Resolving Estimation Ambiguity

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Motivation

Theory versus practice

I. Models offer **conceptual frameworks**, but theory rarely prescribes a specific approach to **estimate them**

Estimation ambiguity:

- Agents know that **different approaches** exist to estimate parameters' *true* value, but they don't know their correct weights

- **Empirical challenges → need to observe**
  I. Selected model
  II. Inputs and assumptions
  III. Estimation methods
  IV. Outcomes

Our paper: how do agents select parameter inputs and estimation methods and how do these choices affect estimation outcomes?

Empirical Setting

- **Estimating a key driver of fin decisions: a firm's required rate of return**

I. **46,000 equity reports** from 2000-2023:

- 4,261 firms
- 4,566 analysts
- 63 countries

II. Focus on **pairs** of analysts for each **firm-date**

Empirical Setting

- **Observe discount rate data**
  I. WACC
  II. WACC inputs

DCF Model - Aixtron

Figures in EUR m	2011e	2012e	2013e	2014e
Sales				
Change				
EBIT				
EBIT-Margin				
Tax rate				
NOPAT				
Depreciation				
in % of Sales				
Change in Liquidity from				
- Working Capital				
- Capex				
Capex in % of Sales				
Other				
Free Cash Flow (WACC-Model)				

Model parameter

Debt ratio		Beta	
Costs of Debt		WACC	
Market return			
Risk free rate		Terminal Growth	

Empirical Setting

- Insights into decision-making:

→ **qualitative discussion** of assumptions, sources, and methods

- **Morgan Stanley, CLF.N, 2008-02-26**: “We assume an 8.3% WACC, based on a 4.8% after-tax cost of debt, a **60-month beta of 1.3**”.

- **HSBC-JBSS3.SA, JBS SA, 2021-03-26**: We assume a beta of 0.8 (based on the **average** of the Bloomberg **2-year daily** beta, **5-year weekly** beta, and **10-year weekly** beta; unchanged)

WACC and CAPM

- **Focus on one class of models: Discount rate and CAPM**

$$\mathbf{WACC} = \mathbf{W_E}\,\underbrace{(\mathbf{r_f} + \boldsymbol{\beta} * \mathbf{ERP})}_{\text{CAPM}} + \mathbf{W_D}\,(\mathbf{1} - \boldsymbol{\tau})\mathbf{r_D}$$

- **CAPM is conceptually simple and common**

  I. **97%** of sell-side equity analysts use the CAPM

  II. Theory offers little guidance for estimation

- **Discount rate → a key parameter in valuation**

  I. Small ΔWACC → large ΔPrice Target

  II. Unbiased predictor of firm future 1-year return, <u>on average</u>

  III. Pivotal for investment decisions

CAPM and Estimation Ambiguity

$$r_E = \boxed{r_f} + \boxed{\beta} * ERP$$

Observed variable

Estimated variable

- Trailing horizon of returns
- Market proxy
- Frequency of returns

- **The risk-free rate is a natural benchmark:**

  I. Beta subject to greater estimation ambiguity

  II. Jointly observe both variables for each cost of equity calculation

CAPM and Estimation Ambiguity

$$r_E = r_f + \boxed{\beta} * ERP$$

Model Version: 1 2 3 4 5 6 7 8 9 10 [...]

P(H=1)= ? P(H={2,3,5})= ? P(H=10)= ?

