Market Design: An Overview

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What is market design?

“A theory of an intelligently guided invisible hand”
Economist, 2007

Market design is a field of economics which attempts to devise practical schemes for resource allocation problems

Mechanism design esp. auction and matching theories underpin the field as a general methodological framework

Market designers aspire to evaluate, interact with, and tinker with real-world systems; research approach spans theoretical to empirical and experimental

Prominent examples of “design”

- Labor market clearinghouses, especially for entry-level professionals (e.g., medicine)
- Spectrum auctions and re-allocation (worldwide)
- Internet markets, ad auctions
- Student assignment systems
- Clearinghouses for exchange of kidneys (and even lungs)
- Tradable permit/ cap-and-trade markets for pollution control
- Design of exchanges for securities, security design for risk-sharing and other financial innovations

Beyond F=ma to bridge building

Consider the design of suspension bridges. Their simple physics, in which the only force is gravity, and all beams are perfectly rigid, is simple, beautiful and indispensable.

But bridge design also concerns metal fatigue, soil mechanics, and the sideways forces of waves and wind. Many questions concerning these complications can't be answered analytically, but must be explored using physical or computational models.

These complications, and how they interact with that part of the physics captured by the simple model, are the concern of the engineering literature. Some of this is less elegant than the simple model, but it allows bridges designed on the same basic model to be built longer and stronger over time, as the complexities and how to deal with them become better understood.

-Roth (2002)

Why design markets?

- First welfare theorem: Markets are efficient under broad set of conditions
  - ✓ no externalities

    e.g., Markets may be missing due to lack of prices
  - ✓ perfect information
  - ✓ perfect competition

- Most market design research starts by documenting market failures

Four broad themes

1) When to use the “market”?
2) Coase theorem: does design matter? what matters?
3) Market clearing without prices
4) Market clearing with prices

Oklahoma Land Rush of 1889

Lotteries vs. the market: When to allow prices?

"Conscription to man the military services in peacetime: The appropriate free market arrangement is volunteer military forces; which is to say, hiring men to serve. There is no justification for not paying whatever price is necessary to attract the required number of men. Present arrangements are inequitable and arbitrary, seriously interfere with the freedom of young men to shape their lives, and probably are even more costly than the market alternative. (Universal military training to provide a reserve for war time is a different problem and may be justified on liberal grounds.)."

Milton Friedman, in *Capitalism and Freedom*

When to use prices?

Basic tradeoff

- Prices: allow people to express preferences, but if market-clearing price used then income determines everything
- Rationing: may lead to over-delivery of goods to those who really do not value them, but allows “true needs” to be met (fairness)

First modeled by Weitzman (1974); Che, Gale and Kim (2014) revist and qualify in presence of resale markets and speculation

Market clearing without prices

One-sided

- House allocation problem (no property rights)
  - ✓ Serial dictatorships
  - ✓ Competitive EEI

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Two-sided

- Marriage problem (one-to-one)
- College admissions (many-to-one)

Match day

History of NRMP

- 1900-1945: Unravelling of Appt. Dates
- 1945-1950: Chaotic Recontracting $\Rightarrow$ centralized mechanism introduced in response
- 1950-197x: High rates of orderly participation (95%) in centralized clearing house
- 197x-198x: Declining rates of participation, particularly among married couples
- 198x-present: Married couples return, following changes in algorithm to accommodate couples and other match variations

From theory to practice

- Study of matching started as “pure” theory: first by David Gale and Lloyd Shapley (1962) who introduced deferred acceptance (DA)
- Roth (1984) is a landmark paper, which observed
  - Since the 1950s, US hospitals have used a clearinghouse to assign graduating medical students to residencies.
  - Students apply and interview at hospitals in the fall, then students and hospitals submit rank-order preferences in February.
  - A computer algorithm is used to assign students to hospitals, and matches are all revealed on a single day: match day.
  - Roth realized that the doctors has independently discovered and were using exactly the Gale and Shapley DA algorithm!
- DA has since been independently discovered several times

Deferred acceptance algorithms

Gale and Shapley defined the following **deferred acceptance algorithm**:

> **Step 1**: Each man proposes to her first choice. Each woman rejects any unacceptable man, and if more than one acceptable proposal is received, she "holds" the most preferred.

