Trade, Formalization Cost, and the Spatial Distribution of Formal and Informal Employment: Evidence from Indonesia

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Abstract:

Informal-sector employment is pervasive in developing economy cities. The informal sector contributes little to public finance and has low productivity due to the lack of access to trade support and export market, which is available in the formal sector. We study cross-region variations in employment formalization within a country in a general equilibrium model where entrepreneurs in each region choose between the formal and informal sectors by weighing the benefit of access to export market and the cost of a local business tax. The model is built on Behrens, Duranton and Robert-Nicoud (2014) to account for (1) skill sorting across regions; (2) occupation selection in terms of employee, informal-sector entrepreneur, and formal-sector entrepreneur; and (3) agglomeration economy arising from home-market effect. We solve for equilibrium employment size and number of entrepreneurs and their skill mix in both formal and informal sectors in individual regions, subject to perfect labor mobility and national aggregate employment and skill endowment constraints. The model can account for the cross-region variations in these equilibrium employment variables observed in Indonesia during the past two decades. The counterfactual analysis shows how employment and skills shift between the employment sectors and regions in response to export improvement and business tax changes.

Key words: *Informal sector; Spatial equilibrium; Formalization cost; Export.*

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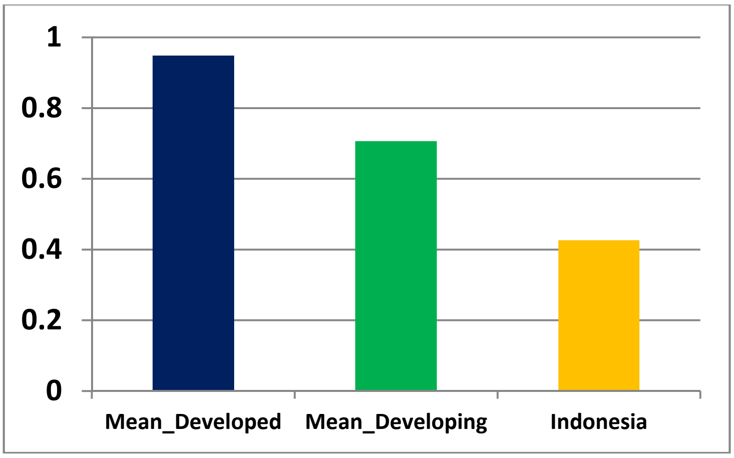
1, Introduction

Public infrastructure is significant for economic development on lowering trade cost and improving efficiency, so that the investment for skills and capital can be more profitable. Without enough infrastructure investment leads to large informal sector (Acoca, Shahana & Susan, 2014), which does not contribute to public finance (2010 UN-HABITAT) and intensifies the shortage of public service. Nearly all developing economies, especially Indonesia, the largest economy among ASEAN countries, are trapped in the high informality, though they have experienced impressive growth and rapid urbanization process in the last few decades (McKinsey Global Institute, November 2014). According to census data from IPUMS international, during the first decade of 21st Century, the average formal employment share is less than 50 percent in Indonesia. It is not only just half of the level in developed countries, but also far lagged behind by other developing economy all over the world.

Figure 1. **Formal employment share of different countries around 2000**



Source: IPUMS international

Facing such a large informal sector, however, there is still policy debate. Part of the researchers emphasize the need and positive impact of the informal sector, arguing that the informal sector is an integral part of urban economy in developing countries (J. Ihrig and K.S. Moe, 2004), because of the following three reason. Firstly, the additional low-skill jobs in the informal sector are necessary to reduce the unemployment ratio caused by the scarce of formal jobs . Secondly, informality supports urbanization through allowing the less skillful new rural immigrants to earn their living before acquiring enough human capital to seek a formal jobs (Lucas, 2004). Thirdly and the most importantly, informal firms provide low-cost goods and service, which are widely accepted and demanded by not only the low-income customers, but also the formal firms for controlling their cost (Porta and Shleifer, 2014; 2010 UN-HABITAT). However, another group of literature focuses on the inefficiency of the informal sector and the distortion caused by it. The productivity of the informal sector is extremely low, compared to the formal sector. The inefficiency of informal firms can be partially explained by their low-skill entrepreneurs and smaller capital–labor ratio (Porta and Shleifer, 2014; Paula and Scheinkman, 2011, N.A. Loayza, 2016). On the contrary, the human capital gap between formal and informal sectors is not obvious on the level of workers (Porta and Shleifer, 2014). Moreover, although the scale of informal economy is huge, informal firms are typically small (Porta and Shleifer, 2014). As a result of not paying taxes, informal firms can not participate in trade, but only sell their goods in the local market, making them unproductive and small. In addition, large informal sector lowers the quality of public service (N.A. Loayza, 1996) and distorts the policy to a greater extent (R. Arnott, 2008). Finally, informality becomes less important as the economy develops. Under the trade liberalization, the economic growth and productivity innovation mainly come from formal sector, while its low efficiency makes informal firms stagnant. (Lucas, 2009; Melitz and Redding, 2014; Perla, Tonetti and Waugh, 2014)

In order to provide new insight of the policy debate, this research put forward a coherent micro-foundation of the literature to study the informal sector. Taking Indonesia as an example, the incentive of formalization and internal labor migration are investigated to shed light on the interaction between the formal and informal employment. This paper intends to examine two hypothesis. Firstly, the development of formal economy is accompanying with expansion of the informal sector. Secondly, cities with better public infrastructure for trade are more preferred by the formal employment, especially large formal firms, rather than the informality. Learning the experience from theoretical framework in Behrens, Duranton & Robert-Nicoud (2014) and Dixit & Stiglizs (1978), a model including location sorting and occupation selection is built to analyze the personal motivation and spatial variety of informality. Taking the setting of heterogeneous skill, constant elasticity of substitution (CES) preference and monopolistic competition market as those in Behrens, Duranton & Robert-Nicoud (2014) and Melitz (2003), the difference on income motivates various occupation selection. The result of occupation selection is similar as the finding in Lucas's (1978) classical theory of entrepreneurial span of control for firms, which predicts that the talent of small-firm managers lies in the middle of the skill distribution, such that the people in the left tail become employees and the people in the right tail run bigger firms. However, the skill variation in our model is reflected on not only the firms' size, but also the choice between the formal sector and informal sector. Based on the fact of huge internal labor flow, individuals are assumed to be free mobility so as to fully understand personal reaction to local condition. Such a theoretical framework seeks to capture the interactions among several factors affecting formalization, including location fundamentals that facilitate trade, increasing return in formalization, complementary between formal and informal employment, differential incentives to participate in the formal economy across skill groups, and local public finance.

The model develops those in Behrens, Duranton & Robert-Nicoud (2014) and Dixit & Stiglizs (1978) in three aspects. Firstly, compared to the closed economy model in Behrens, Duranton and Robert-Nicoud (2014), domestic commodity market is connected to the international market, but not separated into a number of local market. Trade liberalization stimulates the interaction of the formal and informal sector, because it has been considered as an important driving force of promoting resource reallocation from non-exporters to exporters (Lucas, 2009; Perla, Tonetti and Waugh, 2014; Melitz and Redding, 2014). Secondly, there is an additional cost for the allowance to join the international trade. Since this cost is paid and only paid by the formal firms, so it is called "**formalization cost**" in this research. In the real world, formalization cost refers to not only the tax supporting public finance, but also anything bringing limit to the formal firms, like regulation, business environment and premium of land rent. The reason for having formalization cost is the shortage of public infrastructure in the developing countries like Indonesia, compared to the developed countries, so that the formal firms have to pay for using the infrastructure. Formalization cost distinguishes entrepreneurs between formal and informal, because the informal firms with lower marginal production would like to stay in the informal sector to control the fixed cost (Melitz and Redding, 2014). Thirdly, since the scale of international trade is much larger than domestic trade across regions, the model does not analyze inter-city trade separately. By doing so, the computation becomes much easier and the implication of local trade service turns to be more obvious. Unlike the standard iceberg cost in classical NEG model in Fujita, Krugman and Venables' book (2001), trade cost in our model depends on local trade efficiency (Cosar & Demir, 2016), but not the distance from destination, especially when discussing the international trade.

Our theoretical model expands the research of informality. Departing from the literature in the same field before, this paper studies the informality from urban economics perspective by examining the individuals' incentive to be formal and variation of informality across cities (or regions) in an open and emerging economy. In the previous paper, the policy makers and researchers feel interested in informal sector on the policy-level, discussing what is the optimal size of informality or which policy is the best one to control its size (N.A. Loayza, 1996; A. De. Paula and J.A. Scheinkman, 2011; J. Ihrig and K.S. Moe, 2004). However, this group of papers ignores the interaction between informality and other factors, such as trade, labor migration and the development of formal economy, which are included in our theory. Loayza (2016) succeeds in linking informality to labor migration and economy growth. Its research goal is similar to this paper, but focusing on the lower capital-labor ratio in the informal sector. In Loayza's model, formality and informality are exogenously given, rather than the endogenous outcome. With Cobb-Douglas production function in both sectors, the formal firms are set to suffer higher labor cost, while the informal firms borrow capital with higher interest rate. As a result, under the assumption of perfect competitive market structure, the endogenous capital-labor ratio employing by the formal sectors are higher. Compared to Loayza's work, this paper makes new achievement. Firstly, with monopolistic competition and heterogeneous skill setting, we can identify the factors motivating and preventing the labor force entering the formal sector. Secondly, with spatial sorting under free mobility assumption, we are able to explain the distribution pattern of the formal and informal sector across regions based on the difference of local trade infrastructure and formalization cost.

The model in this paper also contributes to the theory of the relation among trade, development, labor migration and heterogeneous firms. On the growth dimension of the NEG model in previous literature, the typical case is production innovation or capital accumulation (Desmet & Rossi-Hansberg, 2010). To be different, our theory emphasizes the role of public infrastructure, which motivates investment in the formal sector by lowering their trade cost. Moreover, unlike innovation and capital accumulation, the impact of better public infrastructure can be diffused through trade, but not just within local market in the classical city size model. Melitz and Redding (2014) studies trade and behavior of heterogeneous firms in the background of U.S.. Under the monopolistic competitive market structure, they come to a conclusion that the firms serving as exporters are more competitive. Firms decide whether to join export sector based on the tradeoff between benefit from external market and transportation cost in ice-berg form. However, their model can not apply in developing countries without enough infrastructure, because it fails to capture the reaction of firms to local public goods and tax. Also, they ignore the spatial pattern of exporters and non-exporters. This paper compensates Melitz and Redding's (2014) work by focusing on the local formalization cost and trade efficiency's impact on spatial variety. Ma and Tang (2016) analyzes the welfare effect of the internal migration in China. There are tradable and non-tradable sector in each cities, and the productivity of each firm is unknown before entering anyone sector. Only after paying operation cost to enter one sector, then the productivity is randomly driven from one Pareto distribution across all the locations. The uncertainty in productivity can not explain the preference of people with various human capital to occupation selection, and the same skill distribution all over the country fails to describe the location preference of different sectors. Our work improves Ma and Tang's (2016) paper with the spatial sorting based on knowledge to personal productivity, similar as those in Behrens, Duranton and Robert-Nicoud (2014).

The model predicts that the city with better trade infrastructure or lower formalization cost, not only attracts workers, but also has a more than proportionately larger number of formal firms. As the local formal economy develops, enterprises in both formal and informal sectors become more profitable, so that more enterprises move in to pursue higher revenue. Taking various methods to become efficient city, however, bringing different result on the spatial distribution pattern. The efficient city with more efficient trade service has higher concentrate extent and larger average size of formal firms. On the other hand, lessening the requirement for the formal sector leads to decreasing formal employment share and smaller average size of formal firms. Based on Indonesia census data from 2000 to 2010, National Labor Force Survey of Indonesia during 1995 to 2015 and other macro data from CEIC dataset, this paper investigates formality and informality on province, county or even individual level, rather than the country level in Porta and Shleifer (2014). The empirical findings are broadly consistent with the propositions in the model. The overall employment size and the total number of firms are found to be positively correlated across locations, showing the same location preference as the model claims. Moreover, the relation between growth of formal employment share and local condition exhibits various pattern during 2005 to 2010 and 2010 to 2015. From 2005 to 2010, the informal sector expands faster in the efficient city, which is similar as what Duranton (2016) finds in Colombia, while the trend become opposite since 2010. The formalization cost in the efficient province is found to be lower from 2005 to 2010, and the trade infrastructure is better in the efficient city between 2010 and 2015, which supports the prediction from our theoretical framework. What's more, the distribution of large formal firms is the same as the expectation from the model. The quantitative estimation shows that every 1% increase of trade efficiency makes local formal-share increase by around 2%, while every 1% decrease of formalization cost causes more than 10% decrease of formal-share. As for the share of large formal firms, 1% decrease of formalization cost makes it decrease by 0.058% in the efficient city. Similarly, controlling local formalization cost, every 1% improvement on trade efficiency promotes large formal firms' share by 0.022%.

Section 2 below presents some stylized facts about formal and informal employment in Indonesia to motivate the theoretical model described and solved in section 3. Section 4 introduces the comparative static validated by further empirical evidence provided in section 5. Section 6 concludes.

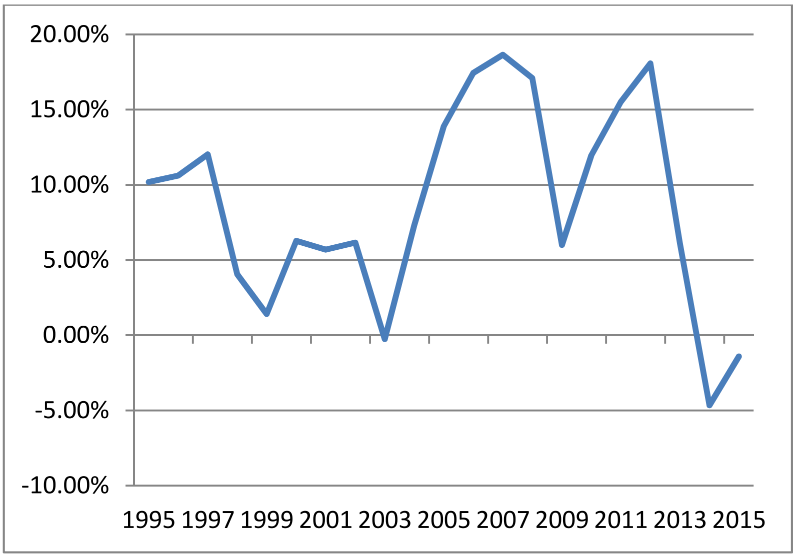
2, Stylized facts of Formality and Informality in Indonesia

The empirical facts presented here are mainly based on the National Labor Force Survey of Indonesia (NLFS) in the last two decades (1995, 2000, 2005, 2010 and 2015) from Badan Pusat Statistik (BPS), Indonesia Census Data in the first decade of 21st Century (2000,2005 and 2010) from IPUMS international and CEIC dataset from the Library of National University of Singapore. For each observation, NLFS provides personal information, such as location, education level, age, employment status, wage, and so on. Census data complements NLFS with personal migration data in the last five years and the information on county level, including urban status, quality of public service, size of employment and educated-share. Both the original census data and NLFS data are dataset on individual level, but the interviewees of various year are different. As a result, in the dynamic analysis, the smallest research unit is county, but not individual. In addition, CEIC dataset fulfills the gap on the macro data of Indonesia, for example, size of trade, provincial road statistics, provincial tax income and direct expenditure on public goods.

In this research, ***formality*** is measured with NLFS data. ***Formal workers*** are defined as wage or salary workers. ***Formal entrepreneurs*** are those with high education (at least graduated from secondary school) or who employ wage or salary workers. All the other labor force not in the formal sector are considered as working in the informal sector. Since this research focuses on employment activities, the individuals not employed or with unknown "class of work" are excluded from analysis.[1] After the adjustment, there are about 9.38 million, 0.45 million and 15.32 million observations in the 2000, 2005 and 2010 Census datasets, respectively; the sample sizes of NLFS from 1995 to 2015 are 367728, 53590, 103414, 513553 and 320344. In the census data, one observation represents 10 individuals, but it is far more than 10 in NLFS of each year and census data of 2005 for their much smaller sample size. Based on the definition of formal workers and employed labor force, ***formal-share*** refers to the share of formal workers to total employed labor force.

As described in the literature, the most obvious difference between the formal and informal sectors is the relative advantage of the formal sector in trade. In the last two decades, the growth of Indonesia's export is shown in ***Figure 2***:

Figure 2. **3-Years Moving Average Growth of Indonesia's Export,1995-2015**



Source: CEIC

Except for several years, the growth of export is positive during this period, and it is higher than 10%, even 20%, in many years. In addition, there is usually significant external shocks in the years with negative growth, such as 1998, 2001 and 2009. Noted that there is Asian Financial Crisis in 1997-1998, Crash of Dot Com Bubble around 2001 and Global Financial Crisis in 2008, so that the decrease of

[1] The observations whose "Class of work" are "Not in the universe" and "Unknown" are dropped. "Not in the universe" means that the individuals are under 10 years-old or not employed, and "Unknown" makes it impossible to identify whether they are formal or informal.

export is temporary. According to the ***Figure 2***, the 3-years moving average of export growth is always bigger than 0 before 2012. In order to capture how the formal-share varies across locations and education level, ***Table 1*** shows the formal-share of different regions and various types of labor force in Indonesia from 1995 to 2015. With the support of expanding global demand and increasing education share, formal employment share increases gradually during this period. In addition, ***Table 1*** also includes the statistics of the urbanization rate and educated share of the formal employment and informal employment. In order to prevent bias for the relative small sample size of NLFS 2000-2010, the data source of ***Table 1*** is Census data 2000-2010 and NLFS 1995, 2015. Based on ***Table 1***, the formal-share rises in the years with strong trade growth, such as 2000 to 2005, while declines in the years when growth of trade slows down, such as 2005-2010. From the forth and fifth rows, formal-share is bigger and grows faster in the urban area significantly. Moreover, the educated individuals (at least graduated from secondary school) are more likely to be formal, compared to the uneducated one. Noted that the decline of formal-share of the educated people from 2010 to 2015 is caused by the increase of educated free-lance (frequently changing employers) and unpaid workers, which belongs to informal employment. It is found in the eighth and ninth rows that the labor force who changes locations during 2000 to 2010 is more likely to be formal, compared to the people who stay in the same county.

Table 1. **Formal-Share, Urbanization-Rate and Educated-Share in Indonesia, 1995-2015**

	1995	2000	2005	2010	2015
Formal-share	39.98%	39.85%	46.04%	42.62%	50.86%
Formal-share (Jawa)	46.54%	45.08%	49.89%	41.59%	53.39%
Formal-share (Exclude Jawa)	30.26%	31.72%	39.87%	45.19%	47.48%
Formal-share (Urban)	63.15%	63.78%	66.71%	55.99%	67.82%
Formal-share (Rural)	28.48%	24.84%	33.29%	28.77%	32.44%
Formal-share (Educated)	95.76%	90.71%	94.28%	95.51%	88.48%
Formal-share (uneducated)	33.42%	26.24%	29.18%	22.58%	26.40%
Formal-share (Stay)	-	38.26%	44.78%	41.21%	-
Formal-share (Migrate)	-	62.57%	73.49%	72.23%	-
Urbanization-Rate	33.18%	38.55%	39.96%	50.87%	52.07%
Urbanization-Rate (Formal)	52.41%	61.70%	57.89%	66.83%	69.43%
Urbanization-Rate (Informal)	20.37%	23.21%	24.65%	39.02%	34.10%
Educated-share	10.53%	21.11%	25.90%	27.47%	39.40%
Educated-share (Formal)	25.23%	48.06%	53.04%	61.57%	68.54%
Educated-share (Informal)	0.74%	3.26%	2.75%	2.15%	9.24%

Source: Census data 2000-2010 and NLFS 1995, 2015

Combining growth of export in ***Figure 2***, increasing urbanization ratio and rising education share in ***Table 1***, it tells us that there is fast development of Indonesia's economy in the last two decades. Under this background, it is more and more profitable to invest in the formal sector, especially for the educated people and the urban area. However, since the development of formal economy is much relying on export, so that there is fluctuation for formal-share when the growth of global demand slows down. ***Table 1*** raises a question: what kind of location is more preferred by the growing formal sector? One potential explanation is that formalization is more likely to happen in relatively developed area, like Jawa region where the capital Jakarta locates. But it is not supported by the result in the second and third rows in ***Table 1*** that formal-share even increases faster in other regions of Indonesia than Jawa. Another hypothesis is that the location sorting decision of formal firms is depending on local conditions, like trade infrastructure and additional cost to be formal, because lower entrance requirement and better infrastructure for trade, such as road and ports, encouraging formality through making it more profitable. We leave this hypothesis to the model section.

Table 2. **Formal and informal employment sizes across counties, 2005-2015**

Note: "F-worker" and ""In-worker" refers to formal and informal employment, respectively

	F-worker 05	In-worker 05	F-worker 10	In-worker 10	F-worker 15	In-worker 15
F-worker 05	1	\	\	\	\	\
In-worker 05	0.6394	1	\	\	\	\
F-worker 10	0.9028	0.5041	1	\	\	\
In-worker 10	0.7042	0.8966	0.6794	1	\	\
F-worker 15	0.8962	0.5267	0.9693	0.7230	1	\
In-worker 15	0.7294	0.8586	0.6944	0.9618	0.7694	1

Source: NLFS.

