A Shadow Rate New Keynesian Model

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Issues caused by ZLB: UMP

Before ZLB

- Federal funds rate is the primary instrument of monetary policy
- Economists rely on it to study monetary policy
  - monetary VAR
  - New Keynesian model

At ZLB

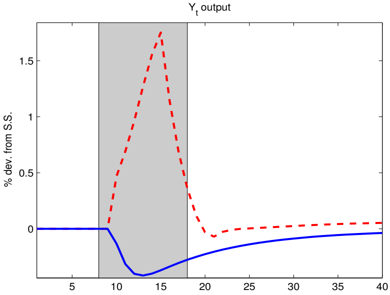
- Unconventional policy tools
  - large-scale asset purchases
  - lending facilities
  - forward guidance

How do we accommodate the ZLB and unconventional monetary policy?

Issues caused by ZLB: counterfactual implications of standard NK models

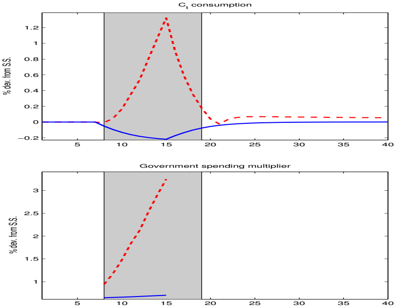
Anomalies at the ZLB without unconventional policy

Negative supply shock



- before: decreases output
- at zlb: increases output

Government spending shock



- before: < 1
- at zlb: > 1

Issues caused by ZLB: computational challenges

The ZLB imposes one of the biggest challenges for solving and estimating these models:

- nonlinearity
- multiple equilibria

Existing methods

- Shortcut
  - greatly simply the solution, but
  - have undesirable economic implications
  - cannot match data
  - hide nonlinear interactions
- Global projection method
  - seriously solve the model, but
  - computationally demanding → estimation impossible

Contributions

- presents new empirical evidence relating the shadow rate with
  - private interest rates
  - Fed's balance sheet
  - Taylor rule
- proposes a New Keynesian model with the shadow rate
  - accommodates both conventional and unconventional policies
- maps unconventional policy tools into the shadow rate framework
  - QE
  - lending facilities
- makes two anomalies disappear
  - a negative supply shock decreases output
  - government-spending multiplier is back to normal
- restores traditional solution and estimation methods

Outline

1. Shadow rate New Keynesian model (SRNKM)
2. Microfoundation I: Mapping QE into SRNKM
3. Microfoundation II: Mapping lending facilities into SRNKM
4. Quantitative analyses

Standard NK model

Definition

A standard New Keynesian model consists of the IS curve

$$y_t = -\frac{1}{\sigma}(r_t - \mathbb{E}_t\pi_{t+1} - s) + \mathbb{E}_t y_{t+1},$$

New Keynesian Phillips curve

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa(y_t - y_t^n),$$

and the Taylor rule with zero lower bound

$$\begin{aligned} s_t &= \phi_s s_{t-1} + (1-\phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right], \\ r_t &= \max(0, s_t). \end{aligned}$$

Long-term interest rate interpretation

$$
\begin{aligned}
y_t &= -\frac{1}{\sigma}\sum_{i=1}^{n}\mathbb{E}_t(r_{t+i-1} - \pi_{t+i} - r) + \mathbb{E}_t y_{t+n} \\
&= -\frac{1}{\sigma} n r_{t,t+n} - \frac{1}{\sigma}\sum_{i=1}^{n}\mathbb{E}_t(-\pi_{t+i} - r) + \mathbb{E}_t y_{t+n}.
\end{aligned}
$$

- long-term rate matters for decision making instead of short rate
- UMP works through long term rates to affect the economy
- this link is missing in standard NK models

Two ways to fill the gap:

- model UMP separately – structural break
- use the shadow rate to capture UMP – no structural break

