The Intangible Borrowing Constraint of Entrepreneurship *
Preliminary and Incomplete

Lian Allub — CAF
Axelle Ferriere — PSE
Luis Franjo — Alicante
Yu Zheng — QMUL

January 15, 2023

Abstract

Entrepreneurship can take different forms, from budding tech startups in Silicon Valley to local Mom-and-pop stores around the neighborhood. We distinguish conceptually two types of investment, which helps understand the heterogeneity within the entrepreneurial sector: intangible investment transforms today's skill input into tomorrow's productivity gain, while tangible investment deepens the physical capital in the production without affecting productivity. The financing of the two types of investment differs consequently, as physical capital can be effectively used as collateral while intangible capital cannot. In this paper, we build a quantitative dynamic general equilibrium model of occupational choice and endogenous productivities, which features agents with heterogeneous abilities and wealth levels choosing between worker and entrepreneur. As an entrepreneur, the agent further decides on tangible and intangible investments in the presence of financial constraints a la Buera et al. (2011). Using this framework, we will be able to discuss not only occupational misallocation due to the financial constraint, but also the misallocation within the entrepreneurial sector between different types of investment and therefore different types of entrepreneurship, and discuss its implication on aggregate output and productivity.

JEL classification:
Keywords: Entrepreneurship, Intangible Investment, Borrowing Constraint

*Footnote here...

1 Introduction

Intangible capital plays an increasingly important role in accounting for the macroeconomic trends in the recent decades in the US (McGrattan and Prescott, 2010, 2014; Farhi and Gourio, 2018; Koh et al., 2020). Intangible capital encompasses a broad notion of productivity-enhancing assets, which include not only the intellectual property products (i.e. software, R&D, and artistic originals) but also investments in "economic competencies," such as spending on strategic planning and product redesign, investments to retain or gain market share, investments in organisational capital through training, and investment in brand equity through advertising and building client lists (Corrado et al., 2005, 2009). Different than tangible capital (i.e. property, plant and equipment), intangibles can be more easily scaled up to apply to the entire operation of a firm but require costs which are often sunk due to the illiquid nature of such asset (Haskel and Westlake, 2017). Though some intangible capital does take a physical form and can be purchased from the market (e.g. standardised software), many of the aforementioned examples are produced in-house by highly skilled employees. Drawing on data from the US private businesses and the corporate sector respectively, Bhandari and McGrattan (2021) and Sun and Xiaolan (2019) show a significant share of intangible investment is borne by employees in the form of sweat equity or delayed wage payments. Intangible capital, in its broad sense, is produced by the human capital and skills of the employees.

Taking the interpretation that intangible investment is inseparable from the employment of key managerial, technology, and marketing talents who work to improve the future productivity of the firm, this paper analyzes the financing of intangible investment in the presence of financial constraints in the entrepreneurial sector. The focus on the entrepreneurial sector is deliberate. While large corporations have access to equity market and have the option to reward their employees with stock awards and options, the entrepreneurial sector does not. At the same time, if the brightest and most radical ideas tend to be born in the entrepreneurial sector, then financing intangible to grow those ideas in the entrepreneurial sector is of first-order importance for future productivity growth. We are interested in the following questions: Among households heterogeneous in wealth and ability, who select into entrepreneurship when investments in both tangible and intangible capital are subject to financial constraints? Among these entrepreneurs, who select into intangible-intensive entrepreneurship, focusing on productivity growth? Are they the highly productive or the wealthy? How does the tightness of the financial constraint affect the investment in intangible capital?

To answer these questions, we propose a dynamic general equilibrium model of occupational choice, endogenous productivity and financial constraints. Starting from a Buera-Kaboski-Shin type of financial constraint for financing tangible working capital, we in addition allow an entrepreneur to invest in intangible capital. We deviate from previous literature in the way we model intangible investment. By hiring skilled workers, an entrepreneur can improve the firm's chance of advancing to a higher level of productivity next period similar to climbing a quality ladder in the endogenous growth literature. To retain stationarity, we however consider only a finite ladder, where entrepreneurs' ideas are reshuffled stochastically due to an exogenous obsolescence shock. We interpret the wage payment to the skilled labor as investment in intangible, which may improve productivity only the next period. The financing of total investments, tangible and intangible, is subject to a working capital constraint, which is a function of current earnings and undepreciated capital as well as current wealth as in Buera et al. (2011).

Because of our timing assumption, tangible investment affects current-period production and tangible capital stock, both of which can be effectively used as collateral in the borrowing. In contrast, intangible investment affects only next period's productivity and therefore has no collateral value in the current period. This implies that even with perfect credit market in the sense of Buera et al. (2011), there is deviation from the first best where an agent with a good business idea but low wealth cannot borrow enough to invest in the first-best level of intangible capital. Imperfect credit markets further exacerbate the under-investment in intangible capital.

Depending on how intangible investment interacts with the financial constraint, we will observe entrepreneur's borrowing to have different sensitivities towards fixed assets and towards earnings. If the special property of intangible capital mainly implies that agents do not select into intangible-intensive entrepreneurship unless they are sufficiently wealthy, then entrepreneurs operating intangible intensive projects will tend to be unconstrained and their borrowing will depend more on the productivity of their ideas than on their fixed assets or wealth. Conversely, if the portfolio of investments is distorted towards tangible investment for most entrepreneurs, then entrepreneurs operating intangible intensive projects will more likely be constrained as these projects entail large sunk costs which needs to be financed together with tangible investment. In this case, the debt of the intangible-intensive entrepreneurs should be more sensitive to their asset or wealth and less sensitive to the quality of the idea. We test these implications with the US corporate data and establish that the empirical evidence suggests the former mechanism to be at work.

We calibrate the full model to the US economy and show that the model can replicate the empirical patterns established from the firm-level data. In particular, using the simulated data we find that the sensitivity of entrepreneur's borrowing to his savings decreases in the amount of intangible investment he makes, whereas the sensitivity of borrowing to the quality of idea as proxied by current earnings increases in the amount of intangible investment. With the calibrated model, we carry out two counterfactual exercises. We first consider a perfect credit market counterfactual, which delivers 40% more output than the benchmark and 7% higher tangible capital to output ratio. However the intangible-to-tangible capital ratio in the entrepreneurial sector is lower than that in the benchmark, suggesting removing credit constraint does not solve the financing challenge brought by the lack of pleadgibility of the intangible capital. We then solve numerically for the social planner's problem for this economy and find...[TBC]

Our paper is related to three strands of literature. The first strand of literature focuses on the macroeconomic implications from rising intangible investment in the US and other advanced economies (Corrado et al., 2005, 2009; McGrattan and Prescott, 2010, 2014; Haskel and Westlake, 2017; Koh et al., 2020; Bhandari and McGrattan, 2021). We propose a framework where investment in intangibles directly affects the future productivity of the firm in a way similar to that modeled in the endogenous growth literature. The second strand of macro literature investigates the aggregate impact on entrepreneurship, inequality and productivity from financial frictions (Quadrini, 2000; Buera et al., 2011; Allub and Erosa, 2019; Buera and Shin, 2013; Midrigan and Xu, 2014). We build on the modeling choice for financial friction in this literature and introduce borrowing for intangible investment, which has distinct characteristic from tangible investment. The third strand of literature from corporate finance studies the financing of the intangible capital and its implications on capital structure and corporate savings (Sun and Xiaolan, 2019; Li, 2020; Eisfeldt et al., 2021). There is also a related literature documenting the increasing presence of earnings-based borrowing constraint relative to asset-based borrowing constraint without specifically linking it to intangibles (see for example Lian and Ma (2021)).

2 Motivating Empirical Evidence

Our sample is constructed from two sources. In order to observe firm-level financing behavior, we ideally need firm-level debts, investments (tangible and intangible), and worker skill composition from all firms, particularly from private firms, which we can map to entrepreneurs' projects in the model. For lack of data on private firms from the US, we start from extracting information from public US firms who are legally required to disclose accounting information.[1] From Compustat, we extract firm-level accounting variables such as intangible assets, fixed assets, non-current liabilities, and earnings before interest, taxes, depreciation and amortisation, for public firms active in the US in 2019 in all industries excluding agriculture (NAICS starting in 11), mining (21), utilities (22), finance and insurance (52), public administration (92) and unclassified (99). We keep all firm observations with known intangible assets and trim the top and bottom 1% of all variables. This gives us a sample with 2,433 firms with non-missing values in all those four key variables. The summary statistics are reported in Panel (a) of Table 2.1. All these variables are skewed to the right with the median values much lower than the mean. In terms of magnitude, the average value of intangible assets, 1,176 thousands USD is about 40% of the average value of fixed assets (property, plant and equipment), 2,838 thousands USD. The particular measure of liabilities we focus on, the non-current liabilities, are obligations on the balance sheet due for more than a year and also known as long-term debt. This accounts for close to 70% of total liabilities in the sample, with the remaining 30% short-term debt.

The second source of data is the IPUMS' 2019 Current Population Survey (CPS) data. We keep all individuals aged between 15 and 64, who are in the labor force, who are either employed or self-employed and we exclude those working in agriculture, forestry and fishing, mining, public administration and others. We extract variables related to the individual's age and educational attainment, the class of worker, and industry. The summary statistics are reported in Panel (b) in Table 2.1. The majority of the individuals in the sample have completed secondary education (56.59%) and about a-third have completed university education (33.07%). The vast majority is employed in the private sector (89.39%), while 6.51% are self-employed unincorporated and 4.10% are self-employed incorporated. In this paper, we define entrepreneurs as those who are self-employed and incorporated. We use the crosswalk between the Census 2017 Industry Code with 2017

[1] We came to be aware of some firm-level data in European countries and our next step will be to revisit these facts in those datasets.

NAICS code published by the Census Bureau to construct a harmonised industry classification for the merged Compustat and CPS samples. At the 2-digit level, we have 77 harmonised industries, with firm and worker/entrepreneur information in each of those industries.