Previous research shows that the informal sector is an integral part of urban economy (J. Ihrig and K.S. Moe, 2004; Lucas, 2004; Porta and Shleifer, 2014; 2010 UN-HABITAT). Thus the reaction of the informal sector to the expanding formal sector is to be further studied. The rapidly increasing urbanization rate from 33.18% to 52.07% in the last two decades, and the accompanying huge internal labor flow among regions, which is more than ten million from 2000 to 2010, making Indonesia be a good example to study the spatial variation of formality. Using census data and NLFS of Indonesia, some evidence about the jointly distribution pattern of the two sectors have been found. Firstly, the size of both sectors on county level are positively correlated, no matter in the same period or in the long-term, according to ***Table 2***. Secondly, the correlation between formal employment and informal employment is becoming bigger and bigger as time goes by. Combining these two pattern, it is found that the formal sector and informal sector intend to grow together. Thirdly, during 2005-2010, informal sector is observed to follow the formal sector, for cor(F-worker 05, In-worker 10) is much bigger than cor(In-worker 05, F-worker 10) (0.7042 versus 0.5041). However, this trend is weaken between 2010 to 2015, because cor(F-worker 10, In-worker 15) turns to be smaller than cor(In-worker 10, F-worker 15) (0.6944 versus 0.7230). The variation of this trend implies that the factors which affects spatial distribution, such as local conditions and global demand, are changing in the period of 2005 to 2015.

Tables 3 shows the estimated Zipf's coefficients of the county employment size distribution from 1995 to 2015.[2] There are two groups of estimates, one for the whole county, another for the urban area. The estimates are derived from the regression of log county employment size on log rank. A smaller absolute value of Zipf's coefficient means more uneven employment size distribution across counties.

Table 3. **Zipf's coefficient of employment size distribution across top 100 counties, 1995-2015**

	Labor	Labor (urban)	Formal	Formal (urban)	Informal	Informal (urban)
Log-Rank 1995	-1.491	-0.990	-1.135	-0.921	-1.349	-1.040
Log-Rank 2000	-2.073	-1.202	-1.071	-1.095	-1.688	-1.138
Log-Rank 2005	-2.240	-1.267	-1.293	-0.996	-1.668	-1.384
Log-Rank 2010	-1.423	-0.920	-0.947	-0.807	-1.301	-1.017
Log-Rank 2015	-1.356	-0.874	-0.977	-0.785	-1.197	-0.941

Source: NLFS.

[2] ***Table 3*** is estimated based on the largest 100 counties each year.

Two patterns can be observed from ***Table 3***. Firstly, comparing to the whole county, concentration trend is more significant in the urban area. Secondly, the concentration degree is higher in the formal sector than informal sector. This characteristic is also supported by the spatial distribution of employment shown in ***Figure 3***,[3] because the formal sector consistently has a flatter upper tail than the informal sector. The first pattern implies that difference in local conditions between urban area and rural area, such as transportation networks and trade infrastructure, significantly affecting location choice of firms, especially for the formal firms. The second pattern shows that the formal firms are more willing to concentrate with each other, possibly for the benefit from agglomeration economies. The reason for these two patterns are tested and discussed further by the theoretical model.

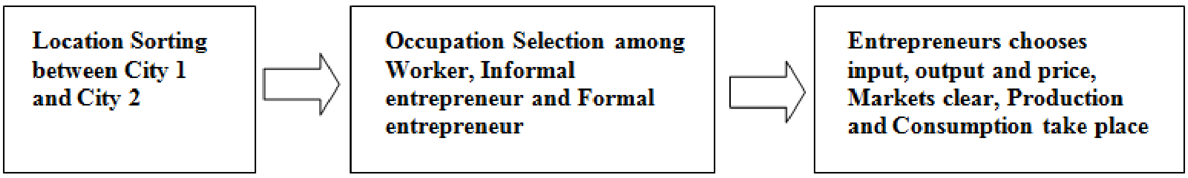
3, The Model

3.1 Model Setting

In this section, we present a stylized open-city model to show the influence of two key policy variables, namely the trade cost and formalization cost, on the general equilibrium formal employment share and the spatial distribution of formal and informal employment across cities.

Endowment and Sequences of location and occupation choices

Figure 4. **Sequences of location and occupation choices**



We consider two cities with inelastic supply of housing in an open economy, varying on exogenous and independent trade efficiency $\boldsymbol{A}$ and formalization cost $\boldsymbol{\theta}$. Each trading with the world market subject to a city-specific trade cost, which is negatively relating to the local trade efficiency $\boldsymbol{A}$. Trade efficiency depends on city location as well as on the city's investment in infrastructure, such as ports and trade-support institutions. The economy is endowed with a continuum of N units of utility-maximizing individuals with heterogeneous skills $\boldsymbol{\varphi}$, such that $\boldsymbol{\varphi} \in [\boldsymbol{\varphi_m}, +\infty]$. The distribution of skill is summarized by the continuously differentiable cumulative Pareto distribution functions $\boldsymbol{G}(\boldsymbol{\varphi})$ over $[\boldsymbol{\varphi_m}, +\infty]$.[4] Every agent initially knows his talent $\boldsymbol{\varphi}$, and sorts for a place to locate accordingly. We assume free labor mobility, but no jointly migration with ex-ante appointment. After making location decision, each individual then selects occupation: worker, informal or formal entrepreneur based on his skill, to maximize personal utility. With a job of worker, everyone supplies one unit of labor, no matter how high his skill level is, and the income depends on the endogenous local wage rate $\boldsymbol{w}$. When working as entrepreneurs, people receive firms' profit as income and their personal productivity is the marginal production of labor, following the framework in Behrens, Duranton & Robert-Nicoud (2014) and Lucas (1978). Being different with informal firms which can not participate in trade, formal firms

[3] ***Figure 3*** is shown in Appendix A.

[4] The probability density function of the Pareto distribution is $\boldsymbol{g}(\boldsymbol{\varphi}) = (\boldsymbol{k}\boldsymbol{\varphi}_m^k)/\boldsymbol{\varphi}^{k+1}$, where $\boldsymbol{\varphi_m} > 0$ is the lower bound of the skill's interval. The parameter $\boldsymbol{k}$ is bigger than 1, guaranteeing that the mean exists.

join trade with a fixed cost. After spatial sorting and occupation selection, entrepreneurs maximize their profit by choosing input, output and price, markets clear, and then production and consumption take place. The sequence of location and occupation choices is exhibited as ***Figure 4***, and the model will be solved backwards.

Population Structure

The size of both cities are endogenous. In the equilibrium, each person should live in one of these two cities. Total population $\boldsymbol{N}$, local population $\boldsymbol{N_c}$ and distribution of productivity $\boldsymbol{g}(\boldsymbol{\varphi})$ in the economy satisfy equation (1) and (2):

$$\boldsymbol{N_c} = \int_{\varphi_m}^{+\infty} \boldsymbol{N_c}(\boldsymbol{\varphi})\, \mathrm{d}\boldsymbol{\varphi} \tag{1}$$

$$\boldsymbol{N}\boldsymbol{g}(\boldsymbol{\varphi}) = \sum_{c=1,2} \boldsymbol{N_c}(\boldsymbol{\varphi}) \tag{2}$$

Equation (1) means that local population can be decomposed into groups differ in productivity. Equation (2) states that the total amount of people with productivity $\boldsymbol{\varphi}$ in the economy is equal to the mass of individual with $\boldsymbol{\varphi}$ between these two cities. Adding up equation (2) across the set of skill leads to $\boldsymbol{N} = \boldsymbol{N_1} + \boldsymbol{N_2}$, which satisfies the full population condition that all agents live in these two cities.

The personal income is discounted by the "labor wedge" $\boldsymbol{\tau}$ from urban friction as those in Desmet & Rossi-Hansberg (2013), which incurs deadweight loss of welfare. The labor wedge $\boldsymbol{\tau_c}$ can be written as:

$$\boldsymbol{\tau_c} = \boldsymbol{b}\boldsymbol{N_c}^{1/2} \tag{3}$$

Where $\boldsymbol{b}$ is a exogenous parameter relates to the commuting cost per mile within city, according to Desmet & Rossi-Hansberg (2013). Higher $\boldsymbol{b}$ is, faster the labor wedge $\boldsymbol{\tau_c}$ increases with local labor size. For simplicity, $\boldsymbol{b}$ is assumed to be homogeneous across goods.

Consumers

In both cities, agents consume all the varieties of goods in the market. Consumers can purchase not only goods produced by local firms, formal or informal, but also goods shipped from the rest of world. Both consumers and producers require one unit of land for accommodation or production, where the land rent is standardized to be zero[5], and do not increase their utility or output by consuming more land. Individuals are risk-neutral so that their personal utility are proportional to consumption bundle, which is equal to nominal income divided by local price index. The elasticity of substitution for any two goods is constant in the personal utility function, which is equal to $(1 + \frac{1}{\varepsilon})$, with $\boldsymbol{\varepsilon} > 0$. Let subscript $\boldsymbol{F}$ and $\boldsymbol{I}$ indicate the formal and informal sectors, respectively, $\boldsymbol{u_c}(\boldsymbol{\varphi})$ and $\boldsymbol{y_c}(\boldsymbol{\varphi})$ indicate the utility and the income of a worker with skill $\boldsymbol{\varphi}$ in city $\boldsymbol{C}$, $\boldsymbol{u_c}(\boldsymbol{\varphi})$. The utility and the budget constraint are given by:

[5] The zero land rent is not only the necessary condition for free mobility of labor and inelastic supply of housing, but also capturing the fact of extremely low land rent in developing countries. This fact is reflected by the extremely high ownership ratio of housing all over Indonesia, which is above 82% during 2005 to 2010, according to the census data. As mentioned in Lucas (2002), Desmet & Rossi-Hansberg (2013) and Behrens, Duranton & Robert-Nicoud (2014), production takes place in the center of cities, so that there is potential premium on land rent for the formal sector. Without losing generality, this model includes the premium in $\boldsymbol{\theta}$, the local additional cost for the formal sector.

$$\boldsymbol{u_c}(\boldsymbol{\varphi}) = \left(\int_{(\boldsymbol{\Omega_{cI}}+\boldsymbol{\Omega_{cF}}+\boldsymbol{\Omega_w})} \boldsymbol{x}(\boldsymbol{j})^{\frac{1}{1+\varepsilon}} \,\mathrm{d}\boldsymbol{j}\right)^{1+\varepsilon} \tag{4}$$

$$(1-\boldsymbol{\tau_c})\boldsymbol{y_c}(\boldsymbol{\varphi}) = \int_{(\boldsymbol{\Omega_{cI}}+\boldsymbol{\Omega_{cF}}+\boldsymbol{\Omega_w})} \boldsymbol{p_c}(\boldsymbol{j})\boldsymbol{x}(\boldsymbol{j}) \,\mathrm{d}\boldsymbol{j}$$

where $\boldsymbol{\Omega_{cI}}$, $\boldsymbol{\Omega_{cF}}$ are endogenous set of local informal sector and local formal sector, while $\boldsymbol{\Omega_w}$ is exogenous set of the imported goods from the world which is used to keep the trade account balance. For simplicity, assuming that the import good with price $\mathbb{P}_{\boldsymbol{w}}$ in the international market, which is exogenous given global price index. $\boldsymbol{y_c}(\boldsymbol{\varphi})$ is nominal income of the agents with skill $\boldsymbol{\varphi}$ in city $\boldsymbol{c}$ and spent only in the consumption goods. $\boldsymbol{x}(\boldsymbol{j})$ stands for personal demand of goods $\boldsymbol{j}$, and $\boldsymbol{p_c}(\boldsymbol{j})$ is the price of goods $\boldsymbol{j}$ in city $\boldsymbol{c}$.

To maximize personal utility, based on the utility function equation (4) and the budget constrain, demand for goods $\boldsymbol{j}$ of an agent with skill $\boldsymbol{\varphi}$ in city $\boldsymbol{c}$ is

$$\boldsymbol{x}(\boldsymbol{j}) = \left[\frac{\boldsymbol{p_c}(\boldsymbol{j})}{\mathbb{P}_{\boldsymbol{c}}}\right]^{-\frac{1+\varepsilon}{\varepsilon}} \frac{(1-\boldsymbol{\tau_c})\boldsymbol{y_c}(\boldsymbol{\varphi})}{\mathbb{P}_{\boldsymbol{c}}} \tag{5}$$

Hence the indirect utility is equal to

$$\boldsymbol{u_c}(\boldsymbol{y_c}(\boldsymbol{\varphi})) = \frac{(1-\boldsymbol{\tau_c})\boldsymbol{y_c}(\boldsymbol{\varphi})}{\mathbb{P}_{\boldsymbol{c}}} \tag{6}$$

Where

$$\mathbb{P}_{\boldsymbol{c}} = \left(\int_{(\boldsymbol{\Omega_{cI}}+\boldsymbol{\Omega_{cF}}+\boldsymbol{\Omega_w})} \boldsymbol{p_c}(\boldsymbol{j})^{-\frac{1}{\varepsilon}} \,\mathrm{d}\boldsymbol{j}\right)^{-\varepsilon} \tag{7}$$

$\mathbb{P}_{\mathrm{c}}$ is the price index of all goods can be access to in city $\boldsymbol{c}$, including oversea goods, local formal and informal goods.

Producers

All firms are monopolistically competitive, having only one manager and producing only one variety goods. The only input for each firm, formal or informal, is labor, and the marginal production is equal to the manager's skill. Compared to the informal firms producing and selling locally, formal firms sell their goods in both local and international market with different prices, because of loss in transportation, which is determined by $\boldsymbol{A_c}$. $\boldsymbol{A_c}$ is trade efficiency in city $\boldsymbol{c}$, where $\boldsymbol{A_c} \in (0,1)$. The higher $\boldsymbol{A_c}$ is, the lower trade cost for local export and import will be. For every unit of formal goods sent from city $\boldsymbol{c}$, only $\boldsymbol{A_c}$ unit arrives its destination. Similarly, for every one unit of goods shipped from the world, only $\boldsymbol{A_c}$ unit arrives city $\boldsymbol{c}$. In order to obtain the same revenue, $\boldsymbol{p_c}(\boldsymbol{j}) = \boldsymbol{A_c}\boldsymbol{p_{cw}}(\boldsymbol{j})$, where $\boldsymbol{p_{cw}}(\boldsymbol{j})$ is the international price of goods produced by the $\boldsymbol{j}$'th formal firm in city $\boldsymbol{c}$. The only input for each firm, formal or informal, is labor. Output of the $\boldsymbol{j}$'th firm in city $\boldsymbol{c}$, $\boldsymbol{X_c}(\boldsymbol{j})$, is equal to[6]

$$\boldsymbol{X_c}(\boldsymbol{j}) = \boldsymbol{\varphi_c}(\boldsymbol{j})\boldsymbol{l_c}(\boldsymbol{j}) \tag{8}$$

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[6] With a production function in the form of equation (8) like those in Behrens, Duranton & Robert-Nicoud (2014), the setting of heterogeneous firms in a constant-elasticity-of-substitution demand system is the sufficient condition for monopolistic competition market structure, because the difference in marginal production of labor prevents the case of perfect competition in the market.

Where $l_c(j)$ is the amount of workers employed by firm j in city c, and $\varphi_c(j)$ is personal productivity of its entrepreneur, taking similar form as those in Behrens, Duranton & Robert-Nicoud (2014) and Lucas (1978). The marginal cost is equal to $d(w_c l_c(j))/dX_c(j) = w_c/\varphi$, which is deceasing with φ, implying that the firms with higher skill management is more competitive for smaller marginal cost. Except for the labor cost measured by efficient labor with endogenous local wage rate w, formal firms have to suffer an additional fixed cost $\theta_c/(1-\tau_c)$. θ_c is additional cost for local formal sector, which mainly comes from tax and incurs welfare loss to the formal entrepreneurs. Although θ is modeled in a form of lump-sum tax, it interprets anything that distorts formalization choice, including regulations, banking service and premium of land rent. In addition, formalization cost automatically increases with urban friction, because the efficiency of public service declines as the city size become larger.[7] As a return, formal firms can join trade:

$$\pi_c(j) = p_c(j)X_c(j) - w_c l_c(j) - \theta_c/(1-\tau_c)\,, j \in \Omega_{cF} \tag{9}$$

$$\pi_c(i) = p_c(j)X_c(j) - w_c l_c(j), j \in \Omega_{cI} \tag{10}$$

Government

The budget constrain of government is

$$\tilde{Y}_c = \frac{\theta_c}{1-\tau_c}\int_{\Omega_{cF}} \mathrm{d}j \tag{11}$$

Where $\tilde{Y}_c$ is the total taxation income of government, equaling to the aggregate formalization cost paid by local formal firms. Government spends its taxation income in employing local and foreign labor force to maintain and improve public service and trade infrastructure. Hence $\tilde{Y}_c$ can be decomposed into two parts, $m\tilde{Y}_c$ and $(1-m)\tilde{Y}_c$, where m is the fraction of expenditure on foreign employees and homogenous across cities. $(1-m)\tilde{Y}_c$ is paid to some labor force to provide public goods. These part of labor force does not participate occupation selection and location sorting, while their utility and consumption behavior is identical. As a result, $(1-m)\tilde{Y}_c$ is indirectly transferred to the local commodity market. On the other hand, $m\tilde{Y}_c$ spent in employing foreign workers is consumed abroad, which can not be injected back into the circular flow of income and expenditure in city c.

3.2 Local production and employment mix

We first solve the workers' choice between an occupation as an employee and that as an entrepreneur given the labor size N_c and skill distribution $G_c(\varphi)$ in the city. Since the informal sector only sells its production locally, while formal firms participate in trade, the aggregate demand of the j'th formal firm and informal firm depend on their price $p_c(j)$

$$X_c(j) = p_c(j)^{-\frac{1+\varepsilon}{\varepsilon}}\left(\mathbb{P}_c^{\frac{1}{\varepsilon}}(1-\tau_c)Y_c + A_c^{\frac{1+\varepsilon}{\varepsilon}}\mathbb{P}_W^{\frac{1}{\varepsilon}}Y_W\right), j \in \Omega_{cF} \tag{12}$$

$$X_c(j) = p_c(j)^{-\frac{1+\varepsilon}{\varepsilon}}\mathbb{P}_c^{\frac{1}{\varepsilon}}(1-\tau_c)Y_c, j \in \Omega_{cI} \tag{13}$$

Where

[7] In effect, it is the government that indexes the formalization cost to urban friction.

$$\boldsymbol{Y_c} = \int_{\boldsymbol{\varphi_m}}^{+\infty} \boldsymbol{y_c}(\boldsymbol{\varphi})\,\mathrm{d}\boldsymbol{\varphi}$$

is aggregate personal disposable income in city $\boldsymbol{c}$. $\mathbb{P}_w$ and $\boldsymbol{Y_w}$ are exogenous price index and total income of the world respectively. Both $\boldsymbol{X_{cF}}(\boldsymbol{j})$ and $\boldsymbol{X_{cI}}(\boldsymbol{j})$ increase with local price index $\mathbb{P}_{\boldsymbol{c}}$, because bigger $\mathbb{P}_{\boldsymbol{c}}$ means advantage of price for all firms in local market. From equation (12) as well as (13), own-price elasticity of demand in both sectors are the same, $-\frac{1+\varepsilon}{\varepsilon}$. However, compared to the informal firms, formal firms' aggregate demand is positively related to local trade efficiency $\boldsymbol{A_c}$, global price index $\mathbb{P}_w$ and global income $\boldsymbol{Y_w}$, for the extra demand from global market. Substituting equation (12) and (13) into equation (9) and (10), yields the profit maximizing price

$$\boldsymbol{p_c}(\boldsymbol{\varphi}) = \frac{(1+\varepsilon)\boldsymbol{w_c}}{\boldsymbol{\varphi}} \tag{14}$$

As a result, the profit-maximizing price is equal to the marginal cost plus markup.