- **Beta horizon as a source of estimation ambiguity**

  I. **Theory**: limited guidance
  - **Analysts faced with a variety of feasible options**

  II. **Practice reason**: Analysts discuss their beta horizon in reports

  III. **Economic importance**: horizon matters significantly for outcomes

Summary: Main Findings

Step 1: Explore the data

Same model, different discount rate

- 140 bps (15.7%) differences in WACC
- 180 bps (17.8%) differences in r^E

Ave. within firm-date

- Beta and ERP
  - Explain 79% of the variance

Estimated variables

- Risk-free rate
  - Remaining 21%

Summary: Main Findings

Same class of models, different discount rate

Different input values

- Analysts use different methodologies to estimate variables
  - Anchor on annual horizons
  - Use 2, 3, or 5-year for beta horizon in 85% of cases
    - Range: 1 to 10-year

Channels of disagreement

Different information

Different models

Different methods

Evidence on the drivers of disagreement within the same models

Summary: Main Findings

Same model, different discount rate

Different estimates of inputs

Method selection drivers

- Institutional norms (1%)
- Individual criteria **(28%)**
  - Gender
  - Race
  - Location
  - Education

Analyst effects swamp brokerage effects

Idiosyncratic criteria

➢ Work-in-progress: Early career influence

Summary: Main Findings

Same model, different discount rate

Different estimates of inputs

Method selection

What are these idiosyncratic criteria

- Methodology is persistent
    - **I. 82% of analysts use 1 method** throughout the sample
        - I. 78% of analysts use single horizon
        - II. 4% aggregate over multiple horizons

Summary: Main Findings

Same model, different discount rate

Different estimates of inputs

Method selection drivers

What are these idiosyncratic criteria

Step 2: Benchmark Empirical Results Against Theories of Ambiguity Resolution

- Results most consistent with behavioral and Bayesian models

Summary: Main Findings

Same model, different discount rate

Different estimates of inputs

Method selection drivers

What are these idiosyncratic preferences

Benchmark results against theory

Step 3: Outcomes
Discount rate disagreement associated with **higher trading volume**

Data

Data

Sell-side equity reports

Sample properties

- **45,992 unique reports**
  I. Firm-year-analyst

- **Hybrid approach**
  I. Numerical data
     I. WACC inputs
  II. Textual information
     I. Methods

Market Data

Refinitiv / Datastream

- **Trading volume**
- **Monthly returns**
- **Shares outstanding**

Firm Data

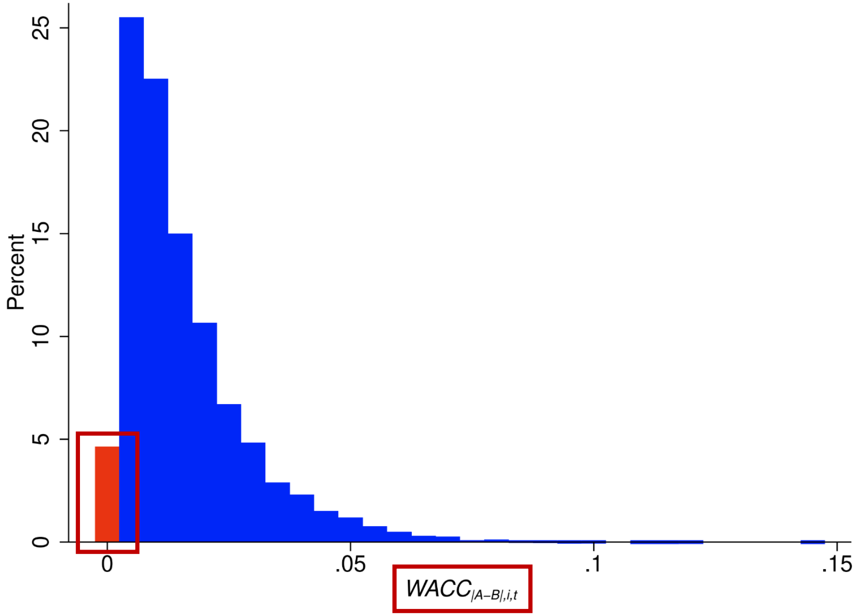
Refinitiv

- **Firm information**

Discount rates differences

- **Unit of observation**
  I. Analyst A- Analyst B-Firm-Date

- **95% of pairs disagree on the discount rate**
  I. 5% use the same value



What drives differences in discount rates?