Table 2.1: Summary Statistics, the U.S. 2019

Panel (a): Compustat data, in thousands 2019 USD					
	Mean	Std. dev.	5th pct.	50th pct.	95th pct.
Intangible assets	1,176	3,334	0	91	5,907
Fixed assets	2,838	6,763	1	475	14,133
Non-current liabilities	1,918	4,741	0	274	9,773
Earnings before interest, taxes, etc.	463	1,225	-44	68	2,320
No. firms per industry			45		
No. firms			2,433		
No. industries			77		

Panel (b): CPS data, in %			
	Age 15-24	Age 25-54	Age 55-64
Age	13.26	70.14	16.60
	Primary or less	Secondary	University
Educational attainment	10.35	56.59	33.07
	Wage/salary, private	Self-employed, unincorporated	Self-employed, incorporated
Class or workers	89.39	6.51	4.10
No. persons per industry		672	
No. persons		51,495	
No. industries		78	

Note: This table reports the summary statistics of key variables from the 2019 Compustat sample of US public firms and the 2019 CPS sample of US individuals. Industries are the 2-digit harmonised industries between the NAICS codes and Census industry codes. Sample selection is explained in the text.

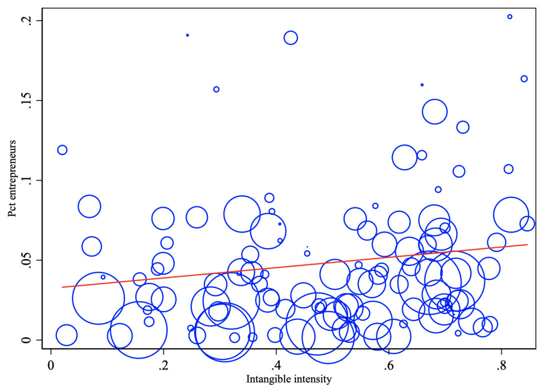
To understand how intangible capital affects the production process and what it implies for the financing, we first construct at the harmonised industry level (4-digit), the percentage of entrepreneurs and the percentage of skilled or college educated workers from the CPS sample and the intangible asset to fixed asset ratio from the Compustat sample. We interpret the intangible asset to fixed asset ratio as the intangible intensity of an industry. Scatter plotting the percentage of entrepreneurs and the percentage of skilled workers against the intangible intensity respectively across these 4-digit industries reveals positive correlations between the two and the intangible intensity (Panel (a) and (b) of Figure 2.1). In sectors with higher intangible intensity, workers tend to be more skilled and there tend to be more presence of entrepreneurs with an incorporated business.

To see the implications on financing from intangibles, we first regress, by industry, firm-level long-term debt on fixed assets and earnings before interests, taxes, depreciation and amortisation, including state fixed effects, for all 2-digit industries with more than 20 observations. The coefficients from these regressions reflect how long-term liability is sensitive to current fixed asset or current earnings. In Panel (c) and (d) of Figure 2.1, we scatter plot these sensitivities against the intangible intensities across 2-digit industries, with the size of the circle indicating the employment size of the industry. Clearly, in industries where intangible is more prominent, firm's long-term borrowing is more sensitive to earnings and less sensitive to fixed asset than in industries with less intangible capital. Both the positive slope in Panel (c) and the negative slope in Panel (d) are statistically significant at 5%. We report the estimated slopes in Figure 2.1 in Table 2.2.

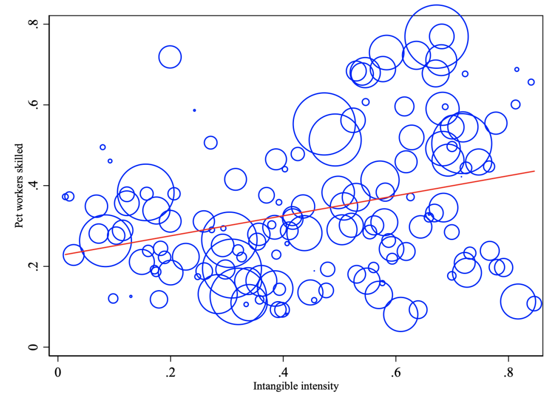
To summarise, the empirical evidence from the US suggests that industries with high intangible capital feature more skilled workers, more incorporated entrepreneurs, higher debt sensitivity to earnings and lower debt sensitivity to fixed asset. Motivated by these observations, we propose a framework where intangible investment by entrepreneurs is done mainly through hiring skilled labor to endogenously affect future productivity subject to a borrowing constrained that only recognised tangible capital as collateral. Before laying out the full dynamic model, we first make use of a two-period toy model to understand what this type of borrowing constraint implies for investing in intangibles.

Figure 2.1: The Relationship of Entrepreneurship, Skill, and Debt Sensitivity with Intangible Intensity, US 2019

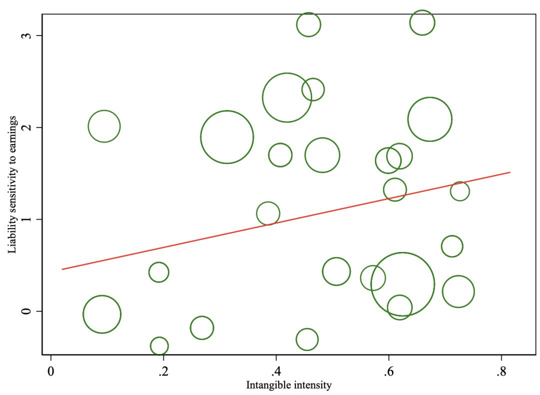
(a) Entrepreneurship and Intangibles



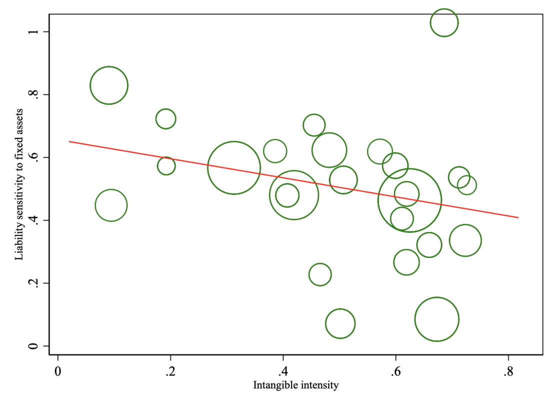
(b) Skill and Intangibles



(c) Debt Sensitivity to Earnings and Intangibles



(d) Debt Sensitivity to Fixed Assets and Intangibles



Note: This figure shows the scatter plots of percentage of entrepreneurs against intangible intensity (Panel (a)), of percentage of skilled workers against intangible intensity (Panel (b)), of debt sensitivity to earnings and intangible intensity (Panel (c)) and of debt sensitivity to fixed assets and intangible intensity (Panel (d)).

Table 2.2: Correlations of Entrepreneurship, Skill, and Debt Sensitivities with Intangible Intensity

	Pct Entrepreneur	Pct Skilled Worker	Sensitivity to Earnings	Sensitivity to Fixed Assets
Intangible Intensity	0.0323^{*}	0.248^{***}		
(4 digit industry)	(1.69)	(3.82)		
Intangible Intensity			1.612^{**}	-0.387^{**}
(2 digit industry)			(2.30)	(-2.62)
R^2	0.024	0.092	0.085	0.106
Observations	116	147	59	60

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: This table reports the regression coefficients of intangible intensity in four regressions: regressing share of entrepreneurs on intangible intensity across 4-digit industries, regressing share of skilled workers on intangible intensity across 4-digit industries, regressing debt sensitivity to earnings on intangible intensity across 2-digit industries, and regressing debt sensitivity to fixed assets on intangible intensity across 2-digit industries.

3 A Two-Period Toy Model

Consider a two-period toy model to gain some intuition. An entrepreneur starts the first period with productivity z_1 and asset a_1. In the first period, he borrows at interest rate R to finance the purchase of capital k_1 and the salaries of skilled labor wh_1. The capital is used in production in the first period and fully depreciates afterwards:

$$y_1 = z_1 k_1^{\alpha}.$$

For simplicity the entrepreneur either hires skilled labor or doesn't, that is h_1 is either 0 or 1. If he does, the productivity increases from z_1 in period 1 to λz_1 in period two, where $\lambda > 1$. We interpret investing in skilled labor as investing in intangible capital which enhances the productivity of tomorrow. As the entrepreneur faces a borrowing constrained a la Buera et al. (2011), his no-default condition specifies that the benefit of default (i.e. keeping $1 - \phi$ fraction of the output this period and losing all his savings) has to be less than the benefit of repaying the debt):

$$(1-\phi)z_1k_1^\alpha \le z_1k_1^\alpha - R(k_1+wh_1) + Ra_1 \quad \Leftrightarrow \quad \phi z_1k_1^\alpha \ge R(k_1+wh_1) - Ra_1.$$

Suppose the borrowing and lending rates are the same and given by R. The constraint can be equivalently expressed as the benefit of repayment (i.e. the part of output that he can keep to himself relative to the case of default) outweighs the benefit of default (i.e. the value of the debt less the lost savings).