Adding up equation (14) over the set of variety of consumption goods in each city, obtaining the local price index:

$$\mathbb{P}_{\boldsymbol{c}} = \frac{(1+\varepsilon)\boldsymbol{w_c}}{\boldsymbol{\Phi_c}^{\varepsilon}} \tag{15}$$

Where

$$\boldsymbol{\Phi_c} = \boldsymbol{\Phi_{cI}} + \boldsymbol{\Phi_{cF}} + \left(\frac{A_c}{\alpha_{wc}}\right)^{\frac{1}{\varepsilon}} \boldsymbol{\Phi_w} \tag{16}$$

$$\boldsymbol{\Phi_{cI}} = \int_{\boldsymbol{\Omega_{cI}}} \boldsymbol{\varphi}(\boldsymbol{j})^{1/\varepsilon}\,\mathrm{d}\boldsymbol{j} \tag{17}$$

$$\boldsymbol{\Phi_{cF}} = \int_{\boldsymbol{\Omega_{cF}}} \boldsymbol{\varphi}(\boldsymbol{j})^{1/\varepsilon}\,\mathrm{d}\boldsymbol{j} \tag{18}$$

$$\boldsymbol{w_w} = \boldsymbol{\alpha_{wc}} \boldsymbol{w_c} \tag{19}$$

Equation (17) and (18) are the definition of sectors' productivity, hence $\boldsymbol{\Phi_c}$ is local aggregate productivity for all kind of goods supplied in city $\boldsymbol{c}$, including local and overseas variety. Rewriting equation (12) and (13) with equation (14), which holds for demand of formal firms $\boldsymbol{X_{cF}}(\boldsymbol{j})$ and informal firms $\boldsymbol{X_{cI}}(\boldsymbol{j})$

$$\boldsymbol{X_{cF}}(\boldsymbol{\varphi}) = \left(\frac{\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}}{\boldsymbol{\Phi_c}}\right)^{1+\varepsilon} \frac{(1-\tau_c)Y_c}{\mathbb{P}_c} + \left[\frac{(\alpha_{wc}A_c\varphi)^{\frac{1}{\varepsilon}}}{\boldsymbol{\Phi_w}}\right]^{1+\varepsilon} \frac{Y_w}{\mathbb{P}_w} \tag{12'}$$

$$\boldsymbol{X_{cI}}(\boldsymbol{\varphi}) = \left(\frac{\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}}{\boldsymbol{\Phi_c}}\right)^{1+\varepsilon} \frac{(1-\tau_c)Y_c}{\mathbb{P}_c} \tag{13'}$$

Noticed that endogenous $\boldsymbol{\Phi_c}$ and exogenous $\boldsymbol{\Phi_w}$ are the aggregate productivity of the firms in local and global commodity market, so that $\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}/\boldsymbol{\Phi_c}$ and $\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}/\boldsymbol{\Phi_w}$ represents the productivity share of a firm in the market. Since bigger $\boldsymbol{\Phi}$ means tougher competition, equation (12') and (13') show that a firm with higher productivity share is more competitive and able to occupy bigger proportion of the market. Holding $\boldsymbol{\Phi_w}$, $\mathbb{P}_w$ and $\boldsymbol{Y_w}$ constant, the wage rate of global labor market $\boldsymbol{w_w}$ is also exogenously given, satisfying $\mathbb{P}_w = \frac{(1+\varepsilon)w_w}{\Phi_w{}^{\varepsilon}}$.[8] Bigger wage ratio α_{wc} means local advantage in labor cost thus export price, which contributes to the increase of external demand. The item $Y_c/\mathbb{P}_c$ and $Y_w/\mathbb{P}_w$ are total demand of city c and the international market respectively, and producers benefit from a market with larger scale of demand.

Combining (12'), (13'), (14) and (15), producers' profit becomes

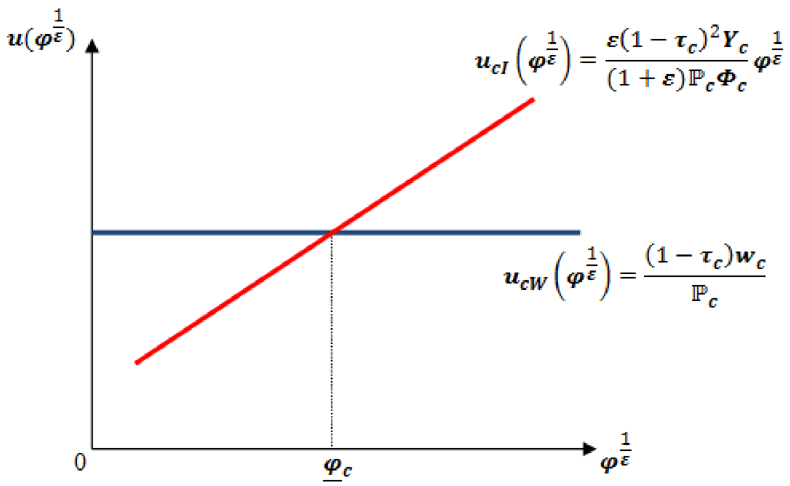
$$\pi_{cF}(\varphi) = \frac{\varepsilon}{1+\varepsilon}\left[\frac{(1-\tau_c)Y_c}{\Phi_c} + \frac{(\alpha_{wc}A_c)^{\frac{1}{\varepsilon}}Y_w}{\Phi_w}\right]\varphi^{\frac{1}{\varepsilon}} - \frac{\theta_c}{1-\tau_c} \tag{20}$$

$$\pi_{cI}(\varphi) = \frac{\varepsilon(1-\tau_c)Y_c}{(1+\varepsilon)\Phi_c}\varphi^{\frac{1}{\varepsilon}} \tag{21}$$

Similar as the implication from the demand functions, profit functions increase with global-local labor cost ratio and the total demand. What's more, higher local trade efficiency A_c not only contributes to larger revenue for the formal producers, but also implies bigger elasticity of income for the global demand Y_w. In addition, $\pi_{cF}(\varphi)$ rises not only with Y_c/Φ_c as $\pi_{cI}(\varphi)$, but also Y_w/Φ_w. Defining $R_c = Y_c/\Phi_c$ and $R_w = Y_w/\Phi_w$, noted that Φ_c is the aggregate productivity of the variety supplied in local market, while Y_c is the total consumption expenditure in city c, so that R_c can be viewed as the return for each unit of productivity in local commodity market. Similarly, after adjustment of global-local labor cost ratio α_{wc} and trade efficiency A_c, $(\alpha_{wc}A_c)^{\frac{1}{\varepsilon}}R_w$ is the return from international market for each unit of formal productivity in city c.

Individual selects occupation to maximize personal utility. When working as a worker, the labor income is equal to w_c, being independent with personal skill. To be an informal entrepreneur, on the other hand, personal income is the profit of the firm as shown in equation (21). The utility tradeoff between workers and informal entrepreneurs is exhibited in ***Figure 5***.

Figure 5. **Utility tradeoff between workers and informal entrepreneurs**



[8] The structure of global market is set to be the same as local market, so that there is no difference on the relation between parameters. However, due to the much larger size, the parameters of global market, like Φ_w, $\mathbb{P}_w$, Y_w and w_w are all exogenous.

Denoting "**entrepreneurship threshold**" $\underline{\boldsymbol{\varphi}}_c$ as the local skill threshold between informal entrepreneurs and workers, and with equation (21), yields

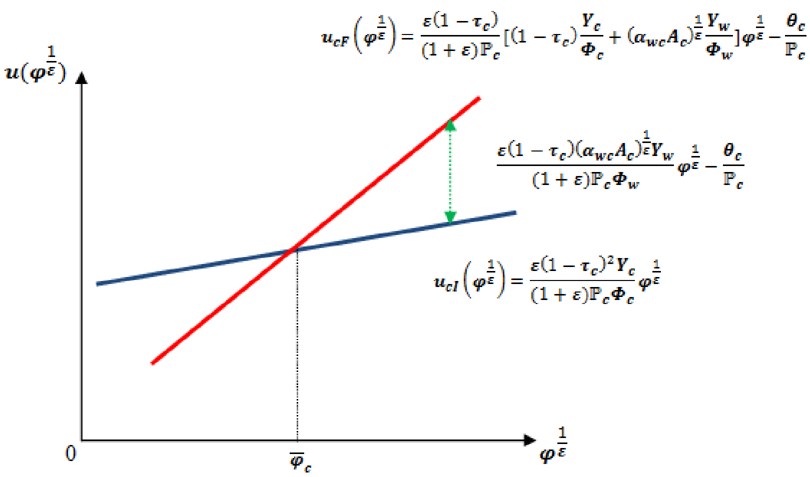
$$\underline{\boldsymbol{\varphi}}_c = \left[\frac{(1+\varepsilon)\boldsymbol{\Phi}_c \boldsymbol{w}_c}{\varepsilon(1-\boldsymbol{\tau}_c)\boldsymbol{Y}_c}\right]^{\varepsilon} \tag{22}$$

The entrepreneurship requirement becomes lower as local total income $\boldsymbol{Y}_c$ increases ($\partial\underline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{Y}_c < 0$), while higher wage rate ($\partial\underline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{w}_c > 0$), tougher local competition ($\partial\underline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{\Phi}_c > 0$) and severer urban friction ($\partial\boldsymbol{\tau}_c/\partial\boldsymbol{\Phi}_c > 0$) make it more difficult to be a entrepreneur. Defining "**formalization threshold**" $\overline{\boldsymbol{\varphi}}_c$ as the skill cutoff between formal and informal entrepreneurs and identifying it by using personal indirect utility (6) with (20) and (21):

$$\boldsymbol{u}_{cF}(\overline{\boldsymbol{\varphi}}_c) - \boldsymbol{u}_{cI}(\overline{\boldsymbol{\varphi}}_c) = \frac{\varepsilon(1-\tau_c)}{1+\varepsilon}\boldsymbol{Y}_w\left(\frac{\alpha_{wc}A_c\overline{\boldsymbol{\varphi}}_c}{\boldsymbol{\Phi}_w}\right)^{\frac{1}{\varepsilon}} - \boldsymbol{\theta}_c = 0 \Rightarrow \overline{\boldsymbol{\varphi}}_c = \left[\frac{\boldsymbol{\theta}_c(1+\varepsilon)\boldsymbol{\Phi}_w}{\varepsilon(1-\tau_c)(w_w A_c)^{\frac{1}{\varepsilon}}\boldsymbol{Y}_w}\right]^{\varepsilon}\boldsymbol{w}_c \tag{23}$$

Comparing the indirect utility function between sectors leads to the left part of (23), meaning that sector selection is a tradeoff between extra revenue from international market and local additional cost for a formal firm, shown as ***Figure 6***:

Figure 6. **Utility tradeoff between informal and formal entrepreneurs**



As a result, Equation (23) implies that local wage rate is the only endogenous variable relates to formalization threshold $\overline{\boldsymbol{\varphi}}_c$. Higher $\boldsymbol{w}_c$ causes labor cost to rise, and therefore, running formal business become less profitable and more difficult. In addition, $\overline{\boldsymbol{\varphi}}_c$ is positively related to global aggregate productivity and adjusted formalization cost $\boldsymbol{\theta}_c/(1-\boldsymbol{\tau}_c)$ ($\partial\overline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{\Phi}_w > 0, \partial\overline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{\tau}_c > 0, \partial\overline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{\theta}_c > 0$), while negatively depending on demand of international market and world wage ($\partial\overline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{Y}_w < 0, \partial\overline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{w}_w < 0$). formalization is easier to take place in a city with more efficient trade service ($\partial\overline{\boldsymbol{\varphi}}_c/\partial\boldsymbol{A}_c < 0$). For a sufficiently small $\boldsymbol{\theta}$,we may have $\overline{\boldsymbol{\varphi}}_c \le \underline{\boldsymbol{\varphi}}_c$, in which case the informal sector disappears. We will focus on the case where $\boldsymbol{\theta}$ is sufficiently high so that $\overline{\boldsymbol{\varphi}}_c > \underline{\boldsymbol{\varphi}}_c$ and the formal and informal sectors coexist.

Replace $\boldsymbol{\Omega}_{cI}$ and $\boldsymbol{\Omega}_{cF}$ with $\boldsymbol{N}_c$, $\boldsymbol{G}_c(\boldsymbol{\varphi})$, $\overline{\boldsymbol{\varphi}}_c$ and $\underline{\boldsymbol{\varphi}}_c$ in (17) and (18) [9]

[9] To guarantee the aggregate productivity of the formal sector $\boldsymbol{\Phi}_{cF}$ exist, $\boldsymbol{k} > 1/\varepsilon$.

$$\boldsymbol{\Phi}_{cI}{}^{\frac{1}{\varepsilon}} = \boldsymbol{N_c} \int_{\underline{\boldsymbol{\varphi}}_c}^{\overline{\boldsymbol{\varphi}}_c} \boldsymbol{\varphi}^{1/\varepsilon}\, \mathrm{d}\boldsymbol{G_c}(\boldsymbol{\varphi}) \tag{17'}$$

$$\boldsymbol{\Phi}_{cF}{}^{\frac{1}{\varepsilon}} = \boldsymbol{N_c} \int_{\overline{\boldsymbol{\varphi}}_c}^{+\infty} \boldsymbol{\varphi}^{1/\varepsilon}\, \mathrm{d}\boldsymbol{G_c}(\boldsymbol{\varphi}) \tag{18'}$$

In the equilibrium, local labor market clears by equalizing labor supply $\boldsymbol{L_c^S}$ and labor demand $\boldsymbol{L_c^D}$. Labor is supplied by low skill agents whose productivity are smaller than $\underline{\boldsymbol{\varphi}}_c$, discounted by the labor wedge $\boldsymbol{\tau_c}$: $\boldsymbol{L_c^S} = (1-\boldsymbol{\tau_c})\boldsymbol{N_c}\int_{\boldsymbol{\varphi_m}}^{\boldsymbol{\varphi_c}} \boldsymbol{d}\,\boldsymbol{G_c}(\boldsymbol{\varphi})$. Labor demand $\boldsymbol{L_c^D}$ can be decomposed with sectors, $\boldsymbol{L_{cI}^D}$ and $\boldsymbol{L_{cF}^D}$. The amount of labor employed by each firm is $\boldsymbol{X(\varphi)/\varphi}$, with equation (12'), (13'), (19) and $\boldsymbol{L_c^S} = \boldsymbol{L_{cI}^D} + \boldsymbol{L_{cF}^D}$, the clear conditions of local labor market is equation (24):

$$\boldsymbol{L_{cI}^D} = \boldsymbol{N_c}\int_{\underline{\boldsymbol{\varphi}}_c}^{\overline{\boldsymbol{\varphi}}_c} \left(\frac{\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}}{\boldsymbol{\Phi_c}}\right)^{1+\varepsilon} \frac{(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{\boldsymbol{\varphi}\mathbb{P}_c}\,\mathrm{d}\boldsymbol{G_c}(\boldsymbol{\varphi}) = \frac{\boldsymbol{\Phi_{cI}}(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{(1+\boldsymbol{\varepsilon})\boldsymbol{w_c}\boldsymbol{\Phi_c}}$$

$$\boldsymbol{L_{cF}^D} = \boldsymbol{N_c}\int_{\overline{\boldsymbol{\varphi}}_c}^{+\infty} \left[\left(\frac{\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}}{\boldsymbol{\Phi_c}}\right)^{1+\varepsilon} \frac{(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{\boldsymbol{\varphi}\mathbb{P}_c} + \left(\frac{(\boldsymbol{\alpha_{wc}A_c\varphi})^{\frac{1}{\varepsilon}}}{\boldsymbol{\varphi\Phi_w}}\right)^{1+\varepsilon} \frac{\boldsymbol{Y_w}}{\boldsymbol{A_c\varphi}\mathbb{P}_w}\right]\mathrm{d}\boldsymbol{G_c}(\boldsymbol{\varphi}) = \frac{\boldsymbol{\Phi_{cF}}}{(1+\boldsymbol{\varepsilon})\boldsymbol{w_c}}\left[\frac{(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{\boldsymbol{\Phi_c}} + \frac{(\boldsymbol{\alpha_{wc}A_c})^{\frac{1}{\varepsilon}}\boldsymbol{Y_w}}{\boldsymbol{\Phi_w}}\right]$$

$$(1+\boldsymbol{\varepsilon})\boldsymbol{w_c}\boldsymbol{N_c}\int_{\boldsymbol{\varphi_m}}^{\boldsymbol{\varphi_c}} \mathrm{d}\,\boldsymbol{G_c}(\boldsymbol{\varphi}) = \frac{(\boldsymbol{\Phi_{cI}}+\boldsymbol{\Phi_{cF}})(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{\boldsymbol{\Phi_c}} + \frac{\boldsymbol{\Phi_{cF}}(\boldsymbol{\alpha_{wc}A_c})^{\frac{1}{\varepsilon}}\boldsymbol{Y_w}}{\boldsymbol{\Phi_w}} \tag{24}$$

The LHS of equation (24) is equal to total local labor income plus markup and therefore is the aggregate value of output in city $\boldsymbol{c}$. It depends on the two part in RHS, which is positively related to $\boldsymbol{R_c} = \boldsymbol{Y_c}/\boldsymbol{\Phi_c}$ and $\boldsymbol{R_w} = \boldsymbol{Y_w}/\boldsymbol{\Phi_w}$, the producer profitability in local and global market, respectively. As we will prove through the lemma 1 later in this section, $\boldsymbol{R_c} = \boldsymbol{Y_c}/\boldsymbol{\Phi_c}$ rises with local aggregate formal productivity $\boldsymbol{\Phi_{cF}}$. Thus total value of local output is an increasing function of $\boldsymbol{\Phi_{cF}}$, implying that more developed formal economy contributes to income growth.

Decomposing $\boldsymbol{Y_c}$ by occupation as $\boldsymbol{Y_c}=\boldsymbol{Y_{cW}}+\boldsymbol{Y_{cI}}+\boldsymbol{Y_{cF}}$, where $\boldsymbol{Y_{cW}}$, $\boldsymbol{Y_{cI}}$ and $\boldsymbol{Y_{cF}}$ is the total income of workers, informal entrepreneurs and formal entrepreneurs in city $\boldsymbol{c}$, respectively, satisfying

$$\boldsymbol{Y_{cW}} = \boldsymbol{w_c}\boldsymbol{N_c}\int_{\boldsymbol{\varphi_m}}^{\boldsymbol{\varphi_c}} \mathrm{d}\,\boldsymbol{G_c}(\boldsymbol{\varphi})$$

$$\boldsymbol{Y_{cI}} = \boldsymbol{N_c}\int_{\underline{\boldsymbol{\varphi}}_c}^{\overline{\boldsymbol{\varphi}}_c} \boldsymbol{\pi_{cI}}(\boldsymbol{\varphi})\,\mathrm{d}\boldsymbol{G_c}(\boldsymbol{\varphi}) = \frac{\boldsymbol{\varepsilon}\boldsymbol{\Phi_{cI}}(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{(1+\boldsymbol{\varepsilon})\boldsymbol{\Phi_c}}$$

$$\boldsymbol{Y_{cF}} = \boldsymbol{N_c}\int_{\overline{\boldsymbol{\varphi}}_c}^{+\infty} \boldsymbol{\pi_{cF}}(\boldsymbol{\varphi})\,\mathrm{d}\boldsymbol{G_c}(\boldsymbol{\varphi}) = \frac{\boldsymbol{\varepsilon}}{1+\boldsymbol{\varepsilon}}\left[\frac{(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{\boldsymbol{\Phi_c}} + \frac{(\boldsymbol{\alpha_{wc}A_c})^{\frac{1}{\varepsilon}}\boldsymbol{Y_w}}{\boldsymbol{\Phi_w}}\right]\boldsymbol{\Phi_{cF}} - \frac{\boldsymbol{m\theta_c N_c}}{1-\boldsymbol{\tau_c}}\int_{\overline{\boldsymbol{\varphi}}_c}^{+\infty} \mathrm{d}\,\boldsymbol{G_c}(\boldsymbol{\varphi})$$

Hence $\boldsymbol{Y_c}$ is expressed by

$$\boldsymbol{Y_c} = \boldsymbol{w_c}\boldsymbol{N_c}\int_{\boldsymbol{\varphi_m}}^{\boldsymbol{\varphi_c}} \mathrm{d}\,\boldsymbol{G_c}(\boldsymbol{\varphi}) + \frac{\boldsymbol{\varepsilon}}{1+\boldsymbol{\varepsilon}}\left[\frac{(\boldsymbol{\Phi_{cI}}+\boldsymbol{\Phi_{cF}})(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{\boldsymbol{\Phi_c}} + \frac{\boldsymbol{\Phi_{cF}}(\boldsymbol{\alpha_{wc}A_c})^{\frac{1}{\varepsilon}}\boldsymbol{Y_w}}{\boldsymbol{\Phi_w}}\right] - \frac{\boldsymbol{m\theta_c N_c}}{1-\boldsymbol{\tau_c}}\int_{\overline{\boldsymbol{\varphi}}_c}^{+\infty} \mathrm{d}\,\boldsymbol{G_c}(\boldsymbol{\varphi}) \tag{25}$$

Combining equation (20) and (21), total local consumption expenditure $\boldsymbol{Y_c}$ can be rewritten as:

$$\boldsymbol{Y_c} = \frac{(\boldsymbol{\Phi_{cI}}+\boldsymbol{\Phi_{cF}})(1-\boldsymbol{\tau_c})\boldsymbol{Y_c}}{\boldsymbol{\Phi_c}} + \frac{\boldsymbol{\Phi_{cF}}(\boldsymbol{\alpha_{wc}A_c})^{\frac{1}{\varepsilon}}\boldsymbol{Y_w}}{\boldsymbol{\Phi_w}} - \frac{\boldsymbol{m\theta_c N_c}}{1-\boldsymbol{\tau_c}}\left(1-\boldsymbol{G_c}(\overline{\boldsymbol{\varphi}}_c)\right) = \boldsymbol{a_{1c}Y_c} + \boldsymbol{a_{2c}Y_w} - \boldsymbol{a_{3c}} \tag{26}$$

Where

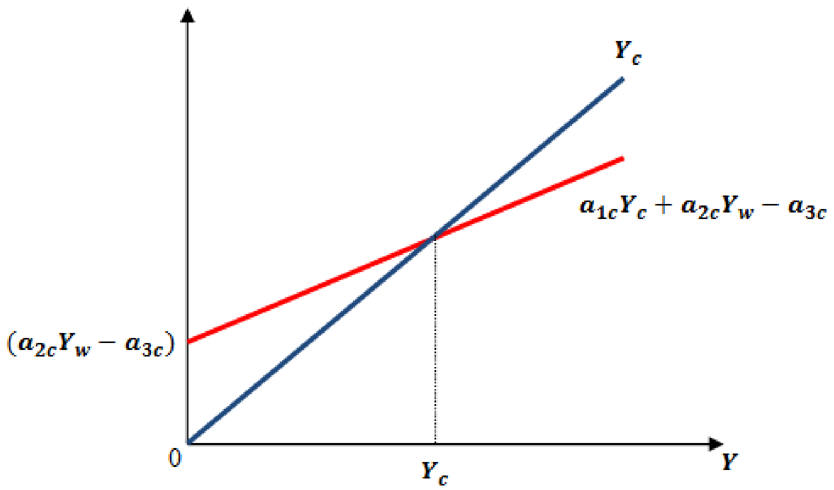
$$a_{1c} = \frac{(\Phi_{cI} + \Phi_{cF})(1-\tau_c)}{\Phi_c} < 1$$

$$a_{2c} = \frac{\Phi_{cF}(\alpha_{wc}A_c)^{\frac{1}{\varepsilon}}}{\Phi_w}$$

$$a_{3c} = \frac{m\theta_c N_c}{1-\tau_c}\left(1 - G_c(\overline{\varphi}_c)\right)$$

Equation (26) states that the aggregate income Y_c in the LHS is equal to total local consumption expenditure in the RHS, which includes an endogenous component $a_{1c}Y_c$ and an autonomous component $(a_{2c}Y_w - a_{3c})$ increasing with world expenditure Y_w. a_{1c} is the proportion of local producers' productivity in local aggregate productivity, discounted by labor wedge, and thus is strictly smaller than 1. $(a_{2c}Y_w - a_{3c})$ is the net profit from trade, equaling total revenue of trade $a_{2c}Y_w$ minus the fraction of formalization cost paid to foreign employees a_{3c}. As a result, the equilibrium Y_c can be described by ***Figure 7***:

***Figure 7*. Aggregate income and consumption expenditure**



The image represents the function of Y_c in each side of equation (26), hence the crossover point is the equilibrium Y_c. As the slope and intercept of the line $(a_{1c}Y_c + a_{2c}Y_w - a_{3c})$ rises, the crossover point is shifted to the right gradually, Y_c is increasing with a_{1c}, a_{2c} and Y_w but decreasing with a_{3c}. As a result, more productive of local producers ($\partial a_{1c}/\partial\Phi_{cI} > 0$, $\partial a_{1c}/\partial\Phi_{cF} > 0$, , $\partial a_{2c}/\partial\Phi_{cF} > 0$), more efficient trade service ($\partial a_{2c}/\partial A_c > 0$), higher global-local labor cost ratio ($\partial a_{2c}/\partial\alpha_{wc} > 0$) and larger expenditure in the international market ($\partial Y_c/\partial Y_w > 0$) contribute to growth of income, while greater urban friction ($\partial a_{1c}/\partial\tau_c < 0$, $\partial a_{3c}/\partial\tau_c > 0$), tougher competition in world market ($\partial a_{2c}/\partial\alpha_{wc} > 0$), higher formalization cost ($\partial a_{3c}/\partial\theta_c > 0$) and bigger fraction of taxation spent abroad ($\partial a_{3c}/\partial m > 0$) bring opposite effect on total income.

