Kyle Meets Friedman: Informed Trading When Anticipating Future Information*

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Abstract

We analyze a dynamic model of an investor who receives private information on an ongoing basis and faces a post-trading disclosure requirement. Characterizing the equilibrium of our trading game between *two* players—the investor and a market maker—can be reduced into a fictitious consumption-saving problem of *one* consumer with a borrowing constraint. Hence, insights from the consumption-saving literature, such as the permanent income hypothesis, can be adapted to shed light on the informed investor's trading strategy and the equilibrium asset prices and market liquidity. Further analysis suggests that these results arise because the informed investor's commitment value is zero.

JEL Classification Numbers: C72, D82, G14.

Keywords: Informed investor trading; Dissimulation; Mixed strategy; Information arrival.

1 Introduction

How asset prices distill investors' information is a central question in economics. Broadly speaking, an investor's information can be revealed in two ways. The first is through trading. For example, as in Kyle (1985), the informed investor's trades move asset prices and partially reveal his information. The second is through disclosure. When the informed investor's trade size is disclosed (for example, due to regulation), it further reveals his information. In this paper, to capture both aspects of information revelation, we analyze a dynamic model of a monopolistic informed investor who faces a post-trading disclosure requirement.

A key feature of our model is that the informed investor receives his private information on an ongoing basis. Much of the literature since Kyle (1985) assumes that the insider receives private information only once. However, corporate insiders and informed investors such as hedge funds usually obtain their information over time. Our sequential-information assumption not only captures this realistic feature, but is also necessary for analyzing some important research questions. For example, the assumption is essential for analyzing how the anticipation of future private information influences the insider's trading decisions and equilibrium prices today.

Our main result is that, under certain conditions, characterizing the equilibrium of this trading model can be reduced into solving a fictitious consumption-saving model. This surprising result is remarkable. In the trading model, the equilibrium prices and trading strategies are determined by the interaction between *two* players: the informed investor and the market maker. In contrast, the fictitious consumption-saving model concerns only *one* consumer. More importantly, this surprising mathematical equivalence suggests that the insights from the well-established consumption-saving literature can be applied to the trading model. We illustrate this point by showing that various insights from the permanent income hypothesis (Friedman, 1957) can be adapted to shed light on the dynamic trading model.

Specifically, we analyze a model that includes both Kyle (1985) and Huddart et al. (2001) as special cases. As in Kyle (1985), we consider an N-period economy with one risky asset and one monopolistic risk-neutral informed investor. The risky asset is a claim to an uncertain cash flow in the final period. The informed investor receives a private signal about the asset's liquidation value each period and trades against noise traders. A risk-neutral market maker sets the price. After each transaction, the informed investor's trade size is, potentially imperfectly, revealed. This could be due to the post-trade disclosure policy modeled in Huddart et al. (2001), where the informed investor perfectly discloses his trade size each period. This could also be due to regulatory filings (such as the 13f filings by mutual funds and hedge funds), which provide imperfect signals about the informed investor's transactions. Finally, as modeled by Yang and Zhu (2020), this could also reflect the fact that some investors (such as high-frequency traders) may have the technology to partially detect the informed investor's trade sizes.

Our first contribution is to show that characterizing the equilibrium in our baseline model, where the post-trade disclosure perfectly reveals the insider's trade sizes, can be reduced into solving the insider's optimal information allocation problem, which can be transformed into a fictitious consumption-saving problem of a consumer with a borrowing constraint. Specifically, in the trading model, the informed investor receives a private signal each period and decides on how much information to "utilize" in the current period and how much to "dissimulate" for future use. By relabeling the informed investor's private signal as "income," the utilized information as "consumption," the dissimulated information as "savings," and the expected trading profit as the "utility from consumption," we transform the information allocation model into a consumption-saving model. One notable feature of the trading model is that the informed investor can dissimulate his current private information for future use, but cannot transfer his future information to utilize today. This asymmetry manifests itself as a borrowing constraint in the consumption-saving model: the consumer can save his current wealth to consume in the future, but cannot borrow against his future income to consume today.

The mathematical equivalence implies that the ideas from the consumption-saving literature can be adapted to our dynamic trading model to shed new light on informed trading and its influences on market liquidity and asset prices. To illustrate this insight, we consider three prominent ideas inspired by the permanent income hypothesis of Friedman (1957) and show how they can be adapted to a trading model.

The first idea is "saving for rainy days." When anticipating times of scarcity, one would consume less today to save for the future. This intuition manifests itself in the trading model as follows. When anticipating less private information in the future, the informed investor would adopt a mixed strategy to dissimulate his current private information for future use. In contrast, when anticipating abundant private information in the future, the informed investor would adopt a pure strategy to utilize all of his current information. These results extend and sharpen the insight in Huddart et al. (2001), who focus on the special case in which the informed investor possesses all his private information in the first period. Anticipating no more private information in the future, the informed investor always adopts a mixed strategy. In our generalized setup, however, the informed investor's strategy depends on his expectation of future private information. If future information is sufficiently abundant, he would adopt a pure strategy to fully utilize his current information.

The second idea is consumption smoothing. One prefers to minimize his consumption fluctuation over time, although perfect smoothing is not always feasible if the consumer faces a borrowing constraint. The counterpart of this idea in our trading model is "information-usage smoothing." The informed investor would like to utilize the same amount of information each period (i.e., "walk down the demand curve" and have the same price impact each period). However, this is not always possible because he can transfer his current information to future periods but not the other way around. We show that, in equilibrium, the informed investor minimizes the variation of his information usage over time, given the timing of his private information. This result implies that the informed investor smooths the variation in market liquidity by minimizing the fluctuation of his price impact over time.

The third idea is precautionary saving, which suggests that one would save more today if he anticipates higher future income uncertainty. To analyze its implications on trading, we extend our baseline model by introducing uncertainty to the size of the investor's future private information. We show that, parallel to the idea of precautionary saving, the investor would save more of his current private information for future use if there is more uncertainty about the size of his future private information.

Why can we transform our baseline model, a *two*-player game between an investor and a market maker, into a consumption-saving model of *one* consumer? As noted earlier, the two-player game is reduced into the insider's information allocation problem, which is then transformed into a consumption-saving problem by relabeling. Hence, what is critical is reducing the two-player game into a single-player one. When is this reduction possible? What is the economic insight behind it?

We show that the reduction result arises if the investor's commitment value is zero. The intuition is as follows. The equilibrium in our baseline model is determined by the actions of two players, the informed investor and the market marker. In each period, the informed investor trades based on his private information and he cannot commit to a trading strategy prior to observing his information. Now, suppose the investor can commit to a strategy, and the market maker sets prices based on that commitment. This commitment model is effectively a one-player game: its equilibrium is essentially determined by the investor's information allocation because the market maker's action is merely a response to the investor's choice. In general, the investor's commitment value is nonnegative because he can always commit to the equilibrium strategy in the baseline model to obtain the same profit. If the commitment happens to have no value, then the baseline model equilibrium will coincide with the commitment equilibrium, which, as noted earlier, can be reduced into the insider's information allocation problem.

We conduct three sets of additional analyses to investigate the above intuition. First, we show that the informed investor's commitment value is indeed zero in our baseline model. Specifically, we consider an alternative setup in which the investor commits to a strategy of adding noise to his demand and chooses the variance of the noise to optimize his trading profits. The rest of the setup remains identical to our baseline model. We find that the equilibrium in this model with commitment coincides with the baseline-model equilibrium, implying a zero commitment value.

Second, we extend our baseline model to incorporate time-varying noise trading intensity and potential information leakage. In both cases, we find that the informed investor's commitment value is zero, and as in the baseline model, one can reduce the equilibrium characterization into the investor's information allocation problem, which can be relabeled into a fictitious consumption-saving problem.

Third, we generalize our baseline model to incorporate potentially imperfect disclosure. Specifically, after the trade each period, the market maker obtains a signal, which is the investor's trade size plus a noise. This setting includes some important scenarios, such as the regulatory filings by mutual funds and hedge funds, which do not fully reveal their transactions. Moreover, this model includes both Kyle (1985) and Huddart et al. (2001) as special cases. As the variance of the noise in the signal approaches zero, the disclosure becomes perfect and the model converges to our baseline model, which is a generalization of the model in Huddart et al. (2001). In the other limiting case, as the noise variance approaches infinity, the disclosure reveals no information and the setting converges to a generalized version of Kyle (1985). We characterize the equilibrium of this model and show that the investor's commitment value may become positive in this setting. Consistent with our intuition, in parameter regions in which the commitment value is zero, we can transform the model into a consumption-saving problem. However, in the parameter region with positive commitment values, this transformation result no longer holds.

Our main contribution is to establish the unexpected mathematical equivalence between a model of informed trading and a consumption-saving model and show that the insight in the consumption-saving literature can be adapted to shed light on the model of informed trading. Hence, our paper adds to the literature on informed trading by corporate insiders and informed investors. This literature is voluminous, and so we discuss most related studies, organized according to the two important features in our setting: mixed strategies and sequential arrivals of private information. In terms of the former, our paper is most related to Huddart et al. (2001), which is the first study to demonstrate that, in a Kyle (1985) model, the informed investor plays a mixed strategy when his trade is perfectly disclosed. Yang and Zhu (2020) investigate the behavior of an informed investor who leaks a signal about the demand to back-runners. The informed investor can play either pure or mixed strategies, and is more likely to play the latter if the information leakage is more severe. Back and Baruch (2004) analyze a variant of Glosten and Milgrom (1985) model and show that an informed investor would adopt a mixed strategy by randomizing over orders to buy, sell, and wait. In addition to the main methodological contribution, our paper complements these studies by generalizing and sharpening the results in Huddart et al. (2001) and characterize the condition for dissimulation in equilibrium.

The second feature—sequential arrivals of private information—is relevant to many settings in practice. The information acquisition process may result from the dynamics of informational events—such as IPOs (e.g., Welch, 1992; Lowry and Schwert, 2002), mergers (e.g. Ferreira and Laux, 2007) and acquisitions (e.g., Denis and Macias, 2013)—or the dynamics of research and learning activities (e.g. Banerjee and Breon-Drish, 2022; Johannes et al., 2014). Numerous studies examine the effects of sequential information acquisition (e.g. Bernhardt and Miao, 2004; Caldentey and Stacchetti, 2010; Chau and Vayanos, 2008; Foucault et al., 2016; Sastry and Thompson, 2019). Disclosure requirement and the ensuing mixed strategy distinguish our analysis from those studies.

The rest of the paper is organized as follows. Section 2 analyzes the baseline model to establish our main result on the equivalence between the trading model and a consumption-saving model. Section 3 examines the reason behind this equivalence result, and Section 4 concludes. All proofs are provided in the Appendix.

2 The Baseline Model

Our baseline model is a generalization of Kyle (1985) and Huddart et al. (2001). In these two classic studies, the informed investor obtains all his private information in the initial period and receives no further private information afterwards. In contrast, our analysis focuses on the sequential arrival of private information.

2.1 Setup

Consider an economy with N periods, denoted by $n = 1, ..., N$. There is one risky asset, which is a claim to the liquidation value F in period N. The ex ante distribution of F is $\mathcal{N}(0, \sigma_F^2)$, with $\sigma_F > 0$. The market is populated by an informed investor, a continuum of noise traders, and a market maker. Everyone is risk neutral. The informed investor can be interpreted as a corporate insider or a sophisticated investor such as a hedge fund that has private information about F. For convenience, we will simply refer to the informed investor as the "insider."

The insider observes private information about the asset's liquidation value and, critical to our analysis, his information arrives over time. Specifically, the liquidation value F has N elements:

$$F = \sum_{n=1}^{N} F_n,$$

where $F_n \sim \mathcal{N}(0, \sigma_{F_n}^2)$, with $\sigma_{F_n} \geq 0$ and is serially independent across time. In each period n, for $n = 1, ..., N$, the insider observes F_n at time n^-, which is before the trading time of the period (see Figure 1). One can think of the asset as a collection of N projects. F_n represents the earnings from project n, which is known to the insider in period n. The assumption of independence is without loss of generality because if the earnings are correlated across projects, we can orthogonalize and redefine them to ensure independence over time. Note that F_n is long-lived information in the sense that it affects the asset's final liquidation value and never becomes public before the final period. The model in Huddart et al. (2001) can be viewed as a special case of our model with $\sigma_{F_1}^2 = \sigma_F^2$ and $\sigma_{F_n}^2 = 0$ for $n > 1$, which implies that the insider obtains all his private information in the first period.

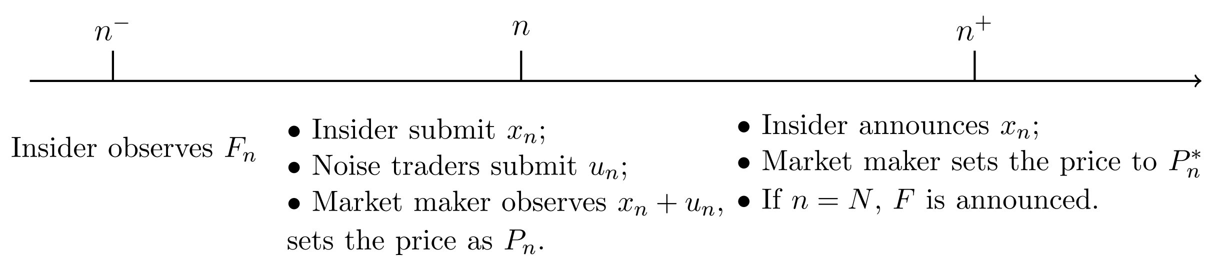

Figure 1. The timeline of the events in period n.