Shadow rate NK model

Definition

The shadow rate New Keynesian model consists of the shadow rate IS curve

$$y_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t \pi_{t+1} - s) + \mathbb{E}_t y_{t+1},$$

New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - y_t^n),$$

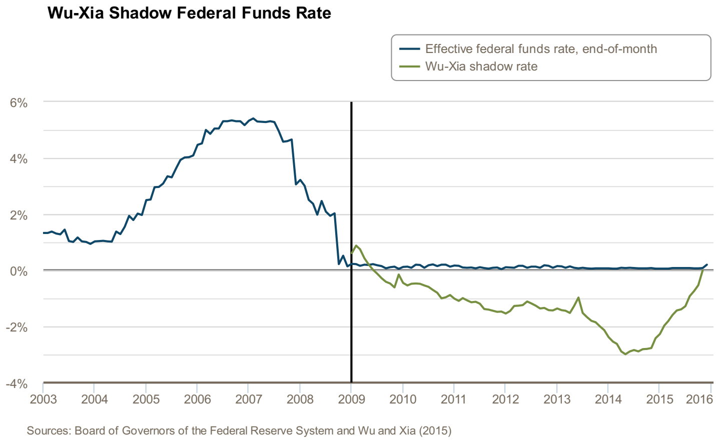
and shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1 - \phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right].$$

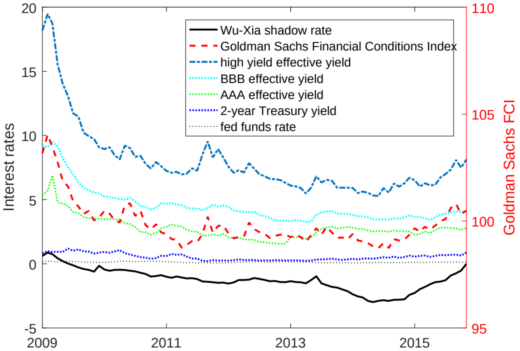
▸ QE ▸ lending facilities

Shadow rate

- Black (1995): $r_t = max(s_t, \underline{r})$

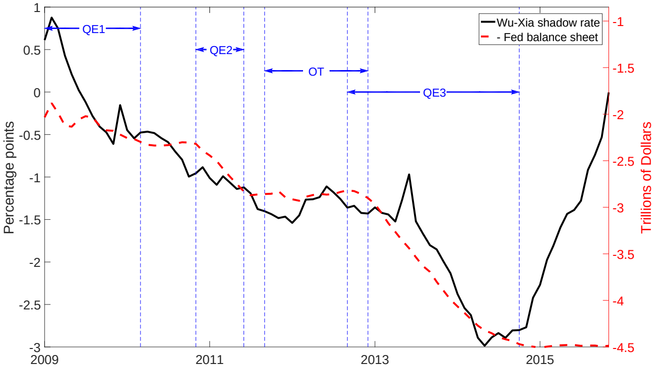


Empirical 1: shadow rate and private rates



- the fed funds rate is at the ZLB
- shadow rate moves in response to unconventional monetary policy
- private rates move with the shadow rate $r_t^B = s_t + rp$
- private rates are the relevant rates for agents

Empirical 2: shadow rate and Fed's balance sheet

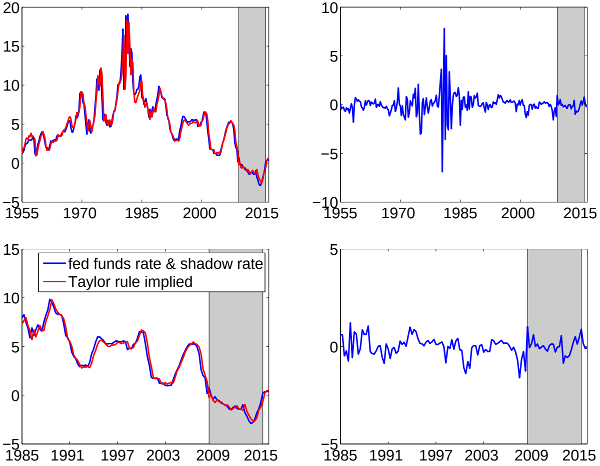


Correlation

- full sample: -0.74
- QE1 - QE3: -0.94

Empirical 3: shadow rate Taylor rule

$$s_t = \beta_0 + \beta_1 s_{t-1} + \beta_2(y_t - y_t^n) + \beta_3 \pi_t + \varepsilon_t$$



