Parallel Digital Currencies and Sticky Prices

Harald Uhlig    Taojun Xie

University of Chicago
National University of Singapore

ABFER Annual Conference
May 24th, 2021
Motivation and Research Question

- Increasing varieties of privately issued digital currencies:
  - Bitcoin
  - Ethereum
  - ...

- **Question**: What happens, when firms price in these currencies, rather than the official currency?

- **Role of money**:
  1. **Unit of account.** Here: currency of pricing.
  3. Store of value.

- **Approach**: an NK model with multiple currencies.
Results Overview

- Exchange rate shocks arise without other sources of uncertainties
- Relative price between sectors becomes state variable. Rich sectoral dynamics.
- In response to a **dollar depreciation**:
  - Considerable persistent **reallocation** between sectors. Large decline in non-dollar sector. Small and temporary aggregate recession.
  - Recession is persistent, if mon pol only reacts to dollar inflation.
  - Increased **flexibility** of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.
  - Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.
- Endogenous currency choice: less flexible sector increases substantially. Slight, persistent aggregate boom, inflation drop.
Literature

Model – Currencies and Prices

- $J$ currencies in total, each with money supply $M_{j,t}$
  - $j = 1$: fiat currency, dollar; $j \neq 1$ parallel currency, bitcoin
  - $E_{j,t}$: price of currency $j$ in dollar
  - $E_{1,t} = 1$
  - $\frac{E_{j,t}}{E_{j',t}}$: price of currency $j$ in currency $j'$

- Firms in sector $j$ set prices in currency $j$, but accept payments in all currencies
  - $V_{j,t}$: set of firms in sector $j$
  - $\upsilon_{j,t}$: measure of sector $j$
  - Sectoral price index $P_{j,t} = \left[ \frac{1}{\upsilon_{j,t}} \int_{V_{j,t}} P_{j,t}(i)E_{j',t}P_{j',t}(i)1^{-\epsilon} di \right]^{1-\epsilon}$
  - General price index $P_t = \left[ \sum_{j=1}^{J} \upsilon_{j,t} (E_{j,t}P_{j,t})1^{-\epsilon} \right]^{1-\epsilon}$
  - General price inflation $\Pi_t = \frac{P_t}{P_{t-1}}$
  - Sectoral relative price $\hat{P}_{j,t} = \frac{E_{j,t}P_{j,t}}{P_t}$
Pricing Sectors

Firm i pricing currency:

Sector 1: Dollar
Sector 2: Bitcoin
Sector 3: Ethereum

0

i: index of firm

Benchmark: Sectors are fixed

Extension: Endogenous currency choice
Households

- **Lifetime utility**
  \[ E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t, N_t) \]

- **Consumption bundle**
  \[ C_t = \left[ \int C_t(i)^{1-\frac{1}{\epsilon}} \, di \right]^{\frac{\epsilon}{\epsilon-1}} \]

- **Liquidity**
  \[ L_t = \sum_{j=1}^{J} L_{j,t}, \text{ where } L_{j,t} = \frac{E_{j,t} M_{j,t}}{P_t} \]

- **Labour supply**
  \[ N_t \]

- **Budget constraint**
  \[ C_t + \frac{B_t}{P_t} + \sum_{j=1}^{J} L_{j,t} = \exp(i_{t-1}) \frac{B_{t-1}}{\Pi_t P_{t-1}} + \sum_{j=1}^{J} \frac{L_{j,t-1}}{\Pi_t} \frac{E_{j,t}}{E_{j,t-1}} + W_t N_t + \Gamma_t \]
Firms

- Production function \( Y_t(i) = A_t N_t(i)^{1-\alpha} \)
- \( 1 - \theta_j \) fraction of firms reset prices in sector \( j \)
- Profit maximization problem

\[
\max_{P^*, j, t} \sum_{\ell=0}^{\infty} \theta_j^\ell E_t \left[ Q_{t,t+\ell} \left( \frac{E_{j,t+\ell} P^*_{j,t}}{P_{t+k}} Y_{t+\ell}(i) - \psi_{t+\ell}(Y_{t+\ell}(i)) \right) \right]
\]

subject to demand function \( Y_{t+\ell}(i) = \left( \frac{E_{j,t+\ell} P^*_{j,t}}{P_{t+\ell}} \right)^{-\epsilon} Y_{t+\ell} \)
Linearised Model

- **Proposition 1**: The nominal exchange rate between any pair of parallel currencies \( j \) and \( j' \) follows a random-walk process:

\[
e_{j,t} - e_{j',t} = E_t (e_{j,t+1} - e_{j',t+1})
\]

