The Intangible Gap

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Abstract

We document a large and rising intangible investment gap between small and large U.S. firms. Smaller firms invest disproportionately in intangible capital, despite facing tighter financing constraints, and this gap has tripled since the 1980s alongside a pronounced increase in intangible investment volatility. We develop a dynamic industry-equilibrium model in which firms invest in both physical and intangible capital under financial frictions. Intangible investment is subject to idiosyncratic quality shocks and can be partially financed internally through equity-based compensation. These features generate an option-like payoff to intangible investment: downside risk is limited by exit, while upside gains scale with realized quality. This mechanism makes intangible capital particularly attractive for small firms with high exit risk. Our analysis reveals that the joint increase in intangible investment volatility and the decline in financing frictions for intangible capital account for approximately 60% of the post-2000 gap in intangible-to-physical capital ratios between small and large firms over the 2001–2023 period.

Keywords: Investment, Intangible, Financial Friction, Size Distribution
JEL Classification: E22, O34, L25, E24, E44

1 Introduction

One of the most prominent ongoing transformations in the US corporate sector is the change in its nature of corporate investment. The introduction of new technology has led to a structural change towards a more intangible-intensive economy (Corrado and Hulten (2010)). Intangible investment rose remarkably (Falato, Kadyrzhanova, Sim, and Steri (2022), Crouzet and Eberly (2019), Corrado, Hulten, and Sichel (2009)) since the 1980s, despite the weak physical capital investment (Alexander and Eberly (2018), Gutiérrez and Philippon (2017), Hall (2015)). At the same time, a key feature of intangible investment is its high and rising uncertainty. Intangible capital—such as R&D, human capital, and organizational capital—typically involves substantial uncertainty in both outcomes and valuation. The investment process itself has also become increasingly costly and volatile. Recent examples highlight this trend: innovation pipelines have shifted toward highly stochastic processes. These innovations generate extremely skewed outcomes—ranging from breakthrough performance to complete training collapse—implying that the variance of innovation shocks has increased dramatically.

In this paper, we document two new stylized facts about firms' intangible investment. First, there is a substantial *intangible gap* between small and large firms: small firms tend to have a higher ratio of intangible over physical capital even though smaller firms tend to be financially constrained. Smaller firms have higher growth rates, higher growth volatility, and their intangible investments are more volatile than physical investments. However, we do not observe a distinct pattern in the physical capital investment between firms over the sample period. Smaller firms contribute to both the level and the volatility of intangible investment substantially. This intangible gap has grown dramatically over time. Second, we document a pronounced increase in both the volatility of intangible investment and the uncertainty in the valuation of intangible capital within the universe of publicly traded firms.

These stylized facts are not direct implications of standard firm-dynamics models—such as Cooley and Quadrini (2001) with two capital inputs—and it is not clear that the rise in intangible intensity alone provides a natural explanation. While a change in the aggregate production function can account for the increasing share of intangible capital in the economy, it does not explain the pronounced heterogeneity in firms' behavior.

We propose an equilibrium model with firm dynamics (Hopenhayn (1992) and Cooley and Quadrini (2001)) to study firms' investment choices on intangible and physical capital over their life-cycles. In this economy, firms make investment decisions in both intangible and physical capital with financial constraints. As standard in the model with firm dynamics, productive firms invest in both intangible and physical capital and grow larger, while small and unproductive firms can exit. Our crucial assumption is that intangible and physical investments face *heterogeneous* investment opportunity sets. Intangible capital accumulation is subject to a random shock, ω, which affects the quality of the intangible output and hence the effective input in the production function. In contrast, physical capital accumulation follows a standard law of motion. We call this shock the intangible *quality* shock. The risk in the intangible investment has an impact on firms' investment decisions. The quality shock is a stochastic adjustment cost, making intangible investment risky. Firms on average deviate from their benchmark intangible investment level.[1] However, the randomness in the intangible capital's productivity introduced additional real option value, the value of the option to exit. Since intangible capital is risky, the option for small firms to exit becomes more valuable. For a given distribution of intangible quality shock, smaller firms invest more in intangible capital relative to physical capital than larger firms.

What drives the difference in the investment opportunity sets for intangible and physical capital? Intangible investments are different from physical investments in nature: intangible investments are considerably riskier than physical investment, and they are unpledgeable so harder to get external financing. In the old economy, firms primarily invested in physical for expansion, e.g., things like buildings and machines which are tangible and easy to estimate its value. However, it is more difficult to foresee the value of intangible capital. A new patent innovation from a sequence of R&D investments due to the uncertainty of the demand. For instance, Large language model (LLM) development has moved away from predictable, incremental transformer training and toward reinforcement-learning pipelines—such as DeepSeek's

[1] The benchmark level is the investment rate without the intangible quality shock. The average intangible investment rate can be higher than the benchmark level if the mean of the quality shock is higher than one.

RLOO—that exhibit significantly greater variance and instability. In addition to patent innovation, other crucial components of intangible capital, e.g., organizational structure, management, and key talents' skill and mobility, are subjects to great uncertainty. The risks of organizational structure becoming obsolete and the leaving of key talents impose challenges in the development of firms' intangible capital. Given the differences, small and large firms may even differ in their motivations for investing in intangibles.

We examine the publicly listed firms in the US for our analysis because of the availability of detailed financial data of the public firms. Different from patent innovations, components of intangible capital might not always be directly measurable, recorded separately in the financial statements[2], they are reported as part of the selling, general and administrative expenses in the cash flow statement, or R&D expenses if the activities are research and development related. Following the literature, we then estimate the accumulated intangible capital using the perpetual inventory method. To capture the dynamics of intangible capital, we construct the relative capital ratio, *hkratio*, for each firm year.

Over the sample period from 1975 to 2023, we observe substantial growth in intangible capital over time. The average *hkratio* grew more than three times since the 1980s. In the meantime, we see much more rapid growth in intangible capital in the smallest firm size group. Since the 1980s, the accumulated investment in intangible capital has increased by more than 7.1 times relative to physical investment on average among the smallest firms. In contrast, the increase is only 2.2 times among the largest firms. However, we didn't find a similar pattern using the balance sheet intangible assets, which indicates that larger firms accumulate intangible assets mostly through acquisition. We also find that the volatility of *hkratio* and the volatility of patent market values rose dramatically over the sample period, consistent with increasing randomness in intangible asset quality.