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Large-scale asset purchases (QE)

The risk premium channel

- government purchases outstanding loans
- decrease interest rates through reducing risk premium
  - Gagnon et al. (2011) and Hamilton and Wu (2012)
- The same mechanism works for government bonds or corporate bonds

Households' problem

Households' utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{{L_t}^{1+\eta}}{1+\eta} \right)$$

budget constraint

$$C_t + \frac{B_t^H}{P_t} = \frac{R_{t-1}^B B_{t-1}^H}{P_t} + W_t L_t + T_t$$

Euler equation

$$C_t^{-\sigma} = \beta R_t^B \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \right]$$

The linear Euler equation

$$y_t = -\frac{1}{\sigma} \left(r_t^B - \mathbb{E}_t \pi_{t+1} - r^B \right) + \mathbb{E}_t y_{t+1}$$

Bond return and policy rate

Define

$$rp_t \equiv r_t^B - r_t$$

Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2012), and Hamilton and Wu (2012) suggest

$$rp_t'(b_t^G) < 0 \Rightarrow rp_t(b_t^G) = rp - \varsigma(b_t^G - b^G)$$

- During normal times, $b_t^G = b^G, r_t = s_t$

$$r_t^B = r_t - rp_t(b_t^G) = r_t + rp = s_t + rp$$

- At the ZLB, $r_t = 0$

$$r_t^B = r_t - rp_t(b_t^G) = rp_t = rp - \varsigma(b_t^G - b^G) = s_t + rp$$

if $s_t = -\varsigma(b_t^G - b^G)$

Shadow rate equivalence for QE

Proposition

The shadow rate New Keynesian model represented by the shadow rate IS curve

$$y_t = -\frac{1}{\sigma}\left(s_t - \mathbb{E}_t \pi_{t+1} - s\right) + \mathbb{E}_t y_{t+1}$$

New Keynesian Phillips Curve, shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1 - \phi_s)\left[\phi_y (y_t - y_t^n) + \phi_\pi \pi_t + s\right].$$

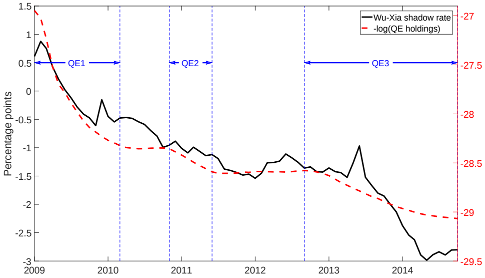
nests both conventional Taylor interest rate rule and QE operation that changes risk premium if

$$\begin{cases} r_t = s_t,\ b_t^G = b^G & \text{for } s_t \geq 0 \\ r_t = 0,\ b_t^G = b^G - \frac{s_t}{\varsigma} & \text{for } s_t < 0. \end{cases}$$

Shadow rate NK model

Quantifying assumption in proposition

$$s_t = -\varsigma(b_t^G - b^G)$$



- linear assumption: correlation $= 0.92$
- $\varsigma = 1.83$
  - Fed increases its bond holdings by 1%, the shadow rate decreases by 0.0183%
  - QE1: 490 billion to 2 trillion $\Rightarrow$ 2.5% decrease in the shadow rate
  - QE3: 2.6 trillion to 4.2 trillion $\Rightarrow$ 0.9% decrease in the shadow rate

