Discussion of

“Kyle Meets Friedman: Informed Trading When Anticipating Future Information”

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Idea of Paper

- My 1985 paper: starting point.
- Huddart, Hughes, and Levine (2001): Adds required disclosure of trades by the informed trader.
- Chau and Vayanos (2008): Add arrivals of new information, without disclosures.
- This paper does both: Has required disclosure of trades and makes information arrival sequential.

Results of this paper:

- Equilibrium solution is surprisingly simple.
- Develop analogy with permanent income hypothesis of Friedman (1953).

Outline of my comments

- Review my 1985 paper.
- Mixed strategy equilibrium is equivalent to delaying arrival of information even further.
- Despite apparent complexity added by using mixed strategies to optimize disclosure, both this paper's model and Huddart, Hughes, Levine model have striking simple solutions.
- Continuous-time extension might be interesting.
- Analogy with exhaustible resource monopolist may be better than analogy with permanent income hypothesis.

1985 paper: One-period model

- Informed trader observes liquidation value $F \sim \mathcal{N}(0, \sigma_F^2)$. Trades quantity $x = X(F)$ to maximize expected profit.
- Noise traders trade exogenous quantity $u \sim \mathcal{N}(0, \sigma_U^2)$.
- Risk neutral, competitive market makers set price $p = P(x + u)$.

Solution is linear:

$$X(F) = \beta \cdot F, \qquad P(y) = \lambda \cdot y,$$

$$\text{where} \qquad \beta = \frac{\sigma_U}{\sigma_F}, \qquad \lambda = \frac{1}{2} \cdot \frac{\sigma_F}{\sigma_U}, \qquad \text{expected profit} = \tfrac{1}{2} \cdot \sigma_F \cdot \sigma_U.$$

Intution for $1/2$: Informed trader exercises monopoly power by incorporating one-half of information into prices.

1985 paper: Continuous-time version

Time unfolds over continuous interval $[0, 1]$.

- Informed trader observes liquidation value $F \sim \mathcal{N}(0, \sigma_F^2)$. Trades quantity $\mathrm{d}x(t)$ to maximize expected profit.
- Noise traders' inventory follows Brownian motion $\mathrm{d}u := \sigma_U \cdot \mathrm{d}B(t)$.
- Risk neutral, competitive market makers set price $p(t) = P(\mathcal{H}(t))$, where $\mathcal{H}(t)$ is the history of past prices and order flow $\mathrm{d}x + \mathrm{d}u$).

Solution is again linear:

$$\mathrm{d}x(t) = \beta(t) \cdot (F - p(t)), \qquad \mathrm{d}p(t) = \lambda \cdot (\mathrm{d}x + \mathrm{d}y),$$

$$\text{where} \qquad \beta(t) = \frac{1}{1-t} \cdot \frac{\sigma_U}{\sigma_F}, \qquad \lambda = \frac{\sigma_F}{\sigma_U}, \qquad \text{expected profit} = \sigma_F \cdot \sigma_U.$$

Expected profit is twice as high in continuous model than in single-period model.

1985 paper: N-period version

- Solution is not so easy any more: Dynamic programming problem whose solution is a system of differences equations. Numerically unstable.
- Market impact parameter λ_n is approximately constant from auction to auction but declines towards the end.
- Solution converges to continuous model for large n.

Comments of 1985 paper

- Amount of remaining information, $\Sigma(t) = \text{var}[F - p(t)]$ decreases linearly from σ_F^2 to 0 in continuous version.
- Informed trader does not "use noise trading as camouflage". Intuition is a Nash equilibrium.
- Even though single-period and N-period models are strictly convex with unique pure strategies, continuous model has a linear (non-strictly convex) objective, so informed trader is indifferent between optimal strategy and other strategies.

Huddart, Hughes, Levine (2001)

Like 1985 paper, except informed trader reveals quantity traded after each round of trading.

- HHL use dynamic programming, similar to N-period model, but need to randomize the quantity traded each period by adding a random variable (mixed strategy).
- This approach makes the dynamic programming problem somewhat more complicated.