In period n, the insider submits a market order to buy x_n shares of the asset in period n. Noise traders have an aggregate demand of u_n shares, with $u_n \sim \mathcal{N}(0, \sigma_u^2)$ and $\sigma_u > 0$, and u_n is independent across n and from F_n. Upon receiving the aggregate order flow from the insider and noise traders, $y_n = x_n + u_n$, the market maker sets the price P_n to his expectation of the liquidation value to execute the trade. That is, the market maker sets the execution price to

$$P_n = E[F|\mathcal{I}_n^M], \tag{1}$$

where $\mathcal{I}_n^M \equiv \{y_1, ..., y_n, x_1, ..., x_{n-1}\}$ is his information set at the time of trading.

As in Huddart et al. (2001), the insider faces a post-trade disclosure requirement. That is, after the transaction in period n but before the next period (denoted by n^+ in Figure 1), the insider publicly discloses his trade size x_n.[1] In response to the disclosure, the market maker adjusts the price from P_n to P_n^*,

$$P_n^* = E[F|\mathcal{I}_{n+}^M], \tag{2}$$

where $\mathcal{I}_{n+}^M \equiv \{y_1, ..., y_n, x_1, ..., x_n\}$ is the market maker's information set after the insider's disclosure. Hence, P_n^* can be viewed as a state variable that tracks the market maker's expectation, and no transaction takes place at this price.

Let $\mathcal{I}_n^I \equiv \{F_1, ..., F_n, P_1, ..., P_{n-1}, P_1^*, ..., P_{n-1}^*\}$ denotes the insider's information set in period n. His objective is

$$\max_{x_n,...,x_N} E\left[\sum_{i=n}^{N} \pi_i|\mathcal{I}_n^I\right], \tag{3}$$

where $\pi_i \equiv x_i(F - P_i)$ is his profit from his period-i trade. Following Kyle (1985), we define an equilibrium as follows:

Definition 1. An equilibrium is defined as the insider's trading strategy and the market maker's pricing rules (x_n, P_n, P_n^*), for $n = 1, ..., N$, such that in period n: (a) the market maker sets prices according to (1) and (2), taking the insider's trading strategies as given; and (b) the insider's strategy $\{x_n, ...x_N\}$ solves (3), taking the market maker's pricing rules as given.

2.2 Equilibrium Characterization

We follow Kyle (1985) and Huddart et al. (2001) to consider linear equilibria. That is, in period n, for $n = 1, ..., N$, the trading strategy and pricing rules are given by

$$x_n = \beta_n(\sum_{i=1}^{n} F_i - P_{n-1}^*) + z_n, \tag{4}$$

$$P_n = P_{n-1}^* + \lambda_n y_n, \tag{5}$$

$$P_n^* = P_{n-1}^* + \gamma_n x_n, \tag{6}$$

[1]This perfect disclosure assumption is made to simplify analysis. In Section 3.4, we relax this assumption so that disclosure imperfectly reveals the insider's trade size.

where $z_n \sim \mathcal{N}(0, \sigma^2_{z_n})$, $P^*_0 \equiv 0$, and the parameters $\{\beta_n, \sigma_{z_n}, \lambda_n, \gamma_n\}$ will be determined in equilibrium.

Intuitively, $\sum_{i=1}^{n} F_i - P^*_{n-1}$ is the difference between the insider's expected liquidation value and the asset price. Conjecture (4) is that the insider's trade is linear in this difference. Moreover, as in Huddart et al. (2001), the insider may play a mixed strategy, i.e., add a noise z_n to his trade. The insider plays a mixed strategy if $\sigma_{z_n} > 0$ and a pure strategy if $\sigma_{z_n} = 0$. The pricing function (5) is that the market maker sets the transaction price based on the aggregate order y_n. After disclosure, as shown in (6), the market maker further adjusts the price based on the disclosed insider order x_n. The following theorem characterizes the equilibrium.

Theorem 1. *There exists a unique linear equilibrium in which the insider's trading strategy and the market maker's pricing rules are given by equations (4)–(6) with*

$$\beta_n = \frac{k_n \sigma_u}{\Sigma_n + k_n^2}, \tag{7}$$

$$\lambda_n = \frac{k_n}{2\sigma_u}, \tag{8}$$

$$\gamma_n = \frac{k_n}{\sigma_u}, \tag{9}$$

$$\sigma^2_{z_n} = \frac{\Sigma_n}{\Sigma_n + k_n^2}\sigma^2_u, \tag{10}$$

for $n = 1, ..., N$, where

$$k_n \equiv \begin{cases} \sqrt{Var(P^*_n - P^*_{n-1})}, & \text{if } 1 \le n \le N-1, \\ \sqrt{Var(F - P^*_{N-1})}, & \text{if } n = N, \end{cases} \tag{11}$$

$$\Sigma_n \equiv Var(\sum_{i=1}^{n} F_i | \mathcal{I}^M_{n+}). \tag{12}$$

Moreover, Σ_n is given by

$$\Sigma_n = \sum_{i=1}^{n} \sigma^2_{F_i} - \sum_{i=1}^{n} k_i^2, \tag{13}$$

and $\{k_1, \cdots, k_N\}$ *are the unique non-negative solution to the following problem:*

$$\max_{k_1,\cdots,k_N} \sum_{i=1}^{N} k_i \tag{14}$$

$$\textit{subject to} \qquad \sum_{i=1}^{n} k_i^2 \leq \sum_{i=1}^{n} \sigma_{F_i}^2, \ \textit{for } n = 1, ..., N. \tag{15}$$

This theorem reveals the key results of our paper. Constructing the equilibrium of our trading game between *two* players, the insider and the market maker, can be reduced into the insider's information allocation problem (14) and (15). To see this, note that the solution to the optimization problem (14) and (15), $k_1, \ldots, k_N$, can pin down all other equilibrium parameters, as shown in (7)–(10) and (13).

Why can we interpret (14) and (15) as the insider's information allocation problem? First, note that the objective function (14) is to maximize the insider's expected total trading profit. Specifically, since the risk-neutral market maker breaks even in equilibrium, the insider's expected profit in period n must be equal to the noise trader's expected loss: $E[\pi_n] = \lambda_n \sigma_u^2$. Substituting (8) into it, we obtain $E[\pi_n] = k_n \sigma_u / 2$. Hence, the objective function (14) is equivalent to $\max \sum_{i=1}^{N} E[\pi_n]$.

Second, the conditions in (15) are the insider's "information budget constraints." Definition (11) suggests that k_n^2 reflects the amount of information revealed in period n. If the insider trades more aggressively, $P_n^* - P_{n-1}^*$ reveals more information, leading to a higher k_n^2. Hence, k_n^2 can be interpreted as the amount of private information "utilized" by the insider in period n. The constraints in (15) are that, for any given period n, the insider's total information usage during periods 1 to n, $\sum_{i=1}^{n} k_i^2$, should be no more than what he has acquired by then, $\sum_{i=1}^{n} \sigma_{F_i}^2$.

Definition (12) shows that Σ_n is the insider's private information that has not yet been revealed after his disclosure in period n. Before the trade in period n, the insider faces constraints on how much private information he can utilize:

$$k_n^2 \leq \Sigma_{n-1} + \sigma_{F_n}^2, \tag{16}$$

for $n = 1, ..., N$, with $\Sigma_0 \equiv 0$. Intuitively, Σ_{n-1} is the insider's unused private information at the beginning of period n. After observing his private signal F_n (which has a variance of $\sigma_{F_n}^2$), his total private information is $\Sigma_{n-1} + \sigma_{F_n}^2$. If he chooses not to utilize all his information ($k_n^2 < \Sigma_{n-1} + \sigma_{F_n}^2$), the unused information Σ_n would be "saved" for future use:

$$\Sigma_n = \Sigma_{n-1} + \sigma_{F_n}^2 - k_n^2. \tag{17}$$

Note that the constraints in (16) and (17) are equivalent to those in (15). They also show that the insider can only transfer his private information in one direction: he can "save" his private information for future use, but cannot "borrow" his future information to use today.

Finally, equation (10) shows that the insider adopts a mixed strategy (i.e., $\sigma_{z_n}^2 > 0$) if and only if $\Sigma_n > 0$. Intuitively, if the insider has an abundance of private information in period n, he would adopt a mixed strategy to save his information for future use: $k_n^2 < \Sigma_{n-1} + \sigma_{F_n}^2$, which is equivalent to $\Sigma_n > 0$. If his current private information is scarce, however, the insider would adopt a pure strategy to utilize all his information: $k_n^2 = \Sigma_{n-1} + \sigma_{F_n}^2$, which is equivalent to $\Sigma_n = 0$.

2.3 Equivalence to a Consumption-Saving Model

We show in this section that the insider's information allocation problem (14)–(15) can be transformed into a dynamic consumption-saving problem. To avoid confusion, we will refer to the decision maker in the consumption-saving problem as a "consumer," which is to contrast with the "insider" described in Section 2.1. Intuitively, for a given period n, the insider receiving his private information (which is measured by $\sigma_{F_n}^2$) corresponds to a consumer receiving an "income." Similarly, the insider's information usage k_n^2 corresponds to the consumer's "consumption." As noted earlier, the insider can save his current private information for future use but cannot borrow his future private information to use today. This feature corresponds to the consumer's borrowing constraint: he can save his current income for future consumption but cannot borrow against his future income to consume today. Guided by the intuition above, we can transform the information allocation problem (14)–(15) into a dynamic consumption-saving problem with a borrowing constraint.

Specifically, let Y_n, C_n, and S_n denote the consumer's income, consumption, and savings in period n, respectively, for $n = 1, ..., N$. The consumer's budget constraints are as follows:

$$C_n \leq S_{n-1} + Y_n, \tag{18}$$

$$S_n = S_{n-1} + Y_n - C_n, \tag{19}$$

with $S_0 = 0$. The constraint in (18) is that the consumption in a given period cannot be more than the consumer's current income and savings. That is, the consumer cannot borrow against his future income to consume. Equation (19) shows that the unconsumed resource $S_{n-1} + Y_n - C_n$ becomes the savings for the next period. Note that this equation implies that the interest rate is zero.

The above discussion suggests that the consumer's income and consumption (Y_n and C_n) correspond to $\sigma^2_{F_n}$ and k^2_n in our trading-game model. Comparison between equations (17) and (19) shows that the consumer's savings S_n corresponds to Σ_n in our trading game. Finally, the correspondence between k^2_n and C_n suggests that k_n is the counterpart of $\sqrt{C_n}$. Hence, the insider's objective function (14) corresponds to the consumer maximizing his utility, where the utility function is

$$u(C_n) = \sqrt{C_n}. \tag{20}$$

Therefore, simply through relabeling, we can transform the maximization problem in (14)–(15) into the following consumption-saving problem:

$$\max_{\{C_n, \cdots, C_N\}} \sum_{i=1}^{N} u(C_i), \tag{21}$$

subject to (18) and (19). Table 1 summarizes the correspondences between the variables in the trading game and those in the consumption-saving problem.

Theorem 2. *The maximization problem of (14)–(15) is equivalent to the consumption-saving problem (21) subject to (18) and (19), if we relabel $\sigma^2_{F_n}$, k^2_n, and Σ_n as Y_n, C_n, and S_n, respectively.*

The above theorem establishes the mathematical equivalence between our trading model in Section 2.1 and a dynamic consumption-saving problem where the consumer has a constant relative risk aversion utility function with a relative risk aversion coefficient of 1/2 and faces a borrowing constraint.

Table 1. Transform the trading game into a consumption-saving problem.

	Trading game	Consumption-saving problem
Variable:		
	Information leakage k_n^2	Consumption C_n
	Expected profit $k_n \sigma_u / 2$	Utility $\sqrt{C_n}$
	Information endowment $\sigma_{F_n}^2$	Income Y_n
	Unused information amount Σ_n	Saving S_n
Friction:		
	Asymmetric information transfer	Borrowing constraint
	$k_n^2 \leq \Sigma_{n-1} + \sigma_{F_n}^2$	$C_n \leq S_{n-1} + Y_n$
	• If $k_n^2 < \Sigma_{n-1} + \sigma_{F_n}^2$: "mixed"	• $C_n < S_{n-1} + Y_n$: "saving"
	• If $k_n^2 = \Sigma_{n-1} + \sigma_{F_n}^2$: "pure"	• $C_n = S_{n-1} + Y_n$: "consuming all"

2.4 Kyle Meets Friedman

The mathematical equivalence in Theorem 2 suggests that we can use the insights from the consumption-saving literature to guide our analysis of the trading model. In this section, we show how the ideas of the permanent income hypothesis (Friedman, 1957), one of the most important insights in the consumption-saving literature, can be adapted to shed light on our dynamic trading model.

The permanent income hypothesis suggests that one's consumption depends on his expectations of future income. Analogously, in our trading model, the insider's information usage in a given period depends not only on his current information but also on the expectation of his future private information. This has direct implications on the insider's trading strategies and the equilibrium market liquidity and asset prices. We consider three prominent ideas inspired by the permanent income hypothesis and show how they shed light on the implications of our trading model.