Our model highlights two central mechanisms that jointly generate the intangible gap in the data. The first is the real option effects created by the intangible quality shock, ω. Because intangible capital is subject to random scaling through ω, firms face a convex payoff struc-

[2] For example, in addition to management skills, organizational structure, founder's influences when firms are young, workers' stock and option incentives are not always reflected in firms' current financial statements. **?** proposed a balance sheet-based measure for employees' stock-based compensation for a long sample period.

ture: high-quality realizations of ω substantially raise effective intangible capital, while low realizations trigger exit and limit losses. This asymmetry makes intangible investment particularly attractive for small firms, which place high value on the option to exit. Consequently, small firms optimally tilt their portfolios toward intangible capital relative to large firms. The second mechanism operates through the relatively relaxed financing frictions for intangibles compared with physical capital. Intangible capital can be financed internally through the labor market—for example, via equity-based compensation—which effectively allows firms to borrow from their workers (Sun and Xiaolan, 2019; Guiso, Pistaferri, and Schivardi, 2012). A higher stock of intangible capital relaxes financing constraints and encourages further intangible accumulation. Together, the real-option mechanism and the financing channel reinforce each other: rising uncertainty increases the option value of intangible investment, while improved financing capacity allows firms—especially smaller ones—to take advantage of this option. This interaction generates a skewed distribution of intangible intensity across firms and amplifies dispersion in investment behavior.

We calibrate the model to match key moments from the pre-2000 U.S. data–specifically, the cross-sectional spread in *hkratio* and the share of employee equity in financing intangible capital. The benchmark parameters successfully replicate several salient features: the pre-2000 H/K spread (the difference in *hkratio* between small and large firms), the share of employee equity in firm value, relative investment rates in H and K, and the average firm exit rate. Importantly, the model also reproduces the monotonic decline in *hkratio* across size groups.

We then conduct comparative statics to assess how rising volatility of intangible quality contributes to the post-2000 rise in intangible intensity. Increasing the volatility of the quality shock by 37.5%—a magnitude consistent with the documented rise in patent-value dispersion—substantially amplifies the real-option value of intangible investment. Small firms raise their intangible-to-physical capital ratios more than large firms. Quantitatively, the model-implied H/K spread increases from 0.443 in the benchmark to 0.812 under the higher volatility calibration. This accounts for approximately 60% of the observed post-2000 H/K spread of 1.374.

Our findings highlight a broader transformation in the U.S. corporate sector: intangible investment is increasingly concentrated among smaller, younger, and lower-valuation firms. These firms bear a substantial share of the economy's innovation risk, undertake more intangible investment, and disproportionately drive the cross-sectional dispersion in investment behavior.

Literature Review Our paper is related to the growing literature on the rise of intangible capital in the US economy. Crouzet and Eberly (2019), Alexander and Eberly (2018), Ayyagari, Demirgüç-Kunt, and Maksimovic (2024) provide links between the increasing importance of intangible capital and the rise of concentration. Falato et al. (2022) studies the relationship between companies' cash holdings and the rise of intangibles. Crouzet and Eberly (2023) evaluates the implications of firms shifting investment to intangibles on market power, valuation, and investment incentives. Our paper focuses on the heterogeneous growth patterns in intangible investment across firm size. We show that frictions in intangible investment must be modeled differently in order to capture the observed patterns of intangible investment under financial constraints.

Our project is related to a vast literature of understanding firm dynamics, size distribution, age, and growth (Lucas and Prescott, 1971; Lucas, 1978; Hopenhayn, 1992; Hopenhayn and Rogerson, 1993). In these models, they characterize the stationary firm size distribution and investment policies. Cooley and Quadrini (2001) study the relationships among firm size, age, and financial borrowing constraints. Small and young firms grow faster, create and destroy more jobs, and are more financially constrained. Clementi and Palazzo (2016) find that the entry and exit margins account for 20% of output growth over ten years. The extensive margin also generates persistence in aggregate time series. We introduce an intangible quality shock in firms' investment decisions and expand the model to better understand the joint distribution of firm size, intangible, and physical capital. Ottonello and Winberry (2024) find that over the firm life cycle, firms increasingly invest in R&D, whereas younger and smaller firms initially concentrate on physical investment. Our findings, in contrast, focus on the level and evolution of the cumulative intangible capital of small versus large firms and on the relative changes in the investment gap over time. We highlight the role of financial frictions, consistent with the existing literature on firms' capital allocation choices, but introduce a novel mechanism: intangible capital is subject to additional quality uncertainty. This feature helps explain both the magnitude and the persistent rise of the intangible gap observed in the data.

Our paper is also closely related to the growing literature that documents and explains the importance and distinctiveness of intangible assets. Eisfeldt and Papanikolaou (2013) develop a model in which organizational capital as a production factor helps explain cross-sectional asset-pricing returns. Gourio and Rudanko (2014) study how customer capital—built through marketing and selling activities—shapes firm investment, profits, and Tobin's q. Sun and Xiaolan (2019) provide a theoretical framework showing how firms use compensation contracts to finance investment in intangible assets. Akcigit and Kerr (2018) build a tractable growth model featuring internal and external innovation to account for firm size distributions. Our paper contributes by focusing on the riskiness of intangible accumulation and its implications for the cross-sectional variation in investment behavior.

2 Stylized Facts on Intangible Investments

2.1 Sample Construction

Our sample is constructed using Compustat fundamental annual from 1975 to 2023. Compustat provides detailed financial information on the publicly traded firms in the US. We restrict our sample to firms that are incorporated in the US. We exclude financial firms (SIC 6000 – 6999), regulated utilities (SIC 4900 – 4999), and firms categorized as public service, international affairs, or non-operating establishments (9000+). We also exclude firms with missing SIC code, negative values of sales, total assets, employees, R&D expenses, capital investment, current long term liabilities and intangible assets. For more detailed data construction, please see Data Appendix for more details.