Jointly analyzing the conditions and factors of local employment mix, Y_c/Φ_c and Φ_{cF}, Φ_{cI} are found to satisfy the following proposition.

Lemma 1 ***Producer productivity and profitability.***
Local producer profitability given by $R_c = Y_c/\Phi_c$ is increasing with formal-sector productivity Φ_{cF} but

decreasing with informal-sector productivity $\boldsymbol{\Phi_{cI}}$.[10] Hence the informality occurs as formal economy develops.

Proof. Based on equation (16) and (26), yields

$$\boldsymbol{R_c = Y_c/\Phi_c} = \frac{(\Phi_{cI}+\Phi_{cF})\frac{(1-\tau_c)Y_c}{\Phi_c}+\Phi_{cF}(\alpha_{wc}A_c)^{\frac{1}{\varepsilon}}\frac{Y_w}{\Phi_w}-\frac{m\theta_c N_c}{1-\tau_c}\left(1-G_c(\overline{\varphi}_c)\right)}{\Phi_{cI}+\Phi_{cF}+\left(\frac{A_c}{\alpha_{wc}}\right)^{\frac{1}{\varepsilon}}\Phi_w} \tag{27}$$

Partially differentiating (27) with $\boldsymbol{\Phi_{cF}}$, $\boldsymbol{\Phi_{cI}}$

$$\frac{\partial R_c/R_c}{\partial\Phi_{cF}/\Phi_{cF}} = \frac{\Phi_{cF}[\left(\frac{A_c}{\alpha_{wc}}\right)^{\frac{1}{\varepsilon}}\frac{(1-\tau_c)\Phi_w Y_c}{\Phi_c}+[\Phi_{cI}+\left(\frac{A_c}{\alpha_{wc}}\right)^{\frac{1}{\varepsilon}}\Phi_w](\alpha_{wc}A_c)^{\frac{1}{\varepsilon}}\frac{Y_w}{\Phi_w}+\frac{m\theta_c N_c}{1-\tau_c}\left(1-G_c(\overline{\varphi}_c)\right)]}{Y_c[\left(\frac{A_c}{\alpha_{wc}}\right)^{\frac{1}{\varepsilon}}\Phi_w+\tau_c(\Phi_{cI}+\Phi_{cF})]} > 0 \tag{28}$$

$$\frac{\partial R_c/R_c}{\partial\Phi_{cI}/\Phi_{cI}} = -\tau_c\Phi_{cI} < 0 \tag{29}$$

Noted that $\boldsymbol{R_c}$ is the return for each unit of productivity of variety supplied in local commodity market. According to the profit function (20) and (21), higher $\boldsymbol{R_c}$ means more profitable for all local producers, formal or informal, hence $\boldsymbol{R_c}$ represents local producer's profitability. Equation (28) and (29) prove that the elasticity of $\boldsymbol{R_c}$ for local aggregate formal productivity $\boldsymbol{\Phi_{cF}}$ is positive, while the elasticity for total informal productivity is negative, so that $\boldsymbol{R_c}$ grows with $\boldsymbol{\Phi_{cF}}$ but reduces with $\boldsymbol{\Phi_{cI}}$. Based on equation (28), the elasticity positively depends on aggregate productivity of formal sector $\boldsymbol{\Phi_{cF}}$, return of productivity in international market $\boldsymbol{R_w}$ and global-local ratio of wage rate $\boldsymbol{\alpha_{wc}}$, implying that the formal sector can contribute to local development more if it is larger and more competitive in trade. On the other hand, equation (29) claims that producer profitability decreases with $\boldsymbol{\tau_c\Phi_{cI}}$, meaning that urban friction and large scale of informality strengthen each other on incurring distortion of the economy.

Using lemma 1, we obtain $\boldsymbol{\partial R_c/\partial\Phi_{cF}} > 0$ thus $\boldsymbol{\partial\pi_{cI}(\varphi)/\partial\Phi_{cF}} > 0$. As the formal economy develops, the aggregate formal productivity $\boldsymbol{\Phi_{cF}}$ rises and thus pushes up consumption expenditure through the revenue from external market. Since the informal producers totally rely on local market, so that informality is more likely to occur in a city with expanding local demand, through "free-ride" behavior that sharing benefit of trade without paying additional cost. The coexistence of formal and informal employment claimed by lemma 1 is supported by ***Table 2***, which shows the positive correlation between formal and informal employment during 1995 to 2015, no matter in the same period or in the long-term. In addition, lemma 1 also sheds light on the second pattern found in ***Table 3*** that formal sector is more concentrated compared to the informal sector. The intuition is that formal sector benefits from agglomeration economies, which motivates concentration.

Let the labor size $\boldsymbol{N_c}$ and skill distribution $\boldsymbol{G_c(\varphi)}$ be given, the endogenous variables set of local employment mix $\{\boldsymbol{\alpha_{wc}}, \boldsymbol{\tau_c}, \boldsymbol{\Phi_c}, \boldsymbol{\Phi_{cI}}, \boldsymbol{\Phi_{cF}}, \mathbb{P}_c, \underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}, \overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}, \boldsymbol{w_c}, \boldsymbol{Y_c}\}$ introduced in this section are shown to be known functions of labor size $\boldsymbol{N_c}$ and skill distribution $\boldsymbol{G_c(\varphi)}$, especially $\boldsymbol{G_c(\overline{\varphi}_c)}$ and $\boldsymbol{G_c(\underline{\varphi}_c)}$, which will be analyzed in the next section of spatial equilibrium.

[10] When discussing $\boldsymbol{\Phi_{cF}}$ and $\boldsymbol{\Phi_{cI}}$, we consider the total scale of them rather than involving the threshold $\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}$, and therefore they are independent with each other.

3.3 Inter-city Equilibrium

Perfectly-Mixed-skill Equilibrium

We turn to the spatial sorting issue in this section. Workers choose a city to maximize their individual utility, which, in equilibrium, equals real income. In the perfectly-mixed-skill equilibrium, free mobility ensures that utility offered by each city for a given skill level is equalized and no one have incentive to deviate[11]:

$$u_{1w}(\varphi) = u_{2w}(\varphi) \Rightarrow \frac{(1-\tau_1)w_1}{\mathbb{P}_1} = \frac{(1-\tau_2)w_2}{\mathbb{P}_2}, \varphi \in [\varphi_m, \underline{\varphi}_c] \tag{30}$$

$$u_{1I}(\varphi) = u_{2I}(\varphi) \Rightarrow \frac{\varepsilon(1-\tau_1)^2 R_1}{(1+\varepsilon)\mathbb{P}_1}\varphi^{\frac{1}{\varepsilon}} = \frac{\varepsilon(1-\tau_2)^2 R_2}{(1+\varepsilon)\mathbb{P}_2}\varphi^{\frac{1}{\varepsilon}}, \varphi \in [\underline{\varphi}_c, \overline{\varphi}_c] \tag{31}$$

$$u_{1F}(\varphi) = u_{2F}(\varphi) \Rightarrow \frac{\varepsilon(1-\tau_1)\varphi^{\frac{1}{\varepsilon}}}{(1+\varepsilon)\mathbb{P}_1}\left[(1-\tau_1)R_1 + (\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w\right] - \frac{\theta_1}{\mathbb{P}_1} = \frac{\varepsilon(1-\tau_2)\varphi^{\frac{1}{\varepsilon}}}{(1+\varepsilon)\mathbb{P}_2}\left[(1-\tau_2)R_2 + (\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w\right] - \frac{\theta_2}{\mathbb{P}_2}, \varphi \in [\overline{\varphi}_c, +\infty) \tag{32}$$

Noted that $R_c = Y_c/\Phi_c$ is the return for each unit of productivity in local commodity market, and $(\alpha_{wc}A_c)^{\frac{1}{\varepsilon}}R_w$ is the adjusted return from international market for each unit of formal productivity in city c. Hence equation (31) and (32) state that both the formal and informal firms benefit as local return for productivity increases, while only the formal firms can be more profitable when global return for productivity rises.

There are two different equilibrium of location sorting: perfectly-mixed-skill equilibrium and separately equilibrium. In the perfectly-mixed-skill equilibrium, the utility of all individuals with $\varphi \in [\overline{\varphi}_c, +\infty)$ are indifferent across locations. We focus on the conditions and propositions of perfectly-mixed-skill equilibrium for formal entrepreneurs in the model part, because it is more convincing in explaining the relation between spatial variation of informality and local condition, such as trade infrastructure and formalization cost, in Indonesia. Furthermore, the prediction of separate equilibrium that formal firms whose managerial skill below or above a cutoff stay in different cities is not supported by the stylized facts. In addition, it is also difficult to apply separate equilibrium when expanding the model to N-cities case. The feasibility of separate equilibrium for formal entrepreneurs is discussed in Appendix B.

Definition 1 (*Perfectly-Mixed-skill Equilibrium*): Perfectly-mixed-skill equilibrium is the allocation $\{N_c^*, G_c^*(\underline{\varphi}_c), G_c^*(\overline{\varphi}_c) \mid c \in \{1, 2\}\}$ and market-clearing price indexes $\{\mathbb{P}_c^*, w_c^*, R_c^* \mid c \in \{1, 2\}\}$, given the parameters set $\{\varepsilon, k, b \mid \varepsilon \in (1/2, +\infty), k > \max(1, 1/\varepsilon), bN^{1/2} \in (1, \sqrt{2})\}$, endowment $\{N, G(\varphi)\}$, policy variables $\{A_c, \theta_c\}$ and world market condition $\{R_w, \alpha_{wc}\}$, such that the endogenous variables $\{\tau_c, \Phi_{cI}, \Phi_{cF}, \underline{\varphi}_c, \overline{\varphi}_c, Y_c\}$ follow the conditions below

(1) α_{wc} and w_c satisfy equation (19);

---

[11] Corner solution in which all workers and firms concentrate in one city is both empirically counterfactual and theoretically meaningless. In addition, it's expected to see how the spatial distribution of formal and informal employment, the formal employment share and the average firm size connect to local condition, such as trade efficiency A and formalization cost θ, from the spatial equilibrium. However, none of these can be studied through the corner solution. Using $\tau_c = b\sqrt{N_c}$, corner solution is irrational in this model, because of a sufficient large total population size N. As a result, the city size of both cities are not equal to zero in the mixed-skill equilibrium.

(2) $\boldsymbol{\Phi}_{cI}$, $\overline{\boldsymbol{\varphi}}_c$ and $\underline{\boldsymbol{\varphi}}_c$ satisfy equation (17');

(3) $\boldsymbol{\Phi}_{cF}$ and $\overline{\boldsymbol{\varphi}}_c$ satisfy equation (18');

(4) $\boldsymbol{\Phi}_c$, $\boldsymbol{\Phi}_{cI}$, $\boldsymbol{\Phi}_{cF}$ and $\boldsymbol{w}_c$ satisfy equation (16);

(5) $\boldsymbol{\Phi}_c$, $\boldsymbol{w}_c$ and $\mathbb{P}_c$ satisfy equation (15);

(6) $\boldsymbol{\Phi}_c$, $\boldsymbol{w}_c$, $\boldsymbol{Y}_c$ and $\underline{\boldsymbol{\varphi}}_c$ satisfy equation (22);

(7) $\boldsymbol{w}_c$ and $\overline{\boldsymbol{\varphi}}_c$ satisfy equation (23);

(8) $\boldsymbol{\Phi}_c$, $\boldsymbol{\Phi}_{cI}$, $\boldsymbol{\Phi}_{cF}$, $\underline{\boldsymbol{\varphi}}_c$, $\boldsymbol{w}_c$ and $\boldsymbol{Y}_c$ satisfy equation (24) (labor market clearing condition);

(9) $\overline{\boldsymbol{\varphi}}_c$, $\boldsymbol{w}_c$ and $\boldsymbol{Y}_c$ satisfy equation (26);

(10) $\boldsymbol{\tau}_c$, $\boldsymbol{w}_c$ and $\mathbb{P}_c$ satisfy equation (28) for $\boldsymbol{c} \in \{1, 2\}$ (utility indifference condition for workers);

(11) $\boldsymbol{\tau}_c$, $\boldsymbol{Y}_c$, $\boldsymbol{\Phi}_c$ and $\mathbb{P}_c$ satisfy equation (29) for $\boldsymbol{c} \in \{1, 2\}$ (utility indifference condition for informal entrepreneurs);

(12) $\boldsymbol{\tau}_c$, $\boldsymbol{Y}_c$, $\boldsymbol{\Phi}_c$, $\boldsymbol{w}_c$ and $\mathbb{P}_c$ satisfy equation (30) for $\boldsymbol{c} \in \{1, 2\}$ (utility indifference condition for formal entrepreneurs).[12]

Where the condition (1)-(9) in definition 1 describe local employment mix, and condition (10)-(12) capture the inter-city perfectly-mixed-skill equilibrium.

Suppose separate equilibrium exists for the workers, meaning that at least there is $\boldsymbol{\varphi}_0 \in [\boldsymbol{\varphi}_m, \underline{\boldsymbol{\varphi}}_c]$ not satisfying equation (28):

$$\boldsymbol{u}_{1w}(\boldsymbol{\varphi}_0) \neq \boldsymbol{u}_{2w}(\boldsymbol{\varphi}_0) \Rightarrow \frac{(1-\boldsymbol{\tau}_1)\boldsymbol{w}_1}{\mathbb{P}_1} \neq \frac{(1-\boldsymbol{\tau}_2)\boldsymbol{w}_2}{\mathbb{P}_2} \tag{30'}$$

Regardless of personal skill, income for workers is local wage rate $\boldsymbol{w}_c$, and therefore, (30') will hold for all workers with $\boldsymbol{\varphi} \in [\boldsymbol{\varphi}_m, \underline{\boldsymbol{\varphi}}_c]$, driving them to move to the city with higher real income. However, bigger size of labor supply raises the labor wedge $\boldsymbol{\tau}_c$, which incurs welfare loss. More importantly, if all workers are in one city, that means all firms have to locate at the same city, too, but it is impossible, because the total population size ***N*** in the economy is sufficient large that corner solution in which all workers and firms concentrate in one city is irrational. As a result, separate equilibrium is infeasible for workers, and equation (30) is robust.

Combining equation (15) and (30), indifference condition on workers' utility across locations leads to

$$\frac{\mathbb{P}_2}{\mathbb{P}_1} = \frac{\boldsymbol{w}_2}{\boldsymbol{w}_1}\frac{1-\boldsymbol{\tau}_2}{1-\boldsymbol{\tau}_1} \Rightarrow \frac{\mathbb{P}_2/(1-\boldsymbol{\tau}_2)}{\mathbb{P}_1/(1-\boldsymbol{\tau}_1)} = \frac{\boldsymbol{w}_2}{\boldsymbol{w}_1} \tag{33}$$

[12] The sufficient condition for the parameters which guarantee the existence of perfectly-mixed-skill equilibrium is provided in Appendix C. In Appendix D, we provide algorithm to show fixed-point mapping of the perfectly-mixed-skill equilibrium and prove its stability.

$$\left(\frac{\Phi_2}{\Phi_1}\right)^{\varepsilon} = \frac{1-\tau_1}{1-\tau_2} \tag{34}$$

In the equilibrium, equation (33) claims that local price index is positively related to wage rate, discounting the benefit from higher wage rate and therefore preventing workers concentrating in the city with bigger return of labor. Equation (34) tells fact that local aggregate productivity, which includes not only local firms, but also the firms provide import goods, is inversely proportional to 1 minus local labor wedge. Since labor wedge is positively depended on population size, the inter-city ratio of aggregate productivity increases with the local city size. It means that although larger labor size raises urban friction and incurs welfare loss, it can be compensated by lower local price index, because of higher aggregate productivity. This implication is supported by equation (33), which states that the price index decreases with labor size.

Similarly, the necessary condition of separate equilibrium for informal entrepreneurs is that at least there is $\varphi_0 \in [\underline{\varphi}_c, \overline{\varphi}_c]$ not satisfying equation (31):

$$u_{1I}(\varphi_0) \neq u_{2I}(\varphi_0) \Rightarrow \frac{(1-\tau_1)\pi_{1I}(\varphi)}{\mathbb{P}_1} \neq \frac{(1-\tau_2)\pi_{2I}(\varphi)}{\mathbb{P}_2} \tag{31'}$$

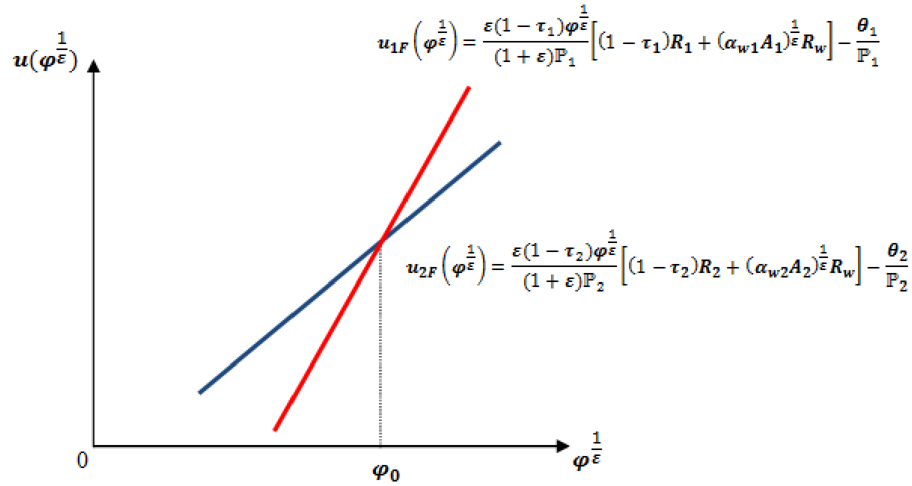
Noticed that the right part of (31') is independent with φ and therefore is satisfied by any $\varphi \in [\underline{\varphi}_c, \overline{\varphi}_c]$. When there is separate equilibrium for the informal entrepreneurs, all informal firms concentrate in one city, while the formal firms are in both cities. However, this case is paradoxical to lemma 1, which claims that informality occurs when formal economy develops. What's more, the concentration of informal employment reduces local producer profitability and drives part of the informal firms to another city. because local producer profitability increases with aggregate formal productivity Φ_{cF}. According to (33) and (34), rewrites (31) with the definition of informal firms' profit in (21), yields

$$\frac{Y_2}{Y_1} = \frac{w_2}{w_1}\left(\frac{1-\tau_1}{1-\tau_2}\right)^{1+\frac{1}{\varepsilon}} \Rightarrow \frac{(1-\tau_2)R_2}{(1-\tau_1)R_1} = \frac{w_2}{w_1} \tag{35}$$

Expanding equation (32) with deduction (33) and (35), we have:

$$\frac{\varepsilon(1-\tau_1)^2 R_1}{(1+\varepsilon)\mathbb{P}_1}\varphi^{\frac{1}{\varepsilon}}\left[\frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2}\right] = \frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2} \tag{36}$$

Figure 8-1. **$u_{1F}\left(\varphi^{\frac{1}{\varepsilon}}\right)$ and $u_{2F}\left(\varphi^{\frac{1}{\varepsilon}}\right)$ (Separate Equilibrium)**



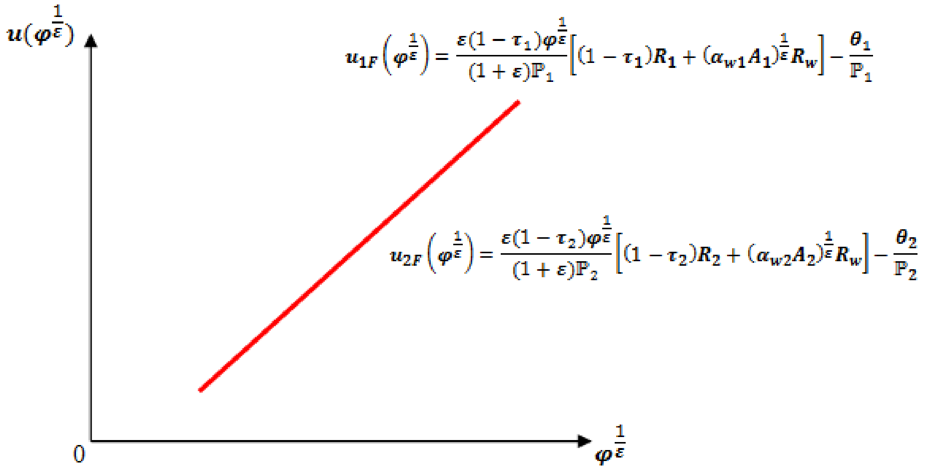
Equation (32) and (36) are not guaranteed to hold for every $\boldsymbol{\varphi} \subset [\overline{\boldsymbol{\varphi}}_c, +\infty)$, except in the mixed-skill equilibrium. If the slope of $\boldsymbol{u_{1F}}\left(\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}\right)$ and $\boldsymbol{u_{2F}}\left(\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}\right)$ are unequal in the equilibrium, personal utility of formal entrepreneurs will rise with skill level by different rate. In this situation, there is separate equilibrium for formal entrepreneurs, which implies that formal firms whose managerial skill below or above a specific cutoff $\boldsymbol{\varphi_0}$ stay in different cities, like ***Figure 8-1*** exhibits.

