Misvaluation of Investment Options

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Singapore, May 2017

Can investors value investment options correctly?

- Central question in financial economics: Can investors correctly value financial assets?
- Behavioral economists argue that various assets are systematically mispriced
- **Our goal**: Understand whether misvaluation of equities is partially due to investors' inability to correctly price investment (growth) options

Important Question

- **Important**: Growth options are the most important component of the firm value
  - Especially now, given the changing nature of the economy and the importance of growth firms: more firms in S&P 500 are growth-oriented than ever before
- But they are **difficult to value**:
  1. Difficult to project cash flows of a growth firm – think Snap Inc.
  2. Risk of the firm changes as options are exercised
- Usual valuation techniques – DCF, multiples – are inappropriate

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3. Compute estimated model values for each firm each month and misvaluation
4. Examine the characteristics of misvaluation and the relation between estimated misvaluation and future returns
   - Such a relation would suggest systematic misvaluation that is gradually corrected over time

Empirical Approach

Model Setup

- A firm is infinitely lived
- Its capital stock at time t is K_t
- It has a continuum of expansion options:
  - For a fixed price η per unit of capital it can purchase and install additional capital
- Capital depreciates at a rate λ per unit of time
- The firm's instantaneous profit is $\pi(K_t, x_t) = (1 - \tau)x_t K_t^\theta$
  - $0 < \theta < 1$ is the curvature of the production function
  - τ is the corporate tax rate
  - Demand x_t follows a geometric Brownian motion process: $dx_t = \mu x_t dt + \sigma x_t dB_t$

Optimal Investment

Solution: Firm Value

- The optimal firm value can be decomposed into 2 parts:
  - Value of Assets In Place:

$$\text{AP} = \frac{x_t K_t^{\theta}(1-\tau)}{r-\mu+\lambda\theta}$$

  - Value of Growth Options:

$$\text{GO} = \left[\frac{(1-\tau)\theta}{\beta_1\left(r-\mu+\lambda\theta\right)}\right]^{\beta_1}\left(\frac{\beta_1-1}{\eta}\right)^{\beta_1-1}\frac{K_t^{\beta_1(\theta-1)+1}}{\beta_1(1-\theta)-1}x_t^{\beta_1}$$

    - ★ subject to the condition: $\theta < 1 - \frac{1}{\beta_1}$
    - ★ β_1 depends on r, λ, θ, μ, σ.

Comparative Statics

Caveats

- The model is not very realistic
  - It ignores financing decision
  - It ignores feedback effects from misvaluation to real option exercise
  - It ignores competition among firms

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- The model is not very realistic
  - It ignores financing decision
  - It ignores feedback effects from misvaluation to real option exercise
  - It ignores competition among firms
- Lack of realism is due to two reasons:
  - We need to be able to solve the model in closed form
  - We want to see how far the simplest possible model can take us before moving to more sophisticated ones

Firm-level Data

- Capital stock, K_{it}
  - PP&E

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    - ★ The model does not accommodate firms with negative EBITDA – we do not estimate the model for these observations (22% of the sample)
- Corporate tax rate, τ
  - 35%
    - ★ Results are robust to using firm-specific John Graham's marginal tax rates

Industry-level Calibration

- Our identifying assumption is that other than capital and the state of the demand process, **firms in the industries have common characteristics**. We use Standard and Poor's Global Industry Classification Standard (GICS).

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- Drift of the demand process, μ: Based on the analysts' long-term growth (LTG) rates forecasts.
- Volatility of demand process, σ: Based on the volatility of seasonally-adjusted quarterly sales over the last 2 years.
- Depreciation rate, λ: Based on depreciation rate of PP&E over the last 5 years

Estimation I

- For given values of
  - Curvature of production function, θ
  - Cost of installing new capital, η

we compute a theoretical firm value of each firm every month, $V_{it}(\theta, \eta)$ and compare it to the firm's pseudo-market value, $\tilde{V}_{it}$, which is the sum of the value of equity and value of debt

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- Firms with ϵ_{it} above/below 1 are overvalued/undervalued

Estimation II

- θ_{jt} and η_{jt} are chosen to minimize for each industry-month mean log misvaluation in the past 12 months:

$$(\hat{\theta}_{jt}, \hat{\eta}_{jt}) = \text{argmin} \sum_{i=t}^{t-12} \sum_{k=1}^{N_{jt}} |\log \epsilon_{kt}|$$

  - This assumes industries are valued correctly on average
  - The optimization is done subject to the constraint: $\theta < 1 - \frac{1}{\beta_1}$
  - If the firm's capital is outside its optimal investment boundary, $x_{it} > X(K_{it})$, we assume that the firm immediately invests optimal amount
    - ★ This happens in 6% of cases; the results are robust to excluding these