2.2 Measures of Intangible Capital

We measure intangible capital as the accumulated stock of intangible investment. The Compustat database provides detailed balance sheet and cash flow statement information that allows us to estimate its book value. We focus on a firm's investment in its intellectual assets and innovation, as well as its organizational structure (including information technology and computer expenses) borrowing from the definition of intangible assets in Brynjolfsson, Hitt, and Yang (2002). According to the existing literature (Corrado, Sichel, and Hulten (2004), Lev and Radhakrishnan (2005), Hulten and Hao (2008), Eisfeldt and Papanikolaou (2013), Falato et al. (2022), etc.), in the Compustat database, the computerized information (e.g., organizational structure) and economic competencies (i.e., knowledge embedded in firm-specific human and structural resources, including brand names) are recognized as a fraction of *Selling, General and Administrative* (SG&A) expenses, and the corresponding line for innovative property is *Research and Development* (R&D) expenses.

We follow Peters and Taylor (2017) to estimate the accumulated R&D capital and organizational capital using the perpetual inventory method.[3] The stock of these capitals are defined as

$$H_{it} = (1-\delta_h)H_{it-1} + I_{H,it}, \tag{1}$$

where H is the variable of interest, i represents firm indicator, t is the year, δ_h is depreciation rate, and I_H is the new investment on H. We also need to estimate the initial stock of H to keep track of the value over time, which is defined as

$$H_0 = \frac{I_0}{g_h + \delta_h},$$

where H_0 and I_0 are initial values, and g_h is the average annual growth rate. We use both SG&A expenditures and R&D expenses to construct our measure of intangible capital, H, for each firm year.

As an alternative measure of intangible capital, we adopt a market-based approach fol-

[3]This is also used by the Bureau of Economic Analysis (BEA)to estimate R&D activities.

lowing (Kogan, Papanikolaou, Seru, and Stoffman, 2017), which infers the value of innovation from stock-market reactions to patent grant announcements. Specifically, we construct a firm-level dollar measure of intangible capital by aggregating the market-implied values of all patents granted to a firm in a given year. This measure captures investors' forward-looking assessments of the economic value of innovation and is available annually from 1926 to 2023.

Sun and Xiaolan (2019) and Eisfeldt, Falato, and Xiaolan (2023) proposed a compensation-based measure for the firm-level human capital, the total value of equity-based compensation. Since companies engaged in innovation and research-intensive activities require the input of high-skilled workers, and they often are paid with equity-based compensation. We adopt this measure to infer the human capital-related component of intangible capital from firms' expenses in their stock-based compensation. We use the cost of stock-based compensation (STKCO) and the value of shares reserved for equity-based compensation as our alternative compensation-based measure of firm-level intangible capital.

We also construct other financial and investment variables at the firm level throughout the entire sample. We obtain data on firms' IPO year to form firms' age based on their founding years. We construct Tobin's q as the ratio of the market value of total assets to the book value of total assets, *ROA* as the profits to total asset ratio. We use debt and equity flows to measure external finance following Covas and Den Haan (2011).

Table 1 reports the summary statistics of the main investment and financial variables by size groups. Firm size is measured by the market value of equity (ME). All variables are deflated using CPI. The statistics in Panel A reveal consistent patterns of firm dynamics that align with the existing literature. Younger firms tend to be smaller with higher growth and standard deviation in labor and total assets. Smaller firms also have higher Tobin's Q but lower profitability. The smallest firms (the bottom 20%), on average, have negative ROA.

Panel B reports investment dynamics across firm size groups and reveals a large and robust intangible investment gap. Across all measures, smaller firms exhibit substantially higher intangible intensity than larger firms. The stock of intangible capital—measured using R&D capital, organizational capital, or the intangible-to-physical capital ratio (PPENT)—is consistently highest among small firms, and this pattern is robust across alternative definitions. A market-based measure yields the same conclusion: patent value relative to total assets is greatest for small firms. Smaller firms also invest more aggressively in intangibles. Patent-related market value, employee stock-based compensation, R&D expenditures, and organizational capital investment (XSGA), each scaled by total assets, are all largest among small firms and decline monotonically with firm size. Advertising expenditures, which are included in selling, general, and administrative expenses, are unlikely to reflect innovation or research activity. Consistent with this interpretation, advertising expense ratios do not differ meaningfully across firm size groups.

Panel C reports measures of firms' financial conditions. Consistent with the existing literature, smaller firms face tighter financial constraints and a higher probability of entry and exit (Cooley and Quadrini, 2001). Across multiple measures, smaller firms in our sample have lower book and market leverage and are systematically more financially constrained than larger firms.

2.3 Stylized Facts

We summarize our findings on the time series behavior of intangible capital and physical capital across firm sizes. We document two central stylized facts. First, there is a large and rising intangible investment gap between small and large firms, driven by disproportionately higher intangible accumulation and investment among small firms rather than differences in physical investment. Second, intangible investment has become increasingly risky over time, with sharply rising volatility in both intangible capital accumulation and market-based valuations, particularly among smaller firms.

2.3.1 Intangible Investment Gap between Small and Large Firms

For each year, firms are classified into five groups based on their sales or market equity. Within each group, we calculate the median intangible-to-physical capital ratio ($hkratio$), where H is calculated using equation (1), and physical capital K is the value of *Property, Plant and Equipment* (PPEGT).

Figure 2 presents the time series of *hkratio* by firm size. There exists substantial growth in intangible capital over time, consistent with Crouzet and Eberly (2023). The average intangible to physical capital ratio was lower than 1 before the 1980s and has risen to almost 2.8 in the 2000s. In the meantime, the growth in intangible capital behaves differently across size groups. We see much more rapid growth in intangible capital in the smallest firm size group. The difference in *hkratio* between small and large firms represents the intangible gap. Since the 1980s, the accumulated investment in intangible capital has increased more than 6 times relative to the physical capital, with an increase of 7.1 times on average among the smallest firms. In contrast, the increase is only 2.2 times among the largest firms.