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Lending facilities

Government injects liquidity to the economy

- Term Asset-Backed Securities Loan Facility in the US
- valuation haircuts in Eurosystem
- credit controls in UK

Combine this with a tax on interest rates

Model features

Entrepreneurs

- produce intermediate goods with labor and capital
- maximize utility
- discount factor $\gamma < \beta$
- borrow from households with a loan-to-value ratio M
- accumulate capital
- use capital as collateral

Government policy at the ZLB

- lending facilities
  - lend directly to entrepreneurs
  - change the loan-to-value ratio from M to M_t
- tax (subsidy) on the interest rate income (payment)

Entrepreneurs' problem

Utility function

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \gamma^t \log C_t^E$$

production function

$$Y_t^E = AK_{t-1}^{\alpha}(L_t)^{1-\alpha}$$

capital accumulation

$$K_t = I_t + (1-\delta)K_{t-1}$$

budget constraint

$$\frac{Y_t^E}{X_t} + \tilde{B}_t = \frac{R_{t-1}^B \tilde{B}_{t-1}}{\mathcal{T}_{t-1}\Pi_t} + W_t L_t + I_t + C_t^E$$

borrowing constraint

$$\tilde{B}_t \leq M_t \mathbb{E}_t \left(\frac{K_t \Pi_{t+1}}{R_t^B} \right)$$

Entrepreneurs' FOCs

Labor demand

$$W_t = \frac{(1-\alpha)AK_{t-1}^{\alpha}L_t^{-\alpha}}{X_t}$$

Euler equation

$$\frac{1}{C_t^E}\left(1 - \frac{M_t \mathbb{E}_t \Pi_{t+1}}{R_t^B}\right) = \gamma \mathbb{E}_t \left[\frac{1}{C_{t+1}^E}\left(\frac{\alpha Y_{t+1}^E}{X_{t+1}K_t} - \frac{M_t}{\mathcal{T}_t} + 1 - \delta\right)\right]$$

Households' problem

Households' utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\eta}}{1+\eta} \right)$$

budget constraint

$$C_t + \tilde{B}_t^H = \frac{R_{t-1}^B \tilde{B}_{t-1}^H}{\mathcal{T}_{t-1} \Pi_t} + W_t L_t + T_t$$

Euler equation

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left(R_t^B \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1} \mathcal{T}_t} \right)$$

labor supply

$$W_t = C_t^{\sigma} L_t^{\eta}$$

Sources of funding

Entrepreneurs' borrowing constraint

$$\tilde{B}_t \leq M_t \mathbb{E}_t \left(\frac{K_t \Pi_{t+1}}{R_t^B} \right)$$

Households lend

$$\tilde{B}_t^H \leq M \mathbb{E}_t \left(\frac{K_t \Pi_{t+1}}{R_t^B} \right)$$

- During normal times $\tilde{B}_t = \tilde{B}_t^H$, and $M_t = M$
- At the ZLB $M_t > M$

Government lends the rest

$$\tilde{B}_t^G = (M_t - M) \mathbb{E}_t \left(\frac{K_t \Pi_{t+1}}{R_t^B} \right)$$

Conventional and unconventional policy

Suppose $R_t^B = R_t RP$

Conventional and unconventional policy tools appear in the model in pairs:

- $R_t / \mathcal{T}_t$ – HH Euler equation, HH&E budget constraints
- R_t / M_t – E borrowing constraint, E Euler equation
- $M_t / \mathcal{T}_t$ – E Euler equation

model

Decreasing R_t is equivalent to increasing $\mathcal{T}_t$ and M_t.

Shadow rate equivalence for lending facilities

Proposition

If

$$\begin{cases} R_t = S_t, \mathcal{T}_t = 1, M_t = M & \text{for } S_t \geq 1 \\ \mathcal{T}_t = M_t/M = 1/S_t & \text{for } S_t < 1, \end{cases}$$

then $R_t/\mathcal{T}_t = S_t$, $R_t/M_t = S_t/M$, $M_t/\mathcal{T}_t = M \ \forall S_t$.