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Properties: strategy-proof, stable, constrained efficient

Student assignment in Boston

“The Soiling of Old Glory” by Stanley J. Forman
1977 Pulitzer Prize for Spot Photography

New student assignment mechanisms

- 2003: **New York City** adopts new centralized mechanism
- 2005: **Boston** changes rules of existing mechanism
- 2007: **England**'s Parliament enacts nationwide ban of ‘First Preference First’ mechanisms
- 2009: **Chicago** abandons mechanism midstream
- 2012: **Denver**, **New Orleans** adopt mechanisms unifying charter and traditional public sectors

✓ 2013, 2014: **Washington DC** and **Newark** follow suit

✓ 2018: **Chicago** adopts unified enrollment

✓ Several other changes worldwide, and many active discussions

The (old) Boston mechanism

The mechanism used by Boston Public until June 2005 worked as follows:

1. For each school a priority ordering is determined according to the following hierarchy:
    - First priority: sibling and walk zone
    - Second priority: sibling
    - Third priority: walk zone
    - Fourth priority: other students

    Students in the same priority group are ordered based on an even lottery.
2. Each student submits a preference ranking of the schools.
3. The final phase is the student assignment based on preferences and priorities:

Immediate acceptance

Round 1: In Round 1 only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as her first choice.

Round k: Consider the remaining students. In Round k only the k^{th} choices of these students are considered. For each school with still available seats, consider the students who have listed it as their k^{th} choice and assign the remaining seats to these students one at a time following their priority order who has listed it as her k^{th} choice until either there are no seats left or there is no student left ho has listed it as her k^{th} choice.

How to rank?

- Consider the following quotation from St. Petersburg Times:

> *Make a realistic, informed selection on the school you list as your first choice. It's the cleanest shot you will get at a school, but if you aim too high you might miss.*
>
> *Here's why: If the random computer selection rejects your first choice, your chances of getting your second choice school are greatly diminished. That's because you then fall in line behind everyone who wanted your second choice school as their first choice. You can fall even farther back in line as you get bumped down to your third, fourth and fifth choices.*

Boston mechanism: empirical facts

Why might parents understand?

- BPS School Guide (2004, p3, quotes in original):

  For a better chance of your "first choice" school...
  consider choosing less popular schools.

- *Advice from the West Zone Parent's Group*:
  Introductory meeting minutes, 10/27/03

  One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a "safe" second choice.

Leveling the Playing Field

- Sophisticated play by some groups of players, unsophisticated play by others
- WZPG Parents, 1/28/2005, Subject: Re: Philbrick School
  I think there are probably 2-3 siblings entering K2 [at the Philbrick]. I know of 2 people who are putting it as a first choice... I don't know what to say— according to last year's numbers, putting it second would be safe, but the year we applied, only first choice people got in. I think it would be okay if your third choice were a VERY safe bet.
- Leveling the playing field is a major argument for strategy-proof mechanisms

Recent protests in Taiwan

"Fill out the school preference form for us" … "[It's] like gambling"