- **Sectoral NKPC**:

\[
\pi_{j,t} = \beta E_t \pi_{j,t+1} + \kappa_j \tilde{y}_t - \lambda_j \hat{p}_j,t
\]

where \( \kappa_j \) and \( \lambda_j \) depend on \( \theta_j \), and

\[
\hat{p}_j,t = \hat{p}_{j,t-1} + \pi_{j,t} + \Delta e_{j,t} - \pi_t
\] (1)

- **Dynamic IS equation**:

\[
\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} \left[ \hat{i}_t - E_t \pi_{t+1} - \hat{r}_t^n \right]
\]
Key Equations in the NK Framework

**Result:** Relative price between sectors becomes state variable. Rich sectoral dynamics.

- With $J$ parallel currencies, the following $(2J + 2)$-equation system summarises dynamics in the economy

\[
\tilde{y}_t = E_t [\tilde{y}_{t+1}] - \sigma^{-1} \left( \hat{i}_t - \nu' E_t [\pi_{t+1}] - r^n_t \right) \tag{2}
\]

\[
\pi_t = \beta E_t [\pi_{t+1}] + \kappa \tilde{y}_t - \lambda \circ \hat{p}_t \tag{3}
\]

\[
\hat{p}_t = \hat{p}_{t-1} + (I - \nu') (\pi_t + \Delta e_t) \tag{4}
\]

\[
\hat{i}_t = \phi_\pi \nu' \pi_t + \phi_y \tilde{y}_t \tag{5}
\]

where $\circ$ is an operator for element-wise multiplication.

- Generalised aggregate inflation:

\[
\pi_t = \beta E_t [\pi_{t+1}] + \nu' \kappa \tilde{y}_t - \nu' (\lambda \circ \hat{p}_t) + \nu' \Delta e_t
\]
Baseline Cases

- **Proposition 2 (homogeneous rigidity):** Between any two sectors $j$ and $j'$ with homogeneous price rigidity $\theta$,
  1. the optimal prices in both sectors are equivalent, $p_{j,t}^* + e_{j,t} = p_{j',t}^* + e_{j',t}$;
  2. the bilateral relative price is an autoregressive process,
     $$s_{jj',t} = \theta (s_{jj',t-1} + \Delta e_{j,t} - \Delta e_{j',t});$$
  3. the inflation differential is linear in bilateral relative price, $\pi_{j,t} - \pi_{j',t} = -\frac{1-\theta}{\theta} s_{jj',t};$
  4. the output-gap differential is linear in bilateral relative price, $\tilde{y}_{j,t} - \tilde{y}_{j',t} = -\epsilon s_{jj',t}.$

- **Proposition 3:** The new Keynesian Philips curve for aggregate inflation is independent of the relative price dynamics if price rigidity is homogeneous across all currency sectors:
  $$\pi_t = \beta E_t [\pi_{t+1}] + \kappa \tilde{y}_t + \nu' \Delta e_t$$

- **Proposition 4 (single flexible sector):** An exchange-rate shock to any non-dollar currency $j$ does not spillover to the other currency sectors if prices are perfectly flexible in sector $j$. 
Monetary Policy

What should monetary policy target?
- Aggregate inflation? Or dollar inflation only?
- Aggregate output gap? Or dollar sector output gap only?

Thus:
- Two sector: dollar vs non-dollar; dollar depreciation shock
- Size of non-dollar sector $\nu = 0.2$
- Taylor rules

\[
\hat{i}_t = \phi_\pi \pi_t + \phi_y \hat{y}_t \quad \text{(AIAO)} \\
\hat{i}_t = \phi_\pi \pi_{1,t} + \phi_y \hat{y}_t \quad \text{(DIAO)} \\
\hat{i}_t = \phi_\pi \pi_{1,t} + \phi_y \hat{y}_{1,t} \quad \text{(DIDO)}
\]
## Parameterization

**Table:** Parameter values in benchmark model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.250</td>
<td>Share of labour input in production function</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.000</td>
<td>Coefficient of risk aversion</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>5.000</td>
<td>Inverse Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.990</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.750</td>
<td>Probability of not adjusting prices in dollar sector</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.750</td>
<td>Probability of not adjusting prices in non-dollar sector</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>9.000</td>
<td>Elasticity of substitution among consumption goods</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.500</td>
<td>Interest-rate reaction to inflation</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.125</td>
<td>Interest-rate reaction to output gap</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.200</td>
<td>Size of non-dollar sector</td>
</tr>
<tr>
<td>$\sigma_{\Delta e}$</td>
<td>0.250</td>
<td>Standard deviation of exchange-rate shock</td>
</tr>
</tbody>
</table>
IRFs to dollar depreciation: Baseline Taylor Rule “AIAO”