Once the investments in tangible and intangible capitals are made, he makes a consumption/saving decision to carry saving a_2 to period 2. In period 2, depending on if he chooses to invest in intangible or not, the productivity z_2 either remains at z_1 or increases to λz_1. He borrows tangible capital to produce subject to the no-default constraint and consumes all the output and wealth. Let the discount factor be β, we can write down the entrepreneur's problem as follows.

$$\begin{aligned}
&\max_{h_1\in\{0,1\},k_1\ge 0,a_2\ge 0,k_2\ge 0} u(c_1)+\beta u(c_2) \\
\text{s.t.}\quad & c_1+a_2 = z_1k_1^\alpha - R(k_1+wh_1)+Ra_1 \\
& \phi z_1k_1^\alpha \ge R(k_1+wh_1)-Ra_1 \\
& c_2 = z_2k_2^\alpha - Rk_2 + Ra_2 \\
& \phi z_2k_2^\alpha \ge Rk_2 - Ra_2.
\end{aligned}$$

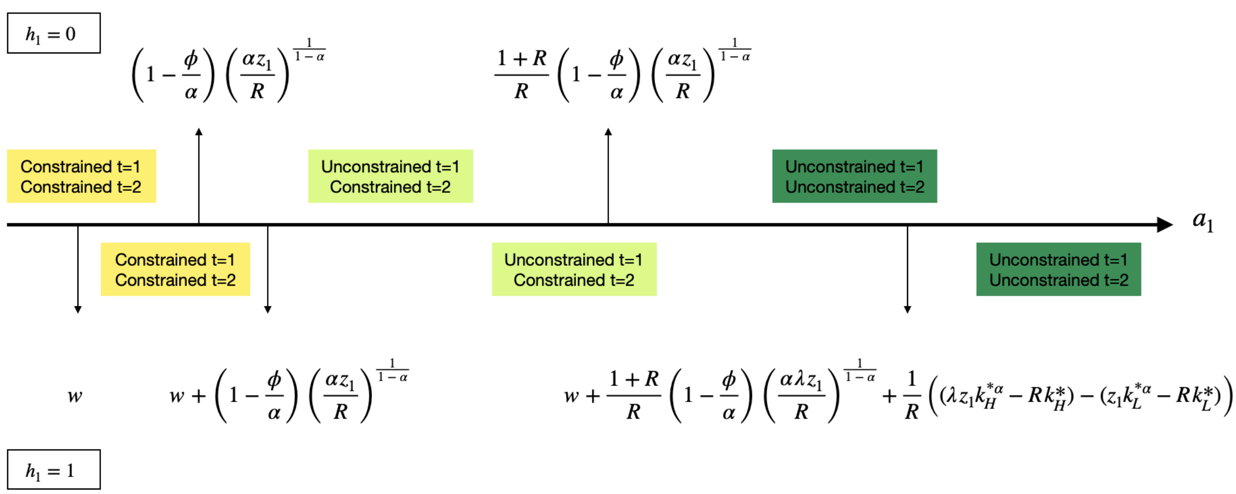
where $z_2 = z_1$ if $h_1 = 0$ and $z_2 = \lambda z_1$ if $h_1 = 1$. For simplicity, suppose $\beta R = 1$ and $u(c) = \log(c)$.

In this toy model, investing in intangible is equivalent to reducing the initial wealth by the cost of intangible investment w in exchange for an improvement in the productivity in period 2. We can solve the entrepreneur's problem conditioning on $h_1 = 0$ or $h_1 = 1$, as these two problems have a similar structure. Fix an initial productivity z_1. If the initial wealth is sufficiently small, then the entrepreneur is financially constrained in both periods. As the initial wealth becomes larger, the financial constraint in the first period becomes non-binding while that in the second period remains binding.[2] When the initial

[2] The result that first-period constraint becomes non-binding earlier than the second-period constraint depends on the assumptions on the utility function and $\beta R = 1$. In general, one of the two constraints becomes non-binding before wealth reaches a sufficiently high level that both constraints become non-binding.

wealth is sufficiently large, he is financially unconstrained in both periods, achieving the efficient size of capital: $k_L^* \equiv= \left(\frac{\alpha z_1}{R}\right)^{\frac{1}{1-\alpha}}$ and $k_H^* \equiv= \left(\frac{\alpha\lambda z_1}{R}\right)^{\frac{1}{1-\alpha}}$ depending on his productivity. Figure 3.1 shows the thresholds in terms of initial wealth a_1 that affect the force of the financial constraints for entrepreneurs who investing in intangible and for those who don't. Evidently, relative to not investing in intangibles (i.e. the scenario above the axis), investing in intangible (i.e. the scenario below the axis) makes the entrepreneur more likely to be financial constrained. This is both because he has a lower wealth after paying for the intangible investment in period one and because his second-period optimal size of capital is larger given a higher productivity level.

Figure 3.1: Binding Financial Constraints As a Function of Initial Wealth a_1, Toy Model



Note: This figure illustrates the conditions on the initial wealth a_1 under which an entrepreneur who does, or does not, invest in intangible is financially constrained in the two periods. Derivation of the toy model is available upon request.

By comparing the indirect utility between investing and not investing in intangible, the entrepreneur chooses h_1 optimally. Intuitively, for the entrepreneur to invest in intangible, the productivity improvement between periods λ must be large enough relative to the value of the cost of the investment, w.[3] We simulate the toy model for various initial (a_1, z_1), where $a_1 \geq w$ so investing in intangible is always feasible, and z is uniformly distributed. The basic patterns of selection into intangible investment and of financial con-

[3] The exact condition under which an unconstrained entrepreneur will choose to invest in intangible is given by $\left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left(\lambda^{\frac{1}{1-\alpha}}-1\right)z_1^{\frac{1}{1-\alpha}} \geq R^2 w$.

straints can be seen in Figure 3.2. The figures depict the selection into intangible investment and the condition of the financial constraints for entrepreneurs with varying a_1 on the x-axis and z_1 on the y-axis under different scenarios. The red curve divides the space into two: The northeastern area represent entrepreneurs with initially higher wealth and higher productivity and they optimally choose to invest in intangible. The southwestern area shows initial characteristics of entrepreneurs who do not invest in intangible. The pattern is intuitive. For a given initial wealth, the initial productivity must be high enough to rationalise the intangible investment, as the benefit of investment is scaled by the initial z_1 while the cost is the same. The yellow, light green and dark green regions illustrate the financial condition. The yellow region is occupied by entrepreneurs who are financial constrained in both periods; the light green by entrepreneurs who are constrained in the second period only; and the dark green by entrepreneurs unconstrained in both periods.

Panel (a) of Figure 3.2 describes an environment with relatively tight financial constraint, $\phi = 0.25$, and a constant $\lambda = 1.08$ meaning investing in intangible improves productivity by 8% in the next period. As we move in the direction of the southeast , financial constraints in generally are loosened. This is because, on one hand larger initial wealth (east) naturally relaxes constraints, and on the other those with lower initial productivity (south) tend not to invest in intangible and stay low in productivity in both periods. The non-monotone part of the contour of the financial conditions which coincides the red dividing line suggests that an entrepreneur who is indifferent between investing in intangible or not sees his financial constraints become more binding as he invests in intangible. For this particular range of initial conditions, almost all entrepreneurs who are always constrained (yellow region) do not invest in intangible, though the majority of those who are always unconstrained (dark green region) also do not invest in intangible. As the sensitivity of debt ($k_1 + wh_1$) to asset (a_1) is higher for more constrained entrepreneurs and zero for the unconstrained, the average sensitivity among entrepreneurs who do not invest in intangible then depends on their composition of the financial conditions. For this set of simulated entrepreneurs, when regressing the first-period debt to initial wealth (a_1) and first-period output (y_1) among those who do not invest in intangible, we get 0.37 and 1.27 as coefficients of the a_1 and y_1. When we do the same for entrepreneurs who invest in intangibles, we get 0.01 and 1.35. We are able to replicate the empirical pattern we find earlier that debt is more sensitive to earnings and less sensitive to collateralisable assets for more intangible intensive firms. But the discussion here should make it clear that to obtain the type of patterns of debt sensitivities that is observed in the data through the lens of this toy model puts much disciplines on what types of entrepreneurs select into intangible investment and under what financial condition. For example, the difference in the debt sensitivities between entrepreneurs who invest in intangible and those who don't is much smaller for Panel (b) in the same figure, where we allow the productivity gain parameter λ to also increase in z_1.

When we allow the initially more productive entrepreneurs to reap more gain from investing in intangible while keeping the average productivity improvement rate the same as in Panel (a) (Figure 3.2(b))[4], the selection into intangible investment changes to include those initially poor, financially constrained and yet highly productive entrepreneurs and to exclude the initially wealthy, unconstrained and yet less productive entrepreneurs. For the initially poor and more productive, the higher return from intangible investment relative to Panel (a) incentivises them to invest in intangible despite being financially constrained always. For the initially wealthy but relatively unproductive ones, the reduced benefit of investing in intangible works in the opposite way. In other words, the selection into investing intangibles is more discriminating on the basis of productivity rather than on the basis of wealth. The debt-to-asset sensitivities among entrepreneurs who invest and do not invest in intangibles are 0.23 and 0.3 respectively and the debt-to-earning sensitivities are 1.27 and 0.97 respectively. The debt sensitivities look much similar between the two groups of entrepreneurs, as the composition of financial conditions (yellow-light green-dark green) within the two groups become more similar.

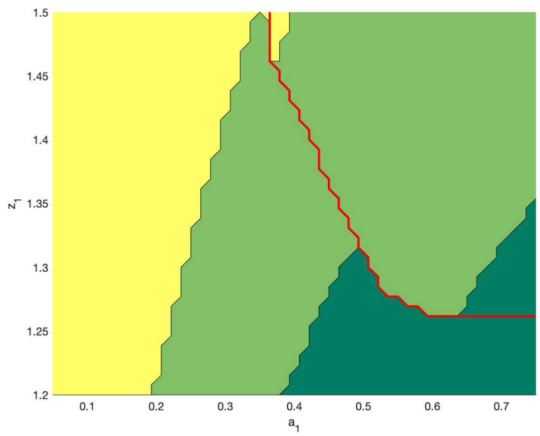
In Panel (c), we relax the borrowing constraint by increasing ϕ from 0.25 to 0.4 while keeping all other parameters the same as in Panel (a). In general, the yellow region shrinks and the dark green region grows, consistent with the relaxation of borrowing constraint. The red line also shifts towards the west, with both groups of entrepreneurs becoming less financial constrained. In Panel (d), we remove the constraint entirely by setting $\phi = 1$. Naturally no one is financially constrained any more. However even in this world we do not achieve efficiency. Notice the slight uptick of the red line for low initial wealth levels. Even though the initially very poor is not financially constrained and they can borrow enough to operate at the optimal capital scale given their productivity, the fact that the investment in intangible will only yield benefit in period 2 presents them with another hurdle. The fact that they cannot effectively borrow from future income means that the steep consumption profile under the intangible investment is undesirable. This

[4]The λ for the lowest z_1 is 1% and for the highest z_1 is 15%, which averages at 8% the same level as the constant λ in Figure 3.2(a).

