Kyle Meets Friedman: Informed Trading When Anticipating Future Information

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How Do Asset Prices Distill Investors' Information?

"Information leakage" via prices and quantities:

- Trading prices, e.g., Kyle (1985).
- Disclosure: Trade quantities are disclosed/detected:
  - Full disclosure, e.g., insider trading laws (Huddart et al., 2001);
  - Partial disclosure, e.g., regulatory filings by mutual funds, ETFs, and hedge funds;
  - Detection of the informed investor's trades (Yang and Zhu, 2020).
- We model both
  - One informed trader
  - **Sequential private information**
  - Post-trading (partial) disclosure

Main Results: Kyle Meets Friedman

- 2-P Model ⇒ Info-usage Problem ⇔ Consumption Problem
  - information usage → consumption
  - information arrival → income
  - cumulated unused information → wealth

- Friedman (1957): permanent income hypothesis
  - Rainy days, Consumption smoothing, Precautionary saving
  - C_t depends on the expectation of lifetime income.

  - Trading, price discovery, and liquidity
  - Trading depends on current and expected future info

- Why can we transform a 2-P model into a 1-P one?
  - The insider's **commitment value** is zero
  - 2-P equilibrium ⇒ 1-P optimization

Model

Setup

- Kyle (1985) is extended with
  - Post-trading disclosure (Huddart et al., 2001)
  - Sequential information arrivals

- N trading periods: $n = 1, ..., N$

- One risky asset with final liquidation value, $F \sim \mathcal{N}(0, \sigma_F^2)$,

$$F \equiv \sum_{n=1}^{N} F_n$$

where $F_n \sim \mathcal{N}(0, \sigma_{F_n}^2)$ and is serially independent.

Players and Information

- Two liquidity demanders:
  - One risk-neutral informed investor: Insider
    - observes F_n in period n
    - trades x_n shares
  - Noise traders demand $u_n \sim \mathcal{N}(0, \sigma_u^2)$
    - Wrong beliefs; hedging; ESG; liquidity (love): private value
- One liquidity supplier: Risk-neutral market maker
  - observes the aggregate order flow: $y_n = x_n + u_n$
  - sets the trading price P_n
  - Bertrand competition or representative aggregation of the rest of the market

Post-Trade Disclosure

- The insider must disclose after his trade $d_n = x_n + \epsilon_n$, with $\epsilon_n \sim \mathcal{N}(0, \sigma_\epsilon^2)$
  - Perfect disclosure (HHL, 2001): $\sigma_\epsilon^2 = 0$
  - Imperfect disclosure: $\sigma_\epsilon^2 > 0$
  - Kyle (1985): $\sigma_\epsilon^2 = \infty$

- **Baseline model**: $\sigma_\epsilon^2 = 0$
  - The market maker's information set in period n:

$$\mathcal{I}_n^M \equiv \{y_1, ..., y_n, x_1, ..., x_{n-1}\}$$
$$\mathcal{I}_{n+}^M \equiv \{y_1, ..., y_n, x_1, ..., x_{n-1}, x_n\}$$

  - HHL (2001) is a special case
    - $\sigma_{F_1} = \sigma_F$.
    - $\sigma_{F_i} = 0$, for $i = 2...N$.

Decisions in Period n

- At the trading time, the market maker sets the price to

$$P_n = E[F|\mathcal{I}_n^M],$$

After disclosure, the market maker adjusts the price to

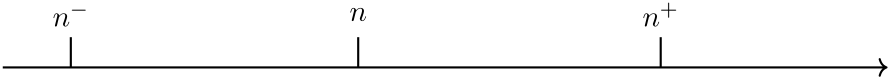
$$P_n^* = E[F|\mathcal{I}_{n+}^M].$$

- The informed investor:

$$\max_{x_n,...,x_N} E\left[\sum_{j=n}^{N} \pi_j|\mathcal{I}_n^I\right],$$

where $\mathcal{I}_n^I \equiv \{F_1, ..., F_n, P_1, ..., P_{n-1}, P_1^*, ..., P_{n-1}^*\}$.

Timeline



The insider observes F_n.

- An insider and noise traders submit x_n and u_n respectively;
- Market maker observes $y_n = x_n + u_n$, sets price as P_n, and fills all demands.