2.4.1 Saving for Rainy Days. The idea of "saving for rainy days" suggests that when anticipating times of scarcity, one would consume less today to save more for the future. This intuition manifests itself in the trading game as follows. In periods with abundant private information, when anticipating less private information in the future, the insider would save his current private information for future use. In contrast, in periods of scarcity of private information, the insider utilizes more or even all his current information. This intuition is illustrated in the example with $N = 2$. The following proposition characterizes the equilibrium.

Proposition 1. *If $N = 2$, the equilibrium is characterized in the following two cases:*

Case 1: If $\sigma_{F_1}^2 > \sigma_{F_2}^2$, the equilibrium is given by

$$\sigma_{z_1}^2 = \frac{\sigma_{F_1}^2 - \sigma_{F_2}^2}{2\sigma_{F_1}^2}\sigma_u^2, \quad \sigma_{z_2}^2 = 0, \quad \beta_1 = \frac{\sigma_F \sigma_u}{\sqrt{2}\sigma_{F_1}^2}, \quad \beta_2 = \frac{\sqrt{2}\sigma_u}{\sigma_F},$$
$$k_i = \frac{\sigma_F}{\sqrt{2}}, \quad \lambda_i = \frac{\sigma_F}{2\sqrt{2}\sigma_u}, \quad \gamma_i = \frac{\sigma_F}{\sqrt{2}\sigma_u}, \quad \textit{for } i = 1, 2.$$

Case 2: If $\sigma_{F_1}^2 \leq \sigma_{F_2}^2$, the equilibrium is given by

$$\sigma_{z_i}^2 = 0, \quad \beta_i = \frac{\sigma_u}{\sigma_{F_i}}, \quad \lambda_i = \frac{\sigma_{F_i}}{2\sigma_u}, \quad \gamma_i = \frac{\sigma_{F_i}}{\sigma_u}, \quad k_i = \sigma_{F_i}, \quad \textit{for } i = 1, 2.$$

This proposition illustrates that the insider's trading strategy depends on the relative sizes of the private information in the two periods ($\sigma_{F_1}^2$ and $\sigma_{F_2}^2$). The intuition can be illustrated in our consumption-saving analogy. Suppose that a consumer's total income across two periods is \$1. He would like to allocate his wealth equally across two periods, i.e., $C_1 = C_2 = \$0.5$. However, due to a borrowing constraint, whether this allocation is feasible depends on the timing of his income in the two periods. Suppose that the consumer receives \$0.7 and \$0.3 in periods 1 and 2, respectively. Anticipating his low income in period 2, the consumer would save some of his period-1 income for period 2 (i.e., save for the rainy day). He would consume \$0.5 out of the first period income \$0.7, and save the remaining \$0.2, so that he can also consume \$0.5 in period 2. In contrast, if he receives \$0.3 in period 1 and \$0.7 in period 2, then he will be forced to consume his income each period (i.e., $C_1 = \$0.3$ and $C_2 = \$0.7$) because he cannot borrow to consume the ideal amount in period 1.

Analogously, in our trading mode, suppose the insider's total private information amount is 1 (i.e., $\sigma_{F_1}^2 + \sigma_{F_2}^2 = 1$). He would like to utilize the same amount each period, i.e, $k_1^2 = k_2^2 = 0.5$. However, whether this is feasible depends on the timing of his information. If more information arrives in the first period, say, $\sigma_{F_1}^2 = 0.7$ and $\sigma_{F_2}^2 = 0.3$ (as in Case 1 of Proposition 1), the insider can achieve the ideal allocation. He adopts a mixed strategy to use $k_1^2 = 0.5$ private information in period 1 and saves the remaining 0.2 for the second period to have $k_2^2 = 0.5$. In contrast, if less information arrives in the first period, for example, $\sigma_{F_1}^2 = 0.3$ and $\sigma_{F_2}^2 = 0.7$ (as in Case 2 of Proposition 1), then the insider adopts a pure strategy to use up his private information in both periods (i.e., $k_1^2 = 0.3$ and $k_2^2 = 0.7$). In summary, analogous to the permanent income hypothesis, the insider's current information usage depends on not only his current information but also his anticipated future information.

These results generalize and sharpen those in Huddart et al. (2001), whose model belongs to Case 1 with $\sigma_{F_1}^2 = \sigma_F^2$ and $\sigma_{F_2}^2 = 0$. Our analysis in Case 1 shows that the dissimulation result in Huddart et al. (2001) holds more generally, i.e., as long as $\sigma_{F_1}^2 > \sigma_{F_2}^2$. However, this result disappears in Case 2, where the insider receives less private information in the first period than in the second ($\sigma_{F_1}^2 \leq \sigma_{F_2}^2$). Anticipating the arrival of more information in the second period, the insider utilizes all his private information available in the first period.

2.4.2 Consumption Smoothing. Consumption smoothing is a key insight in the consumption-saving literature. Its counterpart in the trading model is that the insider would like to smooth his information usage over time. Indeed, in the two-period example in the previous subsection, the insider minimizes the difference between his information usage across the two periods. What is the notion of information-usage smoothing in an N-period model? We formalize it in the following proposition.

Proposition 2. *Denote $\overline{k} \equiv \sum_{i=1}^N k_i/N$ and $\overline{\lambda} \equiv \sum_{i=1}^N \lambda_i/N$. The maximization problem (14) and (15) is equivalent to the following two minimization problems:*
(1) Information-usage smoothing:

$$\min_{\{k_1,\cdots,k_N\}\in\mathbb{R}^N_{\geq 0}} \sum_{i=1}^N (k_i - \overline{k})^2, \tag{22}$$

$$\text{subject to} \quad \sum_{i=1}^n k_i^2 \leq \sum_{i=1}^n \sigma_{F_i}^2, \ \textit{for } n = 1, ...N-1, \tag{23}$$

$$\sum_{i=1}^N k_i^2 = \sum_{i=1}^N \sigma_{F_i}^2. \tag{24}$$

(2) Price-impact smoothing:

$$\min_{\{\lambda_1,\cdots,\lambda_N\}\in\mathbb{R}^N_{\geq 0}} \sum_{i=1}^{N}(\lambda_i - \overline{\lambda})^2, \tag{25}$$

subject to (23) and (24).

Moreover, both k_t and λ_t are weakly increasing over time:

$$k_t \leq k_{t+1} \textit{ and } \quad \lambda_t \leq \lambda_{t+1} \textit{ for } t = 1, ..., N-1. \tag{26}$$

This proposition formalizes the notion that the insider's objective is to minimize the time variation of his information usage over time (as shown by (22)), which is equivalent to minimizing the variation in price impact (as shown by (25)). Intuitively, the insider's private information usage is closely linked to his price impact, and indeed, k_n and λ_n are proportional to each other in our model (see (8)). So, smoothing information usage over time is the same as smoothing price impact.

The proposition also shows that the insider's information usage and price impact are weakly increasing over time. This is a direct consequence from information smoothing in (22). Ideally, the insider would like to keep his information usage k_t constant over time. Hence, $k_t > k_{t+1}$ is never optimal because the insider can increase his expected trading profit by saving his information in period t to use in the next period (i.e., reducing k_t to increase k_{t+1}). On the other hand, $k_t < k_{t+1}$ can be sustained in equilibrium because the insider cannot transfer future private information to utilize today. As noted in (8), since the price impact and information usage are closely related, the insider using information at a weakly increasing rate implies that the price impact is also weakly increasing, $\lambda_t \leq \lambda_{t+1}$. These results are in contrast to those in previous studies, where the price impact is usually either a constant (e.g., Kyle (1985), Huddart et al. (2001)) or tends to decrease over time (e.g. Caldentey and Stacchetti (2010)).[2]

Our results generalize the insight in Kyle (1985) and Huddart et al. (2001), where the information usage variation is minimized to zero (i.e., the insider utilizes his private information at a constant rate). In our model, the total amount of private

[2]One notable exception is Collin-Dufresne and Fos (2016), where the price impact tends to increase over time due to a liquidity-timing option.

information is σ_F^2. Hence, the best possible scenario is to utilize σ_F^2/N each period. However, this is not always feasible, as illustrated in the two-period example in Proposition 1. When is perfect information-usage smoothing possible? We characterize its condition in the following corollary.

Proposition 3. *The necessary and sufficient condition for perfect information-usage smoothing (i.e., $k_n^2 = \sigma_F^2/N$ for $n = 1, \cdots, N$) is*

$$\sum_{i=1}^{n} \sigma_{F_i}^2 \geq \frac{n}{N}\sigma_F^2, \text{ for } n = 1, \cdots, N-1. \tag{27}$$

Under condition (27), the equilibrium in period n has the following properties:

$$\lambda_n = \frac{\sigma_F}{2\sqrt{N}\sigma_u}, \tag{28}$$

$$E[\pi_n] = \frac{\sigma_F \sigma_u}{2\sqrt{N}}, \tag{29}$$

$$U_n = (1 - n/N)\sigma_F^2, \tag{30}$$

where U_n is the uncertainty of the liquidation value conditional on asset prices:

$$U_n \equiv Var\left(F|P_1^*, ..., P_n^*\right).$$

If the inequalities in (27) hold strictly, the insider adopts a mixed strategy in all but the last period.

Condition (27) is such that the insider can always "afford" to utilize σ_F^2/N private information each period. Specifically, if sufficient private information arrives early, the insider always has no less than σ_F^2/N unused information available in each period. Hence, he achieves perfect information smoothing by utilizing σ_F^2/N private information each period. Consequently, his price impact and expected trading profit are also constants over time, as shown in equations (28) and (29), respectively. Since the insider's private information is revealed at a constant rate, as shown in equation (30), the stock price uncertainty decreases linearly over time. If the inequalities in (27) hold strictly, they guarantee that the insider always has more than σ_F^2/N private information in each period. To utilize σ_F^2/N private information each period, the insider dissimulates his private information (i.e., adopt a mixed strategy) in all but the last period.

It is interesting to compare the above results with those in Huddart et al. (2001), where the insider receives all his private information in the first period and, in equilibrium, utilizes the same amount of private information each period. Proposition 3 shows that this result holds more generally under the conditions in (27), which includes the information structure in Huddart et al. (2001) as a special case.

2.4.3 Precautionary Saving. The idea of precautionary saving is that one would reduce his consumption today if he anticipates more income uncertainty in the future. The counterpart of this idea in our trading model is that the insider would utilize less of his current information if he anticipates more uncertainty in his information advantage in the future. To examine this intuition, we introduce uncertainty to the size of the insider's future private information.

For simplicity, we consider the two-period model analyzed in Proposition 1. The only modification is that there is uncertainty in the size of the insider's private information in second period:

$$\sigma_{F_2}^2 = \begin{cases} \overline{\sigma}_{F_2}^2 + \Delta, & \text{with probability } \frac{1}{2}, \\ \overline{\sigma}_{F_2}^2 - \Delta, & \text{with probability } \frac{1}{2}. \end{cases} \tag{31}$$

After period 1, the insider learns the values of $\sigma_{F_2}^2$ and F_2 before his trading in the second period, and the market maker learns the value of $\sigma_{F_2}^2$ when executing the trades. To best illustrate the notion of precautionary saving, we focus on the case $\sigma_{F_1}^2 \geq \overline{\sigma}_{F_2}^2$. We characterize the equilibrium of this modified economy in the appendix and summarize the implications on the trading in the first period below.

Proposition 4. *In the equilibrium of the economy described in this subsection and in the case $\sigma_{F_1}^2 \geq \overline{\sigma}_{F_2}^2$, the insider's information usage in period 1 is increasing in the expectation of his second period information (i.e., $\frac{\partial k_1^2}{\partial \overline{\sigma}_{F_2}^2} > 0$), but is decreasing in the uncertainty (i.e., $\frac{\partial k_1^2}{\partial \Delta} < 0$).*

This proposition shows how the insider's information usage in the first period depends on his expectations of his private information in the second period. First, similar to the intuition of saving for rainy days, the insider utilizes less information if he expects less private information in the second period on average (i.e., if $\overline{\sigma}_{F_2}^2$ is smaller). Second, analogous to the idea of precautionary saving, the insider saves more of his private information in the first period (i.e., k_1 is smaller) if he expects higher uncertainty in the amount of his private information in the second period.

3 What Is Behind the Transformation Result?

Why can we transform our trading model, where the equilibrium is determined by simultaneous moves of *two* players, into a consumption-saving model of *one* consumer? Our earlier analysis offers a clue. For instance, Theorem 1 shows that the two-player trading game can be reduced into the insider's information allocation problem (14) and (15), a one-player game. This information allocation problem is then transformed into a consumption-saving problem simply through relabeling, as shown by Theorem 2. Hence, what is critical is reducing the two-player game into a single-player one.

