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THE PERFORMANCE OF CHARACTERISTIC-SORTED PORTFOLIOS: EVALUATING THE PAST AND PREDICTING THE FUTURE

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Performance of Characteristic-Sorted Portfolios

Annualized Sharpe Ratios for value-weighted portfolios (*t*-stats in parenthesis)

	All	1963 – 1991	1991 – 2019	Difference
Value	0.32	0.42	0.22	**-0.20**
	(2.37)	(2.23)	(1.15)	**-(0.75)**
Investment	0.44	0.44	0.45	**0.02**
	(3.31)	(2.30)	(2.35)	**(0.06)**
Profitability	0.47	0.36	0.57	**0.20**
	(3.48)	(1.89)	(2.96)	**(0.75)**
Size	0.02	0.10	-0.07	**-0.17**
	(0.17)	(0.54)	-(0.37)	**-(0.64)**

The Big Questions

- The **positive** (**frequentist**) questions
  - Do expected returns of characteristic-sorted portfolios vary over time?
  - When we account for time-variation in expected returns, are we still confident that *unconditional* expected returns differ from zero?

- The **normative** (**Bayesian**) questions
  - How much should we tilt our portfolios towards characteristics like value and profitability?
  - To what extent should these tilts change over time as we learn from data?

What Do We Do?

- Propose a **statistical model** that accounts for the possibility that portfolio returns may be persistent
- Consistent with both **rational** and **behavioral** explanations for the abnormal returns associated with characteristics, e.g., the value premium
  - Risk exposures may change over the business cycle
  - Waves of "irrational exuberance" that relate to the introduction of new technology
    - Shiller (2000) and Alti and Titman (2019)
- We apply the model for **value**, **profitability**, **investment** and **size** portfolios
  - Adjust *t*-stats for **OLS** given persistence parameters
  - Estimate model parameters with **maximum likelihood**
  - **Bayesian** analysis with prior beliefs about parameters

Statistical Model

A time-series of market-neutral portfolio returns r_t satisfies

$$r_{t+1} = \mu_t + \epsilon_{t+1}$$
$$\mu_{t+1} = \mu + \lambda(\mu_t - \mu + \delta_{t+1})$$

with normally distributed unexpected return shocks ϵ_{t+1} and expected return shocks δ_{t+1}

- Model implies realized returns are jointly normal with autocorrelations:

$$\text{Corr}(r_t, r_{t-l}) = \lambda^{l-1}\gamma$$

  - γ is the one-period return autocorrelation
  - λ determines the decay rate of autocorrelations over time
    - We express λ in terms of H, the half-life of shocks to μ_t

Note, we are estimating the mean and the persistence of an unobservable variable, μ_t, but we only observe the time-series of r_t

Findings

1. The interpretation of historical mean returns depends on the persistent variation of conditional expected returns
   - With plausible levels of persistent variation standard errors double (relative to iid)
     - There is less independent variation in returns when expected returns are persistent
   - The data tell us very little about the magnitude of persistent variation

⇒ Our inferences are thus ultimately determined by the assumptions

2. Investors' posterior beliefs about expected returns are highly dependent on their priors about persistent variation
   - Applies to conditional as well as unconditional expected returns
   - E.g. Value's conditional Sharpe Ratio in 2020 is **0.29** for investors who believe returns are i.i.d. and **0.05** for investors who believe conditional expected returns vary with a 5-year half life

Autocorrelation Estimates

Estimate autocorrelation using regressions of the form

$$r_t = a + b \cdot \left(\frac{1}{4} \sum_{l=1}^{4} r_{t-l} \right) + \epsilon_t$$

	Value	Investment	Profitability	Size	Pooled	Pooled (no size)
$\hat{b}$	0.19	0.21	0.21	0.47	0.30	0.20
iid *t*-stat	(1.61)	(1.74)	(1.80)	(3.70)	(4.43)	(2.97)