The increase in the intangible gap is likely a result of the difference in investment activity over time. Figure 4 confirms that smaller firms invest more in intangible capital, i.e., R&D or organization capital (XSGA), than larger firms. Although this is true over the entire sample period, the gap of the intangible investment rate between the smallest size group and the largest size group widened, from 1.4% in the year 1980 to 15% in 2023. It seems that more and more investment in intangible is made by smaller firms.[4]

Physical investment has remained broadly stable, with a slight downward trend since the 1980s. Figure 5 illustrates that physical capital investment rates move similarly across firm-size groups, with no meaningful divergence. This suggests that small firms differ from large firms in their *hkratio* primarily in their intangible investment, not in their physical investment activities.

We also want to highlight the distinction between accumulated intangible investment and balance sheet intangible assets. The balance sheet intangible assets (INTAN) reflect the overall value of acquired intangibles, including book values of different types of intangible assets: software, intellectual property, brand, business processes, and goodwill. Over the sample period, the intangible assets to physical capital ratio has been rising (Crouzet and Eberly, 2019) remarkably, but there is no substantial difference across size groups (Figure 6). In fact, larger

[4]One caveat is that our sample is a subset of the US economy since we are not able to examine the intangible investment activities in private firms. However, public firms contribute to more than 40% of US GDP and about one-third of the US employment. It represents a significant portion of the US economy. Although the selection is a general issue, we are likely missing out on some private star firms that are actively engaged in innovation activities. These firms are likely to have small asset values.

firms experience faster growth in balance sheet intangible assets after the mid-1990s. One potential explanation is that intangible assets are outputs of the intangible capital, and they are results of within-firm innovation and acquisition activities. Larger firms accumulate balance sheet intangible assets through acquisition.

2.3.2 Volatile Intangibles

Intangible investment is inherently risky, and its riskiness has increased substantially over time. Figure 7 plots the time series of firm-level *hkratio* volatility by size group, where volatility is measured as the standard deviation of *hkratio* over a rolling five-year window and then averaged within each size group. The figure shows a pronounced rise in intangible volatility among small firms—from 0.15 in 1980 to 2.55 in 2023—while volatility for large firms remains largely stable. This pattern indicates that intangible investment opportunities have become increasingly risky, particularly for small firms, and are notably more volatile than physical investment opportunities.

We also found similar patterns using the market-based measure. Figure 8 further illustrates this pattern using another data source on the value of innovation. The figure plots the standard deviation of the logarithm of the patent-value to total asset ratio, computed within a rolling three-year window. The cross-sectional dispersion in patent value rises sharply from the early 1980s onward, peaks around the early 2000s, and remains elevated thereafter. The increase in the volatility of patent values mirrors the rise in the volatility of intangible investment, as measured by *hkratio*, supporting the view that the riskiness of intangible capital has grown substantially over time.

3 The Model

In this section, we propose an equilibrium model with firm dynamics to study firms' investment choices on intangible and physical capital over their life cycles. We begin by illustrating our main mechanism in a two-period environment and then extend the framework to a full quantitative model.

3.1 A Two-Period Model

There are two periods, $t = 1, 2$. At the beginning of the period $t = 1$, the firm owns intangible capital h_t and produces output $f(h_t) = h_t^{\alpha}$. The firm then decides on the intangible capital of the next period h_{t+1}. We assume the discount rate $\beta = 1$ and intangible capital does not depreciate each period ($\delta_h = 0$).

Intangible capital faces uncertain productivity. The key assumption is that, at the beginning of the second period $t+1$, intangible capital is subject to an intangible *quality* (IQ) shock ω_{t+1} that scales the productivity of the newly installed intangible capital. The effective capital level is $\omega_{t+1}h_{t+1}$. The firm then decides whether to operate or to exit after observing the quality shock ω_{t+1}. Without loss of generality, we assume that the quality shock ω_{t+1} is log-normally distributed, with $\ln(\omega_{t+1}) \sim N(\mu, \sigma^2)$.

We interpret the ω shock as the risk of investing in intangible capital. This risk includes the unexpected departures of firms' key talents, the failure of research and development projects, and the obsolescence of the the existing organizational structure and knowledge. This shock is important for a few reasons. First, we interpret the shock as the changing quality of intangible capital, but it is unknown to all firms before the decision of investments made. The intangible quality shock affects the exit decision. As a result, investing in intangible capital is risky for small and young firms since the quality shock may distort more on their incentives.

The firm exercises its option to operate only if the realized value is nonnegative, yielding the value

$$\max\{(\omega_{t+1}h_{t+1})^{\alpha} - (h_{t+1} - h_t) - \xi, 0\},$$

where $\xi > 0$ denotes a fixed operating cost.

The firm's optimization problem is

$$V_t = \max_{h_{t+1}} E_t\Big[\max\big\{\,(\omega_{t+1}h_{t+1})^{\alpha} - K(h_{t+1}),\ 0\big\}\Big], \tag{2}$$

where

$$K(h_{t+1}) = (h_{t+1} - h_t) + \xi.$$

Let $\bar{\gamma} = E[\omega_{t+1}^{\alpha}] = exp(\alpha\mu + \frac{1}{2}\alpha^2\sigma^2)$. Analogous to the Black-Scholes-Merton framework, the firm's value is

$$V_t = \max_{h_{t+1}} \bar{\gamma} h_{t+1}^{\alpha} N(d_1) - K(h_{t+1}) N(d_2), \tag{3}$$

where $N(\cdot)$ denotes the cumulative distribution function of the standard normal distribution $N(0,1)$, and $d_2 = \frac{\alpha\mu - lnK(h_{t+1}) + \alpha ln h_{t+1}}{\alpha\sigma}$, $d_1 = d_2 + \alpha\sigma$.