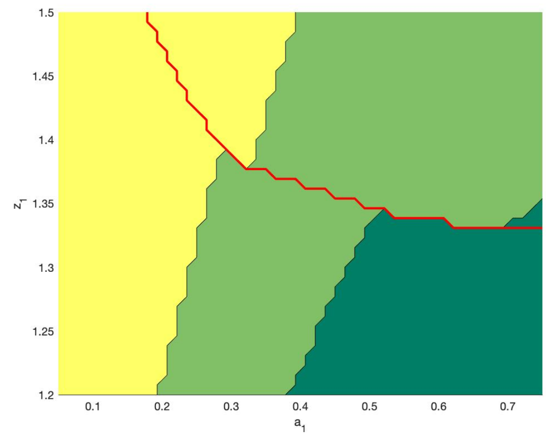
boils down to the fact that future income cannot be collateralised. If only current borrowing can also be based on expected future productivity, can we achieve the first best. As long as this is not feasible, the presence of intangible introduces another source of inefficiency, which tends to distort poor entrepreneurs who would benefit from intangible invest given his initial productivity towards under-investing in intangible. The relative size of this particular source of inefficiency will depend on the distribution of wealth relative to the cost of intangible investment.

In the next section, we present the full dynamic general equilibrium model with occupational choice, investment in tangible and intangible capitals under financial constraints and use it to calibrate to the US economy.

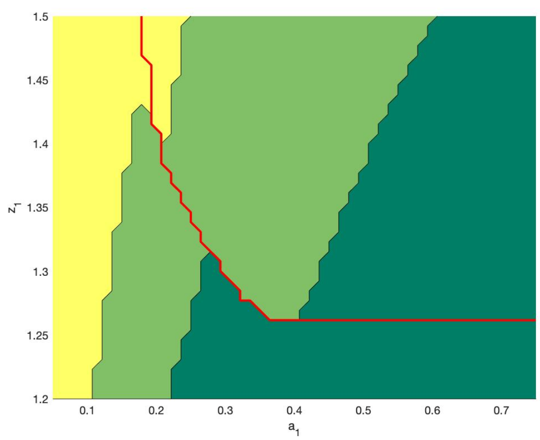
Figure 3.2: Selection into Intangible Investment and Financial Condition, Toy Model



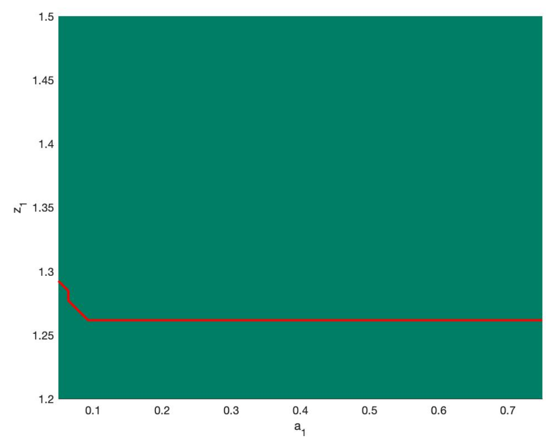
(a) Constant λ



(b) λ Increasing in z_1



(c) Higher ϕ



(d) $\phi = 1$

Note: This figure shows the characteristics of entrepreneurs heterogeneous in (a_1, z_1) that select into intangible investment (i.e. northeast to the red line) and their financial conditions in period 1 and 2 in the toy model. The yellow shaded area denotes an entrepreneur being financially constrained in both periods. The light green area denotes being constrained in period 2 only. The dark green area denotes being unconstrained in both periods.

4 Full Model

4.1 Household Sector

The economy is populated with a measure H of skilled households and a measure L of unskilled households, each endowed with 1 unit of labor. Skilled households can either become an entrepreneur or provide skilled labor for other entrepreneurs. Each skilled household is endowed with a business idea of quality z, which evolves stochastically in a way specified below and can be improved through an investment in intangible capital. Finally, unskilled households inelastically supply one unit of unskilled labor.

The preference of the households is represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^{it} u(c_t) \right\}, \quad i = H, L$$

where β^i is the discount factor for agents with skill level i. The period utility function satisfies the standard assumptions.

4.2 Production

There are two sectors of production: the entrepreneurial sector and the corporate sector.

4.2.1 Entrepreneurial Sector

An entrepreneur with an idea of quality z_n, where $z_n \in \{z_0, z_1, ..., z_N\}$, and asset a, $a \in \mathbf{R}_+$, can start a business. Each period, the quality of the idea evolves stochastically, but the entrepreneur can employ skilled labor to potentially improve upon his initial idea. By hiring h units of skilled labor, which we interpret as investing in intangible, the quality of his idea evolves according to the following transition matrix for productivity, for $0 < n < N$:

$$\begin{aligned}
Pr(z_0|z_n, h) &= \left(1 - \left(\frac{h}{m}\right)^{\gamma}\right) \tau; \\
Pr(z_n|z_n, h) &= \left(1 - \left(\frac{h}{m}\right)^{\gamma}\right) (1 - \tau) + \left(\frac{h}{m}\right)^{\gamma} (1 - \rho); \\
Pr(z_{n+1}|z_n, h) &= \left(\frac{h}{m}\right)^{\gamma} \rho.
\end{aligned}$$

Essentially, there are two lotteries. The first lottery is between z_0 and z_n, with the former state having probability τ and the latter $1-\tau$. The second lottery is between z_n and z_{n+1}, with the former state having probability $1-\rho$ and latter ρ. Investing in intangible can be thought of as varying the probabilities in a compound of the aforementioned two lotteries. A higher h increases the weight put on the second lottery. When $h=0$, only the first lottery is operative and with probability τ the state deteriorates to z_0, which we interpret as losing the business to the lowest quality at which in the equilibrium the majority chooses to be workers. When $h=1$, both lotteries are operative, with the weight on the first lottery being minimized to $\left(1-\left(\frac{1}{m}\right)^{\gamma}\right)$. For any number $m>1$, the worst state z_0 is realized with a minimum probability of $\left(1-\left(\frac{1}{m}\right)^{\gamma}\right)\tau$.

When $n=0$, the idea is of the lowest quality already and intangible investment can only increase the quality, then:

$$
\begin{aligned}
Pr(z_1|z_0,h) &= \left(\frac{h}{m}\right)^{\gamma}\rho; \\
Pr(z_0|z_0,h) &= 1-\left(\frac{h}{m}\right)^{\gamma}\rho.
\end{aligned}
$$

When $n=N$, i.e. the idea is of the highest quality possible but investing in h can still helps to decrease the probability of stepping down to z_0:

$$
\begin{aligned}
Pr(z_0|z_N,h) &= \left(1-\left(\frac{h}{m}\right)^{\gamma}\right)\tau; \\
Pr(z_N|z_N,h) &= 1-\left(1-\left(\frac{h}{m}\right)^{\gamma}\right)\tau.
\end{aligned}
$$

The entrepreneur produces with tangible capital at his ability level:

$$
y = zk^{\alpha}.
$$

At this point, we abstract away from hiring unskilled production labor in the entrepreneurial sector. Capital depreciates at rate δ. In order to operate a business, skilled households must pay an operational fixed cost per period, κ_H.

In making the intangible and tangible investments, the entrepreneur faces a financial friction. He needs to commit all investments up front before the production is completed. If his beginning-of-the-period asset a is not sufficient for the investments, he must borrow.

The borrowing constraint is such that even in the worst scenario, he prefers to repay the debt B. That is, repaying the debt is better than defaulting, in which case he can keep only a fraction $1-\phi$ of the output and undepreciated capital, and lose all other assets:

$$(1-\phi)\left(zk^{\alpha}+(1-\delta)k\right) \leq zk^{\alpha}+(1-\delta)k-RB-(1+r)\kappa_H+(1+r)a.$$

4.2.2 Corporate Sector

There is a corporate sector which employs all unskilled labor and capital that is not used in the entrepreneurial sector to produce the consumption good with a constant return Cobb Douglas production function:

$$Y = AK_c^{\eta}L_c^{1-\eta},$$

where A is total factor productivity and η is the capital share. The capital depreciates at the same rate δ as the capital used in the entrepreneurial sector.

4.3 Household's Problem

The timeline is as follows. At the beginning of a period, a skilled household with an initial entrepreneurial idea of quality z_n and asset a make the following decisions. First, he chooses his occupation: entrepreneur or skilled worker. If he chooses to be an entrepreneur, he then hires skilled workers to develop intangible capital h and invests in tangible capital k subject to the borrowing constraint. Production takes place, debt is repaid, and he makes consumption and saving decision. Then next period business idea, conditional on h, is realized. If, at the beginning of the period, he chooses to be a skilled worker, then he works for another entrepreneur in the idea development stage and receives his wage, w_H. He then makes consumption and saving decision. His idea deteriorates next period to z_0 with probability τ. Regardless of the skilled household's occupation, there is an exogenous probability of death for ideas, κ, and a new draw is made from the invariant distribution of business ideas. Finally, the unskilled household supplies one unit of labor inelastically to the corporate sector in exchange for w_L, and makes the consumption and saving decisions before entering the next period.