- S_t summarizes both conventional and unconventional policies
- Equivalence in the non-linear model

Shadow rate equivalence for lending facilities

Proposition

The shadow rate New Keynesian model represented by the Euler equation

$$c_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t\pi_{t+1} - s) + \mathbb{E}_t c_{t+1}$$

the shadow rate Taylor rule, Phillips curve, ..., nests both conventional Taylor interest rate rule and lending facility – tax policy if

$$\begin{cases} r_t = s_t, \tau_t = 0, m_t = m & \text{for } s_t \geq 0 \\ \tau_t = m_t - m = -s_t & \text{for } s_t < 0. \end{cases}$$

▸ Shadow rate NK model ▸ Detailed model

Quantitative model

Iacoviello (2005, AER) with

- unconventional policy
- technology shock to investigate the impact of negative supply shocks at the ZLB
- government spending to investigate fiscal multiplier at the ZLB
- preference shocks to create ZLB

Methodology

Notations

- standard model: w/o unconventional policy $r_t = 0$
- shadow rate model: w/ unconventional policy $s_t < 0$

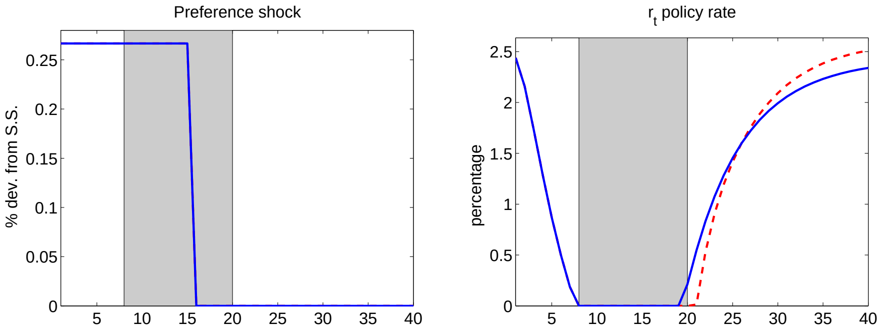
Methodology for standard model:

- piecewise linear – Guerrieri and Iacoviello (2005, JME): toolkit for models with occasionally binding constraints

Methodology for shadow rate model:

- solve linear model with shadow rate
- then use propositions mapping shadow rate into various UMP

Preference shock and the ZLB



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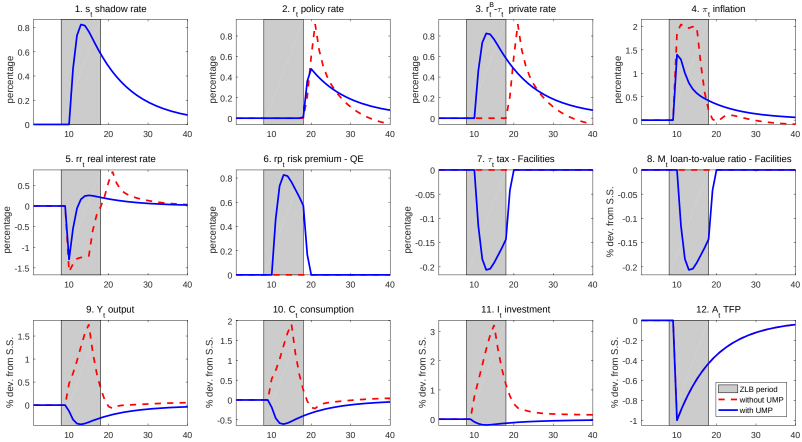
Negative technology shock



Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP

Economic implication 1: negative supply shock

Technology shock

$$a_t \downarrow \;=\; \rho_a a_{t-1} + e_{a,t} \downarrow$$

Phillips Curve

$$\pi_t \uparrow \;=\; \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - y) - \frac{\kappa(1+\eta)}{\sigma+\eta} a_t \downarrow$$