Proposition 1 *The optimal investment h_{t+1}^{*} is strictly increasing in the volatility of the quality shock σ.*

See the appendix for proof. The intuition is that the firm's continuation value is convex in the quality shock. The firm's first-order condition for (2) $\alpha\bar{\gamma}h_{t+1}^{\alpha-1}N(d_2) - N(d_1) = 0$ implies that the marginal benefit of investment is weighted by $N(d_1)$, which captures the upside potential, while the marginal cost is weighted by the survival probability $N(d_2)$. As σ rises, the asymmetry of the payoff structure—where losses are truncated at zero (the abandonment option) while potential gains are unbounded—becomes more pronounced. This leads to a relative increase in the weight on revenues ($N(d_1)$) compared to the weight on costs ($N(d_2)$). This widening 'wedge' effectively discounts the risk of investment. Consequently, the firm is incentivized to aggressively scale up h_{t+1} to capitalize on the increased likelihood of extreme positive realizations of the shock. To further formalize this idea, we can write down the firms problem where there is no option to exit. The firm's problem without option to exit reduces to $V_{0,t} = \max_{h_{t+1}} \bar{\gamma}h_{t+1}^{\alpha} - K(h_{t+1})$ with the optimal investment h_{t+1}^{**} characterized by $\alpha\bar{\gamma}(h_{t+1}^{**})^{\alpha-1} - 1 = 0$. The value of the real option to exit is therefore given by $V_t - V_{0,t}$. The following proposition characterizes how this option value varies with the firm's initial size.

Proposition 2 *The value of the exit option is strictly decreasing in initial intangible capital h_t.*

See the appendix for proof. The relative value of the real option declines as the firm becomes larger, as reflected in a higher stock of existing intangible capital h_t. An increase in h_t effectively pushes the project deeper in the money: continuation becomes the dominant choice across almost all realizations of the quality shock, making the exit option unlikely to be exercised. As a result, the flexibility value embedded in the option to abandon diminishes with firm size.

3.2 A Quantitative Model

Next we extend the two-period model to a dynamic equilibrium model with firm dynamics to study firms' investment choices on intangible and physical capital over their life-cycles. In this economy, firms make investment decisions in both intangible and physical capital with heterogeneous investment opportunity sets.

The Firm There is a continuum of incumbent firms at the beginning of the period t. The production requires the inputs of physical capital, k_t, intangible capital, h_t, and is subject to an idiosyncratic productivity shock, s_t, which follows AR(1) processes. The production function exhibits decreasing returns to scale,

$$F(s_t, k_t, h_t) = s_t k_t^{\alpha_k} h_t^{\alpha_h}. \tag{4}$$

Firms make investment decisions on physical capital, i_k, and intangible capital, i_h. The law of motion of the physical capital takes the form of

$$k_{t+1} = (1-\delta_k) k_t + i_{kt},$$

with an adjustment cost $g_k(i_{kt}) = \frac{\rho_k}{1-\kappa_k}\left(\frac{i_{kt}}{k_t}\right)^{1-\kappa_k} k_t$, where ρ_k and κ_k are the coefficients that control the slope and curvature of the adjustment cost.

Similarly, the law of motion of intangible capital takes the form of

$$\tilde{h}_{t+1} = (1-\delta_h) h_t + i_{ht},$$

where i_{ht} is the total investment, and we assume that there is an adjustment cost of capital, $g_h(i_{ht}) = \frac{\rho_h}{1-\kappa_h}\left(\frac{i_{ht}}{h_t}\right)^{1-\kappa_h} h_t$, where ρ_h and κ_h are the coefficients that control the slope and curvature of the adjustment cost.

Notice that $\tilde{h}_t$ is not the effective intangible capital for the production function (4). At each period t, the quality of intangible capital is subject to some degree of uncertainty. Firms receive a random shock ω_t, immediately after observing s_t, and this shock determines the quality of their intangible capital. The intangible stock, adjusted for its quality, evolves as follows.

$$h_t = \omega_t \tilde{h}_t, \tag{5}$$

where h_t is the amount of effective intangible capital used for production and $\tilde{h}_t$ is the intangible capital at the beginning of period t. ω_t is unknown to all firms before the decision of investments made, but the exit decision is made after the intangible quality shock is realized.

We assume that the intangible *quality* shock (IQ), ω, is independently and identically distributed (i.i.d.), and follows a log-normal distribution. Specifically,

$$\log \omega \sim N\left(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2\right),$$

which ensures that $\omega > 0$, almost surely and $\mathbf{E}[\omega] = 1, \mathrm{Var}(\log \omega) = \sigma_\omega^2$.

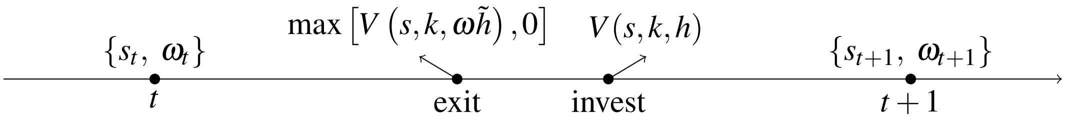

Figure 1: Timing of the Incumbent Firm

This figure illustrates the within-period sequence of the firm's decisions, including the exit choice followed by the investment decision. Given the productivity shock s_t and the quality shock ω_t, the physical capital stock k_t, and the intangible capital stock h_t, firms first decide whether to exit. If they choose to continue operating, they then make their investment decisions based on the observed productivity shocks and the effective intangible stock $\tilde{h}_t$.