4.3.1 Skilled Households

Define the value function of a skilled household with a beginning-of-the-period quality of idea, z_n and asset a as $V_n(a)$. This value function is the maximum of two choice-specific value functions:

$$V_n(a) = \max\{V_n^e(a), V_n^w(a)\}, \quad n = 0, ..., N \tag{1}$$

where $V_n^e(a)$ is the value of this person choosing to be an entrepreneur this period and $V_n^w(a)$ is the value of this person choosing to be a skilled worker this period. Let the optimal occupational choice be denoted $o_n(a) \in \{e, w\}$. The value of being an entrepreneur is further given by the following. For $n = 0$,

$$\begin{aligned} V_0^e(a) = \max_{c,h,k,a'} u(c) + \beta \Bigg((1-\kappa) \Bigg[\left(1 - \left(\frac{h}{m}\right)^{\gamma} \rho \right) V_0(a') \\ + \left(\frac{h}{m}\right)^{\gamma} \rho V_1(a') \Bigg] + \kappa \, \mathbb{E}_{z'} \, V_{z'}(a') \Bigg) \end{aligned} \tag{2}$$

$$\begin{aligned} \text{s.t.} \quad & c + a' = z_0 k^{\alpha} - R(k + w_H h) + (1-\delta)k - (1+r)\kappa_H + (1+r)a \\ & \phi \left(z_0 k^{\alpha} + (1-\delta)k\right) \geq R(k + w_H h) + (1+r)\kappa_H - (1+r)a. \end{aligned}$$

For $n = 1, ..., N-1$:

$$\begin{aligned} V_n^e(a) = \max_{c,h,k,a'} u(c) + \beta \Bigg((1-\kappa) \Bigg[\left(1 - \left(\frac{h}{m}\right)^{\gamma} \right) \tau V_0(a') \\ + \left[\left(1 - \left(\frac{h}{m}\right)^{\gamma} \right) (1-\tau) + \left(\frac{h}{m}\right)^{\gamma} (1-\rho) \right] V_n(a') \\ + \left(\frac{h}{m}\right)^{\gamma} \rho V_{n+1}(a') \Bigg] + \kappa \, \mathbb{E}_{z'} \, V_{z'}(a') \Bigg) \end{aligned} \tag{3}$$

$$\begin{aligned} \text{s.t.} \quad & c + a' = z_n k^{\alpha} - R(k + w_H h) + (1-\delta)k - (1+r)\kappa_H + (1+r)a \\ & \phi \left(z_n k^{\alpha} + (1-\delta)k\right) \geq R(k + w_H h) + (1+r)\kappa_H - (1+r)a. \end{aligned}$$

For $n = N$, the value function takes the following form subject to the same borrowing constraint as above:

$$
\begin{aligned}
V_N^e(a) = \max_{c,h,k,a'} u(c) + \beta \Bigg((1-\kappa) \Bigg[\left(1 - \left(\frac{h}{m}\right)^{\gamma}\right) \tau V_0(a') \\
+ \left[1 - \left(1 - \left(\frac{h}{m}\right)^{\gamma}\right) \tau \right] V_N(a') \Bigg] + \kappa \, \mathbb{E}_{z'} \, V_{z'}(a') \Bigg) \qquad (4) \\
\text{s.t.} \quad c + a' = z_N k^{\alpha} - R(k + w_H h) + (1-\delta)k - (1+r)\kappa_H + (1+r)a \\
\phi \left(z_N k^{\alpha} + (1-\delta)k\right) \geq R(k + w_H h) + (1+r)\kappa_H - (1+r)a.
\end{aligned}
$$

The value of being a skilled worker is given by, for $n = 0, ..., N$:

$$
\begin{aligned}
V_n^w(a) = \max_{c,a'} u(c) + \beta \left((1-\kappa)(\tau V_{n-1}(a') + (1-\tau)V_n(a')) + \kappa \, \mathbb{E}_{z'} \, V_{z'}(a')\right) \qquad (5) \\
\text{s.t.} \quad c + a' = w_H + (1+r)a.
\end{aligned}
$$

The optimal consumption policy function is $c_n^H(a)$ and savings $a_n'^H(a)$. The entrepreneurs' problem will generate investment policy functions that depend on the state (n, a): $h_n(a)$ and $k_n(a)$.

4.3.2 Unskilled Households

There is a measure L of homogenous unskilled households, endowed with a_0. They receive wage w_L from the corporate sector and make consumption and saving decisions. Let the value function be $W(a)$:

$$
\begin{aligned}
W(a) = \max_{c,a'} u(c) + \beta^L W(a') \qquad (6) \\
\text{s.t.} \quad c + a' = w_L + (1+r)a.
\end{aligned}
$$

The optimal consumption policy function is $c^L(a)$ and savings $a'^L(a)$. We shut down idiosyncratic labor income risk among unskilled households, such that their discount factor will determine the interest rate in equilibrium.

4.4 Stationary Equilibrium

A stationary equilibrium is one in which prices, w_H, w_L, and r, are constant, all households optimize, markets clear, and the distribution over states is stationary. More formally, the individual state variables are (n, a) for skilled households and a for unskilled households, and the aggregate states are the distribution over individual states $\Phi(n, a)$ for skilled households and $\Psi(a)$ for unskilled households. Abusing notation, let N represent the set $\{0, 1, ..., N\}$ and $\mathcal{P}(N)$ be the power set of N. Let $A = \mathbf{R}_+$ and $\mathcal{B}(A)$ be the Borel σ-algebra of A. Let $S \equiv N \times A$ and $\mathcal{B}(S) = \mathcal{P}(N) \times \mathcal{B}(A)$ and $\mathcal{M}$ is the set of all probability measures on the measurable space $M = (S, \mathcal{B}(S))$. Let $\mathcal{Q}$ be the set of all probability measures on the measurable space $L = (A, \mathcal{B}(A))$. Define $H : \mathcal{M} \times \mathcal{Q} \to \mathcal{M} \times \mathcal{Q}$ to be the aggregate law of motion, which describes how the aggregate state (Φ, Ψ) evolve to the next period (Φ', Ψ').

Definition 1. *A steady state recursive competitive equilibrium for this economy consists of: (a) Value functions $V_n(a)$, $V_n^e(a)$, $V_n^w(a)$, and $W(a)$, and policy functions, $o_n(a)$, $h_n(a)$, $k_n(a)$, $c_n^E(a)$, $a_n'^E(a)$, $c_n^E(a,h,k)$, $a_n'^E(a,h,k)$, $c_n^H(a)$, $a_n'^H(a)$, $c^L(a)$ and $a'^L(a)$; (b) Interest rates, r and $R = 1 + r$, and wage rates, w_H and w_L; (c) Capital and unskilled labor demand from the corporate sector, K_c and L_c; and capital and skilled labor demand from the entrepreneurial sector, K_e and H_e; (d) A function $M(\Phi, \Psi)$ mapping the skilled households' distribution over states Φ and the unskilled households' distribution over states Ψ into the next period distributions, and their invariant distributions, Φ^* and Ψ^*, such that (a) The policy function $o_n(a)$ solves (1); $c_n^E(a)$, $a_n'^E(a)$, $h_n(a)$, $k_n(a)$, $c_n^H(a)$ and $a_n'^H(a)$ solve (2)-(5); and $c^L(a)$ and $a'^L(a)$ solve (6), all taking prices and borrowing constraints as given. (b) Prices are competitive and functions of the aggregate state (Φ, Ψ). (c) Capital, skilled labor and unskilled labor markets clear:*

$$
\begin{aligned}
&K_e + K_c = \sum_{n=0}^{N} \int \mathbf{1}_{o_n(a)=e} k_n(a) d\Phi(n,a) + K_c = \sum_{n=0}^{N} \int a d\Phi(n,a) + \int a d\Psi(a); \\
&\sum_{n=1}^{N} \int \mathbf{1}_{o_n(a)=e} h_n(a) d\Phi(n,a) + \sum_{n=0}^{N} \int \mathbf{1}_{o_n(a)=e} d\Phi(n,a) = H; \\
&L_c = \int d\Psi(a) = L.
\end{aligned}
$$

(d) The distributions Φ^* and Ψ^* are fixed points of the mapping M.

The aggregate law of motion M is generated by both choices of occupation, intangible and tangible investments, and consumption as well as the exogenous shock to ideas κ. Define the transition function $P_\Phi((n,a),(N,A)) : S \times \mathcal{B}(S) \to [0,1]$ as the probability that a skilled household with current state (n,a) ends up with $n' \in N$ and $a' \in A$ in the next period. Define the transition function $P_\Psi(a, A) : A \times \mathcal{B}(A) \to [0,1]$ as the probability that an unskilled household with current asset a ends up with $a' \in A$ in the next period.

$$\left(\Phi'(N,A), \Psi'(\tilde{A})\right) = M(\Phi(N,A), \Psi(\tilde{A})) = \left(\int P_{\Phi}((n,a),(N,A))d\Phi(n,a), \int P_{\Psi}(a,\tilde{A})d\Psi(a)\right).$$

The steady state recursive competitive equilibrium satisfies

$$\left(\Phi^*(N,A), \Psi^*(\tilde{A})\right) = M(\Phi^*(N,A), \Psi^*(\tilde{A})) = \left(\int P_{\Phi}((n,a),(N,A))d\Phi^*(n,a), \int P_{\Psi}(a,\tilde{A})d\Psi^*(a)\right).$$

The interest rate is given by β^L: $r = \frac{1}{\beta^L} - 1$, which together with L pins down K_c and w_L.

4.5 Algorithm

Initialization. For a given β^L discount factor of the low-skill, compute r.

Value Functions. Guess a w_H. Given w_H and r, compute the optimal capital, $k_n^*(a,h)$, as a function of a, h, and z_n.

