Parallel Digital Currencies and Sticky Prices

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Motivation and Research Question

- Increasing varieties of privately issued digital currencies:
  - Bitcoin
  - Ethereum
  - ...
- **Question:** What happens, when firms price in these currencies, rather than the official currency?
- Role of money:
  1. **Unit of account.** Here: currency of pricing.
  2. Medium of exchange.
  3. Store of value.
- **Approach:** an NK model with multiple currencies.

Results Overview

- Exchange rate shocks arise without other sources of uncertainties
- Relative price between sectors becomes state variable. Rich sectoral dynamics.
- In response to a **dollar depreciation**:
  - Considerable persistent **reallocation** between sectors. Large decline in non-dollar sector. Small and temporary aggregate recession.
  - Recession is persistent, if mon pol only reacts to dollar inflation.
  - Increased **flexibility** of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.
  - Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.
- Endogenous currency choice: less flexible sector increases substantially. Slight, persistent aggregate boom, inflation drop.

Literature

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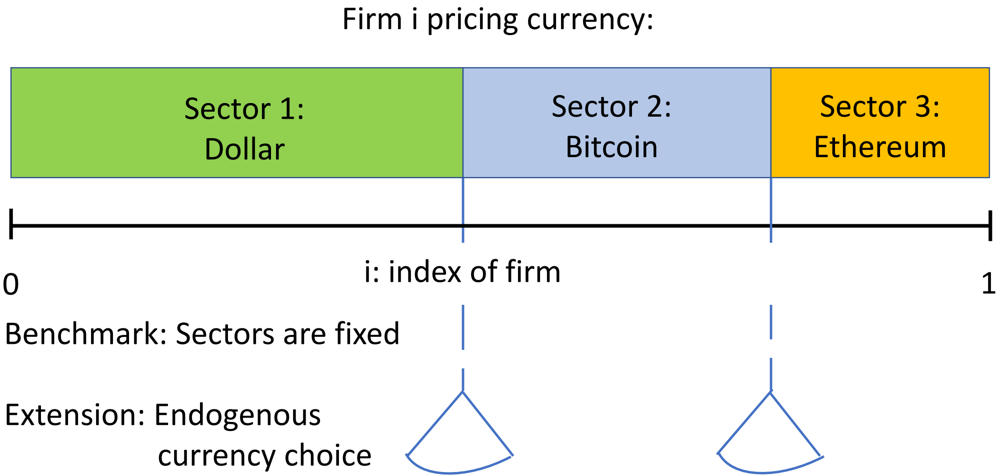
Model – Currencies and Prices

- J currencies in total, each with money supply $M_{j,t}$
  - $j = 1$: fiat currency, *dollar*; $j \neq 1$ parallel currency, *bitcoin*
  - $\mathcal{E}_{j,t}$: price of currency j in dollar
  - $\mathcal{E}_{1,t} = 1$
  - $\frac{\mathcal{E}_{j,t}}{\mathcal{E}_{j',t}}$: price of currency j in currency j'
- Firms in sector j set prices in currency j, but accept payments in all currencies
  - $V_{j,t}$: set of firms in sector j
  - $\upsilon_{j,t}$: measure of sector j
  - sectoral price index $P_{j,t} = \left[\frac{1}{\upsilon_{j,t}} \int_{V_{j,t}} P_{j,t}(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$
  - general price index $P_t = \left[\sum_{j=1}^{J} \upsilon_{j,t} \left(\mathcal{E}_{j,t} P_{j,t}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$
  - general price inflation $\Pi_t = \frac{P_t}{P_{t-1}}$
  - sectoral relative price $\hat{P}_{j,t} = \frac{\mathcal{E}_{j,t} P_{j,t}}{P_t}$

Pricing Sectors



Households

- Lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t, N_t\right)$$

- Consumption bundle $C_t = \left[\int C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$
- Liquidity $L_t = \sum_{j=1}^{J} L_{j,t}$, where $L_{j,t} = \frac{\mathcal{E}_{j,t} M_{j,t}}{P_t}$
- Labour supply N_t
- Budget constraint

$$C_t + \frac{B_t}{P_t} + \sum_{j=1}^{J} L_{j,t} = \frac{\exp(i_{t-1})}{\Pi_t} \frac{B_{t-1}}{P_{t-1}} + \sum_{j=1}^{J} \frac{L_{j,t-1}}{\Pi_t} \frac{\mathcal{E}_{j,t}}{\mathcal{E}_{j,t-1}} + W_t N_t + \Gamma_t$$