Economic implication 1: negative supply shock

Standard model

Monetary policy

$$r_t = \max\{\phi_s s_{t-1} + (1-\phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right], 0\}$$

Real interest rate

$$rr_t = r_t - \mathbb{E}_t[\pi_{t+1}]$$

IS curve

$$y_t = -\frac{1}{\sigma}(rr_t - r) + \mathbb{E}_t y_{t+1}$$

normal times: $\pi \uparrow \rightarrow r \uparrow\uparrow \rightarrow rr \uparrow \rightarrow y \downarrow$

ZLB without UMP: $\pi \uparrow \rightarrow r = 0 \rightarrow rr \downarrow \rightarrow y \uparrow$ Counterfactual

Economic implication 1: negative supply shock

Shadow rate model

Shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1-\phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right]$$

Real interest rate

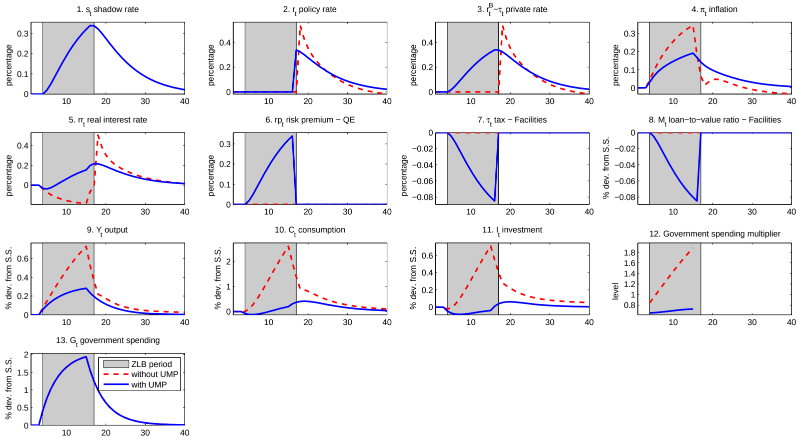
$$rr_t = s_t - \mathbb{E}_t[\pi_{t+1}]$$

Shadow rate IS curve

$$y_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t \pi_{t+1} - r) + \mathbb{E}_t y_{t+1}$$

normal times:	$\pi\uparrow$	$\rightarrow$	$r\uparrow\uparrow$	$\rightarrow$	$rr\uparrow$	$\rightarrow$	$y\downarrow$	
ZLB without UMP:	$\pi\uparrow$	$\rightarrow$	$r=0$	$\rightarrow$	$rr\downarrow$	$\rightarrow$	$y\uparrow$	Counterfactual
ZLB with UMP:	$\pi\uparrow$	$\rightarrow$	$s\uparrow\uparrow$	$\rightarrow$	$rr\uparrow$	$\rightarrow$	$y\downarrow$	Data consistent

Government-spending shock



Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP

details

Economic implication 2: government spending multiplier

Government spending shock

$$g_t \uparrow \quad = \quad (1-\rho_g)g + \rho_g g_{t-1} + e_{g,t} \uparrow$$

Market-clearing condition

$$y_t \uparrow \quad = \quad c_y c_t + g_y g_t \uparrow$$

Phillips Curve

$$\pi_t \uparrow \quad = \quad \beta \mathbb{E}_t \pi_{t+1} + \frac{\kappa}{\delta+\eta}(\sigma(c_t - c) + \eta(y_t \uparrow - y))$$

Economic implication 2: government spending multiplier

Standard model

Monetary policy

$$r_t = \max\{\phi_s s_{t-1} + (1-\phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right], 0\}$$