When is this reduction possible? What is the economic insight behind it? In this section, we show that the reduction result is derived from the fact that the insider's commitment value is zero. To see the intuition, let us modify our baseline model so that the insider can commit to a trading strategy, and the market maker takes the commitment as given when setting prices. The value of this commitment is non-negative because the insider has the option to commit to the strategy in Theorem 1 and hence obtain the same expected profits as in the baseline model. In the case where the insider prefers the trading strategy in Theorem 1, his commitment value is zero, and this commitment equilibrium coincides with the equilibrium in the baseline model. Note that the construction for the commitment equilibrium can be reduced into a one-player game for the insider because the market maker's action is merely a response to the insider's decision. In summary, in the case with a zero commitment value, the equilibrium in the baseline model coincides with the commitment equilibrium, which can be reduced to the insider's information allocation problem.

The remainder of this section investigates this intuition. We introduce the insider's commitment in Section 3.1 and show its value is zero. In Sections 3.2–3.4, we extend the baseline model along various dimensions. In Sections 3.2 and 3.3, as well as some parameter regions in 3.4, the insider's commitment value is zero, and as in the baseline model, one can reduce the equilibrium characterization into solving a one-player problem, which can be transformed into a consumption-saving problem. However, the insider's commitment value is positive in some parameter regions in 3.4, and, as expected, the transformation results do not hold.

3.1 Commitment Value in the Baseline Model

In this subsection, we modify the baseline model so that the insider can commit to a linear trading strategy as specified in equation (4) and choose its parameters $\{\beta_n, \sigma_{z_n}\}_{n=1}^{N}$ before any trading takes place. The commitment is common knowledge. The rest of the model remains the same as in the baseline model in Section 2. Such a committed trading strategy can be interpreted as a predetermined trading plan that specifies the trading rule according to an algorithm in advance. The equilibrium in this model is such that the insider chooses β_n and σ_{z_n} to maximize his expected total trading profit, and the market maker takes commitment (4) as given and sets asset prices according to his expected liquidation value of the asset.

Proposition 5. *The equilibrium in this model with commitment is identical to that characterized in Theorem 1.*

This proposition shows that if we change the setup in Section 2 such that the insider can commit to a trading strategy (4) and the market maker takes the commitment as given to set prices, this modification does not alter the equilibrium in the baseline model. Hence, the proposition formalizes the idea of "mixed strategies as objects of choice" (e.g., Osborne and Rubinstein, 1994). The mixed strategy analyzed in Section 2 can be interpreted as the outcome of an optimization problem where the insider deliberately chooses the optimal amount of noise in his demand to dissimulate his private information.

More importantly, it shows that if the insider in the baseline model can commit to a trading strategy in (4), it would not increase his expected trading profit. That is, the insider's commitment value is zero in the baseline model.[3] This result implies

[3] This is consistent with the recent paper by Bernhardt and Boulatov (2023), who show that commitment has no value in a one-period Kyle model. They also show that in games in which shocks are not normally distributed and so the equilibrium is nonlinear, commitment does have value. Moreover, they consider a Stackelberg setting in which the parameters chosen by the insider are observable to the market maker. We analyze a multi-period setting with mixed strategies. Our

that the equilibrium of the two-player trading game can be reduced into the insider's information allocation problem in (14) and (15).

3.2 Time-Varying Noise Trading

In this extension, we make one modification to the baseline model in Section 2 by introducing time variation in the intensity of noise trading. Specifically, in period n, the aggregate demand from noise traders is u_n shares, with $u_n \sim \mathcal{N}(0, \sigma_{u_n}^2)$, and $\sigma_{u_n} > 0$. Moreover, u_n is independent across n and from F_n. The rest of the model remains identical to the setup in Section 2.

Theorem 3. *1) In the equilibrium of the economy defined in this subsection, the insider's trading strategies and the market maker's pricing rules are given by (4)–(6), where the parameters are characterized by (7)–(10) with σ_u^2 replaced with $\sigma_{u_n}^2$, and $\{k_1, \cdots, k_N\}$ are the unique nonnegative solution to:*

$$\max_{k_1,\cdots,k_N} \sum_{i=1}^{N} k_i \sigma_{u_i} \tag{32}$$

subject to (15).

2) The insider's commitment value is zero, and his information allocation problem (32) and (15) can be transformed into the following consumption-saving problem

$$\max_{C_n,\cdots,C_N} \sum_{i=1}^{N} u(C_i/p_i), \tag{33}$$

*subject to (18) and (19) if we relabel ($\sigma_{F_i}^2$, k_i^2, Σ_i) as **nominal** income, consumption, and saving (Y_i, C_i, S_i), respectively, with $p_i \equiv 1/\sigma_{u_i}^2$ as the price level.*

3) The information allocation problem (32) and (15) is equivalent to

$$\min_{k_1,\cdots,k_N} \sum_{i=1}^{N} \left[\omega_i (k_i' - \overline{k'})^2\right], \tag{34}$$

subject to (23) and (24), where $\omega_i \equiv \sigma_{u_i}^2 / \sum_{j=1}^{N} \sigma_{u_j}^2$, $k_i' \equiv k_i/\sigma_{u_i}$, and $\overline{k'} \equiv \sum_{i=1}^{N} \omega_i k_i'$, and it is also equivalent to

$$\min_{\lambda_1,\cdots,\lambda_N} \sum_{i=1}^{N} \left[\omega_i (\lambda_i - \overline{\lambda})^2\right], \tag{35}$$

result in Proposition 5 holds independent of whether the parameters of the insider's strategy are observable or not.

subject to (23) and (24), where $\overline{\lambda} \equiv \sum_{i=1}^{N} \omega_i \lambda_i$.

This theorem shows that as in the baseline model, the equilibrium construction can be reduced into the insider's information allocation problem. Note that the insider's expected trading profit in period i is given by $k_i \sigma_{u_i}$. Hence, the objective function (32) is to maximize the insider's total expected trading profit, same as its counterpart (i.e., equation (14)) in the baseline model.

Moreover, as in the baseline model, the insider's commitment value is zero and his information usage problem can be transformed into a consumption-saving problem by relabeling variables. The only modification is that income $\sigma_{F_i}^2$, consumption k_i^2, and savings Σ_i are all *nominal* variables with a price level $p_i = 1/\sigma_{u_i}^2$. Intuitively, a higher p_i (i.e., lower $\sigma_{u_i}^2$) implies a lower trading profit for the insider. Analogously, in the consumption-saving model, a higher price level p_i reduces the purchasing power of the consumer's nominal income $\sigma_{F_i}^2$.

Finally, as in the baseline mode, the insider smooths his information usage and price impact in equilibrium. As shown in (34), the insider minimizes the time variation in k_i', which is the insider's normalized information usage k_i/σ_{u_i}. The normalization accounts for the fact that the insider's trading is more profitable when there is more noise trading. Moreover, the observation in period i is weighted by ω_i, noise trading variance in period i divided by the total noise trading variance across N periods. In this model, "perfect" smoothing is achieved if k_i' is a constant over time. That is, if the insider's information usage in a given period is proportional to the noise trading size in that period. Similarly, the objective function (35) is that the insider minimizes the variation in price impact across time. These results are a generalized version of those in the baseline model. If we set $\sigma_{u_i} = \sigma_u$ for $i = 1, ...N$, the two minimization problems in (34) and (35) become those in the baseline model ((22) and (25)).

3.3 Information Leakage

In this extension, we introduce information leakage into the baseline model. Specifically, F may become public with a probability $1 - q \in [0, 1)$ each period. The rest of the economy remains the same as in the baseline model in Section 2. After the information leakage, the trading game is over since the insider has no more incentive to participate. Hence, we will focus on the equilibrium when the information leakage has not yet occurred. We use the same set of variables from the baseline model in Section 2 to represent their counterparts in the current context, assuming no information leakage has occurred. The following theorem characterizes the equilibrium.

Theorem 4. *1) In the equilibrium of the economy defined in this subsection, before the information leakage, the insider's trading strategies and the market maker's pricing rules are given by (4)–(6) with parameters characterized by equations (7)–(10), and* $\{k_1, \cdots, k_N\}$ *are the unique non-negative solution to:*

$$\max_{k_1,\cdots,k_N} \sum_{i=1}^{N} q^{i-1} k_i \tag{36}$$

subject to (15).

2) The insider's commitment value is zero, and his information allocation problem (36) and (15) is equivalent to the following consumption-saving problem:

$$\max_{C_n,\cdots,C_N} \sum_{i=1}^{N} q^{i-1} u(C_i), \tag{37}$$

subject to (18) and (19), if we relabel $\sigma^2_{F_n}$*,* k_n^2*, and* Σ_n *as* Y_n*,* C_n*, and* S_n*, respectively.*

3) The maximization problem defined in (36) and (15) is equivalent to

$$\min_{k_1,\cdots,k_N} \sum_{i=1}^{N} (k_i - q^{i-1}\overline{k})^2, \tag{38}$$

subject to (23) and (24), with $\overline{k} \equiv \sum_{i=1}^{N} q^{i-1} k_i / N$*. It is also equivalent to*

$$\min_{\lambda_1,\cdots,\lambda_N} \sum_{i=1}^{N} (\lambda_i - q^{i-1}\overline{\lambda})^2, \tag{39}$$

subject to (23) and (24), with $\overline{\lambda} \equiv \sum_{i=1}^{N} q^{i-1} \lambda_i / N$*.*

The equilibrium in this case is similar to that in the baseline model. One notable change is the objective function (36), whereby information usage in period i is discounted by q^{i-1}. This is because that, each period, the trading game continues to the next period with a probability q. Once the games stops (i.e., information revelation occurs), the insider can no longer benefit from trading on his information. Hence, the benefit from future information usage is discounted by q each period.

Similarly, as in the baseline model, the insider's commitment value is zero and his information allocation problem can be transformed into a consumption-saving problem through relabeling. As shown in (37), the only adjustment is that the utility in period i is discounted by q^{i-1}.

Finally, also similar to the result in the baseline model, the insider tries to smooth his information usage and price impact. The modifications in the minimization problems (38) and (39) account for the fact that, each period, the trading game continues with a probability q. Hence, the insider achieves "perfect" information-usage smoothing if k_i/q^{i-1} is a constant for all i.

3.4 Imperfect Disclosure

In this subsection, we generalize our baseline model such that the insider's commitment value may be positive. Then, we show that in the parameter region in which the insider's commitment value is zero, as in the baseline model, we can reduce the model into the insider's information allocation problem and transform it into a consumption-saving problem. However, in the parameter region in which the commitment value is positive, these reduction and transformation results no longer hold.

In our analysis so far, disclosure perfectly reveals the insider's trade x_n. In reality, however, disclosure is often imperfect. For example, financial institutions with over \$100 million stock holdings are required to report their holdings at the end of each quarter. Hence, this disclosure reveals their trades imperfectly. Alternatively, as noted in Yang and Zhu (2020), some investors such as high-frequency traders can partially detect informed insiders' trades ex post. To analyze imperfect disclosure, we assume that after the trade in period n, the market maker observes a signal $d_n = x_n + \epsilon_n$, where $\epsilon_n \sim \mathcal{N}(0, \sigma_\epsilon^2)$, and ϵ_n is independent across n and from all other random variables defined earlier. The rest of the model remains the same as in the baseline model in Section 2.

This formulation includes both Kyle (1985) and Huddart et al. (2001) as special cases. As σ_ϵ^2 approaches zero, the disclosure perfectly reveals the insider's trade, and our model converges to a generalized version of the model in Huddart et al. (2001). In the other limiting case, as σ_ϵ^2 approaches ∞, the disclosure does not contain any information, and our model converges to a generalized version of Kyle (1985).

We conjecture a linear equilibrium. That is, in period n, for $n = 1, ..., N$, the trading strategies and the pricing rules are given by

$$x_n = \beta_n(\sum_{i=1}^{n} F_i - P^*_{n-1}) + z_n, \quad \textit{with } z_n \sim \mathcal{N}(0, \sigma^2_{z_n}), \tag{40}$$

$$P_n = P^*_{n-1} + \lambda_n(x_n + u_n), \tag{41}$$

$$P^*_n = P^*_{n-1} + \lambda'_n(x_n + u_n) + \gamma_n(x_n + \epsilon_n), \tag{42}$$

with $P^*_0 \equiv 0$, and $\{\beta_n,\ \lambda_n,\ \lambda'_n,\ \gamma_n,\ \sigma_{z_n}\}$ will be determined in equilibrium.

Theorem 5. *In the equilibrium of the economy with imperfect disclosure, the insider's trading strategies and the market maker's pricing rules are given by (40)–(42), and the parameters for period n, for $n = 1, ..., N$, can be determined as follows.*

If the insider adopts a pure strategy, i.e., $\sigma^2_{z_n} = 0$, the parameters are given by

$$\lambda_n = \frac{\beta_n(\Sigma_{n-1} + \sigma^2_{F_n})}{\beta_n^2(\Sigma_{n-1} + \sigma^2_{F_n}) + \sigma^2_{z_n} + \sigma^2_u}, \tag{43}$$

$$\lambda'_n = \lambda_n - \frac{\gamma_n(\beta_n^2(\Sigma_{n-1} + \sigma^2_{F_n}) + \sigma^2_{z_n})}{\beta_n^2(\Sigma_{n-1} + \sigma^2_{F_n}) + \sigma^2_{z_n} + \sigma^2_u}, \tag{44}$$

$$\gamma_n = \frac{\beta_n(\Sigma_{n-1} + \sigma^2_{F_n})\sigma^2_u}{(\beta_n^2(\Sigma_{n-1} + \sigma^2_{F_n}) + \sigma^2_{z_n})(\sigma^2_u + \sigma^2_\epsilon) + \sigma^2_u\sigma^2_\epsilon}, \tag{45}$$

$$\beta_n = \frac{1 - 2\alpha_n(\lambda'_n + \gamma_n)}{2\lambda_n - 2\alpha_n(\lambda'_n + \gamma_n)^2}, \tag{46}$$

$$\lambda_n - \alpha_n(\lambda'_n + \gamma_n)^2 > 0, \tag{47}$$

where α_n and Σ_n are given by (A.31) and (A.38), respectively. If the insider adopts a mixed strategy, i.e., $\sigma^2_{z_n} > 0$, the parameters are given by (43)–(45) and

$$1 - 2\alpha_n(\lambda'_n + \gamma_n) = 0, \tag{48}$$

$$\lambda_n - \alpha_n(\lambda'_n + \gamma_n)^2 = 0. \tag{49}$$

This theorem provides a way to characterize the equilibrium for the case with imperfect disclosure. However, unlike Theorem 1, it does not show whether the equilibrium construction can be reduced into solving the insider's information allocation problem that can be transformed into a consumption-saving problem. To investigate this issue, we consider the following two examples.