Baseline policy:
\[ \hat{i}_t = \phi \pi_t + \phi_y \hat{y}_t \] (AIAO)

Result:
Considerable persistent reallocation between sectors.
Large decline in non-dollar sector.
Small and temporary aggregate recession.
IRFs to dollar depreciation: Alternative Taylor Rules “DIAO” and “DIDO”

Alternative policies:

$$\hat{i}_t = \phi \pi_{1,t} + \phi_y \tilde{y}_t$$ (DIAO)

$$\hat{i}_t = \phi \pi_{1,t} + \phi_y \tilde{y}_{1,t}$$ (DIDO)

Result:
Persistent aggregate recession.
Heterogeneous rigidity

IRFs to dollar depreciation.

▶ Prices more flexible in non-dollar sector:
  \( \theta_1 = 0.75, \theta_2 \in \{0, 0.5, 0.75\} \).

**Result:**

▶ Flexibility of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.

▶ Subtle: aggregate output persistence.
Volatility

Comparison across parameters. 8-period cummulated IRFs to dollar depreciation. **Result:** more non-dollar rigidity induces larger output drops, higher aggregate inflation.
Different sector shares

IRFs to dollar depreciation. $\theta_1 = 0.75; \theta_2 = 0.5$.

Result:
Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.
Currency Choice

- Firms choose pricing currencies when given opportunity to re-price
- We use a discrete choice model. Let \( \mathcal{U}_{i,j,t} \) denote the utility of a producer, who resets price in period \( t \).

\[
\log (\mathcal{U}_{i,j,t}) = \log (\mathcal{V}_{j,t,t}) + \varepsilon_{i,j,t} \tag{6}
\]

- We assume \( \varepsilon_{j,t} \) to be independent across \( i, j \) and \( t \) as well as any other shocks and to be distributed according to a type I extreme value distribution

\[
F(\varepsilon_{i,j,t}) = \exp (-\gamma_j \exp(-\varepsilon_{i,j,t})) \tag{7}
\]

- \( \mathcal{V}_{j,s,t} \) is the continuation value to the firm when the price has been reset at \( s \leq t \), using currency \( j \) to do so

\[
\mathcal{V}_{j,s,t} = \Xi(p^*_s + e_{j,t} | x_t) + \theta_{j} \mathbb{E}_t [Q_{t,t+1} \mathcal{V}_{j,s,t+1}] \tag{8}
\]

where \( \Xi(p | x_t) \) is the profit function given the state of the economy \( x_t \).
Currency Choice

▶ A firm chooses to price in currency \( j \) if the utility from the optimised value is at least as high as all the other alternatives:

\[
U_{j,t} \geq U_{j',t} \quad \forall j' = 0, \ldots, K.
\] (9)

which holds when

\[
\varepsilon_{j,t} \leq \log \left( \frac{V_{j,t,t}}{V_{j',t,t}} \right) + \varepsilon_{j,t} \quad \forall j' = 1, \ldots, J.
\] (10)

▶ The joint probability from \( J \) currencies gives the probability of a firm pricing in currency \( j \):

\[
Pr_{j,t} = \frac{\gamma_j V_{j,t,t}}{\sum_{j'=1}^{J} \gamma_{j'} V_{j',t,t}}
\] (11)

▶ The law of motion for \( \nu_{j,t} \) is:

\[
\nu_{j,t} = \theta_j \nu_{j,t-1} + Pr_{j,t} \sum_{j'=1}^{K} (1 - \theta_{j'}) \nu_{j',t-1}
\] (12)
Currency choice: transition dynamics

“MIT shock”: dollar sector (“dashed”) or non-dollar sector (“solid”) becomes more flexible.

**Result:**
Less flexible sector increases. Slight, persistent boom, inflation drop.
Conclusion

- Increasing varieties of privately issued digital currencies.

**Question:** What happens, when firms price in these currencies, rather than the official currency?

**Approach:** an NK model with multiple currencies.

**Results:** Relative price between sectors becomes state variable. Rich sectoral dynamics. In response to a dollar depreciation:

- Considerable persistent reallocation between sectors. Large decline in non-dollar sector. Small, temporary aggregate recession with AIAO, persist. w. DIAO, DIDO.
- Increased flexibility of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.
- Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.

Endogenous currency choice: less flexible sector increases substantially. Slight, persistent aggregate boom, inflation drop.