- The insider announces publicly x_n and market maker updates the price to P_n^*;
- If $n = N$, F is announced.

Equilibrium and Equivalence

Linear Equilibrium

- Conjecture and verify a linear equilibrium:

$$x_n = \beta_n(\sum_{i=1}^{n} F_i - P^*_{n-1}) + z_n,$$
$$P_n = P^*_{n-1} + \lambda_n y_n,$$
$$P^*_n = P^*_{n-1} + \gamma_n x_n,$$

where $z_n \sim \mathcal{N}(0, \sigma^2_{z_n})$, $P^*_0 = 0$.

- $\{\beta_n, \lambda_n, \gamma_n, \sigma_{z_n}\}$ are determined in equilibrium.
- Pure strategy: $\sigma^2_{z_n} = 0$, fully reveals the insider's info
- Mixed strategy: $\sigma_{z_n} > 0$, saves info for future use
- $k^2_n \equiv Var(P^*_n - P^*_{n-1})$: info used in period n

Equilibrium Characterization

Theorem (Proof)

There is a unique linear equilibrium with,

$$\beta_n = \frac{k_n \sigma_u}{\Sigma_n + k_n^2}, \lambda_n = \frac{k_n}{2\sigma_u}, \gamma_n = \frac{k_n}{\sigma_u}, \sigma_{z_n}^2 = \frac{\Sigma_n}{\Sigma_n + k_n^2}\sigma_u^2, \qquad (1)$$

where $\Sigma_n = \sum_{i=1}^{n} \sigma_{F_i}^2 - \sum_{i=1}^{n} k_i^2$,
and $\{k_1, \cdots, k_N\} \in \mathbb{R}_{\geq 0}^N$ *are the unique solution to*

$$\max_{\{k_1,\cdots,k_N\} \in \mathbb{R}_{\geq 0}^N} (k_1 + ... + k_N), \qquad (2)$$

$$\textit{subject to} \quad \sum_{i=1}^{n} k_i^2 \leq \sum_{i=1}^{n} \sigma_{F_i}^2, \ \textit{for } n = 1, ..., N. \qquad (3)$$

Equivalence to a Consumption-Saving Problem

- **Reduced to a 1-player Info Usage Problem:**

$$\max_{\{k_1,\cdots,k_N\}\in\mathbb{R}^N_{\geq 0}} k_1 + ... + k_N,$$

$$\text{s.t.} \qquad \sum_{i=1}^{n} k_i^2 \leq \sum_{i=1}^{n} \sigma_{F_i}^2, \text{ for } n = 1, ..., N.$$

- **Equivalent to a Consumption Problem:**

$$\max_{\{C_1,\cdots,C_N\}\in\mathbb{R}^N_{\geq 0}} u(C_1) + ... + u(C_N),$$

$$\text{s.t.} \qquad \sum_{i=1}^{n} C_i \leq \sum_{i=1}^{n} Y_i, \text{ for } n = 1, ..., N.$$

where $u(C) = \sqrt{C}$, CRRA with $RRA = 1/2$.

Transformation by Relabeling

Trading game with disclosure	Consumption-saving problem
Information usage k_n^2	Consumption C_n
Expected profits $k_n \sigma_u / 2$	Utility $\sqrt{C_n}$
Information endowment $\sigma_{F_n}^2$	Income Y_n
Unused information amount Σ_n	Wealth S_n
Asymmetric information transfer	Borrowing constraint
$k_n^2 \leq \Sigma_{n-1} + \sigma_{F_n}^2$	$C_n \leq S_n + Y_n$
• If $k_n^2 < \Sigma_{n-1} + \sigma_{F_n}^2$, "mixed"	• If $C_n < S_n + Y_n$, "save"
• If $k_n^2 = \Sigma_{n-1} + \sigma_{F_n}^2$, "pure"	• If $C_n = S_n + Y_n$, "consume all"

Kyle Meets Friedman

Permanent Income Hypothesis (Friedman, 1957)

1. Saving for rainy days

2. Consumption smoothing

3. Precautionary saving

- Implications on the trading model
  - Asset prices
  - Informativeness
  - Market liquidity

1: Saving for Rainy Days

- Saves more today if expects to be poorer tomorrow
- Save more info today if expects less info tomorrow
- Illustrated in the case of $N = 2$.
  - Saving for rainy days: k_1^2 is increasing in $\sigma_{F_2}^2$.
  - Use all info if expecting more info next period.