In the first example, we consider a case in which the insider's commitment value is positive. Specifically, let $N = 2, \sigma_{F_1} = \sigma_F, \sigma_{F_2} = 0$, and $\sigma_\epsilon^2 = \infty$. Under these parameters, our model becomes the 2-period Kyle (1985) model: all the insider's information arrives before the first trade ($\sigma_{F_1} = \sigma_F, \sigma_{F_2} = 0$) and there is essentially no disclosure requirement ($\sigma_\epsilon^2 = \infty$). We summarize the equilibrium for the cases with and without commitment in Table 2 and leave the details of the derivation to the online appendix.

Table 2. Equilibrium for the case $N = 2, \sigma_{F_1} = \sigma_F, \sigma_{F_2} = 0, \sigma_\epsilon^2 = \infty$.

	k_1^2	k_2^2	$E\pi_1$	$E\pi_2$	$E\pi_1 + E\pi_2$
Without commitment:	$0.308\sigma_F^2$	$0.346\sigma_F^2$	$0.462\sigma_F\sigma_u$	$0.416\sigma_F\sigma_u$	$0.878\sigma_F\sigma_u$
With commitment:	$0.352\sigma_F^2$	$0.324\sigma_F^2$	$0.477\sigma_F\sigma_u$	$0.403\sigma_F\sigma_u$	$0.880\sigma_F\sigma_u$

This table shows that if the insider can commit to his trading strategy and the market maker's response takes the committed strategy as given, then the insider's optimal trading strategy with commitment is different from his equilibrium strategy without commitment. Hence, the insider's expected trading profit with commitment is higher than his expected trading profit without commitment. Consequently, we cannot reduce the construction of the two-player equilibrium into solving the insider's information allocation problem as in Theorem 1.

Specifically, in the equilibrium without commitment, the insider's expected trading profits in the two periods are $E(\pi_1) = \frac{\sigma_u}{\sigma_F} k_1 \sqrt{\sigma_F^2 - k_1^2}$ and $E(\pi_2) = \frac{\sigma_u}{2} k_2$. Hence, one might want to construct the following insider's information allocation problem

$$\max_{k_1, k_2} \frac{\sigma_u}{\sigma_F} k_1 \sqrt{\sigma_F^2 - k_1^2} + \frac{\sigma_u}{2} k_2 \tag{50}$$

$$\text{subject to} \quad k_1^2 + k_2^2 \le \sigma_F^2. \tag{51}$$

However, this maximization problem in (50) and (51) is inconsistent with the equilibrium. To see this point, note that the information allocation problem (50) and (51) is consistent with the commitment equilibrium. However, as shown in Table 2, the commitment equilibrium and the equilibrium without commitment are different. Therefore, the equilibrium without commitment cannot be reduced into the insider's information allocation problem in the form of (50) and (51).

In the second example, we consider a case in which the insider's commitment value is zero. Specifically, we focus on the parameter region in which the insider adopts a mixed strategy in equilibrium in all but the last period, that is, $\sigma_{z_n}^2 > 0$ for $n = 1, ..., N-1$. A necessary and sufficient condition for this case is

$$\frac{\sigma_u^2 - \sigma_\epsilon^2}{\sigma_u^2}(\Sigma_{n-1} + \sigma_{F_n}^2) > \frac{\Sigma_{n-1} + \sigma_{F_n}^2 + \cdots + \sigma_{F_N}^2}{N - n + 1}, \tag{52}$$

for $n = 1, ..., N-1$. The equilibrium in this case is summarized as follows.

Proposition 6. *Under conditions in (52), the equilibrium in period n is given by*

$$\beta_n = \frac{k_n \sigma_u^2}{(\Sigma_n + k_n^2)\sqrt{\sigma_u^2 + \sigma_\epsilon^2}}, \quad \sigma_{z_n}^2 = (\frac{\sigma_u^2 - \sigma_\epsilon^2}{\sigma_u^2} - \frac{k_n^2}{\Sigma_n + k_n^2})\frac{\sigma_u^4}{\sigma_u^2 + \sigma_\epsilon^2}$$
$$\lambda_n = \frac{\sqrt{\sigma_u^2 + \sigma_\epsilon^2}}{2\sigma_u^2}k_n, \quad \lambda_n' = \frac{\sigma_\epsilon^2 k_n}{\sigma_u^2\sqrt{\sigma_u^2 + \sigma_\epsilon^2}}, \quad \gamma_n = \frac{k_n}{\sqrt{\sigma_u^2 + \sigma_\epsilon^2}},$$

for $n = 1, \cdots, N-1$, and the equilibrium in the final period is given by,

$$\beta_N = \frac{\sigma_u}{k_N}, \quad \sigma_{z_N}^2 = 0, \quad \lambda_N = \frac{k_N}{2\sigma_u},$$
$$\lambda_N' = \frac{\sigma_\epsilon^2}{\sigma_u(\sigma_u^2 + 2\sigma_\epsilon^2)}\sqrt{\Sigma_{N-1} + \sigma_{F_N}^2}, \quad \gamma_N = \frac{\sigma_u}{\sigma_u^2 + 2\sigma_\epsilon^2}\sqrt{\Sigma_{N-1} + \sigma_{F_N}^2},$$

where Σ_n is given by (13) and $\{k_1, \cdots, k_N\}$ are the unique non-negative solution to

$$\max_{k_1,\cdots,k_N} \sum_{n=1}^{N-1} k_n + \rho k_N, \tag{53}$$

subject to (15), where $\rho \equiv \sqrt{\frac{\sigma_u^2}{\sigma_u^2 + \sigma_\epsilon^2}}$.

The insider's commitment value is zero, and his information allocation problem (53) and (15) can be transformed into a consumption-saving problem

$$\max_{C_1,\cdots,C_N} \sum_{n=1}^{N-1} u(C_n) + \rho u(C_N), \tag{54}$$

subject to (18) and (19), if we relabel $\sigma_{F_n}^2$, k_n^2, and Σ_n as Y_n, C_n, and S_n, respectively.

In this case, the equilibrium construction can be reduced into the insider's information allocation problem (53) and (15). The only difference relative to the information allocation problem in the baseline model is that the information in the final period, k_N, is discounted by ρ. Similar to the result in Theorem 2, the insider's information allocation problem can be transformed into a consumption-saving problem. The only modification relative to the consumption-saving problem in the baseline model is that the utility from the final period $u(C_N)$ is discounted by ρ.

The intuition is that after trading and disclosure in the final period, in the case with imperfect disclosure (i.e., $\sigma_\epsilon > 0$), the asset price P_N^* does not fully reveal the insider's private information. The "unused" private information will be "wasted" because the insider has no more opportunities to trade before the liquidation value F is publicly announced. In the special case of perfect disclosure (i.e., $\sigma_\epsilon = 0$), P_N^* fully reveals the insider's private information. Hence, we have $\rho = 1$ because there is no waste in the final period. In this case, the objective functions (53) and (54) become those in the baseline model (i.e., (14) and (21)).

4 Conclusion

We analyze a dynamic model of a monopolistic informed investor who receives private information on an ongoing basis and is subject to a post-trading disclosure requirement each period. We show that constructing the equilibrium of this trading model can be reduced into solving the investor's information allocation problem, which can be transformed into a fictitious consumption-saving problem.

This mathematical equivalence implies that the ideas from the consumption-saving literature, such as the permanent income hypothesis of Friedman (1957) can be applied to our trading model. As a illustration, we show that ideas such as "saving for rainy days," "consumption smoothing," and "precautionary saving" can be directly adapted to shed light on our trading model. Our analysis illustrates how the anticipation of future private information affects the informed investor's trading strategy, the equilibrium market liquidity, and prices today.

We show that this equivalence result arises when the informed investor's commitment value is zero. In this case, the two-player game between the informed investor and the market maker can be reduced into the investor's information-allocation problem, which can be turned into to a consumption-saving problem through relabeling.

Appendix: Proofs

Proof of Theorem 1 and Propositions 1 and 5. We conjecture and verify that after the disclosure in period n, for $n = 0, 1, ..., N-1$, the insider's expected future profits have the following quadratic form in the equilibrium:

$$E(\sum_{i=n+1}^{N} \pi_i | \mathcal{I}_{n+1}^I) = \alpha_n (\sum_{i=1}^{n+1} F_i - P_n^*)^2 + \delta_n, \tag{A.1}$$

where α_n and δ_n are constants. Let $\alpha_N \equiv 0$ and $\delta_N \equiv 0$.

Given (5) and (6), the insider's expected profit after period $n-1$ is

$$\begin{aligned} E(\sum_{i=n}^{N} \pi_i | \mathcal{I}_n^I) &= E[x_n(F - P_n) + \alpha_n(\sum_{i=1}^{n+1} F_i - P_n^*)^2 + \delta_n | \mathcal{I}_n^I] \\ &= x_n(\sum_{i=1}^{n} F_i - P_{n-1}^* - \lambda_n x_n) + \alpha_n(\sum_{i=1}^{n} F_i - P_{n-1}^* - \gamma_n x_n)^2 + \delta_n + \alpha_n \sigma_{F_{n+1}}^2. \end{aligned} \tag{A.2}$$

Given the pricing rule (1) and trading strategies (4), we obtain

$$\begin{aligned} P_n &= E(F | \mathcal{I}_n^M) \\ &= P_{n-1}^* + E[\sum_{i=1}^{n} F_i - P_{n-1}^* | \beta_n(\sum_{i=1}^{n} F_i - P_{n-1}^*) + z_n + u_n] \\ &= P_{n-1}^* + \lambda_n(x_n + u_n) \end{aligned}$$

with

$$\lambda_n = \frac{\beta_n(\Sigma_{n-1} + \sigma_{F_n}^2)}{\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_{z_n}^2 + \sigma_u^2}. \tag{A.3}$$

In deriving (A.3), we have used the relationship $E(\sum_{i=1}^{n} F_i - P_{n-1}^*)^2 = \Sigma_{n-1} + \sigma_{F_n}^2$. Similarly, from (2) and (4), we obtain $P_n^* = P_{n-1}^* + \gamma_n x_n$ with

$$\gamma_n = \frac{\beta_n(\Sigma_{n-1} + \sigma_{F_n}^2)}{\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_{z_n}^2}. \tag{A.4}$$

In the following, we prove Theorem 1 for two cases separately.

Case (i): The insider adopts a pure strategy in period n, $\sigma_{z_n}^2 = 0$.

Since the disclosure fully reveals the insider's private information, we have

$$P_n^* = \sum_{i=1}^{n} F_i, \tag{A.5}$$

$$k_n^2 = \Sigma_{n-1} + \sigma_{F_n}^2. \tag{A.6}$$

From (A.6) and (17), we obtain $\Sigma_n = \Sigma_{n-1} + \sigma^2_{F_n} - k^2_n = 0$, which implies (10).

From (4) and (6), we obtain $P^*_n - P^*_{n-1} = \gamma_n \beta_n (\sum_{i=1}^n F_i - P^*_{n-1})$. Substituting (A.5) into this equation, we obtain

$$\gamma_n = \frac{1}{\beta_n}. \tag{A.7}$$

From (A.2), we obtain the first-order condition (FOC):

$$\left(\sum_{i=1}^n F_i - P^*_{n-1} - 2\lambda_n x_n\right) - 2\gamma_n \alpha_n \left(\sum_{i=1}^n F_i - P^*_{n-1} - \gamma_n x_n\right) = 0. \tag{A.8}$$

Substituting $\sum_{i=1}^n F_i - P^*_{n-1} - \gamma_n x_n = \sum\limits_{i=1}^n F_i - P^*_n = 0$ into (A.8), we solve for x_n:

$$x_n = \frac{1}{2\lambda_n}\left(\sum_{i=1}^n F_i - P^*_{n-1}\right). \tag{A.9}$$

Comparing it with (4), we obtain

$$\beta_n = \frac{1}{2\lambda_n}. \tag{A.10}$$

From (A.3) and (A.10), we obtain

$$\beta_n = \frac{\sigma_u}{\sqrt{\Sigma_{n-1} + \sigma^2_{F_n}}}. \tag{A.11}$$

Substituting $k^2_n = \Sigma_{n-1} + \sigma^2_{F_n}$ into it, we obtain (7). Similarly, from (A.3), (A.6) and (A.7), we obtain (8) and (9). Finally, from these expressions, we obtain the expected period-n profits

$$E[\pi_n] = \beta_n(1 - \lambda_n \beta_n)(\Sigma_{n-1} + \sigma^2_{F_n}) = \frac{k_n}{2}\sigma_u.$$

Note that the second-order condition holds if $\lambda_n > 0$ which is equivalent to $k_n > 0$ and hence always holds in equilibrium.