Firms

- Production function $Y_t(i) = A_t N_t(i)^{1-\alpha}$
- $1 - \theta_j$ fraction of firms reset prices in sector j
- Profit maximization problem

$$\max_{P^*_{j,t}} \sum_{\ell=0}^{\infty} \theta_j^{\ell} E_t \left[Q_{t,t+\ell} \left[\frac{\mathcal{E}_{j,t+\ell} P^*_{j,t}}{P_{t+k}} Y_{t+\ell}(i) - \Psi_{t+\ell}\left(Y_{t+\ell}(i)\right) \right] \right]$$

subject to demand function $Y_{t+\ell}(i) = \left(\frac{\mathcal{E}_{j,t+\ell} P^*_{j,t}}{P_{t+\ell}} \right)^{-\epsilon} Y_{t+\ell}$

Linearised Model

- **Proposition 1**: The nominal exchange rate between any pair of parallel currencies j and j' follows a random-walk process:

$$e_{j,t} - e_{j',t} = E_t \left(e_{j,t+1} - e_{j',t+1} \right)$$

- Sectoral NKPC:

$$\pi_{j,t} = \beta E_t \pi_{j,t+1} + \kappa_j \tilde{y}_t - \lambda_j \hat{p}_{j,t}$$

where κ_j and λ_j depend on θ_j, and

$$\hat{p}_{j,t} = \hat{p}_{j,t-1} + \pi_{j,t} + \Delta e_{j,t} - \pi_t \tag{1}$$

- Dynamic IS equation:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} \left[\hat{i}_t - E_t \pi_{t+1} - \hat{r}_t^n \right]$$

Key Equations in the NK Framework

Result: Relative price between sectors becomes state variable. Rich sectoral dynamics.

- With J parallel currencies, the following $(2J+2)$-equation system summarises dynamics in the economy

$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \sigma^{-1}\left(\hat{i}_t - \boldsymbol{\upsilon}'\, E_t\left[\boldsymbol{\pi}_{t+1}\right] - r^n_t\right) \tag{2}$$

$$\boldsymbol{\pi}_t = \beta\, E_t\left[\boldsymbol{\pi}_{t+1}\right] + \boldsymbol{\kappa}\,\tilde{y}_t - \boldsymbol{\lambda}\circ\hat{\boldsymbol{p}}_t \tag{3}$$

$$\hat{\boldsymbol{p}}_t = \hat{\boldsymbol{p}}_{t-1} + \left(\mathbf{I} - \mathbf{1}\,\boldsymbol{\upsilon}'\right)\left(\boldsymbol{\pi}_t + \Delta\boldsymbol{e}_t\right) \tag{4}$$

$$\hat{i}_t = \phi_\pi\,\boldsymbol{\upsilon}'\boldsymbol{\pi}_t + \phi_y\,\tilde{y}_t \tag{5}$$

  where $\circ$ is an operator for element-wise multiplication.

- Generalised aggregate inflation:

$$\pi_t = \beta\, E_t\left[\pi_{t+1}\right] + \boldsymbol{\upsilon}'\boldsymbol{\kappa}\,\tilde{y}_t - \boldsymbol{\upsilon}'\left(\boldsymbol{\lambda}\circ\hat{\boldsymbol{p}}_t\right) + \boldsymbol{\upsilon}'\Delta\boldsymbol{e}_t$$

Baseline Cases

- **Proposition 2 (homogeneous rigidity)**: Between any two sectors j and j' with homogeneous price rigidity θ,
  1. the optimal prices in both sectors are equivalent, $p^*_{j,t} + e_{j,t} = p^*_{j',t} + e_{j',t}$;
  2. the bilateral relative price is an autoregressive process, $s_{jj',t} = \theta \left(s_{jj',t-1} + \Delta e_{j,t} - \Delta e_{j',t} \right)$;
  3. the inflation differential is linear in bilateral relative price, $\pi_{j,t} - \pi_{j',t} = -\frac{1-\theta}{\theta} s_{jj',t}$;
  4. the output-gap differential is linear in bilateral relative price, $\tilde{y}_{j,t} - \tilde{y}_{j',t} = -\epsilon \, s_{jj',t}$.
- **Proposition 3**: The new Keynesian Philips curve for aggregate inflation is independent of the relative price dynamics if price rigidity is homogeneous across all currency sectors:

$$\pi_t = \beta \, E_t \left[\pi_{t+1} \right] + \kappa \, \tilde{y}_t + \boldsymbol{\upsilon}' \Delta \boldsymbol{e}_t$$

- **Proposition 4 (single flexible sector)**: An exchange-rate shock to any non-dollar currency j does not spillover to the other currency sectors if prices are perfectly flexible in sector j.