Drivers of disagreement

$$1 = \frac{cov(rf_{A-B,i,t}, r^E_{A-B,i,t})}{var(r^E_{A-B,i,t})} + \frac{cov(\beta * ERP_{A-B,i,t}, r^E_{A-B,i,t})}{var(r^E_{A-B,i,t})}$$

Dep. variable =	$rf_{A-B,i,t}$				$(\beta \times ERP)_{A-B,i,t}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Full Sample								
$r^E_{A-B,i,t}$	0.227***	0.227***	0.233***	0.214***	0.773***	0.773***	0.785***	0.786***
	(0.033)	(0.032)	(0.019)	(0.040)	(0.033)	(0.032)	(0.040)	(0.040)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	1,498	1,497	1,269	1,117	1,498	1,497	1,119	1,117
F Statistic	46.57	49.93	156.75	29.15	542.02	576.98	386.54	393.63

Key patterns:

I. Disagreement in both set of variables

II. Estimated variables drive > 75% of the disagreement

Drivers of disagreement

$$1 = \frac{cov(rf_{A-B,i,t}, r^E_{A-B,i,t})}{var(r^E_{A-B,i,t})} + \frac{cov(\beta * ERP_{A-B,i,t}, r^E_{A-B,i,t})}{var(r^E_{A-B,i,t})}$$

Panel B: United States Sample								
$r^E_{A-B,i,t}$	0.075	0.083	0.113	0.113	0.925***	0.917***	0.887***	0.887***
	(0.053)	(0.051)	(0.078)	(0.071)	(0.053)	(0.051)	(0.078)	(0.071)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	229	228	160	154	229	228	160	154
F Statistic	2.01	2.70	2.13	2.53	309.10	329.37	130.73	156.88
Panel C: International Sample								
$r^E_{A-B,i,t}$	0.257***	0.256***	0.233***	0.225***	0.743***	0.744***	0.767***	0.775***
	(0.039)	(0.037)	(0.045)	(0.045)	(0.039)	(0.037)	(0.045)	(0.045)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	1,269	1,268	959	957	1,269	1,268	959	957
F Statistic	43.61	47.18	26.46	25.53	364.05	399.78	287.42	301.57

Key patterns:

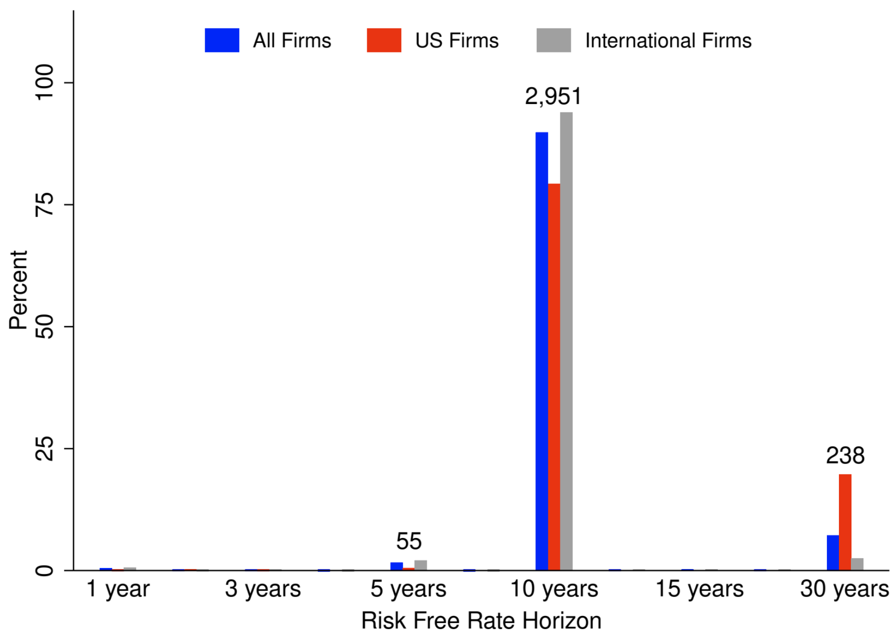
I. Disagreement in both set of variables

