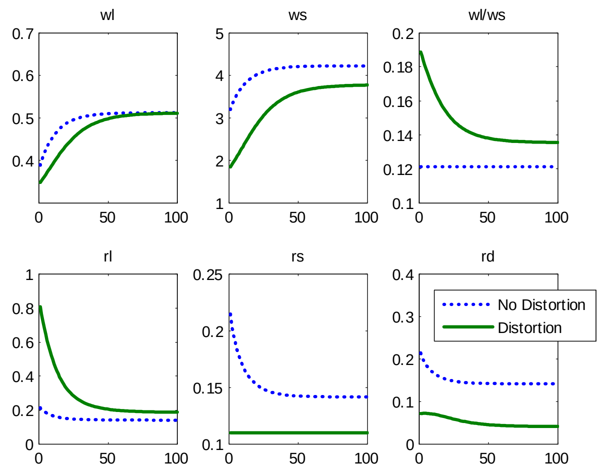


Capital and Labor Allocation With government policy distortion, infrastructure sector enjoys a low interest rate. Therefore, given the same level of initial capital stock, relatively more capital is allocated to the infrastructure sector compared to the case without any distortion. This happens for the entire transition process because the interest rate distortion is permanent. As capital is accumulated in the economy along time, the market interest rate r_l becomes lower and hence the interest rate premium between the government rate r_S and r_l is smaller. Capital will be reallocated from the infrastructure sector to the consumption sector. Unskilled labor allocation between the two sectors follows a similar pattern.



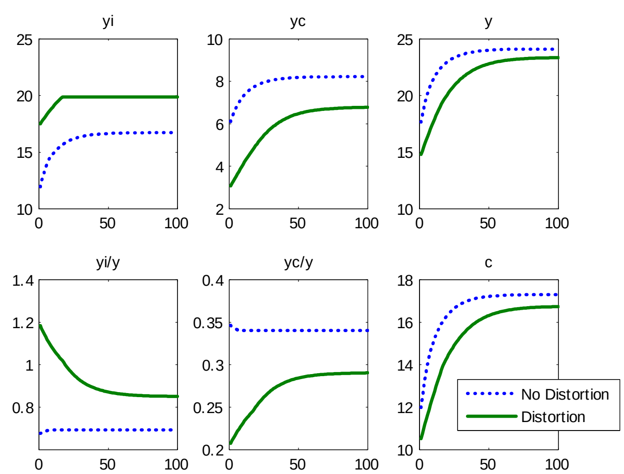


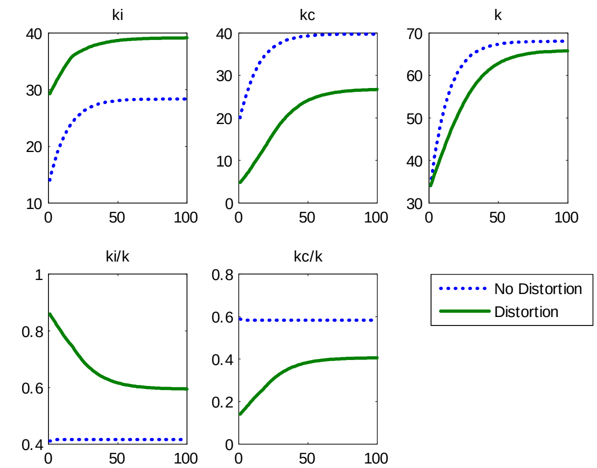
Factor Prices The wage premium is increasing for the distortionary economy as it approaches steady state. This is also caused by the reallocation of resources across sectors. As consumption sector expands, demand for skilled labor increases which drives up the wage for skilled labor and hence put a upward pressure on the wage premium.

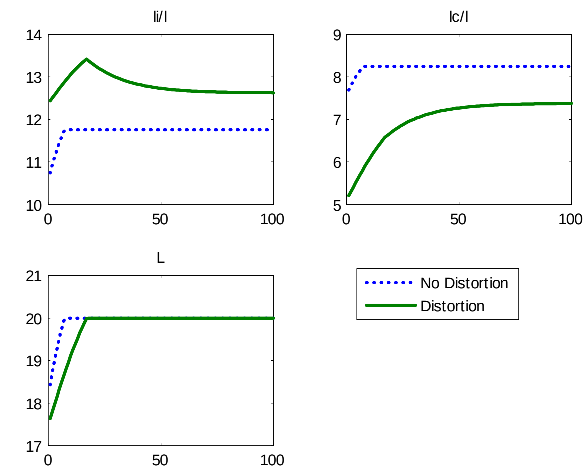


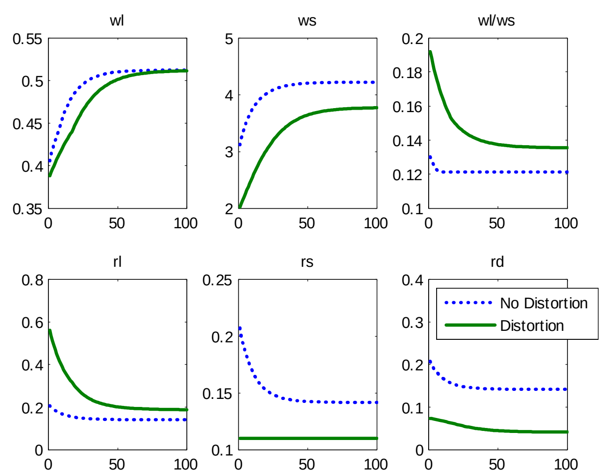
.2 Transition Dynamics with Migration

Under above parameter values, the modeling economy produces 18 periods of migration. We show that the key variables present the similar pattern as those in the economy without migration.




ki
kc
k
ki/k
kc/k
No Distortion
Distortion


li/l
lc/l
L
No Distortion
Distortion


wl
ws
wl/ws
rl
rs
rd
No Distortion
Distortion

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