Monetary Policy

What should monetary policy target?

- Aggregate inflation? Or dollar inflation only?
- Aggregate output gap? Or dollar sector output gap only?

Thus:

- Two sector: dollar vs non-dollar; dollar depreciation shock
- Size of non-dollar sector $\upsilon = 0.2$
- Taylor rules

$$\hat{i}_t = \phi_\pi \, \pi_t + \phi_y \, \tilde{y}_t \qquad \text{(AIAO)}$$

$$\hat{i}_t = \phi_\pi \, \pi_{1,t} + \phi_y \, \tilde{y}_t \qquad \text{(DIAO)}$$

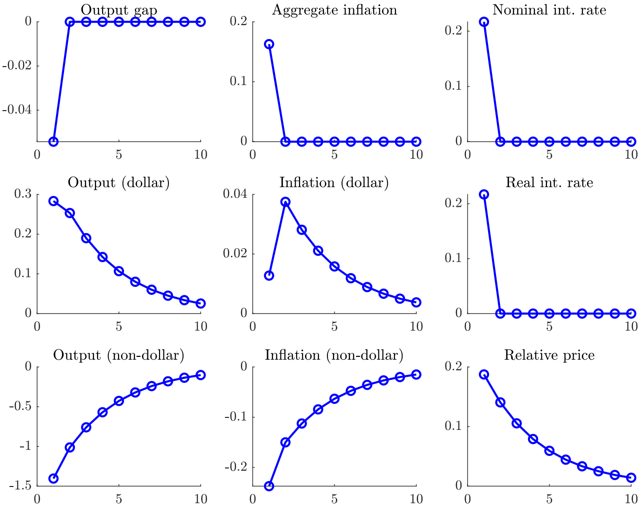
$$\hat{i}_t = \phi_\pi \, \pi_{1,t} + \phi_y \, \tilde{y}_{1,t} \qquad \text{(DIDO)}$$

Parameterization

Table: Parameter values in benchmark model.

Parameter	Value	Description
α	0.250	Share of labour input in production function
σ	1.000	Coefficient of risk aversion
φ	5.000	Inverse Frisch elasticity of labour supply
β	0.990	Discount factor
θ_1	0.750	Probability of not adjusting prices in dollar sector
θ_2	0.750	Probability of not adjusting prices in non-dollar sector
ϵ	9.000	Elasticity of substitution among consumption goods
ϕ_π	1.500	Interest-rate reaction to inflation
ϕ_y	0.125	Interest-rate reaction to output gap
υ	0.200	Size of non-dollar sector
$\sigma_{\Delta e}$	0.250	Standard deviation of exchange-rate shock

IRFs to dollar depreciation: Baseline Taylor Rule "AIAO"

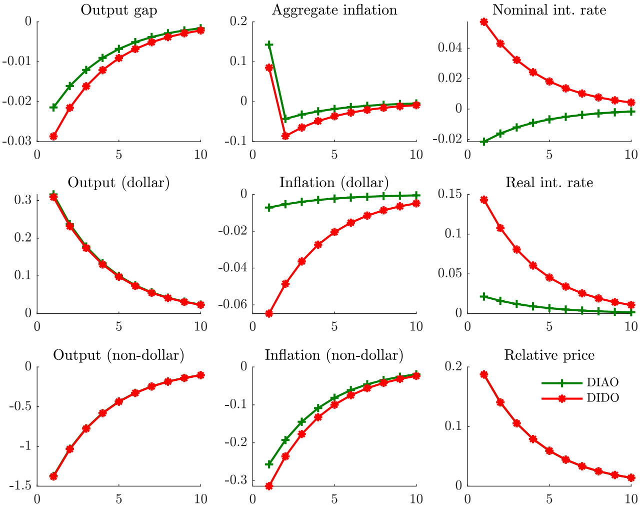


Baseline policy:

$$\hat{i}_t = \phi_\pi \, \pi_t + \phi_y \, \tilde{y}_t \qquad \text{(AIAO)}$$

Result:
Considerable persistent reallocation between sectors. Large decline in non-dollar sector. Small and temporary aggregate recession.