The Case of $N = 2$

- Case 1: If $\sigma_{F_1} > \sigma_{F_2}$, $\qquad$ ($\sigma_{F_2} = 0$ in HHL):

$$\sigma_{z_1}^2 = \frac{\sigma_{F_1}^2 - \sigma_{F_2}^2}{2\sigma_{F_1}^2}\sigma_u^2 \text{ (mixed)}, \quad \sigma_{z_2}^2 = 0 \text{ (pure)},$$

$$\beta_1 = \frac{\sigma_F \sigma_u}{\sqrt{2}\sigma_{F_1^2}}, \quad \beta_2 = \frac{\sqrt{2}\sigma_u}{\sigma_F}, \quad k_1 = k_2 = \sqrt{\frac{\sigma_{F_1}^2 + \sigma_{F_2}^2}{2}},$$

$$\lambda_1 = \lambda_2 = \frac{\sigma_F}{2\sqrt{2}\sigma_u}, \quad \gamma_1 = \gamma_2 = \frac{\sigma_F}{\sqrt{2}\sigma_u}.$$

Saving for rainy days: k_1^2 is increasing in $\sigma_{F_2}^2$.

- Case 2: If $\sigma_{F_1} \leq \sigma_{F_2}$,

$\sigma_{z_1}^2 = \sigma_{z_2}^2 = 0$ (pure strategy),

$$\beta_i = \frac{\sigma_u}{\sigma_{F_i}}, \quad \lambda_i = \frac{\sigma_{F_i}}{2\sigma_u}, \quad \gamma_i = \frac{\sigma_{F_i}}{\sigma_u}, \quad k_i = \sigma_{F_i}, \quad \text{for } i = 1, 2.$$

Consume everything if expecting to be rich tomorrow.

2. Consumption Smoothing

Information smoothing

- In equilibrium, $\{k_1, \cdots, k_N\}$ are the unique solution to

$$\min_{k_1,\cdots,k_N} (k_1 - \overline{k})^2 + ... + (k_N - \overline{k})^2,$$

$$\text{s.t.} \quad \sum_{i=1}^{n} k_i^2 \leq \sum_{i=1}^{n} \sigma_{F_i}^2, \text{ for } n = 1, ..., N-1,$$

$$\sum_{i=1}^{N} k_i^2 = \sum_{i=1}^{N} \sigma_{F_i}^2,$$

where $\overline{k} \equiv (k_1 + ... + k_N)/N$.

- This is equivalent to smoothing $\lambda_1, ..., \lambda_N$.

3. Precautionary Saving

- Save more today if expecting more **uncertainty** tomorrow
- Save more **info** today if expecting more uncertainty
- Illustrated in the case of $N = 2$:

$$\sigma^2_{F_2} = \begin{cases} \overline{\sigma}^2_{F_2} + \Delta, & \text{with probability } \frac{1}{2}, \\ \overline{\sigma}^2_{F_2} - \Delta, & \text{with probability } \frac{1}{2}. \end{cases}$$

- Saving for rainy days: $\frac{\partial k_1^2}{\partial \overline{\sigma}^2_{F_2}} > 0$
- Precautionary saving: $\frac{\partial k_1^2}{\partial \Delta} < 0$

What Is Behind This Transformation?

Answer: The insider's commitment value K is zero.

- 2-P game with commitment $\Longleftrightarrow$ 1-P game.
- $K = 0 \quad \Longleftrightarrow \quad$ Eq. w.o. Comm. = Eq. w. Comm.
- 2-P equilibrium $\Rightarrow$ 1-P problem

Further results:

- In our baseline model: $K = 0$
- 5 additional cases
  - Time varying noise trading: $K = 0$.
  - Potential information leakage: $K = 0$.
  - Partial disclosure: $K = 0$ case, $K > 0$ case.
  - Continuous-time model: $K = 0$.