Financing Investment Firms have access to the financial market by raising equity to finance investments. Given the current capital stock k_t and h_t and the realized shocks, the firm chooses investment, i_{kt} and i_{ht}, equity payout, d_t. A positive d_t means that dividends are distributed, and a negative d_t means that equity will be issued. The budget constraint is

$$i_{kt} + i_{ht} + d_t + g_h(i_{ht}) + g_k(i_{kt}) = F(s_t, k_t, h_t). \tag{6}$$

We assume that the firm's ability to raise external equity is limited[5], However, intangible capital can enhance internal financing capacity through deferred compensation or employee-based financing arrangements (Sun and Xiaolan, 2019). Although intangible capital is less collateralizable for external creditors, it can serve as collateral when firms borrow from high-skilled workers through deferred compensation. And empirically, a large fraction of equity-based compensation effectively substitutes for current wages and thus serves as an internal source of funds for the firm (Eisfeldt, Falato, and Xiaolan, 2023). Formally, we assume that the equity payout choice d_t is subject to the following constraint:

$$d_t \geq -\theta h_t, \tag{7}$$

where $\theta > 0$ governs the extent to which intangible capital relaxes the firm's equity financing capacity. The larger the θ, the greater the collateral value of intangible capital to employees, and firms are less constrained in financing intangible capital. This assumption implies that a higher level of intangible capital h_t expands the firm's ability to issue equity internally (i.e., to set $d_t < 0$), reflecting the mechanism that intangible improves the firm's internal financing capacity through high skilled employee-based financing channels.

Exit Each period, the firm makes exit decisions. The firm exits the market if the continuation value, conditional on the realization of idiosyncratic productivity shock s_t and intangible quality shock ω_t, is negative. All incumbent firms have to pay a fixed cost of production of ξ. Incumbent firms produce consumption goods, make investment decisions on both physical and intangible capital, and choose new equity levels when maximizing the value of the firm $V(s,k,h)$:

$$V(s_t,k_t,h_t) = \max_{d_t,i_{kt},i_{ht}} d_t - \xi + \beta \mathbf{E}\{\max[V(s_{t+1},k_{t+1},\omega_{t+1}\tilde{h}_{t+1}),0]\}, \tag{8}$$

subject to the firm's budget constraint (6) and financing constraint (7).

[5]We also abstract from debt financing in our benchmark model. Consistent with the existing literature (e.g., Cooley and Quadrini (2001)), small firms face more severe credit-market frictions when using physical capital as collateral.

Given productivity shock s_t, physical capital stock k_t, and the intangible capital stock before the quality shock $\tilde{h}_t$, the exit decision involves a reservation rule

$$\eta_t^e\left(s_t,k_t,\tilde{h}_t\right)=\begin{cases}0 & \text{if } \omega\geq\omega^*\left(s_t,k_t,\tilde{h}_t,\right)\\ 1 & \text{o.w.}\end{cases},$$

where

$$\omega^*\left(s_t,k_t,\tilde{h}_t\right)=\inf\left\{\omega\in\Omega:V\left(s_t,k_t,\omega\tilde{h}_t\right)\geq 0\right\}.$$

Entry At the beginning of each period, a total mass of N potential entrants randomly draws an initial idiosyncratic productivity, s_t^0, from the distribution function $\Gamma\left(s_t^0\right)$. After drawing the s_t^0, a potential entrant decides whether to enter the market. If it enters, it pays the entry cost c_e. The new entrant maximizes the expected value of the firm, $V^e\left(s_t^0\right)$, by choosing labor and capital inputs and equity to fund investment

$$V^e\left(s_t^0\right)=\max_{d_t,k_t,h_t} d_t+\beta\mathbf{E}\{\max[V(s_{t+1},k_{t+1},\omega_{t+1}\tilde{h}_{t+1}),0]$$

subject to the budget constraint (6) and financing constraing (7).

At time $t+1$, an entrant will face exactly the same problem as the incumbent. Thus, the firm enters the market if and only if the discounted value of entering exceeds the entry cost

$$V^e\left(s_t^0\right)\geq c_e.$$

This equation determines the threshold, s_e^*, beyond which only firms with idiosyncratic productivity shocks, $s_t^0\geq s_e^*$, enter the market.

Recursive Equilibrium We consider a recursive equilibrium definition. A key element here is the law of the motion of firm size distribution. (Λ_t) are the aggregate state variables in our model. We call Λ_t the aggregate state of the industry, which depends on the current measurements of firms physical and intangible stocks, and idiosyncratic productivity shocks.

Given firms' entry, exit, and default decisions and policy functions, the evolution of the state of the industry Λ satisfies

$$\begin{aligned} & \Lambda\left(s', k', \tilde{h}'\right) \\ = & \int\left\{p^i\left(s'|s\right) \int_{\omega \geq \omega^*(\Lambda)} 1_{\left(k', \tilde{h}'|k, \omega\tilde{h}, s\right)} \Lambda\left(s, k, \tilde{h}\right)\right\} H^{\omega}(d\omega)\, ds \\ & +N \int_{s \geq s^*(\Lambda)} 1_{\left(k, \tilde{h}'\right)} p^i\left(s'|s\right) \Gamma(ds), \end{aligned} \tag{9}$$

where p^i is the conditional probability density functions of idiosyncratic productivity and aggregate shocks, respectively; H^{ω} is the cumulative distribution function of capital shocks; and $1_{\left(k, \tilde{h}'|k, \omega\tilde{h}, s\right)}$ and $1_{\left(k, \tilde{h}'\right)}$ are the indicator functions given incumbents' and entrants' policy functions

$$\begin{aligned} 1_{\left(k', \tilde{h}'|k, \omega\tilde{h}, s\right)} &= 1_{\{\zeta_k^i(k, \omega\tilde{h}, s)=k',\ \zeta_h^i(k, \omega\tilde{h}, s)=\tilde{h}'\}}, \\ 1_{\left(k', \tilde{h}'\right)} &= 1_{\{\zeta_k^e(s)=k',\ \zeta_h^e(s)=\tilde{h}'\}}, \end{aligned}$$

where ζ_k^i, ζ_h^i are the policy functions of incumbents, and ζ_k^e, ζ_h^e are the policy functions of entrants.

A recursive competitive equilibrium consists of the exit decision $\eta^e\left(s, k, \tilde{h}\right)$, the entry decision $s^*(s)$, the optimal decision rules, and the measure of entrants N_t such that incumbents' optimization and entrants' optimization are solved.

The economy is characterized by a certain distribution of firms, μ, over all state variables, k, h and s. In the analysis described in this section, we focus on the invariant distributions of firms. The existence and uniqueness of the invariant distribution therefore depends on the properties of the transition matrix generated by the optimal policy functions, physical and intangible capital, of both incumbents and entrants, which are characterized by the equation (10). The invariant distribution is the fixed point in this contraction mapping.