IRFs to dollar depreciation: Alternative Taylor Rules "DIAO" and "DIDO"



Alternative policies:

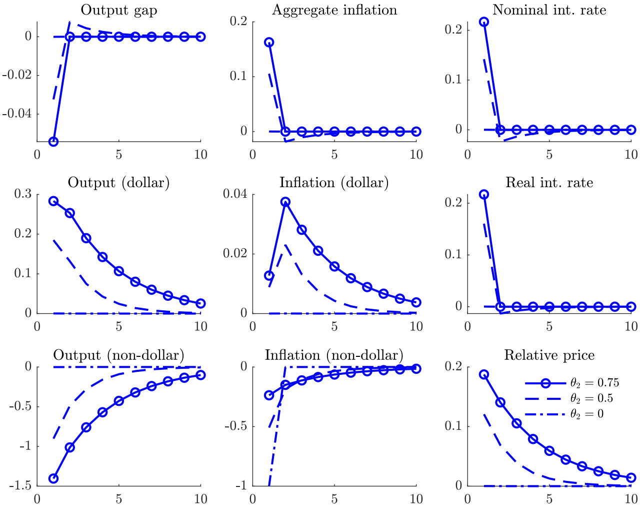
$$\hat{i}_t = \phi_\pi \, \pi_{1,t} + \phi_y \, \tilde{y}_t \qquad \text{(DIAO)}$$

$$\hat{i}_t = \phi_\pi \, \pi_{1,t} + \phi_y \, \tilde{y}_{1,t} \qquad \text{(DIDO)}$$

Result:

Persistent aggregate recession.

Heterogeneous rigidity



IRFs to dollar depreciation.

- Prices more flexible in non-dollar sector: $\theta_1 = 0.75$, $\theta_2 \in \{0, 0.5, 0.75\}$.

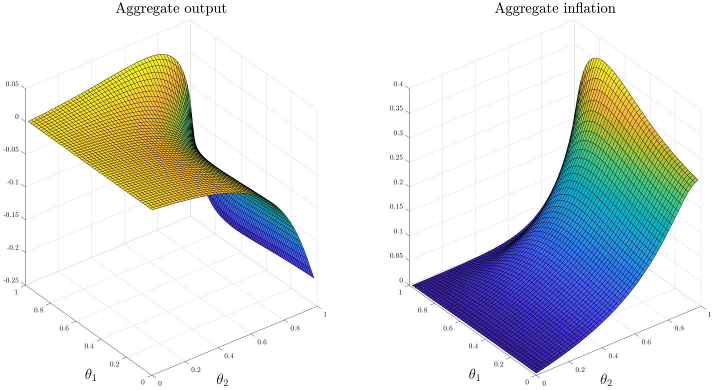
Result:

- Flexibility of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.
- Subtle: aggregate output persistence.

Volatility

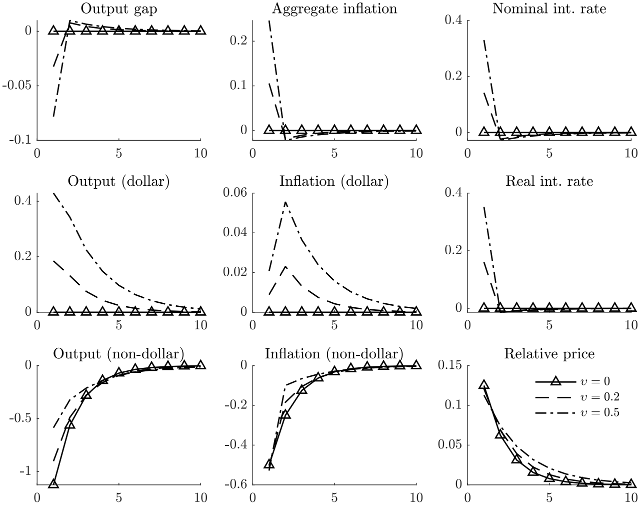
Comparison across parameters. 8-period cummulated IRFs to dollar depreciation.

Result: more non-dollar rigidity induces larger output drops, higher aggregate inflation.



Different sector shares

IRFs to dollar depreciation. $\theta_1 = 0.75$; $\theta_2 = 0.5$.