Case (ii). The informed investor adopts a mixed strategy in period n, i.e., $\sigma^2_{z_n} > 0$. Note that we discuss this case only for $n < N$.

Since the FOC (A.8) holds for all realizations of x_n, we have

$$-\lambda_n + \alpha_n \gamma^2_n = 0, \tag{A.12}$$

$$1 - 2\alpha_n \gamma_n = 0, \tag{A.13}$$

from which, we obtain

$$\gamma_n = 2\lambda_n. \tag{A.14}$$

From (A.3), (A.4) and (A.14), we obtain

$$\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_{z_n}^2 = \sigma_u^2, \tag{A.15}$$

$$\lambda_n = \frac{\beta_n(\Sigma_{n-1} + \sigma_{F_n}^2)}{2\sigma_u^2}. \tag{A.16}$$

From the definition in (11), as well as (4) and (6), we have

$$k_n^2 = \gamma_n^2 Var(x_n) = \gamma_n^2[\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_{z_n}^2].$$

Substituting (A.15) into the above expression, after some algebra, we obtain (9). From (A.14) and (9), we obtain (8). From (A.16) and (8), we obtain (7). Substituting (7) into (A.15), we obtain (10).

Conjecture (A.1) implies that after the disclosure in period $n-1$, for $n = 1, ...N$, the expected future trading profit is

$$E(\sum_{i=n}^{N} \pi_i | \mathcal{I}_n^I) = \alpha_{n-1}(\sum_{i=1}^{n} F_i - P_{n-1}^*)^2 + \delta_{n-1}. \tag{A.17}$$

Comparing the above expression with (A.2), we obtain, for $n = 1, ..., N$,

$$\alpha_{n-1} = \alpha_n(1 - \gamma_n\beta_n)^2 + (1 - \lambda_n\beta_n)\beta_n, \tag{A.18}$$

$$\delta_{n-1} = \delta_n - \lambda_n\sigma_{z_n}^2 + \alpha_n(\sigma_{F_{n+1}}^2 + \gamma_n^2\sigma_{z_n}^2). \tag{A.19}$$

Finally, we derive the results in (14) and (15) in two steps. In the first step, we show that if the insider can commit to his trading strategy, then variable $k_1, ..., k_N$ are determined by (14) and (15) in the commitment equilibrium. In the second step, we show that the commitment equilibrium coincides with the equilibrium without commitment. Hence, (14) and (15) hold in the equilibrium without commitment. Note that this implies that the insider's commitment value is zero and hence proves Proposition 5.

Step 1: Since the market maker is risk neutral and breaks even, the insider's expected profit in period n is equal to the noise trader's expected loss: $E[\pi_n] = \lambda_n\sigma_u^2$. Substituting in the expression of λ_n in equation (8), we obtain

$$E[\pi_n] = k_n\sigma_u/2. \tag{A.20}$$

Hence, the insider's objective is to maximize his expected total profit:

$$\max_{k_1,\cdots,k_N} \sum_{i=1}^{N} (k_i \sigma_u / 2), \tag{A.21}$$

subject to the information constraints (15). This is identical to (14) and (15) .

Step 2: In the equilibrium without commitment, the insider takes the market marker's strategy, λ_n, as given to maximize his expected trading profits,

$$\max_{x_1,\ldots,x_N} \sum_{n=1}^{N} E(\pi_n|\lambda_n),$$

where, as noted earlier,

$$E(\pi_n|\lambda_n) = \beta_n(1 - \lambda_n\beta_n)(\Sigma_{n-1} + \sigma^2_{F_n}) - \lambda_n\sigma^2_{z_n}. \tag{A.22}$$

In the commitment equilibrium, however, the insider takes into account the effect of his strategy $(\beta_n, \sigma^2_{z_n})$ affects the market maker's strategy λ_n. That is, his optimization problem is

$$\max_{x_1,\ldots,x_N} \sum_{n=1}^{N} E(\pi_n|\lambda_n(\beta_n, \sigma^2_{z_n})),$$

where, is as in (A.22), with λ_n being replaced by $\lambda_n(\beta_n, \sigma^2_{z_n})$. In the following, we show that all the results (4)–(6) and (7)–(10) remain unchanged when the insider commits to the demand $x_n = \beta_n(\sum_{i=1}^{n} F_i - P^*_{n-1}) + z_n$.

Suppose the insider adopts a pure strategy in period n, i.e., $\sigma^2_{z_n} = 0$. The market maker's strategy is given by (A.3). In the commitment equilibrium, the insider takes into account the effect of β_n on λ_n. Hence, the his objective is to maximize

$$E\sum_{n}^{N} \pi_i = \beta_n \left(1 - \frac{\beta^2_n(\Sigma_{n-1} + \sigma^2_{F_n})}{\beta^2_n(\Sigma_{n-1} + \sigma^2_{F_n}) + \sigma^2_u}\right)(\Sigma_{n-1} + \sigma^2_{F_n}) + E\sum_{n+1}^{N} \pi_i. \tag{A.23}$$

Note that for any nonzero choice β_n, the price fully reveals the insider's private information after disclosure, i.e., $\Sigma_n = 0$. Hence, β_n does not affect his trading profits after period n, $E\sum_{n+1}^{N} \pi_i$. Therefore, to maximize the total expected future trading profits, the insider just needs to maximize the first term of the right hand side of (A.23), which leads to (A.11), the same choice of β_n as in the equilibrium without commitment. That is, the commitment equilibrium coincides with the equilibrium without commitment in this case.

Suppose the insider adopts a mixed strategy in period n, i.e., $\sigma_{z_n}^2 > 0$. In this case, the FOC (A.8) holds regardless of the value of x_n, leading to (A.12) and (A.13). That is, taking into account his effects on the market maker's strategy does not affect the equilibrium conditions in (A.12) and (A.13). Hence, the commitment equilibrium coincides with the equilibrium without commitment in this case.

The results in Proposition 1 follow directly from setting $N = 2$ in Theorem 1.

□

Proof of Proposition 2. The disclosure in the final period fully reveals the insider's private information. This implies $\sum_{i=1}^N k_i^2 = \sum_{i=1}^N \sigma_{F_i}^2 = \sigma_F^2$. After some algebra, we obtain

$$(k_1 - \overline{k})^2 + ... + (k_N - \overline{k})^2 = \sigma_F^2 - \frac{(k_1 + ... + k_N)^2}{N}. \tag{A.24}$$

Hence, minimizing the left-hand side of the above equation is equivalent to maximizing $k_1 + ... + k_N$. Therefore, maximization problem defined by (14) and (15) is equivalent to the minimization problem (22) subject to (23) and (24). Since $\lambda_n = k_n/(2\sigma_u)$, the minimization problems (22) and (25) are equivalent.

Finally, if (26) does not hold, for example, $k_t > k_{t+1}$, then we can define $k_t' = \sqrt{k_t^2 - \epsilon^2}$, $k_{t+1}' = \sqrt{k_{t+1}^2 + \epsilon^2}$, and $k_m' = k_m$ if m is not t or $t+1$, where $0 < \epsilon^2 < k_t^2 - k_{t+1}^2$. It is easy to check that $(k_1', ..., k_N')$ satisfy the information constraint (15) and $\sum k_t' > \sum k_t$, a contradiction. □

Proof of Proposition 3. Perfect smoothing (i.e., $k_n^2 = \sigma_F^2/N, n = 1, \cdots, N$) is feasible if and only if (27) holds. Moreover, perfect smoothing, if feasible, is always adopted in equilibrium. Hence, (27) is necessary and sufficient for perfect smoothing in equilibrium. If (27) holds, in equilibrium $k_n = \sigma_F/\sqrt{N}$. Equations (28) and (29) follow from (8) and (A.20), respectively. Furthermore, $U_n = \Sigma_n + Var(F_{n+1} + \cdots + F_N) = (1 - n/N)\sigma_F^2$. If (27) holds strictly for $n \leq N - 1$, then with $k_n = \sigma_F/\sqrt{N}$, we have $\Sigma_n = \sum_{i=1}^n (\sigma_{F_i}^2 - \sigma_F^2/N) > 0$ for $n \leq N - 1$ and $\Sigma_N = 0$. Theorem 1 implies $\sigma_{z_n}^2 > 0$ for $n \leq N - 1$ and $\sigma_{z_N}^2 = 0$. □

Proof of Proposition 4. In period 2, as in the baseline model, conditional on the information usage k_2, the insider's expected profits satisfy

$$E(\pi_2|k_2) = \frac{k_2}{2}\sigma_u.$$

Equation (31) implies that k_2^2 has two possible values $k_{2,h}$ and $k_{2,l}$ satisfying

$$k_{2,h}^2 = \sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2 + \Delta - k_1^2, \quad k_{2,l}^2 = \sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2 - \Delta - k_1^2.$$

Hence, the ex ante expectation of the insider's total trading profit is

$$E(\pi_1 + \pi_2) = \frac{\sigma_u}{2}\left(k_1 + \frac{1}{2}\sqrt{\sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2 + \Delta - k_1^2} + \frac{1}{2}\sqrt{\sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2 - \Delta - k_1^2}\right),$$

from which we obtain the first order condition

$$1 - \frac{1}{2}\left(\sqrt{\frac{k_1^2}{\sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2 + \Delta - k_1^2}} + \sqrt{\frac{k_1^2}{\sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2 - \Delta - k_1^2}}\right) = 0. \tag{A.25}$$

Note that the second order condition always holds under the budget constraint. Let k_1^* denoted the unique root of the above first order condition.

Let $f(k_1^2)$ denote the left-hand side of (A.25). Note that $f(\cdot)$ is a decreasing function and $f(k_1^2) = 0$ has a unique solution, which is denoted as k_1^*.

Denote $g(\sigma_{F_2}^2, k_1^2) \equiv \sqrt{\frac{k_1^2}{\sigma_{F_1}^2 + \sigma_{F_2}^2 - k_1^2}}$. We can rewrite $f(k_1^2)$ as

$$f(k_1^2) = 1 - E[g(\sigma_{F_2}^2, k_1^2)].$$

Since $g(\sigma_{F_2}^2, k_2^2)$ is convex in $\sigma_{F_2}^2$, Jensen's inequality implies that

$$f(k_1^2) < 1 - E[g(\overline{\sigma}_{F_2}^2, k_1^2)]. \tag{A.26}$$

Substituting $k_1^2 = \frac{\sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2}{2}$ into the above inequality, we have

$$f(\frac{\sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2}{2}) < 1 - E[g(\overline{\sigma}_{F_2}^2, \frac{\sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2}{2}] = 0. \tag{A.27}$$

Since $f(\cdot)$ is a decreasing function and $f(k_1^{*2}) = 0$, we have

$$k_1^{*2} < (\sigma_{F_1}^2 + \overline{\sigma}_{F_2}^2)/2 \le \sigma_{F_1}^2.$$

Under the condition $\sigma_{F_1}^2 \ge \overline{\sigma}_{F_2}^2$, we have $k_1^{*2} < \sigma_{F_1}^2$. That is, the insider adopts a mixed strategy in period one and $k_1 = k_1^*$ in equilibrium. From (A.25), after some algebra, we obtain $\frac{\partial k_1^2}{\partial \overline{\sigma}_{F_2}^2} > 0$ and $\frac{\partial k_1^2}{\partial \Delta} < 0$. $\square$

Proof of Theorem 3. It is easy to check that Theorem 1 still holds when we replace σ_u^2 with $\sigma_{u_n}^2$ for all expressions. Thus (4)–(6) hold and the parameters satisfy (7)–(10) with σ_u^2 replaced with $\sigma_{u_n}^2$. Moreover, from $E\pi_n = \lambda_n \sigma_{u_n}^2 = \frac{k_n \sigma_{u_n}}{2}$, we obtain the objective function (32).

Similarly, we have

$$\sum_{i=1}^{N} \left(k_i - \frac{\sigma_{u_i}(k_1\sigma_{u_1} + \cdots + k_N \sigma_{u_N})}{\sigma_{u_1}^2 + \cdots + \sigma_{u_N}^2} \right)^2 = \sigma_F^2 - \frac{(k_1\sigma_{u_1} + \cdots + k_N\sigma_{u_N})^2}{\sigma_{u_1}^2 + \cdots + \sigma_{u_N}^2}.$$

Hence, the maximization problem (32) is equivalent to minimizing the left-hand side of the above equation, which is equivalent to both (34) and (35). Finally, the results in part 3) are from relabeling.

□

Proof of Theorem 4. Along the path where information leakage has not occurred, the equilibrium is similar to that in Theorem 1 with the only change that α_n and δ_n are replaced by $q\alpha_n$ and $q\delta_n$, respectively, in the conjecture in (A.2). This is because the probability for a public disclosure is q each period, and hence the insider's future profits are discounted by q each period. For example, his expected period-n profits is

$$E\pi_n = q^{n-1}\lambda_n \sigma_u^2 = q^{n-1} k_n \sigma_u / 2.$$

This leads to his objective function (36).