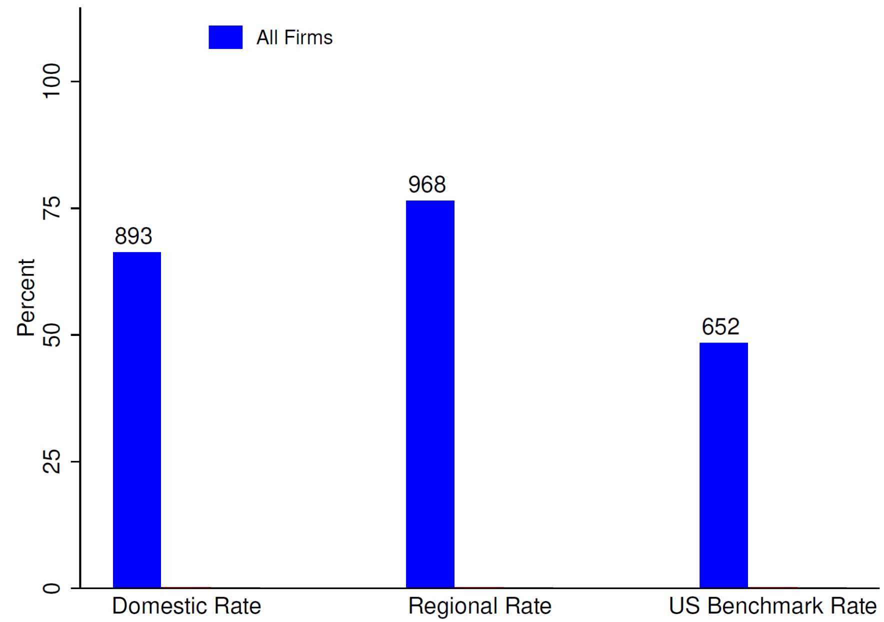
II. Estimated variables drive > 75% of the disagreement