My comment 1: Consider a different approach

Fundamental value F can be expressed as the sum of independently distributed components $v_n \sim \mathcal{N}(0, \sigma_n^2)$. With $N = 3$ periods,

$$F = v_1 + v_2 + v_3, \qquad \text{where} \qquad \Sigma = \text{var}[F] = \sigma_1^2 + \sigma_2^2 + \sigma_3^2.$$

- Components obviously not uniquely defined. Obtained by randomization, like mixed strategy.
- For example, set $v_n := \mathbb{E}[F \mid F + \epsilon_m, m = 1, \ldots, n]$, and vary $\text{var}[\epsilon_n]$ to obtain desired value for $\text{var}[v_n]$.
- Informed trader "pretends" to observe v_n just before trade in period n, uses all of the signal for trading in period n without further mixing.
- Trade in period n is like a one-period 1985 model, with profit $\frac{1}{2} \cdot \sigma_n \cdot \sigma_U$.
- Trader chooses values for σ_n to maximize expected profits $\sum_{n=1}^{N} \frac{1}{2} \cdot \sigma_n \cdot \sigma_U$, subject to the constraint $\sum_{n=1}^{N} \sigma_n^2 = \sigma_F^2$.
- The solution is to make the variances of σ_n^2 all the same: $\sigma_n^2 = \frac{1}{N} \cdot \sigma_F^2$.
- Outcome exactly the same as Huddart, Hughes, Levine (2001), but much simpler.

Information arrival in this paper

This paper generalizes HHL by assuming some information actually does arrive later: $F = \sum_{n=1}^{N} F_n$.

- Informed trader again "pretends" that some of the signals F_n arrive as sums of random variables spread out over later periods, but not earlier, e.g., pretend $F_n = v_{n,n} + v_{n,n+1} + v_{n,n+2}$ with $v_{n,n+i}$ arriving at time $n + i$.
- The expected-profit maximizing strategy tries to equalize the variance of the signal each period, with each period like the one-period version of 1985 paper.
- When this is not possible, some variance is pushed into the future (because the trader cannot move information from the future to the present), which leads to exactly the simple optimization problem in the paper and exactly the solution in the paper.
- While the informed trader pretends to observe information later than it is actually observed, he does not need to commit to this. Thus, ability to commit does not change the equilibrium, consistent with the paper's discussion.

My comment 2

Paper considers various generalizations:

- Time-varying noise trading
- Information leakage (discounting future information)
- Imperfect disclosure: Revealed quantity is noisy.

Here is another one:

- Consider a generalization of the continuous-time version of the 1985 model, with disclosure at a finite number of dates (end of days) and information arrival spread out over time (one month).

Equilibrium with continuous-time generalization

- Informed trader "pretends" that information arrives later, as in the actual model.
- Within each day, the equilibrium looks like the continuous equilibrium: All information is incorporated into prices by the end of the day.
- Thus, disclosure does not move prices, and disclosure does not cost the informed trader anything!
- Profits are the same as in the continuous equilibrium!
- This intuition suggests that the continuous equilibrium in the 1985 model is not unique (!), consistent with the intuition that the linear (not quadratic) objective does not pin down the equilibrium exactly (even though the limit of the discrete time equilibria is pinned down uniquely!).

Comment 3: Analogy with Friedman's permanent income hypothesis

The paper develops an analogy with Friedman's permanent income hypothesis:

- Think of the variance of nonrevealed information $\Sigma_n := \text{var}_n[F]$ as capital stock, whose value (wealth) generates permanent income.
- Since one-period profits are proportional to standard deviation of information used, but information used is in variance units, this is like having a square root utility function of consumption.
- Informed trader smooths consumption, consistent with permanent income hypothesis.

Analogy with an exhaustible resource monopolist

Another equivalent intuition is to think of Σ_n as the stock of an exhaustible resource like oil:

- Inventories increase when new discoveries are made.
- Inventories are reduced when oil is sold to consumers.
- The exhaustible resource is owned by a monopsonist (single informed trader).
- The exhaustible resource does not decay because the monopolist is the sole source of new information. Public information or discounting would make it decay. So would informed trade by competitors.
- The rate of sales depends on the demand function for consumption of the exhaustible resource, again like square root utility.
- Oil cannot be consumed before it is discovered. This could lead to back-loading of consumption.

This analogy yields intuition similar to the paper.

Chau and Vayanos (2008)

Chau and Vayanos (2008) have infinite-horizon model similar to 1985 discrete-time model, except that there is discounting at rate r and arrival of new signals each period. Informed trader does not reveal trades.

- Positive interest rate pushes information revelation towards the present.
- In equilibrium, the informed trader may run his stock of information to near zero.
- This make prices almost full-form efficient, even though trading continues to be profitable.
- The seemingly strange intuition that full-form efficiency is compatible with profitable trading is consistent with the analogy of the permanent income hypothesis or exhaustible resource monopolist when savings or inventories are very low due to continued arrivals of new income or oil discoveries.

Summary

- Nice paper!
- Complicated mixed strategy analysis of HHL could be replaced by pretending to receive information later. Simplified analysis immediately leads to results in paper.
- Continuous trading within “days” is an interesting generalization, which leads to no cost from disclosure.
- Analogy with Friedman’s permanent income hypothesis could be replaced by analogy with exhaustible resource monopolist.
- Paper might consider more discussion of Chau and Vayanos (2008).