1. Guess a value function on grids for assets and ideas $V_n(a)$.
2. Given $V_n(a)$, Golden search on a' to compute $\hat{v}_n(a,h)$, and derive policies $\hat{a'}_n^e(a,h)$ and $\hat{c}_n^e(a,h)$.
3. Given $\hat{v}_n(a,h)$, grid search on h to compute $V_n^e(a)$, and derive policies ${a'}_n^H(a)$, $c_n^H(a)$, $h_n(a)$ and $k_n(a)$.
4. Given $V_n(a)$, Golden search on a' to compute $V_n^w(a)$, and derive policies ${a'}_n^H(a)$ and $c_n^H(a)$.
5. Given $V_n^w(a)$ and $V_n^e(a)$, find the optimal occupational choice, and update $V_n(a)$. For computational reasons, we add a preference shock on the occupational choice. The shock follows a Gumbel distribution with zero mean and small variance.

Iterate until it converges.

Stationary measure. We assume that with probability κ entrepreneurs make a new draw from a Pareto distribution of abilities with shape parameter, ν. Households decisions on ${a'}_n^H(a)$, ${a'}_n^H(a)$, $c_n^H(a)$, $c_n^H(a)$, $h_n(a)$ and $k_n(a)$, together with κ and ν imply the existence of a stationary distribution over individual states, $\Phi^*(n,a)$.

Aggregation.

1. Given w_H, check that skilled labor market clears. If demand is larger than supply, increase w_H, or reverse. Iterate over w_H to clear the skilled labor market.

2. Given a proportion of capital employed in the corporate sector, $\frac{K_c}{K}$, obtain K_c. Find the savings a^L of unskilled agents (assuming a degenerate distribution) such that asset markets clear.

3. Given a proportion of unskilled workers in the economy ($L = L_c$), obtain the capital-to-labor ratio in the corporate sector, $\frac{K_c}{L_c}$. Given r, obtain total factor productivity, A, from the first order condition for capital in the corporate sector.

4. Given K_c, L_c, and A, compute production in the corporate sector, Y_c.

5. From the first order condition for labor in the corporate sector, obtain the wage for unskilled workers, w_L.

5 Quantitative Results

5.1 Calibration

We calibrate the model to the US economy. We set some parameter values to target key macro aggregates and borrow others from the literature as shown in Table 5.1.

Table 5.1: Calibration Results: Parameter Values

Parameter	Description	Source/Target Moments	Value
	Panel (a): Externally calibrated		
σ	CRRA coefficient	Buera et al. (2011)	1.500
α	Production function parameter	Allub and Erosa (2019)	0.406
δ	Depreciation rate	Allub and Erosa (2019)	0.060
ν	Shape Pareto distribution	Buera et al. (2011)	4.840
κ	New skill draw prob.	Allub and Erosa (2019)	0.041
α_c	Corporate capital income share	Quadrini (2000)	0.330
L_c	% of unskilled workers	Barro and Lee (2013)	0.700
m	Fail weight Endogenous prob.		1.500
ρ	Up weight Endogenous prob.		0.500
	Panel (b): Internally calibrated		
β	High skilled discount factor	Capital-output ratio	0.926
β^L	Low skilled discount factor	Interest rate (r)	0.962
ϕ	Tightness of borrowing constraint	Credit-to-GDP ratio	0.220
γ	Curvature Endogenous prob.	Intangible-to-tangible (H_e/K)	0.750
τ	Down weight Endogenous prob.	Firm exit rate	0.300
κ_H	Operational fixed cost	Mass of entrepreneurs	0.400
A	TFP in the corporate sector	% of K in corporations	0.697

We need to calibrate four sets of parameters. First, preferences (β,β_L,σ); second, the production function and parameters governing the borrowing in the entrepreneurial sector (α,γ,m,τ,ρ,κ_H,δ,ϕ); third, the exogenous process for managerial abilities (ν,κ); and fourth, production in the corporate sector (α_c,A,L_c).

Regarding the first set of parameters, we calibrate the discount factor of the unskilled households, β_L, to match an annual interest rate of 4 percent in the US economy as in Buera et al. (2011), and the discount factor of the skilled households, β_H, to match a capital-over-output ratio of about 3. The constant relative risk aversion parameter is set equal to $\sigma = 1.5$, a standard value in the literature of occupation choice (see for example Buera et al. (2011)).

In the entrepreneurial sector, the use of tangible capital involves the calibration of the share of capital in the production function, α, and its depreciation rate, δ, for which we follow Allub and Erosa (2019) and set $\alpha = 0.406$ and $\delta = 0.06$. With respect to the investment in intangible, we need to calibrate the parameters governing the endogenous transition matrix for productivity. We set γ, the parameter that pins down the curvature of the probability function with respect to the intangible investment, such that the model matches the intangible-to-tangible ratio, H_e/K, in the US economy (0.4). We choose τ, which controls the effectiveness of the intangible investment to avoid falling into the worst business idea, to match the establishment exit rate[5] in the US economy, which is equal to 10% as documented in Buera et al. (2011). We set ρ, which governs the effectiveness of the intangible investment to success in improving the business idea, to $\rho = 0.5$. We choose $m = 1.5$ such that it satisfies the restriction $m > 1$ such that there is always a positive probability, conditional on $\tau > 0$ and $\rho > 0$, of improving or worsening the quality of the business idea even if the intangible investment equals zero or one. We calibrate the operational fixed cost, κ_H, to match the mass of entrepreneurs in the US economy which, as reported by the International Labor Organization (2015), equals 10%. Regarding the borrowing constraint parameter, ϕ, we set it to target the credit-to-GDP in the US economy which correspond to the proportion of credit in the non corporate sector as reported in the Financial Stability Report (2019) and equal to 0.263.

We follow Buera et al. (2011) and assume that entrepreneurial abilities follow a Pareto distribution with the shape parameter value, $\nu = 4.84$. We borrow the parameter $\kappa = 0.041$ from Allub and Erosa (2019).

Regarding the corporate sector, we fix the TFP parameter in the production function, A, to match the proportion of capital absorbed by this sector in the US economy which, as reported by Quadrini (2000), equals 60%. We follow Quadrini (2000) and set the same capital share in the corporate sector, $\alpha_c = 0.33$. Finally, we endogenously fix the amount of labor in the corporate sector, L_c, to be equal to the proportion of unskilled workers in the US economy and set $L_c = 0.7$.

Results are provided in Table 5.1, which reports calibrated parameter values, and in Table 5.2, which compares model with data for targeted moments.

[5] We define the exit rate as the proportion of entrepreneurs which do not survive and have to make a new draw plus the proportion of those who drop to the worst business idea.

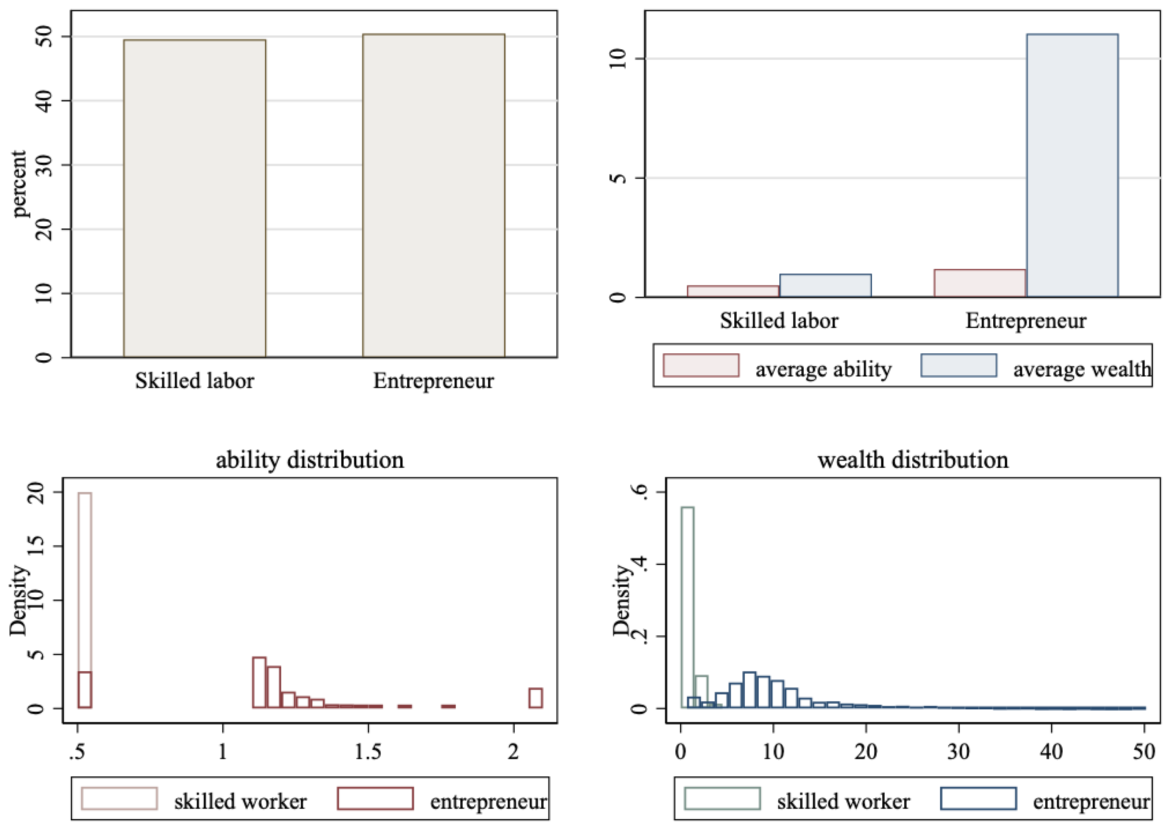
Table 5.2: Calibration Results: Targeted Moments

Moments	Source	Data	Model
Capital-output ratio	Cooley et al. (1995)	3.320	3.328
Credit-to-GDP ratio	Financial Stability Report (May, 2019)	0.263	0.189
Intangible-to-tangible ratio	Compustat	0.400	0.355
Firm exit rate	Buera et al. (2011)	0.100	0.111
Interest rate (r)	Buera et al. (2011)	0.040	0.040
Mass of entrepreneurs	International Labor Organization (2015)	0.100	0.153
% of K in corporations	Quadrini (2000)	0.600	0.600

5.2 Model Performance

Let's first examine the model performance on the occupational choice. Figure 5.1 shows the difference in terms of ability and wealth between skilled agents who select into employment and those who select into entrepreneurship. In the calibrated model, 15.8% of the population are entrepreneurs, who make up roughly half of the skilled population (the top left panel of the figure). This translates to about 32% of the population being skilled, which is close to the 35% of US population aged 25 and over having a bachelor's degree and above, as reported in the most recent CPS. The skilled agents who select into entrepreneurship are clearly wealthier and have better business ideas (the top right panel of the figure). In the model, those with the lowest ability and with little wealth select into employment (the bottom panels). Among the entrepreneurs, we have a dispersion in terms of ability and wealth.