Identification | Industry Misvaluation

Estimation results

- Curvature of production function:
  - Modest variation across industries, ranges from 0.26 to 0.46

Parameter Persistence

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- Curvature of production function:
  - Modest variation across industries, ranges from 0.26 to 0.46
- Cost of acquiring new capital
  - Ranges from 1.00 (equals the value of capital) to 1.36
  - Low estimates in IT sector, high estimates in manufacturing sector

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Estimation results

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  - Modest variation across industries, ranges from 0.26 to 0.46
- Cost of acquiring new capital
  - Ranges from 1.00 (equals the value of capital) to 1.36
  - Low estimates in IT sector, high estimates in manufacturing sector
- Reasonable estimates of mean GO/AP ratios across sectors:

Sector		θ	η	GO/AP
10	Energy	0.30	1.25	32.2%
15	Materials	0.33	1.30	3.4%
20	Industrials	0.32	1.21	11.7%
25	Consumer Discretionary	0.30	1.19	8.3%
30	Consumer Staples	0.31	1.26	9.3%
35	Health Care	0.34	1.07	73.1%
45	Information Technology	0.36	1.04	53.6%
50	Telecommunication Services	0.34	1.17	26.1%

Parameter Persistence

Estimated GO/AP ratio

- Characteristics of the estimated ratio of values of growth options to assets-in-place

Industry GO/AP decile	Industry M/B	Age	R&D	Asset growth	Leverage	Equity beta	St. Dev. returns	NASDAQ
1 (Least growth options)	1.49	33.55	0.010	0.21	0.29	1.03	2.74	0.39
2	1.53	29.38	0.010	0.27	0.28	1.05	3.01	0.40
3	1.59	33.32	0.009	0.21	0.23	1.11	3.03	0.44
4	1.42	34.14	0.020	0.19	0.25	1.03	2.90	0.34
5	1.50	29.50	0.013	0.23	0.25	1.09	3.18	0.46
6	1.71	24.21	0.030	0.26	0.23	1.18	3.28	0.55
7	1.76	23.16	0.034	0.28	0.24	1.21	3.24	0.54
8	1.98	21.83	0.063	0.36	0.19	1.34	3.32	0.63
9	2.06	20.80	0.045	0.34	0.20	1.19	3.27	0.59
10 (Most growth options)	2.45	16.89	0.083	0.44	0.15	1.38	3.52	0.70

Misvaluation Measure

- Characteristics of the estimated misvaluation measure

Misvaluation decile	MV equity	Age	R&D	Asset growth	Invest-ment	Profit-ability	6-month return	St. Dev. of returns	Inst owner	Number analysts	Forecast disp
U	1,853	20.09	0.039	0.296	0.055	0.122	-5.51	3.56	0.44	3.90	0.122
2	2,241	26.70	0.026	0.206	0.053	0.016	-1.32	3.18	0.51	4.17	0.084
3	2,242	29.45	0.023	0.190	0.053	0.016	2.04	2.96	0.53	4.44	0.075
4	2,376	31.72	0.023	0.197	0.054	0.015	4.58	2.84	0.56	4.47	0.063
5	2,382	32.46	0.024	0.206	0.057	0.017	6.96	2.81	0.57	4.67	0.067
6	2,390	32.57	0.025	0.225	0.063	0.020	8.97	2.81	0.58	4.76	0.064
7	2,513	30.63	0.027	0.261	0.075	0.028	10.87	2.89	0.57	4.79	0.055
8	2,473	26.95	0.030	0.314	0.089	0.037	13.47	3.05	0.57	4.75	0.054
9	2,072	21.59	0.039	0.388	0.104	0.052	16.28	3.38	0.54	4.69	0.056
O	1,114	15.25	0.052	0.474	0.114	0.018	17.27	3.93	0.47	3.91	0.099

Misvaluation, Growth Options, and Future Returns

Hypotheses

If:

- Investors cannot price growth firms, **AND**
- It leads to misvaluation in equity markets, **AND**
- Misvaluation is temporary and is gradually corrected

Then:

- **H1a:** Undervalued firms are expected to have higher future returns
- **H1b:** Misvaluation should get corrected at reasonable horizons
- **H2b:** The difference in future returns is larger for GO firms than for AP firms
- **H2a:** The difference in future returns would not exist in a counterfactual model that does not account for the value of GOs