□

Proof of Theorem 5. We conjecture and verify that after the disclosure in period n, for $n = 0, 1, ..., N-1$, the insider's expected future profit still has the quadratic form in (A.1).

After the disclosure in period $n-1$, the insider's expected future profit is

$$\begin{aligned} E[\sum_{i=n}^{N} \pi_i | \mathcal{I}_n^I] = (\sum_{i=1}^{n} F_i - P_{n-1}^* - \lambda_n x_n) x_n + \alpha_n (\sum_{i=1}^{n} F_i - P_{n-1}^* - \lambda_n' x_n - \gamma_n x_n)^2 \\ + \delta_n - \lambda_n \sigma_{z_n}^2 + \alpha_n (\sigma_{F_{n+1}}^2 + {\lambda_n'}^2 \sigma_u^2 + {\gamma_n}^2 \sigma_\epsilon^2). \end{aligned} \tag{A.28}$$

The FOC is

$$(1 - 2\alpha_n(\lambda_n' + \gamma_n))(\sum_{i=1}^{n} F_i - P_{n-1}^*) - (2\lambda_n - 2\alpha_n(\lambda_n' + \gamma_n)^2) x_n = 0 \tag{A.29}$$

and the SOC is (47).

In the case in which the insider adopts a pure strategy, i.e., $x_n = \beta_n(\sum_{i=1}^n F_i - P^*_{n-1})$, we obtain from the FOC that β_n satisfies (46). In the case in which the insider adopts a mixed strategy, $x_n = \beta_n(\sum_{i=1}^n F_i - P^*_{n-1}) + z_n$, the FOC holds for all realizations of x. Hence, we obtain (48) and (49).

Conjecture (A.1) implies that after the disclosure in period $n-1$, for $n = 1, ...N$, the expected future trading profit is

$$E(\sum_{i=n}^{N} \pi_i | \mathcal{I}_n^I) = \alpha_{n-1}(\sum_{i=1}^{n} F_i - P^*_{n-1})^2 + \delta_{n-1}. \tag{A.30}$$

Comparing the above expression with (A.28), we obtain, for $n = 1, ..., N$,

$$\alpha_{n-1} = \alpha_n(1 - (\lambda'_n + \gamma_n)\beta_n)^2 + (1 - \lambda_n\beta_n)\beta_n, \tag{A.31}$$
$$\delta_{n-1} = \delta_n - \lambda_n\sigma^2_{z_n} + \alpha_n(\sigma^2_{F_{n+1}} + {\lambda'_n}^2\sigma^2_u + {\gamma_n}^2\sigma^2_\epsilon + (\lambda'_n + \gamma_n)^2\sigma^2_{z_n}).$$

From projection theorem, with x_n given by (40), we have

$$\begin{aligned} P_n = E(F|\mathcal{I}_n^M) &= P^*_{n-1} + E(F - P^*_{n-1}|x_n + u_n) \\ &= P^*_{n-1} + \lambda_n(x_n + u_n) \end{aligned}$$

with λ_n satisfying (43). Similarly,

$$\begin{aligned} P^*_n &= E[F|P^*_1, \cdots, P^*_{n-1}, x_n + u_n, x_n + \epsilon_n] \\ &= P_n + E[F - P_n|P^*_1, \cdots, P^*_{n-1}, x_n + u_n, x_n + \epsilon_n] \\ &= P_n + E[F - P_n|x_n + \epsilon_n - E(x_n + \epsilon_n|x_n + u_n)] \\ &= P^*_{n-1} + \lambda_n(x_n + u_n) + \gamma_n(x_n + \epsilon_n - E(x_n + \epsilon_n|x_n + u_n)), \end{aligned} \tag{A.32}$$

where

$$\gamma_n = \frac{Cov(\sum_{i=1}^{n} F_i - P_n, x_n + \epsilon_n)}{Var(x_n + \epsilon_n|x_n + u_n)}. \tag{A.33}$$

The numerator and denominator of γ_n are

$$Cov(\sum_{i=1}^{n} F_i - P_n, x_n + \epsilon_n) = \frac{\beta_n(\Sigma_{n-1} + \sigma^2_{F_n})\sigma^2_u}{\beta^2_n(\Sigma_{n-1} + \sigma^2_{F_n}) + \sigma^2_{z_n} + \sigma^2_u}, \tag{A.34}$$

$$Var(x_n + \epsilon_n|x_n + u_n) = \frac{(\beta^2_n(\Sigma_{n-1} + \sigma^2_{F_n}) + \sigma^2_{z_n})\sigma^2_u}{\beta^2_n(\Sigma_{n-1} + \sigma^2_{F_n}) + \sigma^2_{z_n} + \sigma^2_u} + \sigma^2_\epsilon. \tag{A.35}$$

Substituting (A.34) and (A.34) into (A.33), we obtain (45). Comparing (A.32) with (42), we obtain

$$\lambda_n' = \lambda_n - \gamma_n \frac{Cov(x_n + \epsilon_n, x_n + u_n)}{Var(x_n + u_n)},$$

which leads to (44). For $n = 1, \cdots, N$, we have

$$\begin{aligned}
\Sigma_n &= Var(\sum_{i=1}^{n} F_i | P_1^*, \cdots, P_{n-1}^*, x_n + u_n, x_n + \epsilon_n) \\
&= Var[\sum_{i=1}^{n} F_i - P_n | x_n + \epsilon_n - E(x_n + \epsilon_n | x_n + u_n)] \\
&= Var(\sum_{i=1}^{n} F_i - P_n) - \frac{Cov^2(\sum_{i=1}^{n} F_i - P_n, x_n + \epsilon_n)}{Var(x_n + \epsilon_n | x_n + u_n)},
\end{aligned} \tag{A.36}$$

$$\begin{aligned}
Var(\sum_{i=1}^{n} F_i - P_n) &= Var(\sum_{i=1}^{n} F_i | P_1^*, \cdots, P_{n-1}^*, x_n + u_n) \\
&= Var(\sum_{i=1}^{n} F_i - P_{n-1}^* | \beta_n(\sum_{i=1}^{n} F_i - P_{n-1}^*) + z_n + u_n) \\
&= \frac{(\sigma_{z_n}^2 + \sigma_u^2)(\Sigma_{n-1} + \sigma_{F_n}^2)}{\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_{z_n}^2 + \sigma_u^2}.
\end{aligned} \tag{A.37}$$

Substituting (A.37),(A.34), and (A.35) into (A.36), we obtain

$$\begin{aligned}
\Sigma_n = & \frac{1}{\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_{z_n}^2 + \sigma_u^2}[(\sigma_u^2 + \sigma_{z_n}^2)(\Sigma_{n-1} + \sigma_{F_n}^2) \\
& - \frac{(\beta_n(\Sigma_{n-1} + \sigma_{F_n}^2)\sigma_u^2)^2}{(\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_{z_n}^2)(\sigma_u^2 + \sigma_\epsilon^2) + \sigma_u^2\sigma_\epsilon^2}].
\end{aligned} \tag{A.38}$$

□

Proof of Proposition 6. We first construct the equilibrium in the first $N-1$ periods. Specifically, we construct the equilibrium under the conjecture that the insider adopts a mixed strategy in all of the first n periods, and then verify that conditions in (52) are necessary and sufficient for $\sigma_{z_n}^2 > 0$ for $n \leq N-1$, i.e., the insider adopts a mixed strategy for each period.

Suppose the insider adopts a mixed strategy in period n. As in the proof for Theorem 1, the FOC leads to (48) and (49), from which we obtain

$$\alpha_n = \frac{1}{4\lambda_n}. \tag{A.39}$$

Substituting (A.39) and (44) into (48), after some algebra, we obtain

$$\lambda_n = \frac{\gamma_n \sigma_u^2}{\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_{z_n}^2 + \sigma_u^2}. \tag{A.40}$$

Substituting (43) and (45) into (A.40), we obtain

$$\beta_n^2(\Sigma_{n-1} + \sigma_{F_n}^2) = \frac{\sigma_u^2(\sigma_u^2 - \sigma_\epsilon^2)}{\sigma_u^2 + \sigma_\epsilon^2} - \sigma_{z_n}^2. \tag{A.41}$$

Substituting (A.41) into (A.38), we get

$$\Sigma_n = \frac{(\Sigma_{n-1} + \sigma_{F_n}^2)(\sigma_u^2\sigma_\epsilon^2 + \sigma_{z_n}^2(\sigma_u^2 + \sigma_\epsilon^2))}{\sigma_u^4}. \tag{A.42}$$

Substituting (A.42) into $k_n^2 = \Sigma_{n-1} + \sigma_{F_n}^2 - \Sigma_n$, we have

$$k_n^2 = (\Sigma_{n-1} + \sigma_{F_n}^2)\left(1 - \frac{\sigma_u^2\sigma_\epsilon^2 + \sigma_{z_n}^2(\sigma_u^2 + \sigma_\epsilon^2)}{\sigma_u^4}\right). \tag{A.43}$$

Solving for $\sigma_{z_n}^2$ from the above equation, we obtain

$$\sigma_{z_n}^2 = \left(\frac{\sigma_u^2 - \sigma_\epsilon^2}{\sigma_u^2} - \frac{k_n^2}{\Sigma_n + k_n^2}\right)\frac{\sigma_u^4}{\sigma_u^2 + \sigma_\epsilon^2}. \tag{A.44}$$

Substituting (A.44) into (A.41), we get

$$\beta_n = \frac{k_n \sigma_u^2}{(\Sigma_n + k_n^2)\sqrt{\sigma_u^2 + \sigma_\epsilon^2}}. \tag{A.45}$$

Substituting (A.41) and (A.45) into (45), we get

$$\gamma_n = \frac{k_n}{\sqrt{\sigma_u^2 + \sigma_\epsilon^2}}. \tag{A.46}$$

Substituting (A.41) and (A.44) into (43), we get

$$\lambda_n = \frac{\sqrt{\sigma_u^2 + \sigma_\epsilon^2}}{2\sigma_u^2}k_n. \tag{A.47}$$

Substituting (A.47), (A.46), and (A.41) into (44), we get

$$\lambda_n' = \frac{\sigma_\epsilon^2 k_n}{\sigma_u^2\sqrt{\sigma_u^2 + \sigma_\epsilon^2}}.$$

Finally, we show that (52) is the necessary and sufficient condition for $\sigma_{z_n}^2 > 0$ for $n = 1, ..., N-1$. From the expression of $\sigma_{z_n}^2$ (A.44), the necessary and sufficient condition for $\sigma_{z_n}^2 > 0$ is

$$k_n^2 < \frac{\sigma_u^2 - \sigma_\epsilon^2}{\sigma_u^2}(\Sigma_{n-1} + \sigma_{F_n}^2). \tag{A.48}$$

Since in equilibrium, the use the same amount of information each period, we have

$$k_n^2 = \frac{\Sigma_{n-1} + \sigma_{F_n}^2 + \cdots + \sigma_{F_N}^2}{N - n + 1}.$$

Substituting it into (A.48), we get (52).

As in the main model, we can obtain the following:

$$\beta_N = \frac{1}{2\lambda_N}, \quad \lambda_N = \frac{\beta_N(\Sigma_{N-1} + \sigma_{F_N}^2)}{\beta_N^2(\Sigma_{N-1} + \sigma_{F_N}^2) + \sigma_u^2}, \quad \sigma_{z_N} = 0,$$

from which we obtain the final period results in the proposition.

The proof that the commitment value is zero is similar to the proof of Theorem 1. Hence, we obtain (53) from the relationship $E\pi_n = \lambda_n \sigma_u^2$.

Finally, the results in (54) can be obtained from relabeling.

□

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Supplemental Appendix

Appendix A Trading Strategy and Asset prices

To further illustrate the implications in Proposition 3, we consider the following two cases. In Case 1, the insider's information arrives at a decreasing rate, that is

$$\sigma^2_{F_n} > \sigma^2_{F_{n+1}} \text{ for } n = 1, ..., N-1. \tag{A.49}$$

In Case 2, the insider's information arrives at an increasing rate, that is

$$\sigma^2_{F_n} < \sigma^2_{F_{n+1}} \text{ for } n = 1, ..., N-1. \tag{A.50}$$

These two cases are a generalized version of the two cases in the two-period example in Propostion 1. Condition (A.49) in Case 1 is a special case of (27). Hence, as shown in Proposition 3, the insider adopts a mixed strategy in all but the last period and perfectly smooths his information usage over time $k^2_n = \sigma^2_F/N$, for $n = 1, ..., N$. Perfect smoothing is not feasible in Case 2. Since the private information arrives at an increasing rate, the insider does not possess enough private information in early rounds to utilize σ^2_F/N information each period. The equilibrium in this case is summarized in the following proposition.