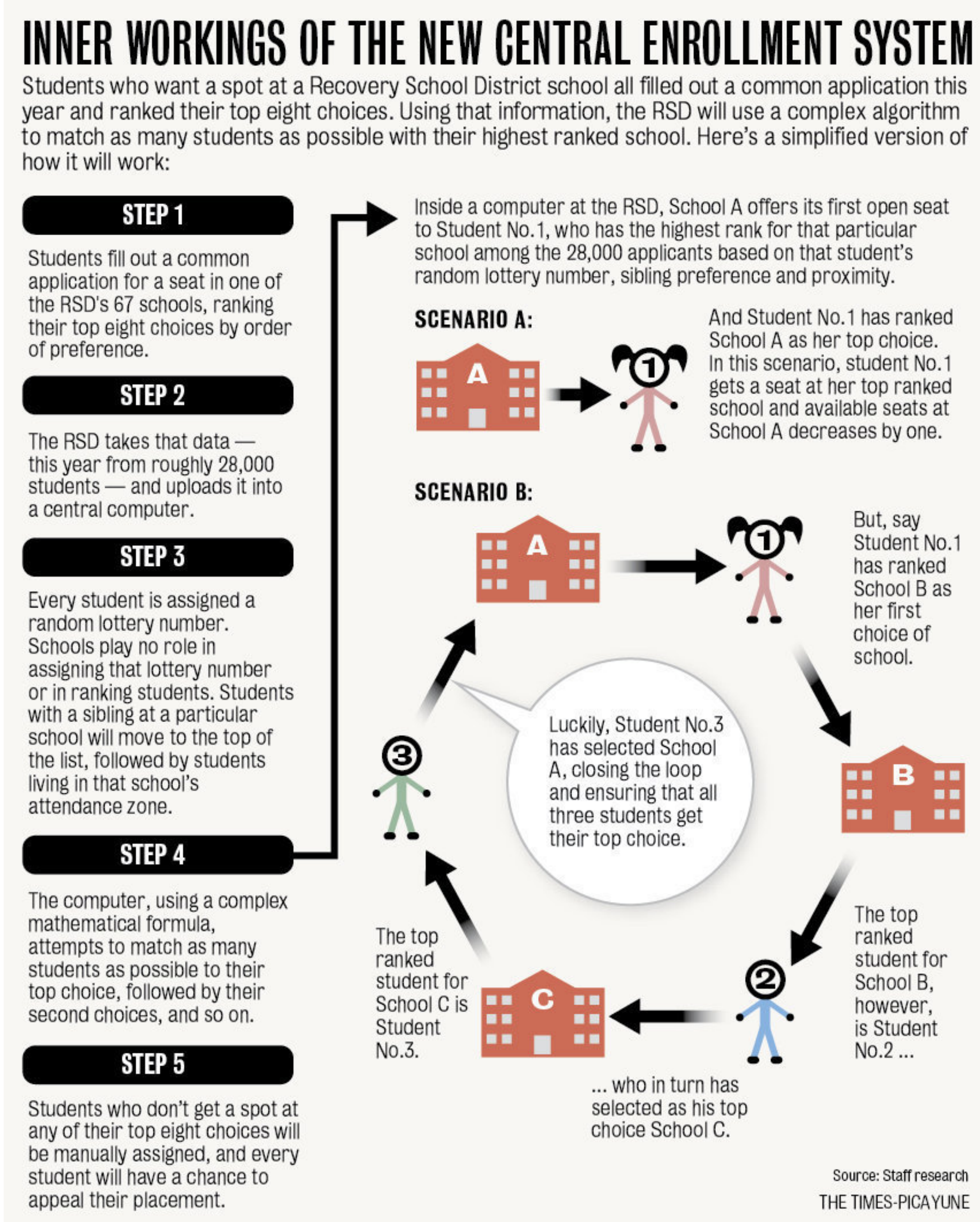
June 21, 2014

"Abolish the ranking order deduction"

Nov 2014

Taiwan's mechanism

- Modify applicant priorities at schools based on the position of that choice on the application form, **deducting points** for lower ranked choices.
- In 2014 Taiwan senior high school admission process, all 15 districts adopted certain deduction rules
  - In Taipei area, 1 point is deducted for a second choice, 2 points for third choices and so on.
  - After the first round centralized allocation result was released, hundreds of parents and teachers in Taipei walked onto the street to protest the new mechanism.
- Same kind of incentive as Boston mechanism!


INNER WORKINGS OF THE NEW CENTRAL ENROLLMENT SYSTEM
Students who want a spot at a Recovery School District school all filled out a common application this year and ranked their top eight choices. Using that information, the RSD will use a complex algorithm to match as many students as possible with their highest ranked school. Here's a simplified version of how it will work:
STEP 1
Students fill out a common application for a seat in one of the RSD's 67 schools, ranking their top eight choices by order of preference.
STEP 2
The RSD takes that data — this year from roughly 28,000 students — and uploads it into a central computer.
STEP 3
Every student is assigned a random lottery number. Schools play no role in assigning that lottery number or in ranking students. Students with a sibling at a particular school will move to the top of the list, followed by students living in that school's attendance zone.
STEP 4
The computer, using a complex mathematical formula, attempts to match as many students as possible to their top choice, followed by their second choices, and so on.
STEP 5
Students who don't get a spot at any of their top eight choices will be manually assigned, and every student will have a chance to appeal their placement.
Inside a computer at the RSD, School A offers its first open seat to Student No. 1, who has the highest rank for that particular school among the 28,000 applicants based on that student's random lottery number, sibling preference and proximity.
SCENARIO A:
A
1
And Student No. 1 has ranked School A as her top choice. In this scenario, student No. 1 gets a seat at her top ranked school and available seats at School A decreases by one.
SCENARIO B:
A
1
But, say Student No. 1 has ranked School B as her first choice of school.
B
2
The top ranked student for School B, however, is Student No. 2 ...
... who in turn has selected as his top choice School C.
C
The top ranked student for School C is Student No. 3.
3
Luckily, Student No. 3 has selected School A, closing the loop and ensuring that all three students get their top choice.
Source: Staff research
THE TIMES-PICAYUNE

Gale's Top Trading Cycles Algorithm

(Described in Shapley & Scarf, attributed to David Gale)

Step 1: Each agent "points to" the owner of his favorite house.
Since there are finite number of agents, there is at least one **cycle**.
Each agent in a cycle is assigned the house of the agent he points to and removed from the market with his assignment.
If there is at least one remaining agent, proceed with the next step.