III. Risk-free rate effect is weaker with US data

Same class of models; different cost of equity

- **Start with the risk-free rate benchmark**





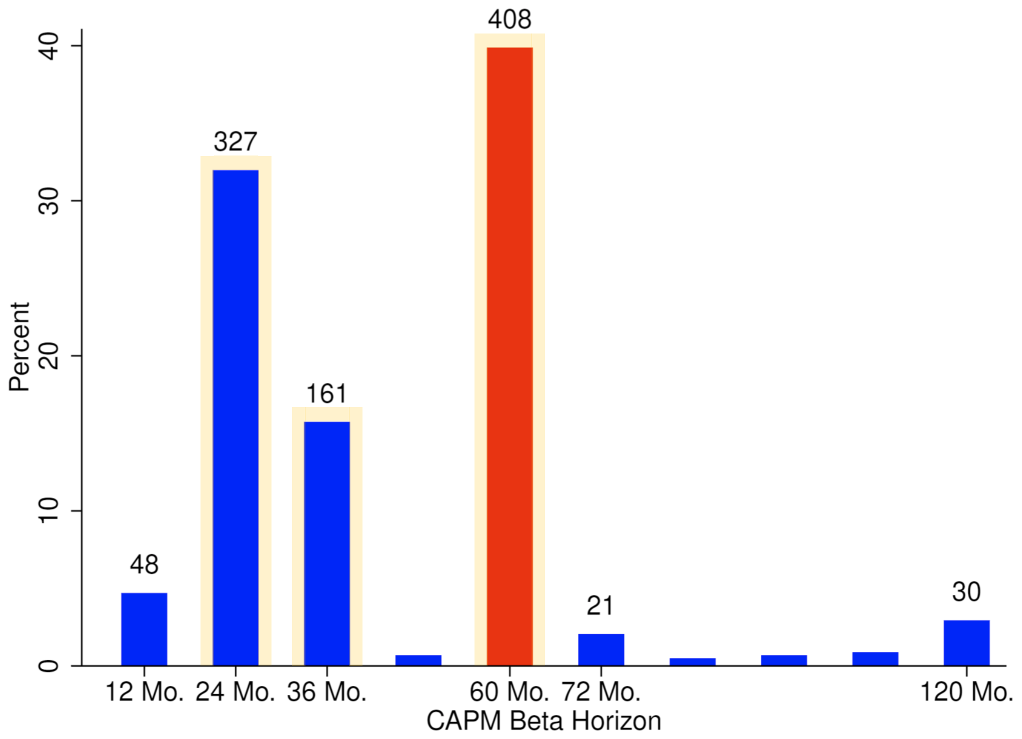
<u>Methodology selection patterns:</u>

<u>Take-away</u>: Helps explain why risk-free rate drives less of the variation for US firms in the decomposition

- US: 99% use US treasury yield
- International: 69% use regional treasury yield

Same class of models; different cost of equity

- **97% of analysts use CAPM when estimating their cost of equity, but…**



Methodology selection patterns:

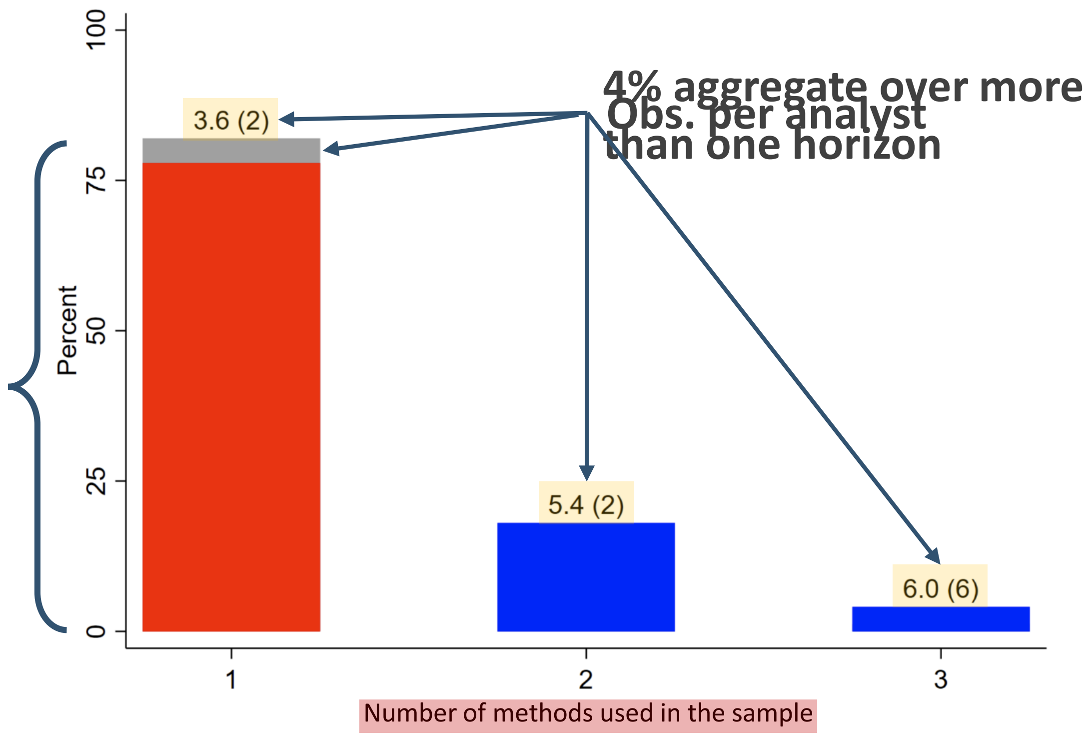
- Anchored on annual horizon
- **2, 3, and 5 years are the main horizons (85%)**
  - Document similar patterns for different **return frequencies**

What drives methodological choices?