Figure 5.1: Ability and Wealth of Skilled Labor and Entrepreneurs, Calibrated Model



Note: This figure shows the share of skilled labor and the complementary share of entrepreneurs, the average ability and wealth of the two groups and the distributions of ability and of wealth for each group.

Next we turn to the entrepreneurs and in particular their financial condition. Panel (a) of Figure 5.2 shows the average wealth of entrepreneurs by ability decile. Note that because the lowest decile and the second lowest decile account for 17% and 14% of entrepreneurs respectively, the third decile in the ability distribution is skipped. Panel (b) of the same figure shows the average ability of entrepreneurs by wealth decile. Among the entrepreneurs, 65% of them are financially constrained (see the red dashed line in Panel (c) of Figure 5.2). Plotting the percentage of constrained entrepreneurs by ability and wealth decide, we confirm the broad tendency that the instance of binding financial constraint increases in ability and decreases in wealth. Note that no entrepreneur in the lowest ability decile is constrained, which implies that relative to their ability level their wealth is sufficient to overcome the financial constraint. This can be confirmed by Panel (a) since the average wealth for the lowest ability group is higher than those for the next seven groups. The lower percentage of constrained entrepreneurs among the highest ability group is consistent with the fact that the average wealth of the highest ability group is the highest.

Figure 5.2: Ability, Wealth, and Financial Condition of Entrepreneurs, Calibrated Model

(a) Average wealth by ability decile

(b) Average ability by wealth decile

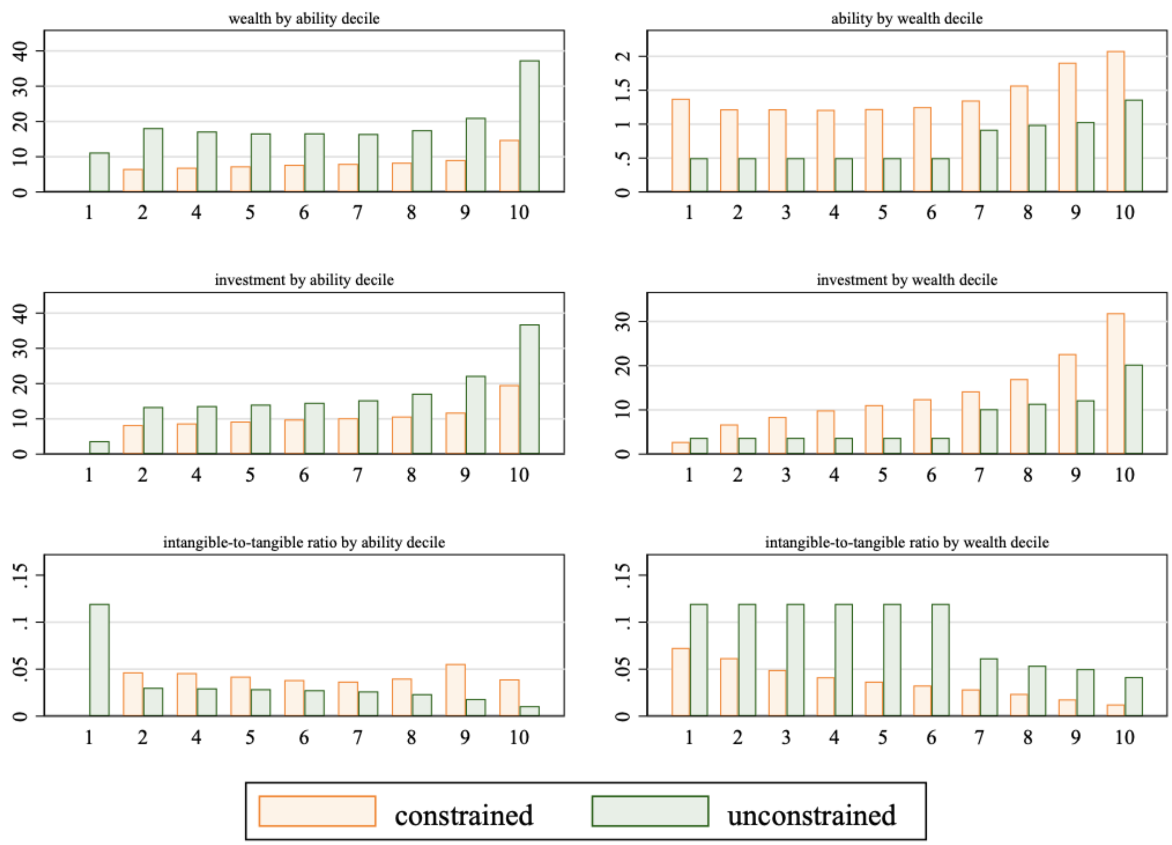
(c) Percent constrained by ability and wealth decile

Note: Panel (a) of the figure shows the average wealth by ability decile among entrepreneurs. Panel (b) shows the average ability by wealth decile among entrepreneurs. Panel (c) shows the fraction of constrained entrepreneurs by ability and wealth deciles. The red dashed line in Panel (c) shows the unconditional percentage of constrained entrepreneurs.

Now let's compare the ability, wealth, level and composition of investment between constrained and unconstrained entrepreneurs (Figure 5.3). Naturally, given ability, unconstrained entrepreneurs are wealthier than the constrained; given wealth, constrained entrepreneurs have higher ability than the unconstrained (the first row of Figure 5.3). Across the ability deciles (the left column of the figure), investment increases in ability and investment by unconstrained entrepreneurs is higher than by constrained entrepreneurs at each ability level. The intangible to tangible capital ratio, on the other hand, declines with ability for the unconstrained, due to the diminishing option value of climbing a step higher on the ability ladder. Notice the high intangible-to-tangible ratio for the lowest ability group, all of which is unconstrained. These skilled agents are keen to invest in intangible to get out of the lowest ability next period and get on the business expansion ladder for the entrepreneurs. Relative to the unconstrained, the constrained entrepreneurs' investment portfolio is more intangible intensive for all ability levels except the lowest one.

Across the wealth decile (the right column of Figure 5.3), investment is roughly increasing in wealth for both constrained and unconstrained entrepreneurs. The unconstrained however invest more heavily in intangible capital relative to tangible than the constrained at every wealth level. Since at every wealth level, the unconstrained has lower ability, their incentive to climb up the quality ladder is more intense than the constrained.

Figure 5.3: Investment and Intangible-to-Tangible Ratio by Ability and Wealth Decile, Calibrated Model



Note: This figure shows the average investment ($w_H h + k$) and the average intangible-to-tangible ratio ($w_H h/k$) by ability and wealth decile.

Using simulated data from the calibrated model, we regress the entrepreneur's debt $b_t = k_t + w_H h_t$ on contemporaneous wealth a_t and earnings y_t, interacted with intangible investment h_t. The results are reported in Table 5.3. Assets correlate always positively with debt, suggesting the presence of an asset-based borrowing constraint. Intangible enters the equation in more subtle ways. Without the interaction effects (column [3]), both earnings and intangibles correlate positively with debt. However after including the interactions (column [4]), the positive correlation between earnings and debt appears to be driven by the strong interaction effect of earnings and intangibles. In other words, investing in intangible appears to increase significantly the sensitivity of debt to earnings. These results suggest that the model has the capacity to generate similar patterns of debt sensitivities we observe from the data.

Table 5.3: Debt Sensitivity to Assets and Earnings, Simulated Data

	[1]	[2]	[3]	[4]
Assets	0.674***	0.239***	0.225***	0.921***
	(225.30)	(182.08)	(171.06)	(10.27)
Earnings		3.184***	3.168***	-0.887***
		(577.36)	(587.17)	(-3.96)
Intangibles			2.170***	-2.459***
			(49.31)	(-10.85)
Assets × Intangibles				-0.698***
				(-7.78)
Earnings × Intangibles				4.058***
				(18.13)
R^2	0.504	0.935	0.938	0.939
Observations	98480	98480	98480	98480

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: This table reports the regression of debt on assets, earnings, intangible investment and their interactions, using the simulated data from the calibrated model.

5.3 Counterfactual

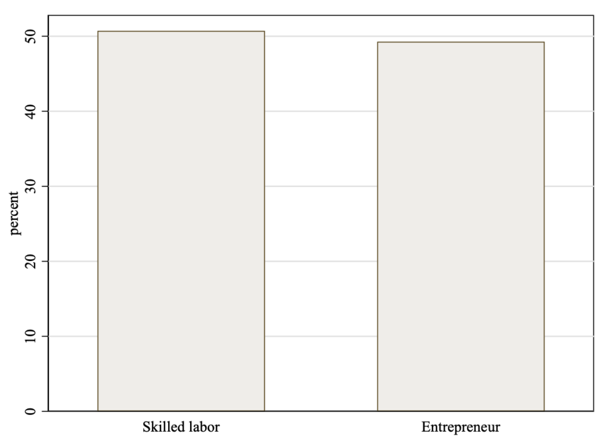
In this section, we conduct two counterfactual exercises. We start by examining the perfect credit market case and evaluate its contribution to aggregate output. Then we establish the first best and interpret the difference between the perfect credit market case and the first best as stemming from the lack of pledgibility of intangible capital.