Result:

Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.

Currency Choice

- Firms choose pricing currencies when given opportunity to re-price
- We use a discrete choice model. Let $\mathcal{U}_{i,j,t}$ denote the utility of a producer, who resets price in period t.

$$\log\left(\mathcal{U}_{i,j,t}\right) = \log\left(\mathcal{V}_{j,t,t}\right) + \varepsilon_{i,j,t} \tag{6}$$

- We assume $\varepsilon_{j,t}$ to be independent across i, j and t as well as any other shocks and to be distributed according to a type I extreme value distribution

$$F\left(\varepsilon_{i,j,t}\right) = \exp\left(-\gamma_j \exp(-\varepsilon_{i,j,t})\right) \tag{7}$$

- $\mathcal{V}_{j,s,t}$ is the continuation value to the firm when the price has been reset at $s \leq t$, using currency j to do so

$$\mathcal{V}_{j,s,t} = \Xi\left(p^*_{s,j} + e_{j,t} \,|\, x_t\right) + \theta_j \mathbb{E}_t\left[Q_{t,t+1}\mathcal{V}_{j,s,t+1}\right] \tag{8}$$

where $\Xi\left(p \,|\, x_t\right)$ is the profit function given the state of the economy x_t.

Currency Choice

- A firm chooses to price in currency j if the utility from the optimised value is at least as high as all the other alternatives:

$$\mathcal{U}_{j,t} \geq \mathcal{U}_{j',t} \ \forall j' = 0, ..., K. \tag{9}$$

which holds when

$$\varepsilon_{j',t} \leq \log\left(\frac{\mathcal{V}_{j,t,t}}{\mathcal{V}_{j',t,t}}\right) + \varepsilon_{j,t} \ \forall j' = 1, ..., J. \tag{10}$$

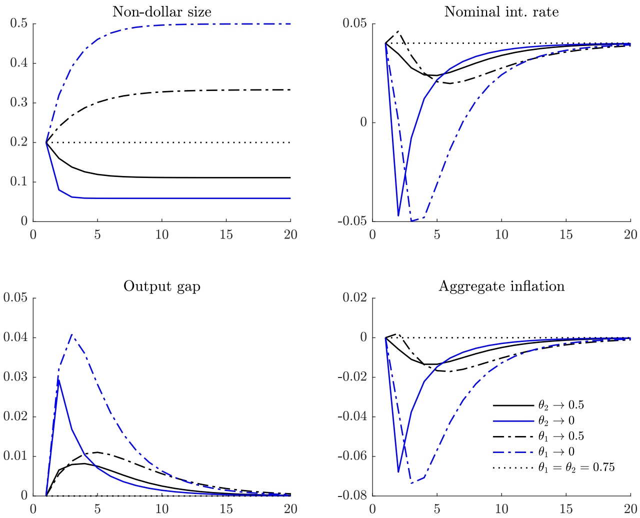
- The joint probability from J currencies gives the probability of a firm pricing in currency j:

$$\mathrm{Pr}_{j,t} = \frac{\gamma_j \, \mathcal{V}_{j,t,t}}{\sum_{j'=1}^{J} \gamma_{j'} \, \mathcal{V}_{j',t,t}} \tag{11}$$

- The law of motion for $\upsilon_{j,t}$ is:

$$\upsilon_{j,t} = \theta_j \, \upsilon_{j,t-1} + \mathrm{Pr}_{j,t} \sum_{j'=1}^{K} \left(1 - \theta_{j'}\right) \, \upsilon_{j',t-1} \tag{12}$$

Currency choice: transition dynamics



"MIT shock": dollar sector ("dashed") or non-dollar sector ("solid") becomes more flexible.

Result:
Less flexible sector increases. Slight, persistent boom, inflation drop.

Conclusion

- Increasing varieties of privately issued digital currencies.
- **Question:** What happens, when firms price in these currencies, rather than the official currency?
- **Approach:** an NK model with multiple currencies.
- **Results:** Relative price between sectors becomes state variable. Rich sectoral dynamics. In response to a dollar depreciation:
  - Considerable persistent reallocation between sectors. Large decline in non-dollar sector. Small, temporary aggregate recession with AIAO, persist. w. DIAO, DIDO.
  - Increased flexibility of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.
  - Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.

  Endogenous currency choice: less flexible sector increases substantially. Slight, persistent aggregate boom, inflation drop.