CAPM Beta Trailing Horizon Methodology

- **What do the analysts in our sample say that they do?**

Always use the same horizon(s)



Methodology selection patterns:

Take-away: Persistent choice in methodology, cross-sectional disagreement

- **Robust to using only 4+ forecasts, multi-year, multi-firm, …**

What drives methodological choices?

What drives analysts' decisions:

$$\beta_{i,a,t} = \alpha_i + \alpha_{\mathbf{Brokerage}} + \alpha_{\mathbf{Analyst}}$$

- Firm characteristics
- **Institutional norms** (Brokerage house "cookbook")
- **Personal criteria**

	Sum of Squares (% of Model)	Degrees of Freedom	Adjusted Partial R^2
	(1)	(2)	(3)
Panel A: Full Sample			
Firm Indicators	78%	1,947	0.38
Brokerage Indicators	2%	36	0.01
Analyst Indicators	19%	1,120	0.28
Observations		6,411	
R^2		0.78	
Adjusted R^2		0.57	

Results

- **Analyst effects** matter more for beta choices

Methodological Choices and Related Theories

Resolving Estimation Ambiguity: Theory

- **Robust Methodological Choices**
  I. Max-Min (Gilboa and Schmeidler, 1989, Hansen and Sargen, AER 2001)
    - Select a method that would be optimal under a justifiable worst-case scenario

	Max-Min	**Bayesian selection**	**Behavioral**
Decision criteria	E.g.: Highest Beta Across Horizons		
Persistency	Time-varying		
Individual-specific	No		
Nb. of horizons	One		

Resolving Estimation Ambiguity: Theory

- **Robust Methodological Choices**

- **Bayesian selection**
  I. Bayesian selection criteria to average across different specifications (Giacomini et al., 2019, 2022)

	Max-Min	**Bayesian selection**	**Behavioral**
Decision criteria	E.g.: Highest Beta Across Horizons	Average Over Multiple Horizons	
Persistency	Time-varying	Time-varying*	
Individual-specific	No	Yes*	
Nb. of horizons	One	Multiple*	

Resolving Estimation Ambiguity: Theory

- **Robust Methodological Choices**
- **Aggregation Theories**
- **Behavioral Theories**
  I. Familiar strategies (Heath and Tversky, 1991; Fox and Tversky, AER 1995)
  II. Anchoring and simplification

	Max-Min	**Bayesian selection**	**Behavioral**
Decision criteria	E.g.: Highest Beta Across Horizons	Average Over Multiple Horizons	Familiarity and anchoring
Persistency	Time-varying	Time-varying*	Persistent
Individual-specific	No	Yes*	Yes
Nb. of horizons	One	Multiple*	Not specified

Evidence So Far

- **Methodological choices are persistent over time**
- **Driven by idiosyncratic individual-specific criteria**
- **Only 3-4% of individuals use more than one horizon**

	~~Max-Min~~	**~~Bayesian selection~~**	**Behavioral**
Decision criteria	E.g.: Highest Beta Across Horizons	Average Over Multiple Horizons	Familiarity and anchoring
Persistency	✗ Time-varying	✗ Time-varying*	✓ Persistent
Individual-specific	✗ No	✓ Yes*	✓ Yes
Nb. of horizons	✓ One	✗ Multiple*	✓ Not specified

Conclusion

- **Large variation in estimated discount rates, even using the same model**
- **Methodological choices persistent and specific to analysts**
- **Agents appear to resolve ambiguity by adopting one model and applying it across settings**

Appendix

		Discount Rate$_{a,i,t}$		
		Above Consensus	Below Consensus	Total
$TGR_{a,i,t}$	Above Consensus	29.5% 7,037	25.1% 6,004	54.6% 13,041
	Below Consensus	19.7% 4,706	25.7% 6,136	45.4% 10,842
	Total	49.2% 11,743	50.8% 12,140	100.0% 23,883