We propose the existence and uniqueness of the invariant distribution with some weaker conditions than those of Cooley and Quadrini (2001).

Proposition 3 *An invariant measure of firms, μ^*, exists. μ^* is unique, and the sequence of measures generated by the transition functions Ω, $\{\Omega^n(\mu_0)\}_{n=0}^{\infty}$, converges weakly to μ^* from* *any initial arbitrary* μ_0.

3.3 Investment Decisions

We now analyze the firm's intangible and physical capital investment decisions. We can write down the investment Euler equation for k and h:

$$(1+\lambda_t)q_{k,t} = \beta \mathbf{E}_t \left[\left((1+\lambda_{t+1}) \left(\alpha_k \frac{F_{t+1}}{k_{t+1}} + (1-\delta_k) q_{k,t+1} + \frac{\rho_k}{2} \left(\frac{i_{k,t+1}}{k_{t+1}} \right)^2 \right) \right) \cdot \mathbf{1}_{\{\omega_{t+1} > \omega^*_{t+1}\}} \right], \tag{10}$$

$$\begin{aligned}(1+\lambda_t)q_{h,t} = \beta \mathbf{E}_t \Bigg[\omega_{t+1} \cdot \Bigg((1+\lambda_{t+1}) \left(\alpha_h \frac{F_{t+1}}{h_{t+1}} + (1-\delta_h) q_{h,t+1} + \frac{\rho_h}{2} \left(\frac{i_{h,t+1}}{h_{t+1}} \right)^2 \right) \\ + \lambda_{t+1}\theta \Bigg) \cdot \mathbf{1}_{\{\omega_{t+1} > \omega^*_{t+1}\}} \Bigg],\end{aligned} \tag{11}$$

where $q_{k,t}$ and $q_{h,t}$ are the marginal cost of investment, and λ_t is the Lagrange multiplier associated with the financial constraint $d_t \geq -\theta h_t$. The left-hand side of each Euler equation captures the marginal cost of purchasing and adjusting capital, while the right-hand side represents the expected marginal benefit of an additional unit of investment at time t. In the absence of intangible-quality risk ω_t and when financial frictions do not bind $\lambda_t = 0$, firms behave symmetrically with respect to physical and intangible investment. Both types of capital enter the production function with decreasing returns, and smaller firms naturally have incentives to accumulate capital. In this benchmark case, investment sensitivities for physical and intangible capital are similar, unless one imposes assumptions about systematic differences in firm composition or technology that mechanically generate stronger intangible investment among small firms.[6]

A key distinction between physical and intangible investment emerges once we introduce the quality shock ω_{t+1} and the endogenous exit option. With these features, the Euler equation for physical capital retains the standard structure: the marginal investment cost $(1+\lambda_t)q_{k,t}$

[6] Even if sensitivities differ modestly, one would need additional assumptions—such as a changing composition toward intangible-intensive small firms—to match the empirical patterns.

equals the expected discounted marginal return, conditional on firm survival $\omega_{t+1} > \omega^*_{t+1}$. In contrast, the marginal return to intangible investment includes an additional multiplicative term ω_{t+1}, which makes the payoff to intangible capital strictly convex in the quality realization. This convexity creates a real option: the firm incurs investment costs today but retains the ability to exit tomorrow if realized quality is sufficiently low. The survival indicator $\mathbf{1}_{\{\omega_{t+1}>\omega^*_{t+1}\}}$ truncates low-quality states, allowing the firm to walk away from unfavorable outcomes while disproportionately benefiting from high-quality realizations amplified by ω_{t+1}. Because large firms rarely exit, their continuation value is almost always positive, and the abandonment option contributes little to their Euler equation. Small firms, whose continuation values lie close to zero, face a much tighter exit threshold. For them, intangible investment resembles a one-sided bet: downside losses are eliminated through exit, while upside gains scale with ω_{t+1}. Consequently, the real option embedded in intangible capital makes such investment particularly attractive for young and small firms.

The magnitude of this option value is increasing in the volatility of the quality shock σ_ω. When $\omega_{t+1} \sim N(\mu_\omega, \sigma^2_\omega)$, multiplying the marginal return of intangible investment by ω_{t+1} is equivalent to evaluating the return under a tilted lognormal distribution $\tilde{\omega}_{t+1}$ whose log-mean is shifted to the right by exactly σ^2_ω.[7] A higher σ_ω therefore disproportionately increases the weight placed on high–quality realizations, raising the expected marginal return to intangible investment. This mechanism explains why the value of the real option embedded in intangible capital is strictly increasing in the dispersion of the shock: uncertainty expands the upside with-

[7] A useful property of the lognormal distribution helps interpret how the quality shock affects the marginal return to intangible capital in the Euler equation. Suppose $\log \omega \sim N(\mu_\omega, \sigma^2_\omega)$ so that $E[\omega] = 1$. For any measurable function $g(\omega)$, the expectation weighted by ω can be written as

$$E[\omega\, g(\omega)] = \int \omega g(\omega) f(\omega) d\omega.$$

Since $\omega f(\omega)$ is itself a probability density under the normalization $E[\omega] = 1$, tilting by ω is equivalent to evaluating $g(\cdot)$ under a shifted lognormal distribution. Specifically, $\omega f(\omega)$ corresponds to a lognormal distribution whose log-mean shifts from μ_ω to $\mu_\omega + \sigma^2_\omega$. Hence,

$$E[\omega g(\omega)] = E_{\ln\tilde{\omega}\sim N(\mu_\omega+\sigma^2_\omega,\, \sigma^2_\omega)}\left[g(\tilde{\omega})\right].$$

Intuitively, multiplying by ω overweights high–quality realizations by shifting the log-mean of the shock distribution to the right by σ^2_ω. As volatility σ_ω increases, the effective distribution tilts toward high-quality states, amplifying the convex option value embedded in intangible investment.

out changing the downside, because the downside is endogenously cut off by the exit threshold. The policy functions plotted in Figures 10 illustrate these mechanisms quantitatively. Figure 10 compares the optimal intangible-to-physical capital ratio under low and high quality-shock volatility in the absence of financial frictions. The figure shows that in the environment of large intangible uncertainty (σ_ω), the H'/K' ratio is much larger for smaller firms (measured as low k or low h).