On the other hand, in the case of mixed-skill equilibrium, the function images of $\boldsymbol{u_{1F}}\left(\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}\right)$ and $\boldsymbol{u_{2F}}\left(\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}\right)$ are perfectly coincident in the interval $[\overline{\boldsymbol{\varphi}}_c, +\infty)$, as shown in ***Figure 8-2***, implying that their slope and intercept are equal. Hence $\frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2} = \mathbf{0}$ and $\frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2} = 0$ are true simultaneously is both the sufficient and necessary condition of mixed-skill equilibrium, yields

$$\frac{w_2}{w_1} = \left(\frac{A_2}{A_1}\right)^{\frac{1}{1+\varepsilon}} \tag{37}$$

$$\frac{1-\tau_2}{1-\tau_1} = \frac{\lambda_1}{\lambda_2} \tag{38}$$

Figure 8-2. **$u_{1F}\left(\varphi^{\frac{1}{\varepsilon}}\right)$ and $u_{2F}\left(\varphi^{\frac{1}{\varepsilon}}\right)$ (Perfectly-Mixed-skill equilibrium)**



Suppose there is an interval of $\boldsymbol{\varphi_0} \subset [\overline{\boldsymbol{\varphi}}_c, +\infty]$ not satisfying equation (32) and (36), so that with equation (33) and (35), equation (36) can be rewritten as

$$u_{1F}(\varphi) > u_{2F}(\varphi) \Rightarrow \frac{\varepsilon w_w^{\frac{1}{\varepsilon}} Y_w}{(1+\varepsilon) w_1^{\frac{1}{\varepsilon}} \Phi_w} \varphi_0^{\frac{1}{\varepsilon}} \left[A_1^{\frac{1}{\varepsilon}} - \left(\frac{w_1}{w_2}\right)^{1+\frac{1}{\varepsilon}} A_2^{\frac{1}{\varepsilon}}\right] > \frac{\theta_1}{(1-\tau_1)} - \frac{w_1\theta_2}{w_2(1-\tau_2)}, \varphi \in \varphi_0 \tag{36'}$$

In this situation, individuals whose $\boldsymbol{\varphi} \in \boldsymbol{\varphi_0}$ deviate the mixed-skill equilibrium and concentrate to city 1. As the number of formal firms increases, there is immigration flow of workers looking for jobs, hence the labor wedge rises in city 1 but reduces in city 2. Moreover, the inter-city ratio of wage rate $\boldsymbol{w_1/w_2}$ becomes higher because of employment reallocation caused by the formal firms with managerial skill in the interval of $\boldsymbol{\varphi_0}$. As a result, the LHS become smaller faster than RHS until both sides is equal, adjusting the economy to the perfectly-mixed-skill equilibrium again.

Where $\boldsymbol{\lambda_c} = \frac{A_c^{1/(1+\varepsilon)}}{\theta_c}$ is the benefit to cost ratio of formalization, which reflects the efficiency of the local government in providing trade supporting services**.** With equation (37) and (38), rewriting equation (33)-(35):

$$\frac{\mathbb{P}_2}{\mathbb{P}_1} = (\frac{A_2}{A_1})^{\frac{1}{1+\varepsilon}} \frac{1-\tau_2}{1-\tau_1} = \frac{\theta_2}{\theta_1} \tag{33'}$$

$$\left(\frac{\Phi_2}{\Phi_1}\right)^{\varepsilon} = \frac{\lambda_2}{\lambda_1} \tag{34'}$$

$$\frac{Y_2}{Y_1} = \left(\frac{\lambda_2}{\lambda_1}\right)^{1+\frac{1}{\varepsilon}} (\frac{A_2}{A_1})^{\frac{1}{1+\varepsilon}} \tag{35'}$$

From equation (33')-(35'), (37), (38), inter-city ratio of nominal wage $\boldsymbol{w_c}$, labor wedge τ_c which relates to population size $\boldsymbol{N_c}$, aggregate productivity $\boldsymbol{\Phi_c}$, price index $\mathbb{P}_c$ and total income $\boldsymbol{Y_c}$ are decided by the relative size of three local policy variables: formalization cost $\boldsymbol{\theta_c}$, trade efficiency $\boldsymbol{A_c}$ and governance efficiency $\boldsymbol{\lambda_c}$. Price index is proportional to trade efficiency but decreasing with city size. In the equilibrium, these two opposite effect counteract with each other, making the price index be proportional to the formalization cost. More efficient governance enhances the equilibrium city size and aggregate productivity, while higher local trade cost lower wage rate. Inter-city total income ratio is jointly related to local governance efficiency and trade cost, which increases with the former but informal goods.

Propositions of Perfectly-Mixed-skill Equilibrium

With the local employment mix in section 3.1 and mixed-skill distribution in section 3.2, several important propositions of the equilibrium are found. Firstly, proving the existence and uniqueness of the local employment mix $\{\boldsymbol{\alpha_{wc}}, \boldsymbol{\tau_c}, \boldsymbol{\Phi_c}, \boldsymbol{\Phi_{cI}}, \boldsymbol{\Phi_{cF}}, \mathbb{P}_c, \underline{\boldsymbol{\varphi}}_c, \overline{\boldsymbol{\varphi}}_c, \boldsymbol{w_c}, \boldsymbol{Y_c}\}$, when the labor size $\boldsymbol{N_c}$ and skill distribution $\boldsymbol{G_c}(\boldsymbol{\varphi})$ are given.

Proposition 1 (Existence and Uniqueness) ***Given the city size N_c and local productivity distribution function $G_c(\varphi)$, the local employment mix exists and is unique.***

Proof. Using equation (17'), (18'), (22), (23) and (24), deduction (33') and (35') to eliminate $\boldsymbol{\Phi_c}$, $\boldsymbol{\Phi_{cI}}$, $\boldsymbol{\Phi_{cF}}$, $\boldsymbol{Y_c}$, $\overline{\boldsymbol{\varphi}}_c$ and obtain an implicit solution for $\underline{\boldsymbol{\varphi}}_c$:

$$\int_{\varphi_m}^{\underline{\varphi}_c} \mathrm{d}\, G_c(\varphi) = \frac{1}{\varepsilon \underline{\varphi}_c^{\frac{1}{\varepsilon}}} \int_{\underline{\varphi}_c}^{+\infty} \varphi^{\frac{1}{\varepsilon}} \,\mathrm{d}G_c(\varphi) + \frac{Y_w (w_w A_c)^{\frac{1}{\varepsilon}}}{(1+\varepsilon) w_c^{1+\frac{1}{\varepsilon}} \Phi_{\mathrm{w}}^{\frac{1}{\varepsilon}}} \int_{\left[\frac{\theta_c(1+\varepsilon)}{\varepsilon(1-\tau_c)Y_w}\right]^{\varepsilon} \frac{w_c \Phi_w}{w_w A_c}}^{+\infty} \varphi^{\frac{1}{\varepsilon}} \,\mathrm{d}G_c(\varphi) \tag{39}$$

When $\underline{\boldsymbol{\varphi}}_c = \boldsymbol{\varphi_m}$, the LHS of equation (37) is equal to 0, while the RHS is positive. When $\underline{\boldsymbol{\varphi}}_c \to +\infty$, since $\boldsymbol{\varepsilon} > 0$, the LHS of equation (37) is strictly increasing with $\underline{\boldsymbol{\varphi}}_c$, starting from 0 and become positive. On the other hand, the RHS is monotonically decreasing to 0 from positive, because $\underline{\boldsymbol{\varphi}}_c \sim \boldsymbol{w_c}$, According to equation (22). As a result, there is a unique solution of $\underline{\boldsymbol{\varphi}}_c$, so does the whole equilibrium of local employment mix. In addition, proposition 1 does not rely on the condition of mixed-skill equilibrium for formal entrepreneurs and therefore is robust.

Proposition 2 ***The skill threshold for formal-sector entrepreneur and that for informal-sector entrepreneur are invariant across cities.***

Proof. With equation (23), (33') to (35'), (37) and (38), the inter-city ratio of formalization cutoff is

$$\overline{\varphi}_1 / \overline{\varphi}_2 = \left[\frac{(1-\tau_2)\theta_1}{(1-\tau_1)\theta_2}\right]^{\varepsilon} \frac{w_1 A_2}{w_2 A_1} = 1 \tag{40}$$

Using equation (22), (33) and (35), the ratio between $\underline{\boldsymbol{\varphi}}_1$ and $\underline{\boldsymbol{\varphi}}_2$ is figured out

$$\underline{\boldsymbol{\varphi}}_1/\underline{\boldsymbol{\varphi}}_2=\left(\frac{w_1(1-\tau_2)Y_2\boldsymbol{\Phi}_1^{\frac{1}{\varepsilon}}}{w_2(1-\tau_1)Y_1\boldsymbol{\Phi}_2^{\frac{1}{\varepsilon}}}\right)^{\varepsilon}=1 \tag{41}$$

Proposition 2 states that the skill threshold for informal-sector entrepreneur and that for formal-sector entrepreneur are invariant across cities, given the total population size, skill distribution and free mobility assumption. The reason of proposition 2 is that, for the formal firms, benefit from higher local trade efficiency is offset by higher labor cost (wage rate), and the impact of better governance efficiency (bigger $\boldsymbol{\lambda_c}$) is fully covered by the loss caused by more congestion (labor wedge) and tougher local competition.

In addition, any variation on the skill thresholds across locations is contradictory to the indifferent conditions (31) and (32), which imply $\boldsymbol{u_{1I}(\varphi)}=\boldsymbol{u_{2I}(\varphi)}$ and $\boldsymbol{u_{1F}(\varphi)}=\boldsymbol{u_{2F}(\varphi)}>\boldsymbol{u_{1I}(\varphi)}=\boldsymbol{u_{2I}(\varphi)}$. Suppose $\overline{\boldsymbol{\varphi}}_1<\overline{\boldsymbol{\varphi}}_2$, then for any $\boldsymbol{\varphi}\in[\overline{\boldsymbol{\varphi}}_1,\overline{\boldsymbol{\varphi}}_2]$, such that $\boldsymbol{u_{1F}(\varphi)}>\boldsymbol{u_{1I}(\varphi)}$ and $\boldsymbol{u_{2F}(\varphi)}<\boldsymbol{u_{2I}(\varphi)}$, leading to paradox. As a result of proposition 1, the equilibrium can be simplified through $\underline{\boldsymbol{\varphi}}_1=\underline{\boldsymbol{\varphi}}_2=\underline{\boldsymbol{\varphi}}$ and $\overline{\boldsymbol{\varphi}}_1=\overline{\boldsymbol{\varphi}}_2=\overline{\boldsymbol{\varphi}}$.

When total population size and skill distribution across cities are given, proposition 1 shows the existence and uniqueness of local employment mix. In order to facilitate the further study to the proposition of equilibrium, we have the following simplification

$$\frac{N_1\int_{\varphi_m}^{\underline{\varphi}}\mathrm{d}\,G_1(\varphi)}{N_2\int_{\varphi_m}^{\underline{\varphi}}\mathrm{d}\,G_2(\varphi)}=\frac{N_1G_1(\underline{\varphi})}{N_2G_2(\underline{\varphi})}=(\boldsymbol{\lambda}_1/\boldsymbol{\lambda}_2)^{\frac{1}{\varepsilon}} \tag{42}$$

Equation (42) means that the workers relatively concentrate to a city with better governance efficiency. LHS of equation (42) is the relative size of labor supply across cities, which is equal to the labor demand in equilibrium. This equation is reasonable because of three findings. Firstly, according to the demand function of goods (12') and (13'), demand of labor $\frac{X(\varphi)}{\varphi}\sim\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}$. Secondly, equation (34') shows that $\boldsymbol{\Phi_c}^{\varepsilon}\sim\boldsymbol{\lambda_c}$, where $\boldsymbol{\Phi_c}$ is the aggregation of productivity $\boldsymbol{\varphi}^{\frac{1}{\varepsilon}}$. Thirdly, the pattern that better local governance allows more labor supply is consistent with our empirical observation, shown later in ***Table 4***. To be convenience, taking "**efficient city**" to represent the city with higher $\boldsymbol{\lambda_c}$ below.

Proposition 3 ***The relative number of formal firms is bigger than the relative size of workers in efficient city.***
Proof. Substituting equation (24) into (25), rewrites (26) as

$$\boldsymbol{Y_c}=(1+\boldsymbol{\varepsilon})\boldsymbol{w_cN_cG_c}(\underline{\boldsymbol{\varphi}})-\frac{m\theta_cN_c}{1-\tau_c}\left(1-\boldsymbol{G_c}(\overline{\boldsymbol{\varphi}})\right) \tag{26'}$$

With equation (26') and (35'), yields

$$\frac{Y_2}{Y_1}=\left(\frac{\lambda_2}{\lambda_1}\right)^{1+\frac{1}{\varepsilon}}\left(\frac{A_2}{A_1}\right)^{\frac{1}{1+\varepsilon}}=\frac{(1+\varepsilon)w_2N_2G_2(\underline{\varphi})-[m\theta_2/(1-\tau_2)]N_2\left(1-G_2(\overline{\varphi})\right)}{(1+\varepsilon)w_1N_1G_1(\underline{\varphi})-[m\theta_1/(1-\tau_1)]N_1\left(1-G_1(\overline{\varphi})\right)} \tag{43}$$

Using equation (42) and the RHS of (43)

$$\frac{(1+\varepsilon)w_2N_2G_2(\underline{\varphi})}{(1+\varepsilon)w_1N_1G_1(\underline{\varphi})}=\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}}\left(\frac{A_2}{A_1}\right)^{\frac{1}{1+\varepsilon}} \tag{44}$$

Combining equation (38) and (44) leads to

$$\begin{cases} \frac{[m\theta_2/(1-\tau_2)]N_2(1-G_2(\overline{\varphi}))}{[m\theta_1/(1-\tau_1)]N_1(1-G_1(\overline{\varphi}))} \le \left(\frac{\lambda_2}{\lambda_1}\right)^{1+\frac{1}{\varepsilon}} (\frac{A_2}{A_1})^{\frac{1}{1+\varepsilon}} \Rightarrow \frac{N_2(1-G_2(\overline{\varphi}))}{N_1(1-G_1(\overline{\varphi}))} \le \left(\frac{\lambda_2}{\lambda_1}\right)^{1+\frac{1}{\varepsilon}} < \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}}, if\ \lambda_1 \ge \lambda_2 \\ \frac{[m\theta_2/(1-\tau_2)]N_2(1-G_2(\overline{\varphi}))}{[m\theta_1/(1-\tau_1)]N_1(1-G_1(\overline{\varphi}))} > \left(\frac{\lambda_2}{\lambda_1}\right)^{1+\frac{1}{\varepsilon}} (\frac{A_2}{A_1})^{\frac{1}{1+\varepsilon}} \Rightarrow \frac{N_2(1-G_2(\overline{\varphi}))}{N_1(1-G_1(\overline{\varphi}))} > \left(\frac{\lambda_2}{\lambda_1}\right)^{1+\frac{1}{\varepsilon}} > \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}}, if\ \lambda_1 < \lambda_2 \end{cases} \tag{45}$$

Where $N_c \int_{\overline{\varphi}}^{+\infty} d\,G_c(\varphi)$ is the number of formal firms in city c, so that $N_2(1-G_2(\overline{\varphi}))/N_1(1-G_1(\overline{\varphi}))$ is the relative number of formal firms. From equation (42) and (45), it can be found out that both the size of workers and number of formal firms are proportional to local governance efficiency, showing identical location preference. Furthermore, according to equation (45), the relative number of formal firms is bigger than the relative size of workers in the efficient city, implying that the concentration magnitude is higher for the formal entrepreneurs.

The proposition 3 driven from our model show that the efficient city with better governance efficiency $\lambda_c = \frac{A_c^{1/(1+\varepsilon)}}{\theta_c}$ enjoys advantage of trade in an open economy. As a result, the efficient city attracts more firms to locate in and more workers to look for jobs. The formal firms have stronger preference to higher λ_c than the workers, because they benefit from agglomeration economies, based on lemma 1. In addition, it is found that there is incentive of "free-ride" behavior in the informal sector, so that the informality takes place as formal economy develops. Higher local λ_c means better trade service and/or lower formalization cost. More efficient service for trade makes formal firms more profitable, and less requirement to be formal allows more people to enter the formal sector. Both of these two impact contribute to the expansion of formality. As the formal economy develops, the local aggregate productivity of the formal sector Φ_{cF} increases, which pushes up the demand of labor in city c. What's more, since larger Φ_{cF} implies higher Y_c/Φ_c, which is the producer profitability in the local market, more firms move in city c to pursue higher revenue.

4, Comparative Static Analysis

Based on the mixed-skill equilibrium and its proposition, we do comparative static analysis to check the policy impact of formalization cost θ_c, trade efficiency A_c and governance efficiency λ_c on the spatial distribution of the informal and formal employment.

With positive correlation of their size observed in section 2, the concentration degree of the formal and informal sector are discussed in the propositions below. Taking advantage of equation (24) and (37), yields the following expression:

$$(\frac{A_2}{A_1})^{\frac{1}{1+\varepsilon}} = \frac{(1-\tau_2)R_2\left[(\Phi_{2I}+\Phi_{2F})+\Phi_{2F}(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}\frac{R_W}{(1-\tau_2)R_2}\right]\Big/N_2G_2(\underline{\varphi})}{(1-\tau_1)R_1\left[(\Phi_{1I}+\Phi_{1F})+\Phi_{1F}(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}\frac{R_W}{(1-\tau_1)R_1}\right]\Big/N_1G_1(\underline{\varphi})} \tag{46}$$

Considering equation (33'), (35') and (42):

$$\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} = \frac{N_2G_2(\underline{\varphi})}{N_1G_1(\underline{\varphi})} = \frac{(\Phi_{2I}+\Phi_{2F})+\Phi_{2F}(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}\frac{R_W}{(1-\tau_2)R_2}}{(\Phi_{1I}+\Phi_{1F})+\Phi_{1F}(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}\frac{R_W}{(1-\tau_1)R_1}} \tag{46'}$$

According to equation (16) and (34'), we have:

$$\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} = \frac{\boldsymbol{\Phi}_{2I}+\boldsymbol{\Phi}_{2F}+\left(\frac{A_2 w_2}{w_W}\right)^{\frac{1}{\varepsilon}}\boldsymbol{\Phi}_W}{\boldsymbol{\Phi}_{1I}+\boldsymbol{\Phi}_{1F}+\left(\frac{A_1 w_1}{w_W}\right)^{\frac{1}{\varepsilon}}\boldsymbol{\Phi}_W} \tag{47}$$

Denote

$$\boldsymbol{\Phi}_I = \boldsymbol{N}\int_{\underline{\varphi}}^{\overline{\varphi}} \boldsymbol{\varphi}^{\frac{1}{\varepsilon}}\,\mathrm{d}\boldsymbol{G}(\boldsymbol{\varphi}) = \boldsymbol{\Phi}_{1I} + \boldsymbol{\Phi}_{2I} \tag{48}$$

$$\boldsymbol{\Phi}_F = \boldsymbol{N}\int_{\overline{\varphi}}^{+\infty} \boldsymbol{\varphi}^{\frac{1}{\varepsilon}}\,\mathrm{d}\boldsymbol{G}(\boldsymbol{\varphi}) = \boldsymbol{\Phi}_{1F} + \boldsymbol{\Phi}_{2F} \tag{49}$$

Where $\boldsymbol{\Phi}_I$ and $\boldsymbol{\Phi}_F$ represents the aggregate productivity of the informal and formal sector in the economy, respectively. Using equation (48) and (49), rewriting (47) as:

$$\frac{\boldsymbol{\Phi}_{2I}+\boldsymbol{\Phi}_{2F}}{\boldsymbol{\Phi}_{1I}+\boldsymbol{\Phi}_{1F}} = \frac{\lambda_2^{\frac{1}{\varepsilon}}(\boldsymbol{\Phi}_I+\boldsymbol{\Phi}_F)+\boldsymbol{\Phi}_W\left(\frac{A_1^{\frac{1}{1+\varepsilon}}w_2}{w_W}\right)^{\frac{1}{\varepsilon}}\left[\left(\frac{A_1}{\theta_2}\right)^{\frac{1}{\varepsilon}}-\left(\frac{A_2}{\theta_1}\right)^{\frac{1}{\varepsilon}}\right]}{\lambda_1^{\frac{1}{\varepsilon}}(\boldsymbol{\Phi}_I+\boldsymbol{\Phi}_F)+\boldsymbol{\Phi}_W\left(\frac{A_2^{\frac{1}{1+\varepsilon}}w_1}{w_W}\right)^{\frac{1}{\varepsilon}}\left[\left(\frac{A_2}{\theta_1}\right)^{\frac{1}{\varepsilon}}-\left(\frac{A_1}{\theta_2}\right)^{\frac{1}{\varepsilon}}\right]} \tag{47'}$$

Based on equation (37), $\boldsymbol{A}_1^{\frac{1}{1+\varepsilon}}\boldsymbol{w}_2 = \boldsymbol{A}_2^{\frac{1}{1+\varepsilon}}\boldsymbol{w}_1$. Jointly analyzing (46') and (47'), leads to the following proposition 4 and 5 about concentration magnitude of different sectors and size distribution of formal firms across cities.