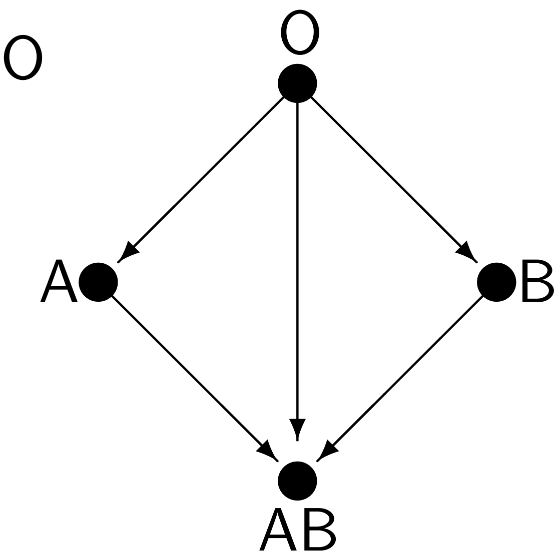
Step t: Each remaining agent points to the owner of his favorite house among the remaining houses.
Every agent in a cycle is assigned the house of the agent he points to and removed from the market with his assignment.
If there is at least one remaining agent, proceed with the next step.

Kidney exchange

- Transplantation is the best treatment for kidney failure
  - Improves quality and length of life
  - Each transplant is estimated to save Medicare hundreds of thousands of dollars
- Almost 100K patients are waiting on the kidney list
  - List has been growing, and thousands die while waiting
- Two sources of kidney transplants
  1. $\sim$12K receive a deceased donor transplant each year
  2. $\sim$6K receive a kidney from a living donor
- Potential for many more living donor transplants
  - Biological compatibility prevents many direct donations

Medical constraint: Blood type compatibility

- There are four blood types: A, B, AB and O
- In the absence of other complications:



✓ type O kidneys can be transplanted into any patient

✓ type A kidneys can be transplanted into type A or type AB patients

✓ type B kidneys can be transplanted into type B or type AB patients

✓ type AB kidneys can only be transplanted into type AB patients

A barrier to organ markets

- In principle, shortage of organs can be solved using monetary incentives (Becker and Elias, '07)

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- Section 301, National Organ Transplant Act (NOTA), 42 U.S.C. 274e 1984:
  - "it shall be unlawful for any person to knowingly acquire, receive or otherwise transfer any human organ *for valuable consideration* for use in human transplantation"
- Near universal norm (exception: Republic of Iran)
  - Societies often constrain transactions (Roth, '07)
  - Concerns about inadequate protections against exploitation and coercion

Double coincidence of wants: kidney exchange

- Chapter I (Jevons, 1876):
  - "The first difficulty in barter is to find two persons whose disposable possessions mutually suit each other's wants. ... to allow of an act of barter, there must be a *double coincidence*, which will rarely happen."
  - "Sellers and purchasers can only be made to fit by the use of some commodity... which all are willing to receive... This common commodity is called a *medium, of exchange*..."