Result 1: prior-year returns positively predict next-quarter, strongest for size, others marginal

Result 2 (in paper): cannot reject highly persistent variations ($H = 10$)

Model-Based Standard Errors

- Any λ, γ, and sample size T implies a standard error correction for OLS estimates of μ:

$$SE(\hat{\mu}^{OLS}) = \frac{\sigma_r}{\sqrt{T}}\sqrt{1 + 2\gamma\frac{\lambda^T + T(1-\lambda) - 1}{T(1-\lambda)^2}} \approx \frac{\sigma_r}{\sqrt{T}}\sqrt{1 + \frac{2\gamma}{1-\lambda}}$$

  - Same intuition as Newey-West: ↑ autocorrelation ⇒ ↑ standard errors
  - Newey-West doesn't work in this context if H/T is large
  - Analytically derived from the model with assumed parameters

Model-Based Standard Errors

	Value	Investment	Profitability	Size
$\hat{\mu}$	4.37	4.72	4.91	0.35
Unadjusted t-stat	(2.39)	(3.34)	(3.54)	(0.17)
Newey-West t-stat (10 lags)	(2.35)	(2.90)	(3.33)	(0.12)
Newey-West t-stat (20 lags)	(2.35)	(3.04)	(3.71)	(0.11)
Newey-West t-stat (40 lags)	(2.41)	(3.23)	(3.48)	(0.12)
Model t-stat ($H = 2.5$, $\gamma = 2.5\%$)	(1.83)	(2.56)	(2.71)	(0.13)
Model t-stat ($H = 2.5$, $\gamma = 5\%$)	(1.54)	(2.15)	(2.28)	(0.11)
Model t-stat ($H = 2.5$, $\gamma = 10\%$)	(1.23)	(1.71)	(1.81)	(0.09)
Model t-stat ($H = 5$, $\gamma = 2.5\%$)	(1.58)	(2.21)	(2.34)	(0.11)
Model t-stat ($H = 5$, $\gamma = 5\%$)	(1.26)	(1.77)	(1.87)	(0.09)
Model t-stat ($H = 10$, $\gamma = 2.5\%$)	(1.34)	(1.87)	(1.98)	(0.10)
Model t-stat ($H = 10$, $\gamma = 5\%$)	(1.03)	(1.44)	(1.53)	(0.07)

Result: t-stats half as large for reasonable H and γ

Maximum Likelihood Hypothesis Testing

Estimate (H, γ) jointly with μ using max likelihood, test $\mu = 0$ restriction using **likelihood ratio test**

	Const μ	Evolving μ				
		$\hat{H}$ (years)		$\hat{\gamma}$(%)		
	$\mu = 0$ p-value	Unrest.	$\mu = 0$	Unrest.	$\mu = 0$	$\mu = 0$ p-value
Value	1.8%	14.9	20.0	-0.6%	2.7%	**10.2%**
Investment	0.1%	9.4	14.7	-0.9%	5.3%	**5.7%**
Profitability	0.0%	20.0	20.0	-0.4%	4.5%	**5.7%**
Size	86.3%	1.3	2.2	15.4%	14.8%	**37.8%**

Result: evidence against $\mu = 0$ weaker when allowing time-variation

- Data could reflect persistent but temporary μ_t (large H and γ)

Summary of Frequentist Evidence

- If we fix reasonable model parameters, standard errors can be 50% to 100% larger than Newey-West

- If we estimate model parameters freely, $\mu = 0 \ \& \ H \gg 0$ fits data well enough for $\mu = 0$ p-value to be above 5% even when iid p-value is 0.1%

- When many different assumptions, all consistent with the data, lead to substantially different conclusions, natural to use a **Bayesian** approach that integrates across reasonable parameter values weighted by priors