Proposition 4

(1) If an economy in which formal workers are more spatially concentrated to the efficient city than the informal workers, then the efficient city must have lower trade cost;

(2) If an economy in which informal workers are more spatially concentrated to the efficient city than the formal workers, then the efficient city must have lower formalization cost;

(3) If the efficient city have higher formalization cost, then formal workers are more spatially concentrated to the efficient city than the informal workers;

(4) If the efficient city have higher trade cost, then informal workers are more spatially concentrated to the efficient city than the formal workers;

Proof. Proving (1) and (2) first. Assuming city 2 is the efficient city, obtaining $\boldsymbol{\lambda}_2 > \boldsymbol{\lambda}_1$ based on the definition of efficient city. According to equation (47'),

$$\begin{cases} \frac{\boldsymbol{\Phi}_{2I}+\boldsymbol{\Phi}_{2F}}{\boldsymbol{\Phi}_{1I}+\boldsymbol{\Phi}_{1F}} \geq \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}}, \boldsymbol{if}\ \boldsymbol{A}_1\boldsymbol{\theta}_1 \geq \boldsymbol{A}_2\boldsymbol{\theta}_2 \\ \frac{\boldsymbol{\Phi}_{2I}+\boldsymbol{\Phi}_{2F}}{\boldsymbol{\Phi}_{1I}+\boldsymbol{\Phi}_{1F}} < \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}}, \boldsymbol{if}\ \boldsymbol{A}_1\boldsymbol{\theta}_1 < \boldsymbol{A}_2\boldsymbol{\theta}_2 \end{cases} \tag{50}$$

Considering these two cases with equation (46'), noted that $(\boldsymbol{\alpha}_{w1}\boldsymbol{A}_1)^{\frac{1}{\varepsilon}}\frac{Y_w/\boldsymbol{\Phi}_w}{(1-\tau_1)Y_1/\boldsymbol{\Phi}_1} = (\boldsymbol{\alpha}_{w2}\boldsymbol{A}_2)^{\frac{1}{\varepsilon}}\frac{Y_w/\boldsymbol{\Phi}_w}{(1-\tau_2)Y_2/\boldsymbol{\Phi}_2}$ in the equilibrium, because of the condition described in equation (36), obtains

$$\begin{cases} \frac{\Phi_{2F}}{\Phi_{1F}} \leq \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}}, if\ A_1\theta_1 \geq A_2\theta_2 \\ \frac{\Phi_{2F}}{\Phi_{1F}} > \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}}, if\ A_1\theta_1 < A_2\theta_2 \end{cases} \quad (51)$$

Combining relation shown in (50) and (51), yields

$$\begin{cases} \frac{\Phi_{2I}}{\Phi_{1I}} \geq \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} \geq \frac{\Phi_{2F}}{\Phi_{1F}}, if\ A_1\theta_1 \geq A_2\theta_2 \\ \frac{\Phi_{2I}}{\Phi_{1I}} < \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} < \frac{\Phi_{2F}}{\Phi_{1F}}, if\ A_1\theta_1 < A_2\theta_2 \end{cases} \quad (52)$$

When $A_1\theta_1 \geq A_2\theta_2$, since $\lambda_2 > \lambda_1$, $\frac{\Phi_{2I}}{\Phi_{1I}} \geq \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} \geq \frac{\Phi_{2F}}{\Phi_{1F}}$ represents that the informal workers are more concentrated than the formal workers in city 2, because the demand of labor $\frac{X(\varphi)}{\varphi} \sim \varphi^{\frac{1}{\varepsilon}}$.[13] Noted that $\lambda_2 > \lambda_1 \Rightarrow \frac{A_2{}^{1/(1+\varepsilon)}}{\theta_2} > \frac{A_1{}^{1/(1+\varepsilon)}}{\theta_1}$, then $A_1\theta_1 \geq A_2\theta_2$ leads to $\theta_1 \geq \theta_2$, meaning formalization cost in city 2 must be lower than city 1. In another situation, when $A_1\theta_1 < A_2\theta_2$, since $\lambda_2 > \lambda_1$, $\frac{\Phi_{2I}}{\Phi_{1I}} < \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} < \frac{\Phi_{2F}}{\Phi_{1F}}$ represents that the concentration magnitude is larger in the formal sector than the informal sector. Noted that $\lambda_2 > \lambda_1 \Rightarrow \frac{A_2{}^{1/(1+\varepsilon)}}{\theta_2} > \frac{A_1{}^{1/(1+\varepsilon)}}{\theta_1}$, then $A_1\theta_1 < A_2\theta_2$ leads to $A_1 < A_2$, which means trade cost in the efficient city must be lower. The conclusion in the case $\lambda_1 > \lambda_2$ is the same, so that (1) and (2) of proposition 4 are proven.

When $\theta_2 \geq \theta_1$, yields $A_1\theta_1 < A_2\theta_2$ because of $\lambda_2 > \lambda_1$. Based on equation (52), $\frac{\Phi_{2I}}{\Phi_{1I}} < \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} < \frac{\Phi_{2F}}{\Phi_{1F}}$ holds in this case, representing the formal workers prefer city 2 more, compared to the informal workers. When $A_1 \geq A_2$, obtaining $A_1\theta_1 \geq A_2\theta_2$ due to $\lambda_2 > \lambda_1$. Using equation (52), $\frac{\Phi_{2I}}{\Phi_{1I}} \geq \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} \geq \frac{\Phi_{2F}}{\Phi_{1F}}$ holds in this case, meaning the informal workers prefers city 2 more, compared to the formal workers. The conclusion from the case $\lambda_1 > \lambda_2$ is identical, which proves (3) and (4) of proposition 4.

Proposition 5

(1) If the efficient city has higher trade cost, then the average size of formal firms is smaller in the efficient city;

(2) If the average size of formal firms is larger in the efficient city, then the efficient city must have lower trade cost.

Proof. Assuming city 2 is the efficient city, obtaining $\lambda_2 > \lambda_1$ based on the definition of efficient city. Higher trade cost in the efficient city means $A_1 \geq A_2$, so that $\theta_1 > \theta_2$ because of $\lambda_2 > \lambda_1$. As a result, $A_1\theta_1 \geq A_2\theta_2$, leading to $\frac{\Phi_{2F}}{\Phi_{1F}} \leq \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}}$, with (45), obtains

[13] So that L_{cI}^D and L_{cF}^D, the labor demand in the formal and informal sector of city c, satisfying

$L_{cI}^D = N_c \int_{\underline{\varphi}}^{\overline{\varphi}} \frac{X(\varphi)}{\varphi} \mathrm{d}G_c(\varphi) \sim N_c \int_{\underline{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\varepsilon}} \mathrm{d}G_c(\varphi) = \Phi_{cI}$ and $L_{cF}^D = N_c \int_{\overline{\varphi}}^{+\infty} \frac{X(\varphi)}{\varphi} \mathrm{d}G_c(\varphi) \sim N_c \int_{\overline{\varphi}}^{+\infty} \varphi^{\frac{1}{\varepsilon}} \mathrm{d}G_c(\varphi) = \Phi_{cF}$, respectively.

$$\frac{\Phi_{2F}}{\Phi_{1F}} \leq \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} < \left(\frac{\lambda_2}{\lambda_1}\right)^{1+\frac{1}{\varepsilon}} < \frac{N_2(1-G_2(\overline{\varphi}))}{N_1(1-G_1(\overline{\varphi}))} \Rightarrow \frac{\Phi_{2F}}{N_2(1-G_2(\overline{\varphi}))} \leq \frac{\Phi_{1F}}{N_1(1-G_1(\overline{\varphi}))} \tag{53}$$

Where $\frac{\Phi_{cF}}{N_c(1-G_c(\overline{\varphi}))}$ can be taken as the average size of formal firms in city c, because the size of formal firms depends on $\frac{X(\varphi)}{\varphi} \sim \varphi^{\frac{1}{\varepsilon}}$, the amount of labor employed. Equation (53) claims that the average size of formal firms is smaller in the efficient city with higher trade cost. In other words, the share of larger formal firms in city 1 is bigger than those in city 2, while it is opposite for the smaller formal firms. The potential explanation is that, under free mobility condition, the less skillful formal entrepreneurs prefer the city with lower formalization cost to diminish their fixed cost, which pulls the average size of formal firms in efficient city down. The part (1) of proposition 5 is proven.

If the average size of formal firms is larger in the efficient city, then

$$\frac{\Phi_{2F}}{N_2(1-G_2(\overline{\varphi}))} > \frac{\Phi_{1F}}{N_1(1-G_1(\overline{\varphi}))} \Rightarrow \frac{\Phi_{2F}}{\Phi_{1F}} > \frac{N_2(1-G_2(\overline{\varphi}))}{N_1(1-G_1(\overline{\varphi}))} > \left(\frac{\lambda_2}{\lambda_1}\right)^{1+\frac{1}{\varepsilon}} > \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{\varepsilon}} \tag{54}$$

Consequently, $A_1\theta_1 < A_2\theta_2$ because of equation (49). Noted that $\lambda_2 > \lambda_1 \Rightarrow \frac{A_2{}^{1/(1+\varepsilon)}}{\theta_2} > \frac{A_1{}^{1/(1+\varepsilon)}}{\theta_1}$, then it must have $A_1 < A_2$, which proves part (2) of proposition 5. The explanation for this proposition is that the more skillful formal entrepreneurs prefer the city with lower trade cost since they rely more on export, which can be the reason of pushing the average size of formal firms in city 2 up.

Taking various methods to improve governance efficiency brings different result on the aspect of spatial distribution, according to proposition 4 and 5. Promoting governance through investment on infrastructure to lower trade cost makes the share of larger firms in the formal sector bigger, and concentration degree of the formal sectors higher, compared to the informal workers. In stead of improving the quality of trade service due to the constrain of public finance, if local government tries to encourage private investment with lower fixed cost, like cutting some tax and providing better business environment, then more smaller formal firms, rather than the larger formal firms will move in, and the magnitude of concentration will be higher in the informal sector than the formal sector. In addition, the effect of decreasing formalization cost leads to assemble of the informal sector can be used to explain the equation (31'), which claims that local price index is proportional to the formalization cost, because of the cheaper goods and service from the informal firms.

5, Empirical Evidences of Policy Impact

In this section, the predictions of model are tested, including the same location preference between formal employment and informal employment, better trade infrastructure promotes the expansion of the formal sector and lower tax incurs more informal firms in the efficient city. The statistics of provincial road and tax income between 2005 and 2015 from CEIC dataset make it possible to find out evidence supporting the model. There are 31 provinces in the statistics of Indonesia during 2005 to 2015. For each province, we use the provincial density of "good road" to represent trade efficiency A_c in the model, because longer and wider domestic road is found to have positive impact on trade (Cosar & Demir, 2016). The reason for using provincial density of road is to discuss the impact of road after controlling provincial area, and "good road" is the road which are used for transportation most frequently. What's more, in order to be consistent with model's setting, formalization cost θ_c is estimated with tax income divided by the number of formal firms in every province. Finally, the ratio between trade efficiency and formalization cost is the simulation of local governance efficiency λ_c.

Table 4 and ***Table 5*** provide evidence of lemma 1 and proposition 3, which claims that workers, informal firms and formal firms all prefer cities with better governance, and the concentration extent is larger for the formal firms, compared to the workers. All the correlation between local λ_c and workers/informal firms/formal firms in ***Table 4*** are positive, and the correlation of formal firms are bigger than those of workers. These facts supports the prediction of lemma 1 and 4. The higher concentration degree of formal firms is also proven by the fact that the standard deviation across counties of formal firms' number's natural logarithm is larger than those of workers' natural logarithm during 2005 to 2015, based on ***Table 5***.

Table 4. Governance efficiency $\boldsymbol{\lambda_c}$ and workers/firms, 2005-2015

	2005	2010	2015
Cor(worker, $\boldsymbol{\lambda_c}$)	0.3535	0.4722	0.3316
Cor(formal firm, $\boldsymbol{\lambda_c}$)	0.3907	0.4826	0.3877
Cor(informal firm, $\boldsymbol{\lambda_c}$)	0.2560	0.4551	0.3474

Source: CEIC and NLFS

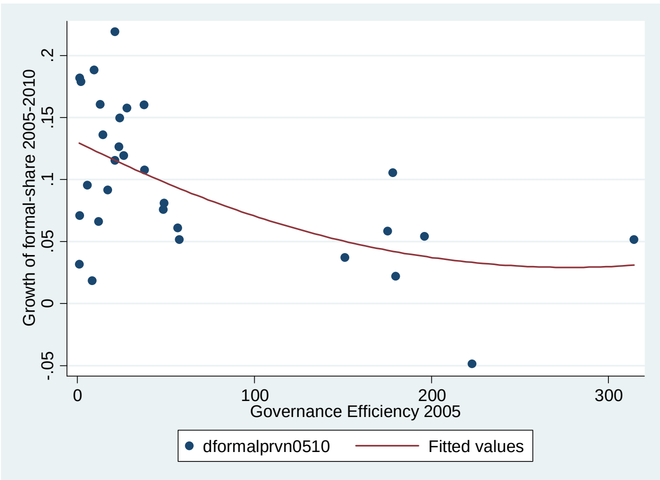
Table 5. Dispersion of Employment, 2005-2015

	2005	2010	2015
S.D. of labor size (ln scale)	1.4196	1.9303	1.7522
S.D. of workers (ln scale)	1.1857	1.1465	1.1005
S.D. of number of informal firm (ln scale)	1.0546	1.0975	1.0050
S.D. of number of formal firm (ln scale)	1.2684	1.2197	1.1046

Source: CEIC and NLFS

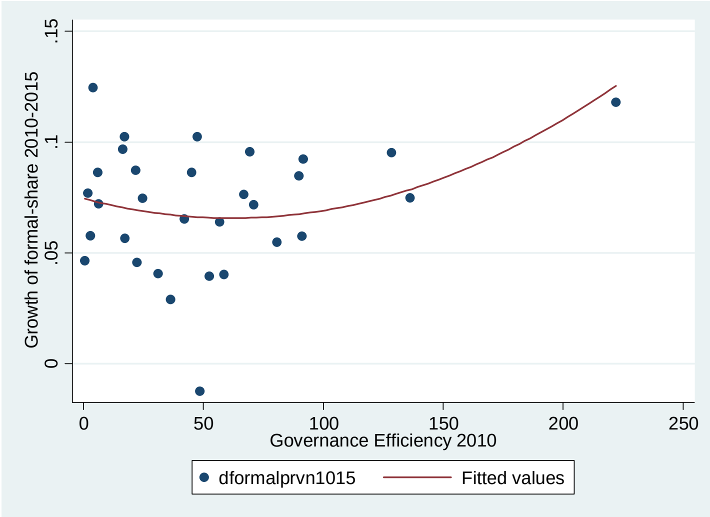
In order to test the policy impact predicted in proposition 4 and 5, figuring out the relation between local governance efficiency and relative concentration magnitude of the formal employment first, which can be observed through the growth of formal-share:

Figure 9. Governance efficiency and growth of formal-share,2005-2010



Source: CEIC and NLFS

Figure 10. **Governance efficiency and growth of formal-share,2010-2015**



Source: CEIC and NLFS

The relation between local governance efficiency and growth of formal employment share is different in the last decade. In the first half, the formal-share's growth is smaller in the efficient provinces with higher λ_c, meaning that the concentration extent is larger in the informal sector than the formal sector. However, the situation changes in the second half. The positive relation implies that formal sector prefers the efficient provinces more in this period. ***Table 6*** provides us more insight about the contrast between these two pattern during 2005 to 2010 and 2010 to 2015:

Table 6. **A_c, θ_c and λ_c five years ago and provincial growth of formal-share, 2005-2010 and 2010-2015**

	2005-2010	2010-2015
Mean(λ_c)	100.17	92.595
Mean(A_c)	0.4301	0.2736
Mean(θ_c)	0.00390	0.00299
Mean(λ_c, GF>=0.95)	15.66	187.42
Mean(A_c, GF>=0.95)	0.0494	0.4432
Mean(θ_c, GF>=0.95)	0.00295	0.00270
Mean(λ_c, GF<=0.05)	222.94	48.498
Mean(A_c, GF<=0.05)	0.1479	0.1220
Mean(θ_c, GF<=0.05)	0.00066	0.00252

Source: CEIC and NLFS

Where GF is the cumulative distribution function of formal-share's growth, so that the provinces whose growth of formal employment share satisfying GF>=0.95/GF<=0.05 are those with the largest/smallest formal-share's growth. Combining ***Figure 9*** and the second column of ***Table 6***, growth of formal-share is negatively depending on λ_c from 2005 to 2010. In other words, the informal sector expands faster than the formal sector in the efficient provinces in this period. Comparing A_c, θ_c and λ_c of the provinces in the group with GF<=0.05 to the average level, though the trade efficiency is much smaller than the average level, their λ_c are still much bigger, because θ_c of the efficient provinces are extremely low, only equal to one sixth of the average formalization cost. On the other hand, according to ***Figure 10***, the formal sector develops much faster than the informal sector in the efficient provinces from 2010 to 2015. Comparing $\boldsymbol{A_c}$, $\boldsymbol{\theta_c}$ and $\boldsymbol{\lambda_c}$ of the provinces in the group with GF>=0.95 to the average level, though the formalization cost is nearly equal to the average level, their $\boldsymbol{\lambda_c}$ are still much bigger, because of the much higher $\boldsymbol{A_c}$ in the efficient provinces. The situation in the efficient provinces during 2005-2010 and 2010-2015 are perfectly consistent with the prediction of proposition 4.

Table 7 and ***Table 8*** are estimated to quantify the impact of $\boldsymbol{A_c}$, $\boldsymbol{\theta_c}$ and $\boldsymbol{\lambda_c}$ on the formal employment share. [14] Since there is hysteresis of the impact in the real world, ***Table 7*** focuses on the correlation between current formal-share and the local characteristics $\boldsymbol{A_c}$, $\boldsymbol{\theta_c}$, $\boldsymbol{\lambda_c}$ and number of formal firms five years ago. From ***Table 7***, compared to the $\boldsymbol{\lambda_c}$ and number of formal firms, $\boldsymbol{A_c}$ and $\boldsymbol{\theta_c}$ five years ago are more significant in affecting the current formal employment share. Based on this finding, a regression with current formal-share as dependent variable and ln($\boldsymbol{\theta_c}$), ln($\boldsymbol{A_c}$) five years ago as independent variables is done with the data during 2005-2010 and 2010-2015. The result of this regression is shown in ***Table 8***. [15] After controlling the trade efficiency $\boldsymbol{A_c}$, every 1% increase of $\boldsymbol{\theta_c}$ is predicted to raise formal-share by more than 10%. That means, if a province having the same trade efficiency as the others would like to be the efficient province through lowering its formalization cost, then every 1% decrease of $\boldsymbol{\theta_c}$ will incur more than 10% decrease of formal-share. On the other hand, After controlling the formalization cost $\boldsymbol{\theta_c}$, every 1% increase of $\boldsymbol{A_c}$ is expected to cause the formal-share increase by about 2% in the period of 2005 to 2015, though the effect is not as statistically significant as those of $\boldsymbol{\theta_c}$. That means, if a province having the same formalization cost as the others becomes efficient province through improving its infrastructure to lower the trade cost, then every 1% increase of $\boldsymbol{A_c}$ raise formal-share by around 2%.