Real interest rate

$$rr_t = r_t - \mathbb{E}_t[\pi_{t+1}]$$

IS curve

$$c_t = -\frac{1}{\sigma}(rr_t - r) + \mathbb{E}_t c_{t+1}$$

normal times:	$\pi \uparrow y \uparrow$	$\rightarrow$	$r \uparrow\uparrow$	$\rightarrow$	$rr \uparrow$	$\rightarrow$	$c \downarrow$	$\rightarrow$	$\Delta y < \Delta g$
ZLB without UMP:	$\pi \uparrow y \uparrow$	$\rightarrow$	$r = 0$	$\rightarrow$	$rr \downarrow$	$\rightarrow$	$c \uparrow$	$\rightarrow$	$\Delta y > \Delta g$

Shadow rate NK model and Anomaly 2

Shadow rate model

Shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1 - \phi_s)\left[\phi_y (y_t - y_t^n) + \phi_\pi \pi_t + s\right]$$

Real interest rate

$$rr_t = s_t - \mathbb{E}_t[\pi_{t+1}]$$

Shadow rate IS curve

$$c_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t \pi_{t+1} - r) + \mathbb{E}_t c_{t+1}$$

normal times:	$\pi \uparrow y \uparrow$	$\rightarrow$	$r \uparrow\uparrow$	$\rightarrow$	$rr \uparrow$	$\rightarrow$	$c \downarrow$	$\rightarrow$	$\Delta y < \Delta g$
ZLB without UMP:	$\pi \uparrow y \uparrow$	$\rightarrow$	$r = 0$	$\rightarrow$	$rr \downarrow$	$\rightarrow$	$c \uparrow$	$\rightarrow$	$\Delta y > \Delta g$
ZLB with UMP:	$\pi \uparrow y \uparrow$	$\rightarrow$	$s \uparrow\uparrow$	$\rightarrow$	$rr \uparrow$	$\rightarrow$	$c \downarrow$	$\rightarrow$	$\Delta y < \Delta g$

Conclusion

We build a shadow rate NK model, capturing

- the conventional interest rate rule at normal times
- unconventional monetary policy at the ZLB

The shadow rate policy can be implemented by

- QE
- lending facilities

Economic implications

- a negative supply shock is not stimulative
- government-spending multiplier is as usual

Model solution

- the ZLB is not associated with a structural break

Lending facilities

$$\begin{aligned} c_t &= -\frac{1}{\sigma}(r_t^B - \tau_t - \mathbb{E}_t\pi_{t+1} - r - rp) + \mathbb{E}_t c_{t+1} \\ \Rightarrow c_t &= -\frac{1}{\sigma}(s_t - \mathbb{E}_t\pi_{t+1} - r) + \mathbb{E}_t c_{t+1} \end{aligned}$$

$$\begin{aligned} C^E c_t^E &= \alpha\frac{Y}{X}(y_t - x_t) + Bb_t - R^B B(r_{t-1}^B + b_{t-1} - \tau_{t-1} - \pi_{t-1}) - Ii_t + \Lambda_1 \\ \Rightarrow C^E c_t^E &= \alpha\frac{Y}{X}(y_t - x_t) + Bb_t - R^B B(s_{t-1} + rp + b_{t-1} - \pi_{t-1}) - Ii_t + \Lambda_1 \end{aligned}$$

$$\begin{aligned} b_t &= \mathbb{E}_t(k_t + \pi_{t+1} + m_t - r_t^B) \\ \Rightarrow b_t &= \mathbb{E}_t(k_t + \pi_{t+1} + m - s_t - rp) \end{aligned}$$

Lending facilities

$$
\begin{aligned}
0 &= \left(1-\frac{M}{R^B}\right)(c_t^E - \mathbb{E}_t c_{t+1}^E) + \frac{\gamma\alpha Y}{XK}\mathbb{E}_t(y_{t+1} - x_{t+1} - k_t) \\
&+ \frac{M}{R^B}\mathbb{E}_t(\pi_{t+1} - r_t^B + m_t) + \gamma M(\tau_t - m_t) + \Lambda_2 \\
\Rightarrow 0 &= \left(1-\frac{M}{R^B}\right)(c_t^E - \mathbb{E}_t c_{t+1}^E) + \frac{\gamma\alpha Y}{XK}\mathbb{E}_t(y_{t+1} - x_{t+1} - k_t) \\
&+ \frac{M}{R^B}\mathbb{E}_t(\pi_{t+1} - s_t - rp + m) - \gamma Mm + \Lambda_2
\end{aligned}
$$