H1a: Future returns

Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
1 (Most undervalued)	1.05	0.38	0.36	0.44	0.63	0.69	0.90	0.90
	(3.45)	(1.96)	(1.85)	(2.25)	(3.24)	(3.54)	(4.67)	(4.66)
10 (Most overvalued)	0.15	-0.69	-0.43	-0.53	-0.19	-0.28	-0.41	-0.41
	(0.40)	(-3.22)	(-2.08)	(-2.55)	(-0.92)	(-1.39)	(-1.85)	(-1.86)
Difference 1-10	0.90	1.07	0.79	0.97	0.82	0.98	1.31	1.31
	(1.91)	(3.70)	(2.79)	(3.40)	(2.89)	(3.45)	(4.46)	(4.48)

Robustness | Factor Loadings

H1a: Fama-MacBeth Regressions

Intercept	0.90	0.90	1.06
	(2.69)	(3.35)	(3.24)
log(B/M)	0.36		0.23
	(5.53)		(3.59)
log(ME)	-0.05		-0.07
	(-1.42)		(-2.06)
Investment	-0.99		-0.88
	(-6.76)		(-6.02)
Profitability	0.53		0.35
	(4.37)		(2.70)
Return [-1,0)	0.01		0.01
	(5.13)		(5.47)
Return [-12,-1)	-0.04		-0.04
	(-8.60)		(-8.48)
Log (misvaluation)		-0.45	-0.38
		(-6.95)	(-7.34)

H1b: Misvaluation is corrected over time

		Forward misvaluation decile									
Horizon	Misvaluation decile	1	2	3	4	5	6	7	8	9	10
1 month	1	**0.82**	0.12	0.02	0.01	0.01	0.01	0.00	0.01	0.01	0.01
	10	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.08	**0.87**
3 months	1	**0.67**	0.16	0.04	0.03	0.02	0.02	0.01	0.01	0.02	0.02
	10	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.04	0.14	**0.73**
6 months	1	**0.53**	0.17	0.07	0.04	0.03	0.03	0.03	0.03	0.03	0.04
	10	0.03	0.02	0.02	0.02	0.03	0.03	0.04	0.07	0.18	**0.54**
1 year	1	**0.37**	0.17	0.09	0.07	0.05	0.05	0.05	0.05	0.05	0.06
	10	0.04	0.04	0.04	0.04	0.05	0.06	0.08	0.12	0.21	**0.32**
2 years	1	**0.26**	0.15	0.10	0.08	0.07	0.06	0.06	0.06	0.07	0.07
	10	0.06	0.06	0.06	0.06	0.07	0.08	0.09	0.13	0.18	**0.20**
3 years	1	**0.22**	0.14	0.11	0.09	0.08	0.07	0.07	0.07	0.08	0.07
	10	0.07	0.08	0.07	0.08	0.08	0.09	0.10	0.12	0.16	**0.15**

H2a: GO/AP Split

GO/AP tercile	Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
Lowest	Most undervalued	1.03	0.36	0.16	0.34	0.17	0.33	0.50	0.49
		(3.08)	(1.51)	(0.68)	(1.50)	(0.73)	(1.44)	(1.97)	(2.02)
	Most overvalued	0.72	-0.06	0.12	-0.03	0.17	0.04	-0.20	-0.19
		(1.31)	(-0.12)	(0.25)	(-0.06)	(0.35)	(0.07)	(-0.38)	(-0.38)
	Difference	0.32	0.42	0.04	0.37	0.00	0.30	0.69	0.69
		(0.49)	(0.79)	(0.07)	(0.69)	(-0.00)	(0.54)	(1.21)	(1.21)
Highest	Most undervalued	1.11	0.42	0.51	0.59	0.83	0.88	1.05	1.05
		(3.29)	(1.78)	(2.16)	(2.46)	(3.48)	(3.69)	(4.48)	(4.48)
	Most overvalued	0.21	-0.62	-0.27	-0.34	0.07	0.01	-0.26	-0.26
		(0.54)	(-2.51)	(-1.18)	(-1.44)	(0.32)	(0.02)	(-1.04)	(-1.04)
	Difference	0.91	1.04	0.79	0.93	0.75	0.88	1.31	1.31
		(1.78)	(3.04)	(2.37)	(2.76)	(2.29)	(2.65)	(3.82)	(3.82)