Table 7. **Formal-share and local characteristics five years ago, 2005-2010 and 2010-2015**

	2005-2010	2010-2015
Cor(formal share, $\boldsymbol{\lambda_c}$ [5])	0.1926	0.1157
Cor(formal share, $\boldsymbol{A_c}$ [5])	0.5890	0.2718
Cor(formal share, $\boldsymbol{\theta_c}$ [5])	0.7407	0.5136
Cor(formal share, formal firm[5])	-0.0404	0.0847

Source: CEIC and NLFS

Table 8. **Formal-share and local $\boldsymbol{A_c}$ and $\boldsymbol{\theta_c}$ five years ago, 2005-2010 and 2010-2015**

Formal-share=α_0+α_1*ln($\boldsymbol{\theta_c}$ [5])+α_2* ln($\boldsymbol{A_c}$ [5])		
	2005-2010	2010-2015
α_1	0.1006***	0.1385***
α_2	0.0208**	0.0175

Source: CEIC and NLFS

Table 9 is the statistics of $\boldsymbol{A_c}$, $\boldsymbol{\theta_c}$ and large formal firms' proportion in total number of formal firms ("large-share" for short) for the provinces in the group of E>=0.95, where E is the cumulative distribution function of provincial $\boldsymbol{\lambda_c}$. According to NLFS of Indonesia, large formal firms refer to those employing more than 20 workers, and the statistics is only available in 2005 and 2010's survey.

[14] "[5]" represents that this is a variable five years ago.

[15] *** p<0.01, ** p<0.05, * p<0.1

Table 9. **A_c, θ_c and Large formal firms share of provinces with highest λ_c, 2005 and 2010**

	2005	2010
Mean(large-share)	0.00106	0.00144
Mean(A_c)	0.4301	0.2736
Mean(θ_c)	0.00390	0.00299
Mean(large-share, E>=0.95)	0.00189	0.00189
Mean(A_c, E>=0.95)	5.4008	0.4596
Mean(θ_c, E>=0.95)	0.01643	0.00328

Source: CEIC and NLFS

Compared to the average level, large formal firms' share is higher in provinces with E>=0.95, and so are the trade service A_c and formalization cost θ_c, supporting part (2) of proposition 5 driven from the model. Similar as the regression in ***Table 8***, the regression "Large-share=β_0+β_1*ln(θ_c [5])+ β_2* ln(A_c [5])" is estimated, where β_1=0.00058 and β_2=0.00022, and both of them are statistically significant. These coefficients imply that, controlling local trade efficiency, 1% decrease of formalization cost makes the large formal firms' share decrease by 0.058% in the efficient city. Similarly, controlling local formalization cost, every 1% improvement on trade efficiency promotes large formal firms' share by 0.022%.

Formal firms, especially for those with larger size, preferring efficient cities with more efficient trade service, while the informal sector and formal firms whose managers are near the skill threshold of formalization would like to locate in the efficient city with lower entrance requirement of the formal sector. The evidence driven from Indonesia's data is broadly consistent with the pattern predicted by proposition 4 and 5. For the provinces relying on trade, they usually enjoy advantage in geography and already have better transportation network to support the demand for export and import. Moreover, their incentive to maintain and improve the public infrastructure is much stronger. Lower trade cost attracts formal firms, especially the larger formal firms, to run their business there, because their products with lower price can be more competitive in the global market. What's more, under globalization and trade liberalization, the demand from international market increases significantly, as shown in ***Figure 2***. As a result, higher trade efficiency not only means more profitable currently, but also brings expectation of faster growth on income in the future, because of the bigger elasticity to the scale of international demand. In addition, as formal economy develops, the expanding base of public finance further strengthens the willingness and ability of local government to provide better public goods. It implies that there is increasing return of formality, which provides a potential mechanism to support formalization in the long term.

On the other hand, for the provinces in shortage of public infrastructure, typically their public finance is also constrained, seen in the second row of ***Table 10***:

Table 10. **A_c, θ_c and direct expenditure on public goods and service, 2005-2010 and 2010-2015**

	2005-2010	2010-2015
Cor(Expenditure, θ_c [5])	0.7425	0.1042
Cor(A_c, Expenditure [5])	0.4329	0.2730

Source: CEIC

In order to promote the economy growth, these provinces improve local λ_c through cutting down the tax rate to encourage establishing new firms and creating job opportunities. With less tax, individuals whose skill are close to the formalization threshold $\overline{\varphi}_c$ are expected to come and run their own formal firms. However, less tax income implies that government direct expenditure on public goods and service will become lower, bringing negative effect on local trade efficiency in the future. The potential welfare loss caused by lower expectation of $\boldsymbol{A_c}$ makes the formal firms with relative large size not prefer the provinces with small $\boldsymbol{\theta_c}$, even though local $\boldsymbol{\lambda_c}$ is higher than other provinces with better trade infrastructure. According to lemma 1, the increase in the formal sector also incurs new informal firms, so that the concentration magnitude of the informal employment is larger than the formal employment in the provinces with small $\boldsymbol{\theta_c}$, as observed in Indonesia's case from 2005 to 2015 and predicted by proposition 4 and 5 in the theoretical model.

6, Conclusion

Inadequate public infrastructure raises the cost to run formal business and incurs high informality. Large informal sector constrains public finance and intensifies the infrastructure shortage in developing countries, such as Indonesia. Policy debate in the existing literature focuses on how to raise the labor market participation in the formal sector. To provide new insight of this issue, this paper put forward a coherent micro-foundation to study the formal and informal employment through examining their spatial variation with Indonesia's data. The elementary empirical work implies that formal employment share increases in the whole economy but still varies considerably across region, and the development of formal economy is accompanying with large scale of informal sector. In order to explain the coexistence of formality and informality and study the reaction of formal-share's growth to local trade efficiency and formalization cost, a theoretical model is put forward to investigate the motivation of occupation selection and location sorting of the formal and informal sector, based on the theoretical framework in Behrens, Duranton and Robert-Nicoud (2014) and Dixit & Stiglizs (1978). It is found that the efficient city (or province/region) with higher governance efficiency $\boldsymbol{\lambda_c}$, meaning better local trade efficiency $\boldsymbol{A_c}$ or lower formalization cost $\boldsymbol{\theta_c}$, not only attracts workers, but also encourages more formal firms. As the local formal economy develops, firms in both formal and informal sectors become more profitable, so that more enterprises move in to pursue higher revenue. Taking various methods to become efficient city, however, bringing different result on the spatial distribution pattern. The formal share's growth and average size of formal firms is bigger in the efficient city with more efficient trade service. On the other hand, lowering the formalization cost leads to slower growth of formal employment share and smaller average size of formal firms. Based on Indonesia census data from 2000 to 2010, National Labor Force Survey of Indonesia during 1995 to 2015 and other macro data from CEIC dataset, this paper analyzes formality and informality on province, county or even individual level. The empirical findings are broadly consistent with the propositions in the model. The quantitative estimation shows that every 1% increase of trade efficiency raises local formal-share by around 2%, while every 1% decrease of formalization cost causes more than 10% decrease of formal-share. As for the share of large formal firms, 1% decrease of formalization cost makes it decrease by 0.058% in the efficient city. Similarly, controlling local formalization cost, every 1% improvement on trade efficiency promotes large formal firms' share by 0.022%.

There are at least three potential direction waiting to be focused on in the future research. Firstly, the implication from the framework of inter-city trade worth studying separately, because domestic demand will become more and more significant as the economy grows. What's more, the expected multiply effect caused by the inter-city trade may lead to increasing return in formal economy, which sustains formalization in the long term. In addition, the framework of N-cities to study inter-city trade makes it possible to do counterfactual analysis with Indonesia's data, so that we can quantify the impact of factors on determining formal and informal employment distribution. Secondly, ***Table 10*** implies that there is positive correlation between current tax income, government expenditure on public goods and the trade efficiency in the future. In addition, suppose there is endogenous relation between $\boldsymbol{A_c}$ and $\boldsymbol{\theta_c}$, it can be used to simplify the analysis of equation (47') and improve proposition 4 and 5 in the model. Most importantly, endogenous $\boldsymbol{A_c}$ and $\boldsymbol{\theta_c}$ capture the linkage of formality to development, on the perspective of promoting the quality and quantity of public goods and infrastructure. Thirdly, compared to the given skill distribution assumption, introducing over-lapping generation setting and personal learning behavior, as those mentioned in Lucas (2004) strengthens the model in understanding the incentive of human capital accumulation and its effect on spatial sorting.

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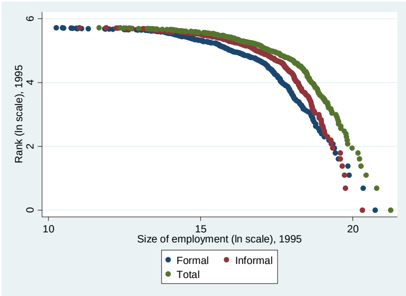
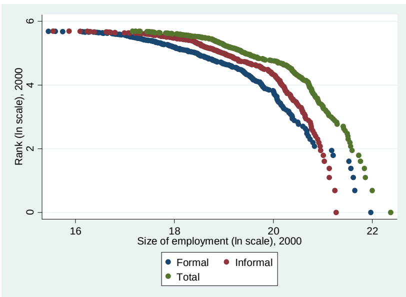
Appendix A: Spatial Distribution of Formal and Informal Employment across counties in Indonesia, 1995-2015

Figure 3-1. **The spatial Distribution of Employment across counties, 1995**



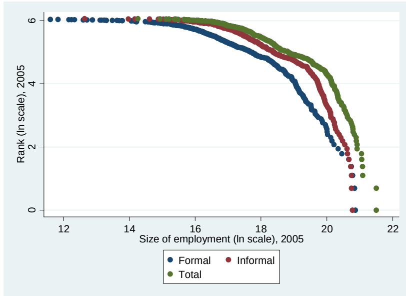
Source: NLFS

Figure 3-2. **The spatial Distribution of Employment across counties, 2000**



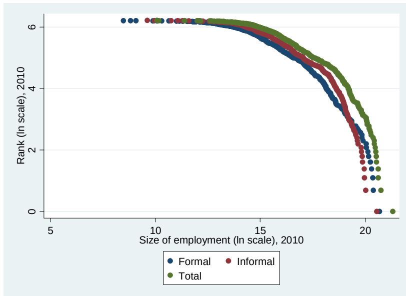
Source: NLFS

Figure 3-3. **The spatial Distribution of Employment across counties, 2005**



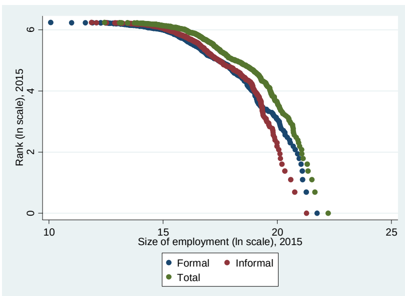
Source: NLFS

Figure 3-4. **The spatial Distribution of Employment across counties, 2010**



Source: NLFS

Figure 3-5. **The spatial Distribution of Employment across counties, 2015**



Source: NLFS

Appendix B: Separate Equilibrium of Spatial Sorting

In this section, we discuss the possibility and potential propositions of ***separate equilibrium of sorting***, which is different with ***perfectly-mixed-skill equilibrium*** analyzed in the model part. Proved in section 3.3, separate equilibrium is only possible for formal entrepreneurs. Denote

$$\boldsymbol{H(\varphi) = u_{1F}(\varphi) - u_{2F}(\varphi)} \tag{A1}$$

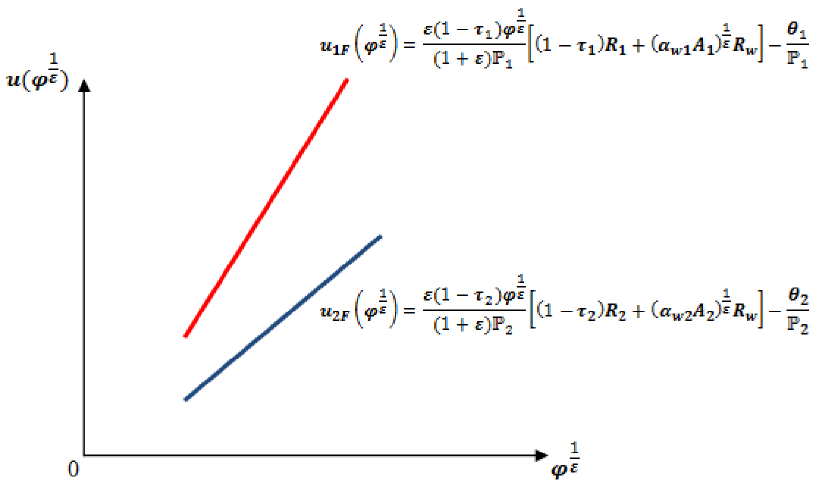
Expanding (A1) with equation (33)-(35):

$$\boldsymbol{H(\varphi)} = \frac{\varepsilon(1-\tau_1)^2 Y_1}{(1+\varepsilon)\mathbb{P}_1 \Phi_1}\varphi^{\frac{1}{\varepsilon}}\left[(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}\frac{Y_w/\Phi_w}{(1-\tau_1)Y_1/\Phi_1} - (\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}\frac{Y_w/\Phi_w}{(1-\tau_2)Y_2/\Phi_2}\right] - \left(\frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2}\right) \tag{A1'}$$

Mixed-skill equilibrium requires equation (36) holds for any $\varphi \subset [\overline{\varphi}_{c'}, +\infty)$, so that the case in which $\frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2} = 0$ and $\frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2} = 0$ are true simultaneously is both sufficient and necessary condition of perfectly-mixed-skill equilibrium. When $\frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2} = 0$ and $\frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2} \neq 0$, shown as ***Figure 8-3***, obtains

$$\begin{cases} H(\varphi) > 0, if \ \frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2} > 0, \varphi \subset [\overline{\varphi}_{c'}, +\infty) \\ H(\varphi) < 0, if \ \frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2} < 0, \varphi \subset [\overline{\varphi}_{c'}, +\infty) \end{cases} \tag{A2}$$

Figure 8-3. **$u_{1F}\left(\varphi^{\frac{1}{\varepsilon}}\right)$ and $u_{2F}\left(\varphi^{\frac{1}{\varepsilon}}\right)$ (Different Slope, same Intercept)**



Relation (A2) claims that, if real formalization cost is the same across cities, which means $\frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2} = 0$, all formal firms will sort for location in which is able to maximize the benefit from trade. The case in (A2) is possible in the theory, but it is not consistent with the real world, such as Indonesia, because formal-share of various statistic groups in ***Table 1*** all increase during 1995 to 2015.

We then turn to another situation in which the symbol of $\frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2}$ and $\frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2}$ are identical. The function images of $u_{1F}\left(\varphi^{\frac{1}{\varepsilon}}\right)$ and $u_{2F}\left(\varphi^{\frac{1}{\varepsilon}}\right)$ are exhibited in ***Figure 8-1***. Studying the case in which both of them are positive first, we have

$$\begin{cases} \frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2} > 0 \Rightarrow \left(\frac{w_2}{w_1}\right)^{\frac{1+\varepsilon}{\varepsilon}}\left(\frac{A_1}{A_2}\right)^{\frac{1}{\varepsilon}} > 1 \\ \frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2} > 0 \Rightarrow \frac{\theta_1\mathbb{P}_2}{\theta_2\mathbb{P}_1} > 1 \end{cases} \tag{A3}$$

Replacing the (A1') with equation (23), (33) and (35), yields

$$\begin{cases} \boldsymbol{H}(\overline{\boldsymbol{\varphi}}_1) = \frac{\boldsymbol{\theta}_1}{\mathbb{P}_1}[\frac{\boldsymbol{\theta}_2\mathbb{P}_1}{\boldsymbol{\theta}_1\mathbb{P}_2} - \left(\frac{\boldsymbol{w}_1}{\boldsymbol{w}_2}\right)^{\frac{1+\varepsilon}{\varepsilon}}\left(\frac{A_2}{A_1}\right)^{\frac{1}{\varepsilon}}] \\ \boldsymbol{H}(\overline{\boldsymbol{\varphi}}_2) = \frac{\boldsymbol{\theta}_2}{\mathbb{P}_2}[\left(\frac{\boldsymbol{w}_2}{\boldsymbol{w}_1}\right)^{\frac{1+\varepsilon}{\varepsilon}}\left(\frac{A_1}{A_2}\right)^{\frac{1}{\varepsilon}} - \frac{\boldsymbol{\theta}_1\mathbb{P}_2}{\boldsymbol{\theta}_2\mathbb{P}_1}] \end{cases} \tag{A4}$$

Using the equation (33) and the LHS of (39):

$$\overline{\boldsymbol{\varphi}}_1/\overline{\boldsymbol{\varphi}}_2 = \left[\frac{\boldsymbol{\theta}_1\mathbb{P}_2}{\boldsymbol{\theta}_2\mathbb{P}_1}\left(\frac{\boldsymbol{w}_1}{\boldsymbol{w}_2}\right)^{\frac{1+\varepsilon}{\varepsilon}}\left(\frac{A_2}{A_1}\right)^{\frac{1}{\varepsilon}}\right]^{\varepsilon} \tag{A5}$$

According to (A4) and (A5), if $\overline{\boldsymbol{\varphi}}_1/\overline{\boldsymbol{\varphi}}_2 < 1 \Rightarrow \left(\frac{w_1}{w_2}\right)^{\frac{1+\varepsilon}{\varepsilon}}\left(\frac{A_2}{A_1}\right)^{\frac{1}{\varepsilon}} < \frac{\theta_2\mathbb{P}_1}{\theta_1\mathbb{P}_2}$, then $\boldsymbol{H}(\overline{\boldsymbol{\varphi}}_1) > 0$. Since $\frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2} > 0$, noticed that $\boldsymbol{H}(\boldsymbol{\varphi})$ is a monotonic increasing function of $\boldsymbol{\varphi}$. Now that $\overline{\boldsymbol{\varphi}}_1 < \overline{\boldsymbol{\varphi}}_2$ and $\boldsymbol{H}(\overline{\boldsymbol{\varphi}}_1) > 0$, the cutoff of separate equilibrium $\boldsymbol{\varphi}_0$ is smaller than $\overline{\boldsymbol{\varphi}}_1$. Hence for all $\boldsymbol{\varphi} \subset [\overline{\boldsymbol{\varphi}}_c, +\infty)$, such that $\boldsymbol{H}(\boldsymbol{\varphi}) > 0$. In this case, the sorting behavior of the formal entrepreneurs is similar as those described by (A2), in which all formal firms select the same city.

If $\overline{\boldsymbol{\varphi}}_1/\overline{\boldsymbol{\varphi}}_2 > 1 \Rightarrow \left(\frac{w_1}{w_2}\right)^{\frac{1+\varepsilon}{\varepsilon}}\left(\frac{A_2}{A_1}\right)^{\frac{1}{\varepsilon}} > \frac{\theta_2\mathbb{P}_1}{\theta_1\mathbb{P}_2}$, then $\boldsymbol{H}(\overline{\boldsymbol{\varphi}}_1) < 0$ and $\boldsymbol{H}(\overline{\boldsymbol{\varphi}}_2) < 0$. Based on the definition of $\boldsymbol{H}(\boldsymbol{\varphi})$, $\boldsymbol{H}(+\infty) > 0$. As a result, there is $\boldsymbol{\varphi}_0 \subset [\overline{\boldsymbol{\varphi}}_c, +\infty)$, such that $\boldsymbol{H}(\boldsymbol{\varphi}_0) = \mathbf{0}$. Now the separate equilibrium implies that the individuals with skill $\boldsymbol{\varphi} \subset [\overline{\boldsymbol{\varphi}}_2, \boldsymbol{\varphi}_0)$ chooses city 2 to run their formal firms, while the other group of individual whose $\boldsymbol{\varphi} \subset [\boldsymbol{\varphi}_0, +\infty)$ selects city 1. At this situation, the average formal firms' size is larger in city 1 than city 2. The case in which the symbol of $\frac{(\alpha_{w1}A_1)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_1)R_1} - \frac{(\alpha_{w2}A_2)^{\frac{1}{\varepsilon}}R_w}{(1-\tau_2)R_2}$ and $\frac{\theta_1}{\mathbb{P}_1} - \frac{\theta_2}{\mathbb{P}_2}$ are negative leads to similar conclusion.

Appendix C: The Parameter Range of Mixed-skill equilibrium

Based on the conditions and propositions of mixed-skill equilibrium put forward in the model part, we identify the parameters' range in which the mixed-skill equilibrium exists.