Bayesian Estimation

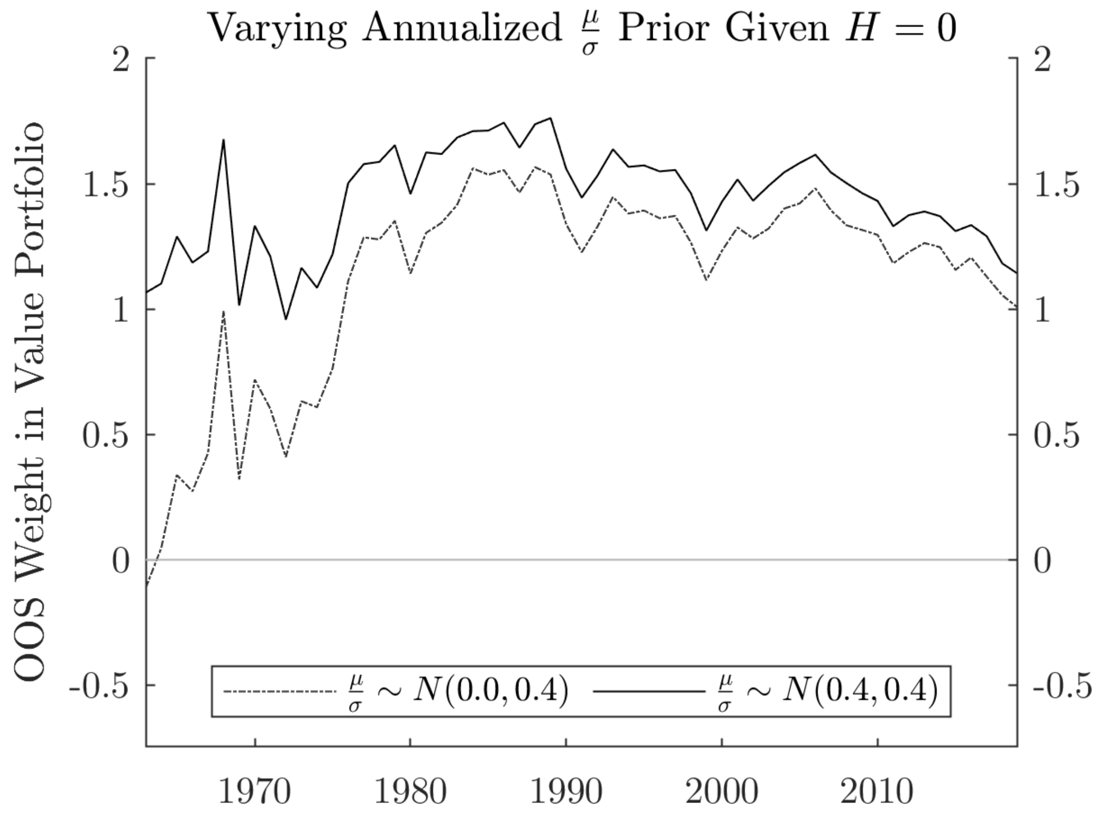
1. Specify priors over model parameters, e.g., μ and H
2. Compute posterior beliefs about model parameters based on priors + data
   - Using full sample
   - Using past returns only at the end of each calendar year
3. Using posterior distribution of parameters, compute informative moments
   - Conditional and unconditional expected returns & Sharpe Ratios
   - Optimal portfolio allocations for CRRA investors