5.3.1 Perfect Credit Market

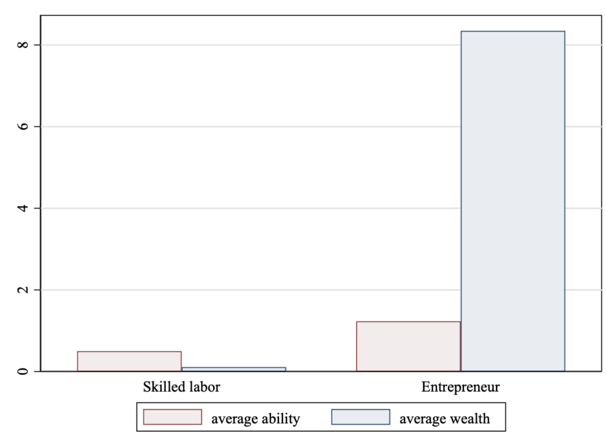
We set the tightness of borrowing constraint ϕ to 1, keeping other parameters as in the benchmark and solve for the counterfactual equilibrium under a perfect credit market. The aggregate output under the perfect credit market is about 40% higher than under the benchmark; the tangible capital to output ratio is 7.2% higher than under the benchmark; and the intangible-to-tangible capital ratio in the entrepreneurial sector is 16.3% lower than that in the benchmark. Having perfect credit market certainly helps the accumulation of tangible capital and therefore increases aggregate output. However it does not remove the financing friction specific to intangible capital, which cannot be collateralised.

Among the skilled agents, 80% take up entrepreneurship whereas 20% select into employment under perfect credit market (Figure 5.4). The wealth gap between entrepreneurs and skilled workers is 8.4 : 0.55, which is wider than that in the benchmark, 11 : 1. The ability gap between entrepreneurs and skilled workers, 1.23 : 0.5, is also slightly wider than that in the benchmark, 1.19 : 0.5.

Figure 5.4: Ability and Wealth of Skilled Labor and Entrepreneurs, Perfect Credit Market Counterfactual



(a) Percent of skill labor and entrepreneur

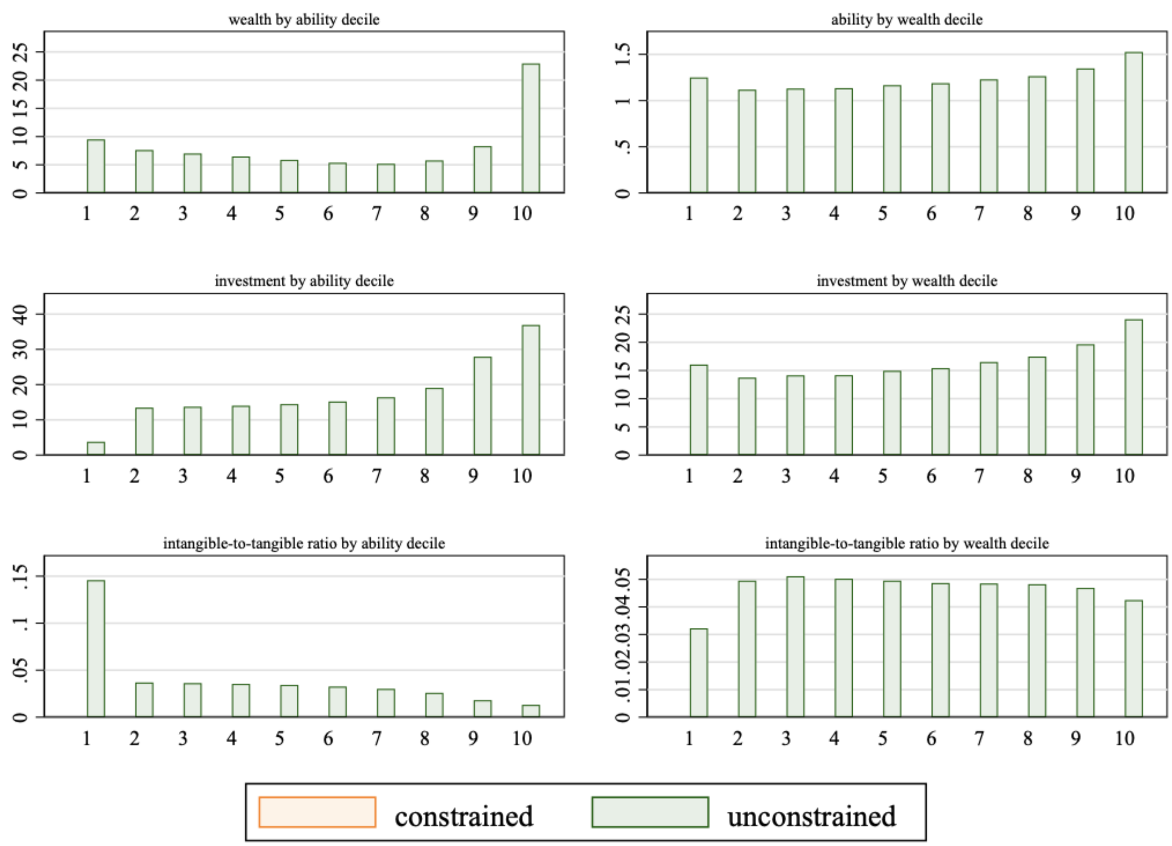


(b) Average ability and wealth

Note: This figure shows the share of skilled labor and the complementary share of entrepreneurs, the average ability and wealth of the two groups, in the counterfactual of perfect credit market.

In the case of perfect credit market, no agent is financially constrained and one might expect that the investment decision together with its composition should only depend on ability and no longer depend on wealth. Figure 5.5 illustrates the opposite. While across ability levels, investment increases and intangible intensity decreases as expected, across wealth levels, it is noticeable that the investment is skewed towards tangible capital for the lowest wealth group. This result echoes the observation made in Section 3 that even with perfect credit market, the nature of intangible capital skewed the investment towards tangible investment for those with lower wealth. In the next section, we will verify and quantify this source of distortion by computing the first best allocation of the economy.

Figure 5.5: Investment and Intangible-to-Tangible Ratio by Ability and Wealth Decile, Perfect Credit Market Counterfactual



Note: This figure shows the average investment ($w_H h + k$) and the average intangible-to-tangible ratio ($w_H h/k$) by ability and wealth decile, in the counterfactual of perfect credit market.

5.3.2 First Best

Work in progress...

6 Conclusion

TBA

References

Allub, Lian and Andrés Erosa, "Financial Frictions, Occupational Choice and Economic Inequality," *Journal of Monetary Economics*, 2019, *107* (C), 63–76.

Barro, Robert J and Jong Wha Lee, "A New Data Set of Educational Attainment in the World, 1950–2010," *Journal of Development Economics*, 2013, *104*, 184–198.

Bhandari, Anmol and Ellen R. McGrattan, "Sweat Equity in U.S. Private Business," 2021, *136* (2), 727–781.

Buera, Francisco J and Yongseok Shin, "Financial frictions and the persistence of history: A quantitative exploration," *Journal of Political Economy*, 2013, *121* (2), 221–272.

Buera, Francisco, Joseph P Kaboski, and Yongseok Shin, "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 2011, *101* (5), 1964–2002.

Cooley, Thomas F, Edward C Prescott et al., "Economic growth and business cycles," *Frontiers of business cycle research*, 1995, *1*, 38.

Corrado, Carol, Charles Hulten, and Daniel Sichel, "Measuring Capital and Technology: An Expanded Framework," in "Measuring Capital in the New Economy," University of Chicago Press, 2005, pp. 11–46.

_ , _ , and _ , "Intangible Capital and US Economic Growth," *Review of Income and Wealth*, 2009, *55* (3), 661–685.

Eisfeldt, Andrea L., Antonio Falato, and Mindy Z. Xiaolan, "Human Capitalists," 2021. NBER Working Papers 28815.

Farhi, Emmanuel and Francois Gourio, "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia," *Brookings Papers on Economic Activity Fall*, 2018, pp. 147–250.

Haskel, Jonathan and Stian Westlake, *Capitalism without Capital: the Rise of the Intangible Economy*, Princeton University Press, 2017.

Koh, Donya, Raul Santaeulalia-Llopis, and Yu Zheng, "Labor Share Decline and Intellectual Property Products Capital," *Econometrica*, 2020, *88* (6), 2609–2628.

Li, Ye, "Fragile New Economy: Intangible Capital, Corporate Savings Glut, and Financial Instability," 2020. Fisher College of Business Working Paper Series.

Lian, Chen and Yueran Ma, "Anatomy of Corporate Borrowing Constraints," *Quarterly Journal of Economics*, 2021, *136* (1), 229–291.

McGrattan, Ellen R. and Edward C. Prescott, "Unmeasured Investment and the Puzzling U.S. Boom in the 1990s," *American Economic Journal: Macroeconomics*, 2010, *2* (4), 88–123.

_ **and** _ , "A Reassessment of Real Business Cycle Theory," *American Economic Review Papers and Proceedings*, 2014, *104* (5), 177–182.

Midrigan, Virgiliu and Daniel Yi Xu, "Finance and misallocation: Evidence from plant-level data," *American economic review*, 2014, *104* (2), 422–58.

Quadrini, Vincenzo, "Entrepreneurship, Saving and Social Mobility," *Review of Economic Dynamics*, January 2000, *3* (1), 1–40.

Sun, Qi and Mindy Z. Xiaolan, "Financing Intangible Capital," *Journal of Financial Economics*, 2019, *133* (3), 564–588.