Denote $\boldsymbol{T} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2$, using equation (32), obtains:

$$\begin{cases} \boldsymbol{\tau}_1 = \frac{(\boldsymbol{T}-1)\lambda_2+\lambda_1}{\lambda_1+\lambda_2} \\ \boldsymbol{\tau}_2 = \frac{(\boldsymbol{T}-1)\lambda_1+\lambda_2}{\lambda_1+\lambda_2} \end{cases} \Rightarrow \frac{\tau_1}{\tau_2} = \frac{{N_1}^{1/2}}{{N_2}^{1/2}} = \frac{(\boldsymbol{T}-1)\lambda_2+\lambda_1}{(\boldsymbol{T}-1)\lambda_1+\lambda_2} \tag{A6}$$

Since $\boldsymbol{N}_1 + \boldsymbol{N}_2 = \boldsymbol{N}$, $\boldsymbol{T} = (\boldsymbol{b}{\boldsymbol{N}_1}^{1/2} + \boldsymbol{b}{\boldsymbol{N}_2}^{1/2}) \in [\boldsymbol{b}\boldsymbol{N}^{1/2}, \sqrt{2}\boldsymbol{b}\boldsymbol{N}^{1/2}]$. In order to reach mixed-skill equilibrium, the corner solution in which all firms and workers stay in one city have to be irrational, such that $\boldsymbol{b}\boldsymbol{N}^{1/2} > 1$. On the other hand, in order to guarantee there is $\{\boldsymbol{N}_1, \boldsymbol{N}_2\}$ satisfying $\boldsymbol{\tau}_1 < 1$ and $\boldsymbol{\tau}_2 < 1$, the upper bound of $\boldsymbol{b}\boldsymbol{N}^{1/2}$ is $\sqrt{\mathbf{2}}$. Denote

$$\boldsymbol{R}(\boldsymbol{T}) = \frac{(\boldsymbol{T}-1)(\lambda_2+\lambda_1)+(2-\boldsymbol{T})\lambda_1}{(\boldsymbol{T}-1)(\lambda_2+\lambda_1)+(2-\boldsymbol{T})\lambda_2} \tag{A7}$$

When $\boldsymbol{T} \to \mathbf{1}, \boldsymbol{R}(\boldsymbol{T}) \to \frac{\lambda_1}{\lambda_2}$; When $\boldsymbol{T} \to \mathbf{2}, \boldsymbol{R}(\boldsymbol{T}) \to \mathbf{1}$. Since $\boldsymbol{R}(\boldsymbol{T})$ is a continuous strictly monotonic function of $\boldsymbol{T}$, rewriting (A10) as:

$$\boldsymbol{R(\beta)} = \left(\frac{\lambda_1}{\lambda_2}\right)^{\boldsymbol{\beta}} \tag{A7'}$$

Where $\boldsymbol{T} \to \boldsymbol{\beta}$: $(\mathbf{1},\mathbf{2}) \to (\mathbf{0},\mathbf{1})$. With (A13'), we have

$$\frac{\boldsymbol{N_1}}{\boldsymbol{N_2}} = \left(\frac{\lambda_1}{\lambda_2}\right)^{2\boldsymbol{\beta}} \tag{A8}$$

If perfectly-mixed-skill equilibrium exists, then (45) and proposition 3 must hold. Decomposing (A8), yields

$$\frac{\boldsymbol{N_1}}{\boldsymbol{N_2}} = \frac{\boldsymbol{N_1 G_1}(\underline{\boldsymbol{\varphi}})+\boldsymbol{N_1}(1-\boldsymbol{G_1}(\overline{\boldsymbol{\varphi}}))+\boldsymbol{N_1}(\boldsymbol{G_1}(\overline{\boldsymbol{\varphi}})-\boldsymbol{G_1}(\underline{\boldsymbol{\varphi}}))}{\boldsymbol{N_2 G_2}(\underline{\boldsymbol{\varphi}})+\boldsymbol{N_2}(1-\boldsymbol{G_2}(\overline{\boldsymbol{\varphi}}))+\boldsymbol{N_2}(\boldsymbol{G_2}(\overline{\boldsymbol{\varphi}})-\boldsymbol{G_2}(\underline{\boldsymbol{\varphi}}))} \tag{A9}$$

With (45), denote $\frac{N_1(1-G_1(\overline{\varphi}))}{N_2(1-G_2(\overline{\varphi}))} = \left(\frac{\lambda_1}{\lambda_2}\right)^{\boldsymbol{\eta}}$, where $\boldsymbol{\eta} > 1 + \frac{1}{\varepsilon}$. The existence of mixed-skill equilibrium requires $\{\mathbf{2}\boldsymbol{\beta}|\boldsymbol{\beta} \in (\mathbf{0},\mathbf{1})\} \cap (\frac{1}{\varepsilon},\boldsymbol{\eta}) \neq \emptyset$. As a result, the sufficient condition of mixed-skill equilibrium is that there is $\boldsymbol{T_0} \in (\mathbf{1},\mathbf{2})$, such that $\mathbf{2}\boldsymbol{\beta}(\boldsymbol{T_0}) \in (\frac{1}{\varepsilon},\boldsymbol{\eta})$. Now that we have $\mathbf{2}\boldsymbol{\beta} \in (\mathbf{0},\mathbf{2})$ and $\boldsymbol{\varepsilon} > 0$, the parameters' range of mixed-skill equilibrium is $\boldsymbol{\varepsilon} \in (\mathbf{1/2},+\infty)$ and $\boldsymbol{bN^{1/2}} \in (\mathbf{1},\sqrt{\mathbf{2}})$. Since the elasticity of substitution for any two goods is equal to $(\mathbf{1}+\frac{1}{\varepsilon})$, $\boldsymbol{\varepsilon} \in (\mathbf{1/2},+\infty)$ implies that the mixed-skill equilibrium exists when the substitution elasticity is less than 3.

Appendix D: The Fixed-Point Mapping of Model

In this section, we provide algorithm to show fixed-point mapping of the perfectly-mixed-skill equilibrium in definition 1.

Since $\boldsymbol{N_1} + \boldsymbol{N_2} = \boldsymbol{N}$, with equation (38), $\{\boldsymbol{N_c}, \boldsymbol{\tau_c}\}$ for $\boldsymbol{c} \in \{1, 2\}$ are solved. For simplicity, wage rate in city 1 $\boldsymbol{w_1}$ is normalized to 1. Using equation (23), $\overline{\boldsymbol{\varphi}}$ is figured out:

$$\overline{\boldsymbol{\varphi}} = \left[\frac{\boldsymbol{\theta_1}(1+\boldsymbol{\varepsilon})\boldsymbol{\Phi_w}}{\boldsymbol{\varepsilon}(1-\boldsymbol{\tau_1})(\boldsymbol{w_w A_1})^{\frac{1}{\varepsilon}}\boldsymbol{Y_w}}\right]^{\boldsymbol{\varepsilon}} \tag{A10}$$

Where $\overline{\boldsymbol{\varphi}}_1 = \overline{\boldsymbol{\varphi}}_2 = \overline{\boldsymbol{\varphi}}$, according to proposition 2 in section 3.3.

Based on the inter-city ratio (47'), the definition of $\boldsymbol{\Phi_I}^{\frac{1}{\varepsilon}}$, $\boldsymbol{\Phi_F}^{\frac{1}{\varepsilon}}$ in equation (48), (49) respectively and $\boldsymbol{A_1}^{\frac{1}{1+\varepsilon}}\boldsymbol{w_2} = \boldsymbol{A_2}^{\frac{1}{1+\varepsilon}}\boldsymbol{w_1}$ because of equation (36), yields

$$\boldsymbol{\Phi_{1I}} + \boldsymbol{\Phi_{1F}} = \frac{\boldsymbol{\lambda_1}^{\frac{1}{\varepsilon}}(\boldsymbol{\Phi_I}+\boldsymbol{\Phi_F})}{(\boldsymbol{\lambda_1}^{\frac{1}{\varepsilon}}+\boldsymbol{\lambda_2}^{\frac{1}{\varepsilon}})} + \frac{\boldsymbol{\Phi_w}\left(\frac{\boldsymbol{A_2}^{\frac{1}{1+\varepsilon}}}{\boldsymbol{w_w}}\right)^{\frac{1}{\varepsilon}}\left[\left(\frac{\boldsymbol{A_2}}{\boldsymbol{\theta_1}}\right)^{\frac{1}{\varepsilon}}-\left(\frac{\boldsymbol{A_1}}{\boldsymbol{\theta_2}}\right)^{\frac{1}{\varepsilon}}\right]}{(\boldsymbol{\lambda_1}^{\frac{1}{\varepsilon}}+\boldsymbol{\lambda_2}^{\frac{1}{\varepsilon}})} \tag{A11}$$

According to (A11) and the Pareto distribution of skill $\boldsymbol{g(\varphi)} = (\boldsymbol{k\varphi_m^k})/\boldsymbol{\varphi^{k+1}}$ for all $\boldsymbol{\varphi} \in [\boldsymbol{\varphi_m}, +\infty)$ in the whole economy, replacing $(\boldsymbol{\Phi_{1I}}^{\frac{1}{\varepsilon}} + \boldsymbol{\Phi_{1F}}^{\frac{1}{\varepsilon}})$ into equation (16)

$$\boldsymbol{\Phi_1} = \frac{\boldsymbol{\lambda_1}^{\frac{1}{\varepsilon}}\boldsymbol{Nk\varphi_m^k}}{(\boldsymbol{\lambda_1}^{\frac{1}{\varepsilon}}+\boldsymbol{\lambda_2}^{\frac{1}{\varepsilon}})(\boldsymbol{k}-\frac{1}{\varepsilon})}\underline{\boldsymbol{\varphi}}^{\frac{1}{\varepsilon}-\boldsymbol{k}} + \frac{\boldsymbol{\Phi_w}}{\boldsymbol{w_w}^{\frac{1}{\varepsilon}}}\left[\boldsymbol{A_1}^{\frac{1}{\varepsilon}} + \frac{\left(\boldsymbol{A_2}^{\frac{1}{1+\varepsilon}}\right)^{\frac{1}{\varepsilon}}\left[\left(\frac{\boldsymbol{A_2}}{\boldsymbol{\theta_1}}\right)^{\frac{1}{\varepsilon}}-\left(\frac{\boldsymbol{A_1}}{\boldsymbol{\theta_2}}\right)^{\frac{1}{\varepsilon}}\right]}{(\boldsymbol{\lambda_1}^{\frac{1}{\varepsilon}}+\boldsymbol{\lambda_2}^{\frac{1}{\varepsilon}})}\right] \tag{A12}$$

Transferring equation (22) as

$$\boldsymbol{Y_1} = \frac{(1+\varepsilon)\boldsymbol{\Phi_1}}{\varepsilon(1-\tau_1)}\underline{\boldsymbol{\varphi}}^{-\frac{1}{\varepsilon}} \tag{A13}$$

Expanding equation (26) with the relative size of workers in (42)

$$\boldsymbol{Y_1} = \frac{\lambda_1^{\frac{1}{\varepsilon}}(1+\varepsilon)N}{(\lambda_1^{\frac{1}{\varepsilon}}+\lambda_2^{\frac{1}{\varepsilon}})}\left(1-\varphi_m^k\underline{\boldsymbol{\varphi}}^{-k}\right) - \frac{m\theta_1 N_1(1-G_c(\overline{\boldsymbol{\varphi}}))}{1-\tau_c} \tag{A14}$$

In the real world, the fraction $\boldsymbol{m}$ of taxation income spent abroad is small. In addition, formal entrepreneurs only take a little part in the labor force that their proportion $(1-\boldsymbol{G_c}(\overline{\boldsymbol{\varphi}}))$ is always lower than 3.5% in Indonesia during 1995 to 2015, based on NLFS. Hence the item $\boldsymbol{m}(1-\boldsymbol{G_c}(\overline{\boldsymbol{\varphi}}))$ is approximately equal to zero. Combining (A12) to (A14), we have

$$\gamma_1\underline{\boldsymbol{\varphi}}^{\frac{1}{\varepsilon}-k} + \gamma_3 = \gamma_2\underline{\boldsymbol{\varphi}}^{\frac{1}{\varepsilon}} \tag{A15}$$

Where the coefficients γ_1, γ_2 and γ_3 consist of parameters, exogenous variables and endogenous variables which are figured out, satisfying

$$\gamma_1 = \frac{\lambda_1^{\frac{1}{\varepsilon}}(1+\varepsilon)[k+(\varepsilon k-1)(1-\tau_1)]N\varphi_m^k}{(\lambda_1^{\frac{1}{\varepsilon}}+\lambda_2^{\frac{1}{\varepsilon}})(\varepsilon k-1)(1-\tau_1)} > 0 \tag{A16}$$

$$\gamma_2 = \frac{\lambda_1^{\frac{1}{\varepsilon}}(1+\varepsilon)N}{(\lambda_1^{\frac{1}{\varepsilon}}+\lambda_2^{\frac{1}{\varepsilon}})} > 0 \tag{A17}$$

$$\gamma_3 = \frac{(1+\varepsilon)\boldsymbol{\Phi_w}}{\varepsilon(1-\tau_1)w_w^{\frac{1}{\varepsilon}}}\left[A_1^{\frac{1}{\varepsilon}} + \frac{\left(A_2^{\frac{1}{1+\varepsilon}}\right)^{\frac{1}{\varepsilon}}\left[\left(\frac{A_2}{\theta_1}\right)^{\frac{1}{\varepsilon}}-\left(\frac{A_1}{\theta_2}\right)^{\frac{1}{\varepsilon}}\right]}{\left(\lambda_1^{\frac{1}{\varepsilon}}+\lambda_2^{\frac{1}{\varepsilon}}\right)}\right] \tag{A18}$$

Denoting $\boldsymbol{S_1}(\underline{\boldsymbol{\varphi}}) = \gamma_1\underline{\boldsymbol{\varphi}}^{\frac{1}{\varepsilon}-k} + \gamma_3$ and $\boldsymbol{S_2}(\underline{\boldsymbol{\varphi}}) = \gamma_2\underline{\boldsymbol{\varphi}}^{\frac{1}{\varepsilon}}$, so that both $\boldsymbol{S_1}(\underline{\boldsymbol{\varphi}})$ and $\boldsymbol{S_2}(\underline{\boldsymbol{\varphi}})$ are continuous monotonic function of $\underline{\boldsymbol{\varphi}}$. Noticed that $k>\max(1, 1/\varepsilon)$ and $\varepsilon \in (1/2, +\infty)$, when $\underline{\boldsymbol{\varphi}} \to \varphi_m$, $\boldsymbol{S_1}(\underline{\boldsymbol{\varphi}}) > \boldsymbol{S_2}(\underline{\boldsymbol{\varphi}})$[16]; when $\underline{\boldsymbol{\varphi}} \to +\infty$, $\boldsymbol{S_1}(\underline{\boldsymbol{\varphi}}) \to \gamma_3$, $\boldsymbol{S_2}(\underline{\boldsymbol{\varphi}}) \to +\infty$. Hence there is and only is one $\underline{\boldsymbol{\varphi_0}} \in [\varphi_m, +\infty)$, such that $\boldsymbol{S_1}(\underline{\boldsymbol{\varphi_0}}) = \boldsymbol{S_2}(\underline{\boldsymbol{\varphi_0}})$, which is equivalent to proposition 1 in section 3.3.

Using equation (16) and (34')

$$\boldsymbol{\Phi_{1I}} + \boldsymbol{\Phi_{1F}} = \boldsymbol{\Phi_1} - \left(\frac{A_1}{w_w}\right)^{\frac{1}{\varepsilon}}\boldsymbol{\Phi_w} \tag{A19}$$

$$\boldsymbol{\Phi_{2I}} + \boldsymbol{\Phi_{2F}} = \frac{\lambda_2^{\frac{1}{\varepsilon}}}{\lambda_1^{\frac{1}{\varepsilon}}}\boldsymbol{\Phi_1} - \left(\frac{A_2}{w_w}\right)^{\frac{1}{\varepsilon}}\boldsymbol{\Phi_w} \tag{A20}$$

Using (A19), (A20) and (24), obtain

[16] When $\underline{\boldsymbol{\varphi}} = \varphi_m$, $\gamma_1 = \frac{\lambda_1^{\frac{1}{\varepsilon}}(1+\varepsilon)kN\varphi_m^k}{(\lambda_1^{\frac{1}{\varepsilon}}+\lambda_2^{\frac{1}{\varepsilon}})(\varepsilon k-1)(1-\tau_1)} > 0$ and $\gamma_2 = 0$, implying that $\boldsymbol{S_1}(\underline{\boldsymbol{\varphi}}) > \boldsymbol{S_2}(\underline{\boldsymbol{\varphi}})$.

$$\boldsymbol{\Phi_{1F}} = \{\frac{\lambda_1^{\frac{1}{\varepsilon}}(1+\varepsilon)N\left(1-\varphi_m^k\underline{\varphi}^{-k}\right)}{\left(\lambda_1^{\frac{1}{\varepsilon}}+\lambda_2^{\frac{1}{\varepsilon}}\right)} - \left[\boldsymbol{\Phi_1} - \left(\frac{A_1}{w_w}\right)^{\frac{1}{\varepsilon}}\boldsymbol{\Phi_w}\right]\frac{(1-\tau_1)Y_1}{\boldsymbol{\Phi_1}}\}\Big/(\boldsymbol{w_w A_1})^{\frac{1}{\varepsilon}}\frac{\boldsymbol{Y_w}}{\boldsymbol{\Phi_w}} \tag{A21}$$

$$\boldsymbol{\Phi_{2F}} = \{\frac{\lambda_2^{\frac{1}{\varepsilon}}(1+\varepsilon)Nw_2\left(1-\varphi_m^k\underline{\varphi}^{-k}\right)}{\left(\lambda_1^{\frac{1}{\varepsilon}}+\lambda_2^{\frac{1}{\varepsilon}}\right)} - \left[\frac{\lambda_2^{\frac{1}{\varepsilon}}}{\lambda_1^{\frac{1}{\varepsilon}}}\boldsymbol{\Phi_1} - \left(\frac{A_2}{w_w}\right)^{\frac{1}{\varepsilon}}\boldsymbol{\Phi_w}\right]\frac{(1-\tau_2)Y_2}{\boldsymbol{\Phi}_2}\}\Big/(\boldsymbol{\alpha_{w2} A_2})^{\frac{1}{\varepsilon}}\frac{\boldsymbol{Y_w}}{\boldsymbol{\Phi_w}} \tag{A22}$$

We conclude the algorithm by defining the solve of general equilibrium:

Definition 2 (*Solve of perfectly-mixed-skill equilibrium*): Given the parameters set $\{\boldsymbol{\varepsilon}, \boldsymbol{k}, \boldsymbol{b} \mid \boldsymbol{\varepsilon} \in (1/2, +\infty), \boldsymbol{k} > \max(1, 1/\boldsymbol{\varepsilon}), \boldsymbol{b}\boldsymbol{N}^{1/2} \in (1, \sqrt{2})\}$, endowment $\{\boldsymbol{N}, \boldsymbol{G}(\boldsymbol{\varphi})\}$, policy variables $\{\boldsymbol{A_c}, \boldsymbol{\theta_c}\}$ and world market condition $\{\boldsymbol{Y_w}, \boldsymbol{\Phi_w}, \boldsymbol{w_w}\}$, the solve of general equilibrium $\{\boldsymbol{N_c}, \boldsymbol{G_c}(\underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}), \boldsymbol{G_c}(\overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}), \boldsymbol{\tau_c}, \boldsymbol{\alpha_{wc}}, \boldsymbol{\Phi_c}, \boldsymbol{\Phi_{cI}}, \boldsymbol{\Phi_{cF}}, \mathbb{P}_c, \underline{\boldsymbol{\varphi}}_{\boldsymbol{c}}, \overline{\boldsymbol{\varphi}}_{\boldsymbol{c}}, \boldsymbol{w_c}, \boldsymbol{Y_c} \mid \boldsymbol{c} \in \{1, 2\}\}$ in definition 1 satisfies

(1) $\boldsymbol{w_1}$ is normalized to be 1;

(2) $\underline{\boldsymbol{\varphi}}$ is identified by equation (A15);

(3) $\overline{\boldsymbol{\varphi}}$ is identified by equation (A10);

(4) $\boldsymbol{\Phi_1}$ is identified by $\underline{\boldsymbol{\varphi}}$ and equation (A12);

(5) $\boldsymbol{Y_1}$ is identified by $\underline{\boldsymbol{\varphi}}$, $\boldsymbol{\Phi_1}$ and equation (A13);

(6) $\{\boldsymbol{\Phi_2}, \boldsymbol{Y_2}, \boldsymbol{w_2}\}$ is identified by $\{\boldsymbol{\Phi_1}, \boldsymbol{Y_1}, \boldsymbol{w_1}\}$ and equation (33'), (35'), (37);

(7) $\mathbb{P}_c$ is identified by $\boldsymbol{\Phi_c}$, $\boldsymbol{w_c}$ and equation (15), for $\boldsymbol{c} \in \{1, 2\}$;

(8) $\boldsymbol{\alpha_{wc}}$ is identified by $\boldsymbol{w_c}$ and equation (19), for $\boldsymbol{c} \in \{1, 2\}$;

(9) $\{\boldsymbol{\Phi_{1F}}, \boldsymbol{\Phi_{2F}}\}$ is identified by $\underline{\boldsymbol{\varphi}}$, $\boldsymbol{\Phi_c}$, $\boldsymbol{Y_c}$, $\boldsymbol{w_c}$, $\boldsymbol{\alpha_{wc}}$ and equation (A21), (A22);

(10) $\boldsymbol{\Phi_{cI}}$ is identified by $\boldsymbol{\Phi_c}$, $\boldsymbol{\Phi_{cF}}$, $\boldsymbol{\alpha_{wc}}$ and equation (16), for $\boldsymbol{c} \in \{1, 2\}$;

(11) $\{\boldsymbol{N_c}, \boldsymbol{\tau_c}\}$ is identified by $\boldsymbol{N_1} + \boldsymbol{N_2} = \boldsymbol{N}$ and equation (38), for $\boldsymbol{c} \in \{1, 2\}$;

(12) $\{\boldsymbol{G_c}(\underline{\boldsymbol{\varphi}}), \boldsymbol{G_c}(\overline{\boldsymbol{\varphi}})\}$ is identified by $\boldsymbol{N}$, $\boldsymbol{G}(\boldsymbol{\varphi})$ and equation (42), (45), for $\boldsymbol{c} \in \{1, 2\}$;