Commitment Game

Reformulate the game by changing the insider's strategy space:

- In period 0, the insider commits to linear trading strategy:

$$x_n = \beta_n(\sum_{i=1}^{n} F_i - P^*_{n-1}) + z_n, \text{ with } z_n \sim \mathcal{N}(0, \sigma^2_{z_n})$$

- In period 0, the insider chooses $\{\beta_n, \sigma_{z_n}\}_n$
  - For example, predetermined plans implemented by algorithms

Time-varying Noise Trading Intensity

- Noise trading intensity varies over time $\sigma^2_{u_i}$
- $(\sigma^2_{F_i},\, k_i^2,\, \Sigma_i) \rightarrow (Y_i,\, C_i,\, S_i)$: **nominal** quantities.
- Price level: $p_i \equiv 1/\sigma^2_{u_i}$

$$\max_{C_n,\cdots,C_N} \sum_{i=1}^{N} u(C_i/p_i),$$

$$\text{s.t.} \quad \sum_{i=1}^{n} C_i \leq \sum_{i=1}^{n} Y_i, \text{ for } n = 1, ..., N.$$

Potential Information Leakage

- Information is leaked with a probability q each period
- $(\sigma^2_{F_i},\, k^2_i,\, \Sigma_i) \rightarrow (Y_i,\, C_i,\, S_i)$

$$\max_{\{C_n,\cdots,C_N\}} \sum_{i=1}^{N} q^{i-1} u(C_i),$$

$$\text{s.t.} \quad \sum_{i=1}^{n} C_i \leq \sum_{i=1}^{n} Y_i, \text{ for } n = 1, ..., N.$$

Continuous-time Limit

- Continuous-time limit as trading frequency approaches ∞
- $(\sigma_F^2(t),\, k^2(t),\, \Sigma(t)) \to (Y(t),\, C(t),\, S(t))$.

$$\begin{aligned} &\max_{C(t)\geq 0} \int_0^1 u(C(t))dt, \\ \text{s.t.} \quad & C(t)dt \leq S(t) + Y(t)dt, \\ & dS(t) = (Y(t) - C(t))dt. \end{aligned}$$

Partial Disclosure

- In period n, MM gets d_n: $d_n = x_n + \epsilon_n$, with $\epsilon_n \sim \mathcal{N}(0, \sigma_\epsilon^2)$
  - Huddart et al. (2001): $\sigma_\epsilon = 0$.
  - Kyle(1985): $\sigma_\epsilon = \infty$.

- $N = 2$ and $\sigma_\epsilon = \infty$, $K > 0$, transformation doesn't work

- σ_ϵ is small: $K = 0$ and $(\sigma_{F_i}^2,\, k_i^2,\, \Sigma_i) \to (Y_i,\, C_i,\, S_i)$,

$$\max_{\{C_n,\cdots,C_N\}} \sum_{i=1}^{N-1} u(C_i) + \rho u(C_N),$$

$$\text{s.t.} \quad \sum_{i=1}^{n} C_i \leq \sum_{i=1}^{n} Y_i, \text{ for } n = 1, ..., N,$$

where $\rho \equiv \sqrt{\frac{\sigma_u^2}{\sigma_u^2 + \sigma_\epsilon^2}}$.

What Drives Commitment Value in Kyle-type Models?

- Our exercises suggest:
  - 0 commitment value $\Rightarrow$ equivalence
  - + commitment value $\Rightarrow$ non-equivalence
- What drives commitment value in the first place?
- Normal distributions + risk-neutrality:
  - Time dimension:
    - One period: 0
    - Continuous time: 0
    - $T = 2$: +
  - Disclosure:
    - Perfect or precise disclosure: 0
    - Imprecise disclosure: +
- One-period Kyle models (Bernhardt and Boulatov, 2023):
  - Symmetric Bernoulli distribution of asset value: +
  - Risk-averse insider: +

Conclusion

- A model with a sequence of information arrival and post-trade (partial) disclosure.
- Equilibrium computation is equivalent to solving a consumption-saving model.
- Ideas transported from permanent income hypothesis: Information usage today depends on the expectation of future information.
  - Saving for rainy days
  - Consumption smoothing
  - Precautionary saving
- Zero commitment value drives the equivalence result.
  - Future research: What drives the commitment value?

Reference

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