H2a: M/B Split

M/B tercile	Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
Lowest	Most undervalued	1.17	0.58	0.41	0.53	0.34	0.45	0.58	0.57
		(3.99)	(2.78)	(2.12)	(2.75)	(1.73)	(2.29)	(2.65)	(2.70)
	Most overvalued	0.61	-0.16	0.01	-0.18	0.04	-0.13	-0.31	-0.31
		(1.12)	(-0.33)	(0.03)	(-0.37)	(0.08)	(-0.27)	(-0.62)	(-0.61)
	Difference	0.56	0.74	0.40	0.70	0.30	0.58	0.89	0.88
		(0.91)	(1.42)	(0.78)	(1.38)	(0.58)	(1.11)	(1.61)	(1.61)
Highest	Most undervalued	1.01	0.33	0.43	0.55	0.74	0.84	0.97	0.97
		(3.00)	(1.38)	(1.81)	(2.30)	(3.12)	(3.51)	(4.11)	(4.10)
	Most overvalued	0.02	-0.85	-0.44	-0.48	-0.09	-0.14	-0.32	-0.32
		(0.05)	(-3.53)	(-1.98)	(-2.17)	(-0.40)	(-0.64)	(-1.33)	(-1.33)
	Difference	0.99	1.17	0.87	1.03	0.83	0.98	1.29	1.29
		(1.93)	(3.48)	(2.68)	(3.16)	(2.58)	(3.03)	(3.84)	(3.84)

H2b: Assets-in-Place-Only Model

Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
1 (Most undervalued)	1.15	0.52	0.23	0.43	0.13	0.31	0.27	0.30
	(2.28)	(1.11)	(0.52)	(0.94)	(0.29)	(0.67)	(0.55)	(0.64)
10 (Most overvalued)	0.63	-0.28	-0.16	-0.21	0.48	0.42	0.27	0.26
	(1.06)	(-0.55)	(-0.32)	(-0.39)	(0.93)	(0.80)	(0.50)	(0.49)
Difference 1-10	0.52	0.80	0.39	0.64	-0.35	-0.10	0.00	0.04
	(0.67)	(1.16)	(0.58)	(0.92)	(-0.50)	(-0.15)	(-0.00)	(0.05)

Conclusion

Conclusion

- Investors misprice growth-oriented firms because **GOs are hard to value**
- Important to understand **why**:
  - What tools do analysts use for these firms?
- **Policy implication**: Market participants need to be aware of this fact.
  - Investment community need to discuss more appropriate tools how to value growth firms
- Redistribution of wealth from regular investors (index funds) to more sophisticated hedge funds

Solution: Optimal Investment

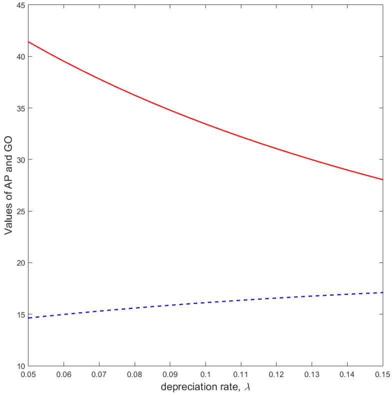
- Optimal investment policy:
- Purchase and install an infinitesimally small amount of additional capital as soon as x_t reaches investment threshold $X(K_t)$
- The investment threshold is

$$X(K_t) = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu + \lambda\theta)\eta K_t^{1-\theta}}{(1-\tau)\theta}$$

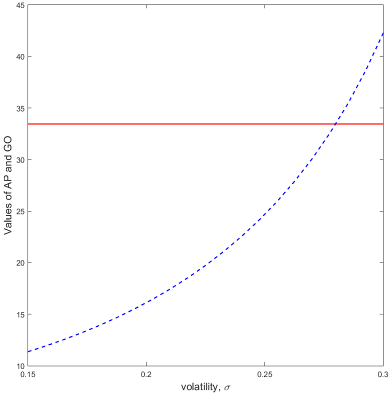
  - Increases in K_t: a better capitalized firm optimally waits longer, until a higher realization of x_t is reached

Back to Empirical Approach

Comparative statics



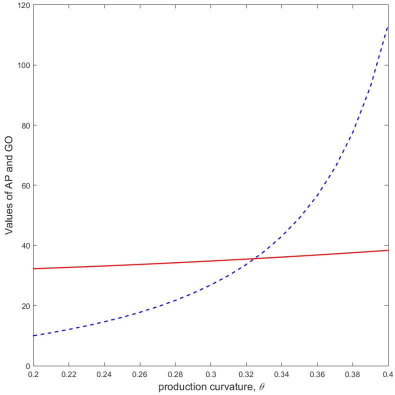
(a) GO and AP as functions of λ



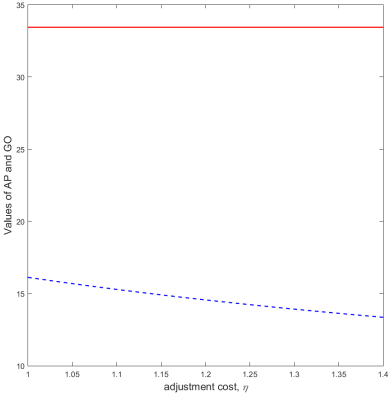
(b) GO and AP as functions of σ

Back to Firm Value

Comparative statics



(a) GO and AP as functions of θ



(b) GO and AP as functions of η

Back to Firm Value

Identification

- The cost of installing new capital, η, is identified through the value of growth options only:

$$\left[\frac{(1-\tau)\theta}{\beta_1\left(r-\mu+\lambda\theta\right)}\right]^{\beta_1}\left(\frac{\beta_1-1}{\eta}\right)^{\beta_1-1}\frac{K_t^{\beta_1(\theta-1)+1}}{\beta_1(1-\theta)-1}x_t^{\beta_1}$$

- The curvature of production function, θ, is identified from the value of assets-in-place:

$$\frac{x_t K_t^{\theta}(1-\tau)}{r-\mu+\lambda\theta}$$

as well as through the value of GOs.

Back to Estimation

Estimation: Accounting for industry misvaluation

- The assumption in the above estimation procedure is that industries are correctly priced on average at any point in time
- This is too restrictive – industries may be misvalued
- To estimate it, we follow the procedure in Rhodes-Kropf, Robinson and Viswanathan (2005)

Back to Estimation

Estimation: Accounting for industry misvaluation

- Estimate RRV regression in year t in each industry:

$$\log(\mathsf{ME}_{it}) = \alpha_0 + \alpha_1 \log(\mathsf{BE}_{it}) + \alpha_2 \log(|\mathsf{NI}_{it}|) + \alpha_3 \mathbb{I}(\mathsf{NI}_{it} < 0) + \alpha_4 \mathsf{Lev}_{it} + \epsilon_{it}$$

  - Predicted firm value from this regression is the current firm value assuming it's valued as an average firm in the industry
- Compute the same but for years $[t-5, t-1]$
  - Predicted firm value from this regression is the current firm value assuming it's valued as an average firm in the industry give the data
- Industry misvaluation in a given year is the misvaluation of a median firm in the industry in that year
- We then minimize the following objective function:

$$(\hat{\theta}_{jt}, \hat{\eta}_{jt}) = \mathsf{argmin} \sum_{\tau=t-12}^{t-1} \left(\sum_{\upsilon=1}^{N_{jt}} |\log \epsilon_{\upsilon\tau}| - \mathsf{median}(\log \epsilon_{j\tau}) \right)$$

Back to Estimation

Parameter Persistence

- The θ and η estimates are relatively stable

Lag	% abs. change in θ	% abs. change in η
1	4.8%	2.3%
3	9.5%	4.3%
6	14.4%	6.5%
12	21.0%	9.4%
24	27.0%	12.6%
36	28.3%	13.8%

Back to Estimation Results

H1a: Robustness

- Robust to various definitions of undervaluation/overvaluation
  - Quintiles of misvaluation
  - Binary classification (undervalued vs overvalued)
- Robust to equally-weighted returns
- Robust to changes in estimation assumptions
  - Estimation using only firms with above-industry-median analyst coverage
  - Estimation using SIC or NAICS industry definitions
  - Estimation using John Graham's marginal tax rates
- Robust to various return horizons (with gradual decay)
- Stronger in NBER recessions than in expansions

Back to Main Results

H1a: Factor Loadings

Model	Misvaluation Decile	MKT	SMB	HML	RMW	CMA	r_{ME}	$r_{I/A}$	r_{ROE}	MOM
FF5+MOM	Most undervalued	0.97	-0.10	0.17	-0.35	-0.46				-0.10
		(20.78)	(-1.58)	(1.96)	(-4.22)	(-3.58)				(-2.34)
	Most overvalued	1.16	0.18	-0.33	-0.47	-0.16				0.15
		(23.58)	(2.67)	(-3.65)	(-5.40)	(-1.17)				(3.42)
	Difference	-0.18	-0.29	0.51	0.12	-0.30				-0.24
		(-2.69)	(-3.02)	(3.99)	(0.99)	(-1.63)				(-4.09)
Q+MOM	Most undervalued	0.92					0.10	-0.50	-0.42	0.00
		(20.95)					(1.50)	(-4.95)	(-4.93)	(0.03)
	Most overvalued	1.21					0.18	-0.50	-0.25	0.19
		(24.40)					(2.45)	(-4.39)	(-2.62)	(3.56)
	Difference	-0.29					-0.08	0.00	-0.17	-0.19
		(-4.44)					(-0.85)	(0.02)	(-1.29)	(-2.65)

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