Proposition 7. *Under the conditions in (A.50), the insider adopts a pure strategy in every period and the equilibrium in period n, for $n = 1, ..., N$, has the following properties:*

$$k_n = \sigma_{F_n}, \tag{A.51}$$

$$\lambda_n = \frac{\sigma_{F_n}}{2\sigma_u}, \tag{A.52}$$

$$E[\pi_n] = \frac{\sigma_{F_n}\sigma_u}{2}, \tag{A.53}$$

$$U_n > (1 - n/N)\sigma^2_F, \ \textit{if } n < N. \tag{A.54}$$

Anticipating more private information in the future, as shown in (A.51), the insider utilizes all his private information (i.e., adopts a pure strategy) each period. It has been noted in the literature that a monopolistic insider has the incentive to minimize the price impact by either breaking down his order into small ones (Kyle, 1985) or by adding noise to his order Huddart et al. (2001) to "go down" the market maker's demand curve. Proposition 7 shows that the anticipation of future private information expedites the insider's usage of his private information. It generalizes the results in the two-period example in Proposition 1 and shows that when private information arrives at an increasing rate, the insider chooses to fully utilize his private information each period. Moreover, since the insider utilizes information at an increasing rate, his price impact and expected trading profits also increase over time, as shown in equations (A.52) and (A.53). Finally, relative to the equilibrium in Case 1, the insider utilizes less private information and hence the stock price informativeness is lower (i.e., U_n is higher) in all but the final periods (see (30) and (A.54)).

To further illustrate the equilibrium, we analyze a numerical example of Cases 1 and 2. Specifically, we set $N = 10$, $\sigma_F^2 = 1$, and $\sigma_u^2 = 0.1$. The insider's private information arrives at a linearly decreasing rate in Case 1:

$$\sigma_{F_n}^2 = \frac{2(N-n+1)}{N(N+1)}\sigma_F^2, \tag{A.55}$$

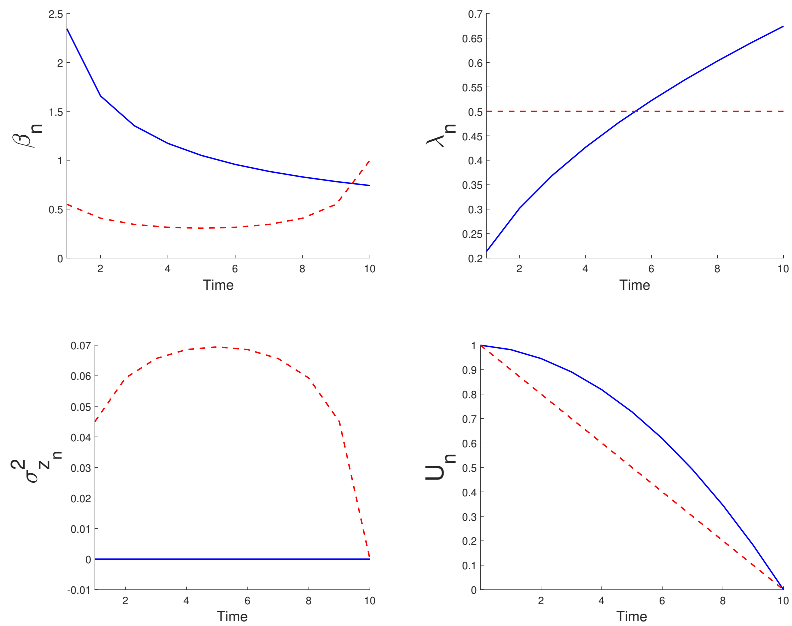
and at a linearly increasing rate in Case 2:

$$\sigma_{F_n}^2 = \frac{2n}{N(N+1)}\sigma_F^2. \tag{A.56}$$

The equilibria in these two cases are summarized in Figure 2. The upper left panel plots the trading intensity β_n against the trading period n. The dashed line and solid line represents Cases 1 and 2, respectively. In all but the final period, the insider trades less aggressively in Case 1 than in Case 2. This is because, in Case 1, the insider anticipates less private information in later periods and hence trades less aggressively to save his information for future use. In contrast, when anticipating more private information in Case 2, the insider would exploit his current information more aggressively in early periods.

The upper right panel plots the price impact. As shown in Proposition 3, in Case 1 (illustrated by the dashed line), the insider utilizes the same amount of information each period, leading to a constant price impact. The solid line shows that the price impact increases over time in Case 2. This is because the insider's information ar-

Figure 2. Equilibrium under Monotonic Information Arrivals
This figure plots the trading intensity β_n, price impact λ_n, the noise in the insider's demand $\sigma^2_{z_n}$, and price uncertainty U_n respectively, for the case with a decreasing information arrival rate as in equation (A.55) (dashed line) and the case with an increasing information arrival rate as in equation (A.56) (solid line). Parameter values: $\sigma^2_F = 1, \sigma^2_u = 0.1$, and $N = 10$.



rives at an increasing rate and, as shown in Proposition 7, the insider utilizes all his information each period.

The lower left panel reports how the insider dissimulates his private information. The dashed line shows that in Case 1, the case with a decreasing information arrival rate, the insider adopts a mixed strategy (i.e., $\sigma^2_{z_n} > 0$) in all but the last period. In contrast, as shown by the solid line, the insider always adopts a pure strategy (i.e., $\sigma^2_{z_n} = 0$) in Case 2.

Finally, the lower right panel plots U_n, the uncertainty about the liquidation value conditional on asset price history till period n, against time n. The dashed line shows that in Case 1, the insider utilizes the same amount of information each period and hence the uncertainty decreases linearly. In Case 2, where the insider's possesses less private information in earlier periods. Although all private information is revealed each period, the uncertainty still decreases more slowly than in Case 1 (i.e., the solid line is above the dashed line).

Proof of Proposition 7. We prove this by contradiction. Suppose the insider does not play a pure strategy in all periods. Let $t1$ denote the first period when the insider adopts a mixed strategy, i.e.,

$$t1 \equiv \min\{n, \sigma_{z_{t1}}^2 > 0\}. \tag{A.57}$$

Theorem 1 implies that the information received in period $t1$ is not used up:

$$k_{t1}^2 < \sigma_{F_{t1}}^2. \tag{A.58}$$

Moreover, since all private information is used after the final period, there exists a $t2$ such that $t2 > t1$ and more information is used than received in period $t2$:

$$k_{t2}^2 > \sigma_{F_{t2}}^2. \tag{A.59}$$

From (A.50), (A.58), and (A.59), we have $k_{t1}^2 < \sigma_{F_{t1}}^2 < \sigma_{F_{t2}}^2 < k_{t2}^2$.

This leads to a contradiction: the insider can increase his total trading profit by slightly increasing his information usage in $t1$ and decrease decrease his usage in $t2$. Specifically, if k_{t1} and k_{t2} are replaced by $k_{t1}' = \sqrt{k_{t1}^2 + \epsilon^2}$ and $k_{t2}' = \sqrt{k_{t2}^2 - \epsilon^2}$ respectively, with $\epsilon^2 \in (0, k_{t2}^2 - \sigma_{F_{t2}}^2)$, and $k_m' = k_m, m \neq t1, t2$, then

$$\sum_{n=1}^{N} k_n' > \sum_{n=1}^{N} k_n. \tag{A.60}$$

Therefore, the insider adopts a pure strategy in all periods, leading to (A.51)–(A.53), as well as $U_n = \sigma_{F_{n+1}}^2 + \cdots + \sigma_{F_N}^2$, which leads to (A.54). □

Appendix B The 2-Period Example in Section 3.4

Consider the model in Section 3.4 with $N = 2, \sigma_{F_1} = \sigma_F, \sigma_{F_2} = 0, \sigma_\epsilon^2 = \infty$.

The equilibrium without commitment. The equilibrium was derived in Proposition 1 of Huddart et al. (2001), and we summarize it below.

$$\beta_1 = (2\lambda_2 - \lambda_1)/(\lambda_1(4\lambda_2 - \lambda_1)), \tag{A.61}$$

$$\beta_2 = 1/(2\lambda_2), \tag{A.62}$$

$$\lambda_1 = \beta_1 \Sigma_1/\sigma_u^2, \tag{A.63}$$

$$\lambda_2 = \beta_2 \Sigma_2/\sigma_u^2, \tag{A.64}$$

$$\Sigma_1 = (1 - \lambda_1\beta_1)\sigma_F^2, \tag{A.65}$$

$$\Sigma_2 = (1 - \lambda_2\beta_2)\Sigma_1, \tag{A.66}$$

$$\lambda_2/\lambda_1 \approx 0.901. \tag{A.67}$$

From (A.61), (A.65), and (A.67),

$$k_1^2 = \sigma_F^2 - \Sigma_1 = \frac{2\lambda_2 - \lambda_1}{4\lambda_2 - \lambda_1}\sigma_F^2 \approx 0.308\sigma_F^2. \tag{A.68}$$

From (A.62), (A.66) and (A.68),

$$k_2^2 = \Sigma_1 - \Sigma_2 = \lambda_2\beta_2\Sigma_1 = \frac{1}{2}(\sigma_F^2 - k_1^2) \approx 0.346\sigma_F^2. \tag{A.69}$$

Now, we solve for $E\pi_n$. Projection theorem gives

$$\lambda_1 = \frac{\beta_1\sigma_F^2}{\beta_1^2\sigma_F^2 + \sigma_u^2}. \tag{A.70}$$

We can rewrite λ_1 as

$$\lambda_1 = \frac{\lambda_1\beta_1\sigma_F^2}{\lambda_1\beta_1^2\sigma_F^2 + \lambda_1\sigma_u^2}. \tag{A.71}$$

From (A.61), (A.67), and (A.71),

$$\lambda_1 = \frac{\sigma_F}{\sigma_u}\sqrt{(\lambda_1\beta_1) - (\lambda_1\beta_1)^2} \approx 0.4617\frac{\sigma_F}{\sigma_u}. \tag{A.72}$$

Then from (A.67)

$$\lambda_2 \approx 0.4159 \frac{\sigma_F}{\sigma_u}. \tag{A.73}$$

Finally, from $E\pi_n = \lambda_n \sigma_u^2$, we have

$$E\pi_1 \approx 0.4617 \sigma_u \sigma_F, \quad E\pi_2 \approx 0.4159 \sigma_u \sigma_F. \tag{A.74}$$

Commitment equilibrium. In this commitment equilibrium, the insider commits to the following strategy in period $n = 1, 2$,

$$x_n = \beta_n (F - P_{n-1}). \tag{A.75}$$

The market maker takes the commitment as given to set the price as

$$P_n = E[F | x_1 + u_1, \cdots, x_n + u_n] = P_{n-1} + \lambda_n (x_n + u_n), \tag{A.76}$$

where

$$\lambda_n = \frac{\beta_n (\Sigma_{n-1} + \sigma_{F_n}^2)}{\beta_n^2 (\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_u^2}. \tag{A.77}$$

Note that, in this economy with commitment, the insider takes into account of the effect of his choice of β_n on the market maker's response in (A.77).

From the projection theorem, we have

$$\Sigma_n = \frac{(\Sigma_{n-1} + \sigma_{F_n}^2) \sigma_u^2}{\beta_n^2 (\Sigma_{n-1} + \sigma_{F_n}^2) + \sigma_u^2}. \tag{A.78}$$

Substituting $\Sigma_n = \Sigma_{n-1} + \sigma_{F_n}^2 - k_n^2$ into the above equation, we obtain

$$\beta_n^2 = \frac{k_n^2 \sigma_u^2}{(\Sigma_{n-1} + \sigma_{F_n}^2)(\Sigma_{n-1} + \sigma_{F_n}^2 - k_n^2)}. \tag{A.79}$$

Hence, choosing β_n is equivalent to choosing k_n^2.

Substituting (A.79) into the expression of (A.77), we can get

$$\lambda_n = \frac{1}{\sigma_u} \sqrt{\frac{(\Sigma_{n-1} + \sigma_{F_n}^2 - k_n^2) k_n^2}{\Sigma_{n-1} + \sigma_{F_n}^2}}. \tag{A.80}$$

In equilibrium, the insider's expected trading profit in period n is

$$E\pi_n = \lambda_n \sigma_u^2 = \sigma_u \sqrt{\frac{(\Sigma_{n-1} + \sigma_{F_n}^2 - k_n^2)k_n^2}{\Sigma_{n-1} + \sigma_{F_n}^2}}. \tag{A.81}$$

In period 2, the insider's objective function is

$$\max_{k_2^2} E\pi_2. \tag{A.82}$$

Solving it, we obtain

$$k_2^2 = \frac{\Sigma_1}{2} = \frac{\sigma_F^2 - k_1^2}{2}, \tag{A.83}$$

$$E\pi_2 = \frac{\sigma_u}{2}\sqrt{\sigma_F^2 - k_1^2}. \tag{A.84}$$

From (A.81) and (A.84), the maximization problem in period 1 is

$$\max_{k_1^2} E(\pi_1 + \pi_2) = \max_{k_1^2} \sigma_u \sqrt{\frac{(\sigma_F^2 - k_1^2)k_1^2}{\sigma_F^2}} + \frac{\sigma_u}{2}\sqrt{\sigma_F^2 - k_1^2}. \tag{A.85}$$

Hence, we obtain the following:

$$k_1^2 \approx 0.352\sigma_F^2, \quad k_2^2 \approx 0.324\sigma_F^2, \quad E\pi_1 \approx 0.477\sigma_F\sigma_u, \quad E\pi_2 \approx 0.403\sigma_F\sigma_u. \tag{A.86}$$