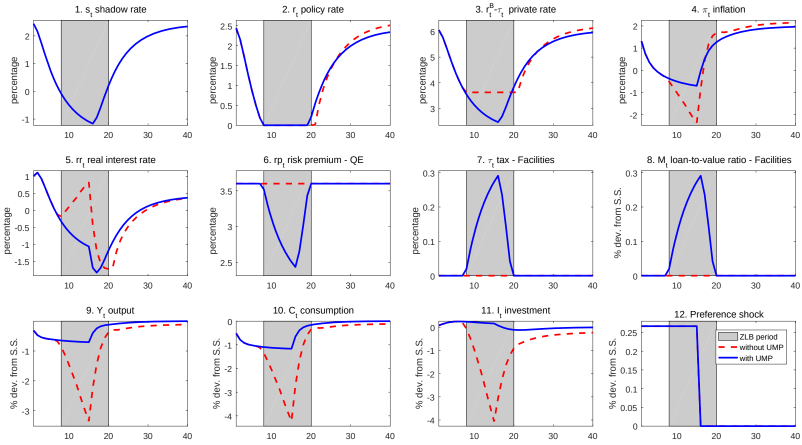
Back

Calibration

para	description	source	value
β	discount factor of patient households	Iacoviello (2005)	0.99
β^I	discount factor of impatient households	Iacoviello (2005)	0.95
γ	discount factor of entrepreneurs	Iacoviello (2005)	0.98
j	steady-state weight on housing services	Iacoviello (2005)	0.1
η	labor supply aversion	Iacoviello (2005)	0.01
μ	capital share in production	Iacoviello (2005)	0.3
ν	housing share in production	Iacoviello (2005)	0.03
δ	capital depreciation rate	Iacoviello (2005)	0.03
X	steady state gross markup	Iacoviello (2005)	1.05
θ	probability that cannot re-optimize	Iacoviello (2005)	0.75
α	patient households' wage share	Iacoviello (2005)	0.64
M^E	loan-to-value ratio for entrepreneurs	Iacoviello (2005)	0.89
M^I	loan-to-value ratio for impatient households	Iacoviello (2005)	0.55
r_R	interest rate persistence	Iacoviello (2005)	0.73
r_Y	interest rate response to output	Iacoviello (2005)	0.27
r_Π	interest rate response to inflation	Iacoviello (2005)	0.13
$\frac{G}{Y}$	steady-state government-spending-to-output ratio	Fernandez-Villaverde et al. (2015)	0.20
ρ_a	autocorrelation of technology shock	Fernandez-Villaverde et al. (2015)	0.90
ρ_g	autocorrelation of government-spending shock	Fernandez-Villaverde et al. (2015)	0.80
ρ_β	autocorrelation of discount rate shock	Fernandez-Villaverde et al. (2015)	0.80
σ_a	standard deviation of technology shock	Fernandez-Villaverde et al. (2015)	0.0025
σ_g	standard deviation of government-spending shock	Fernandez-Villaverde et al. (2015)	0.0025
σ_β	standard deviation of discount rate shock	Fernandez-Villaverde et al. (2015)	0.0025
ξ_p	price indexation	Smets and Wouters (2007)	0.24
Π	steady-state inflation	2% annual inflation	1.005
B^G	steady-state government bond holdings	no gov. intervention in private bond market	0
$\mathcal{T}$	steady-state tax/subsidy on interest rate income/payment	no tax in normal times	1
rp	steady-state risk premium	3.6% risk premium annually	1.009

Back

Preference shock and the ZLB



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