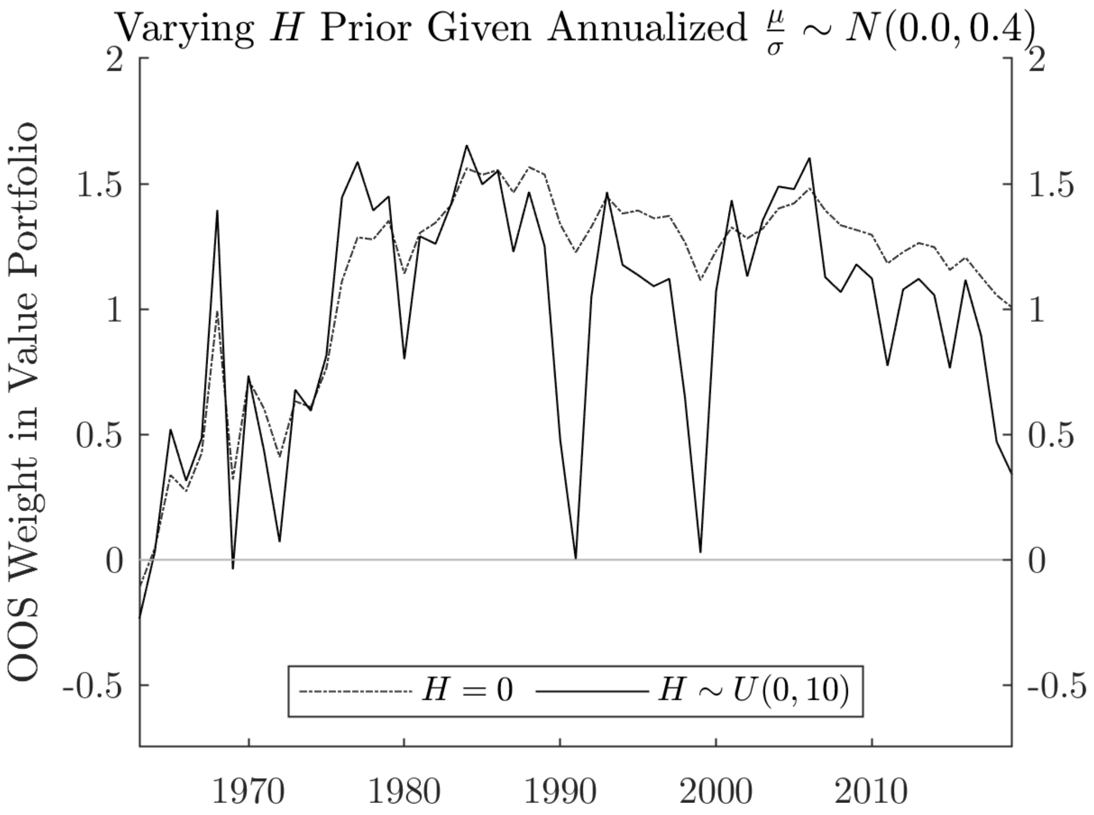
Bayesian Findings

Investor believes that returns are i.i.d. ($H = 0$)	Investor considers possibility that conditional expected returns are time-varying ($H > 0$)

• Priors about unconditional expected returns matter little after observing 56 years of data
• Posteriors about unconditional expected returns are measured more precisely
• Posteriors in about expected returns depend equally on the entire history, e.g. in 2020:
  • Tilts towards profitable value firms
  • No size tilt
	• Priors about unconditional expected returns still matter after observing 56 years of data
	• Posteriors about unconditional expected returns are very uncertain
	• Posteriors about conditional expected returns follow recent trends, e.g. in 2020:
	  • Tilts towards large and profitable firms
	  • No value tilt

Out-of-sample learning and timing for Value





Result 1: disagreement about μ resolves quickly when $H = 0$

Result 2: disagreement about H leads to large, permanent differences in the intensity of timing

Conclusions

1. Characteristic-sorted portfolios are likely to exhibit persistent time-variations in expected returns, ignoring this will produce **false positives**
   - Potential explanation for "factor zoo"
   - Can also produce **false negatives**, e.g., there may be a large unconditional size premium
2. Cannot precisely detect the degree of persistent variation in expected returns
   - Our estimates indicate that Bayesians will **follow trends** anyway
3. **Priors matter much more if returns are potentially persistent**
   - Disagreement and motive for trading
4. These issues are relevant for the evaluation of any portfolios
   - Evidence of superior mutual fund performance
   - Returns of stocks of firms headquartered in some cities outperform stocks from other cities