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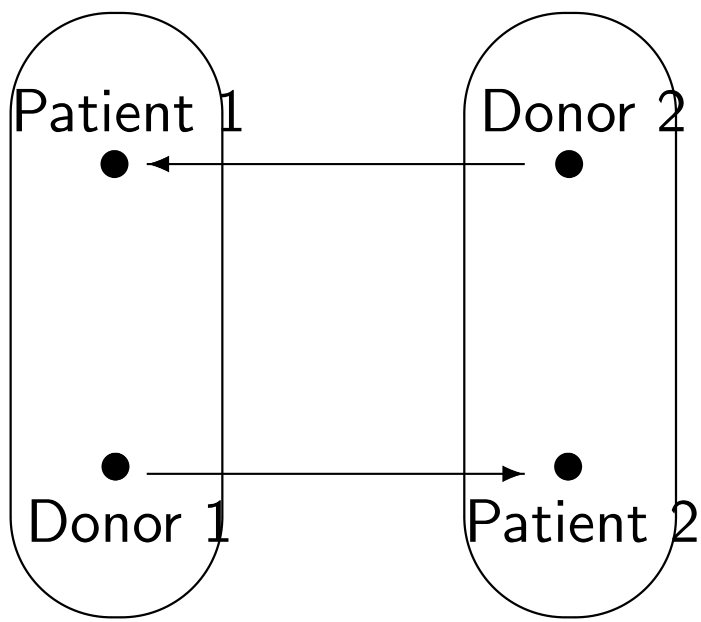
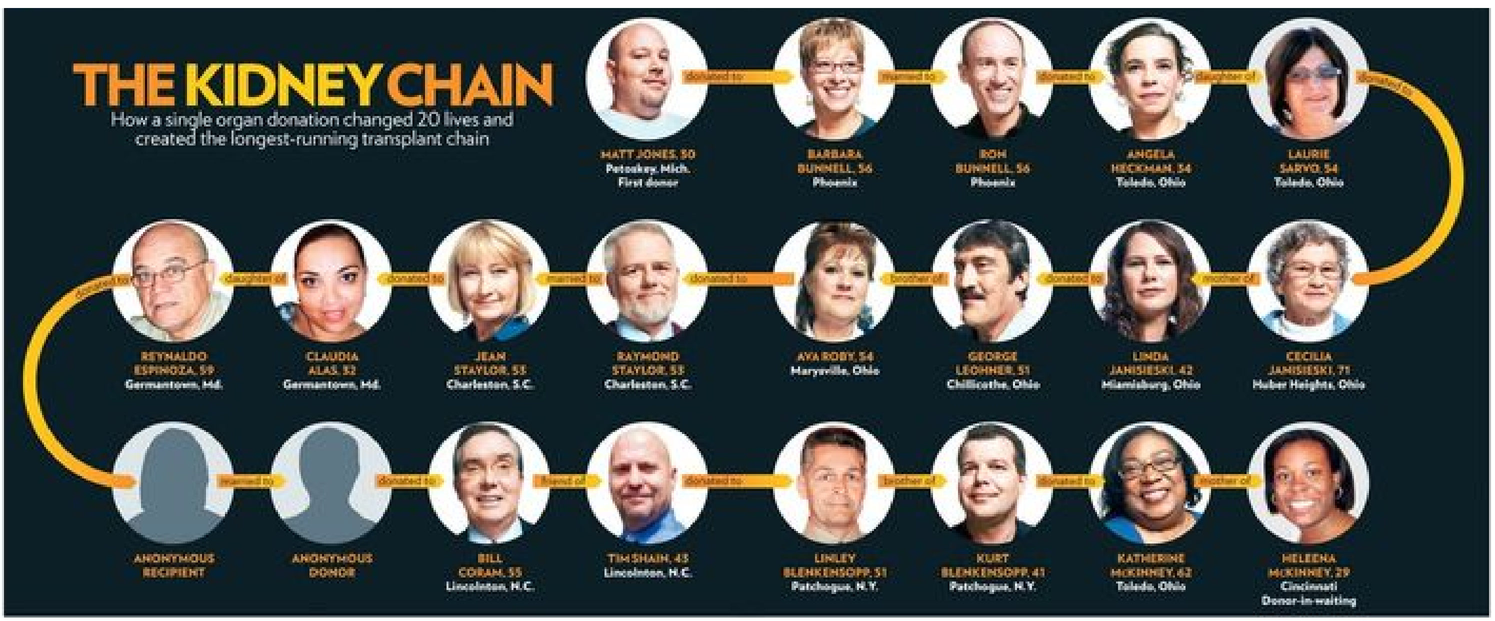
- Organized Kidney Exchange (Pairwise)



Chains

- Non-simultaneous altruistic donor chains
  - Vast majority of transplants in large exchanges
  - Typically four to five donors long, although long chains are possible and useful



Market clearing with prices

Single-Unit Auctions

- ✓ 2nd price auction
- ✓ Open vs. sealed format

Multi-Unit Auctions

- ✓ Uniform vs. Discriminatory
- ✓ Vickrey-Clarke-Groves

Two-sided exchanges

- ✓ Assignment markets
- ✓ "Funny money"./ Script systems

Matching meets Auctions

- ✓ Kelso-Crawford
- ✓ Matching with Contracts

New frontiers

- Financial market design
- Market-based schemes for environmental regulation
- Open-access markets – uber, airbnb, wireless, feedingAmerica
- Transportation and congestion management through smart pricing