Financial frictions further strengthen the attractiveness of intangible capital. When the financing constraint binds ($\lambda_{t+1} > 0$), the firm attaches a shadow value to internal liquidity. Intangible capital relaxes the financing constraint through θ, as reflected in the additional term $\lambda_{t+1}\theta$ in its Euler equation. This creates a complementarity between the real option and financial frictions: constrained firms benefit more from states where the firm survives and receives a high–quality intangible realization, because those states not only generate higher marginal product but also loosen the financing constraint going forward. Consequently, constrained firms place an even higher value on intangible capital relative to physical capital, amplifying the endogenous shift toward intangible investment among small and financially constrained firms. Figure 11 demonstrates that when financial frictions are present, the same increase in volatility substantially elevates the H'/K' ratio over a wider range of firm sizes, particularly for small firms near the exit margin.

It is worth noting that the assumption that intangible capital plays a unique role in relaxing firms' financial constraints may appear to favor younger firms' tilt toward intangible investment. Although this assumption is grounded in empirical evidence, the model's qualitative and quantitative results still hold even if physical capital can also be financed through the internal labor market. The key mechanism is that younger firms benefit from the real option embedded in the quality shock; however, if they are financially constrained, one would not expect large movements in intangible investment simply because of limited resources. It is the combination of a less stringent financial constraint and the option value associated with intangible capital that generates the observed intangible investment gap in the data. Figure 12 shows that the volatility effect across firm size becomes smaller when the financial constraint is severe (θ is small).

These figures map directly into the implications of the Euler equations for physical and intangible capital investment. Together, the theory and the policy functions imply that intangible investment is disproportionately attractive for (i) small firms with high exit risk, (ii) firms operating in environments with high σ_ω, and (iii) financially constrained firms. The quality shock introduces a convex return structure that generates an option-like payoff, and the interaction between uncertainty and financing constraints produces strong cross-sectional variation in intangible investment intensity.

4 Calibration and Quantitative Analysis

In this section, we bring the model to the data. We calibrate the quantitative model with the intangible quality shock introduced in Section 3.2, and then quantify how intangible capital uncertainty, together with financial frictions, shapes intangible intensity across the firm-size distribution.

4.1 Calibration

We now describe our choices of parameter values, which we report in Table 3. We set $\beta = 0.96$,which corresponds to an annual discount rate of roughly 4%. $\alpha_h = 0.4$ and $\alpha_k = 0.3$ are chosen to match the median H/K ratio of 1.36 in the data over the period 1975 to 2000. To isolate the impact of the volatility of the intangible quality shock, we impose the same depreciation rate and adjustment cost for physical and intangible capital, setting $\delta_h = \delta_k = 0.1$, $\rho_h = \rho_k = 0.1$. The investment adjustment cost and depreciation rate are chosen to match the investment rate of physical and intangible capital in the data. We set the fixed production cost to $\xi = 2.45$, which helps match the targeted exit rate for U.S. firms. The idiosyncratic productivity shock follows an AR(1) process in logs with persistence parameter ρ_s and standard deviation of innovations σ_s. We set $\rho_s = 0.8$ and $\sigma_s = 0.1$, similar to models in the literature on investment with firm-level productivity shocks (see Hennessy and Whited (2005)).

We calibrate the key parameters governing intangible investment frictions, the standard deviation of the quality shock, σ_ω, and the intangible capital financing capacity, θ, using cross-sectional moments that directly capture the dispersion in intangible investment across firm sizes. Specifically, we target the H/K spread, defined as the difference in the intangible-to-physical capital ratio between firms in the smallest and largest size quintiles, as well as the employee-equity-to-market-equity ratio, which reflects the extent to which firms rely on equity-based compensation to fund intangible investment. These two moments jointly calibrate σ_ω and θ: a higher σ_ω increases the option value of intangible investment and amplifies the H/K spread, while a higher θ relaxes intangible financing constraints and raises the share of employee equity in firm value.

Table 4 reports the model's fit relative to these empirical moments. The benchmark moments are calculated from the sample period from 1975 to 2000. Panel A compares the model moments under $(\theta = 0.8, \sigma_\omega = 0.2)$ with their data counterparts, while Panel B evaluates the size distribution of intangible intensity, showing that the model successfully reproduces the cross-sectional pattern of H/K ratios across firm-size groups. The spread in the H/K ratio between the bottom 15% and the top 75% of firms is 0.467 in the data, and the model generates a comparable gap of 0.50 while also closely matching the level of the H/K ratio within each size quintile.

4.2 Quantitative Implications

We evaluate the model's quantitative performance and examine how changes in intangible quality shocks shape the dispersion of intangible investment across firms.

Given the sharp rise in the uncertainty of intangible investment and output documented in Section 2.3.2, we ask how much of the increase in the intangible gap since 2000 can be attributed to the rise in the quality shock.

We begin with the benchmark calibration $(\theta = 0.8, \sigma_\omega = 0.2)$ reported in Table 4, which matches key cross-sectional moments in the data before 2000. The benchmark successfully reproduces the pre-2000 H/K spread, the share of employee equity in firm value, average investment rates, and the average firm exit rate. Empirically, the intangible gap increased to 1.374—nearly tripling since 2001—and the quality-uncertainty parameter σ_ω rose by 37.5%, consistent with the increase in the normalized standard deviation of patent values documented in Figure 8.

Table 5 reports comparative statics that vary σ_ω while holding θ fixed. As the volatility parameter increases from 0.2 to 0.275, the intangible gap expands. A higher σ_ω amplifies the real-option value of intangible investment, disproportionately raising intangible intensity among small firms and widening the dispersion in H/K ratios across size groups. Quantitatively, increasing σ_ω generates a substantial rise in the intangible gap. The model-implied H/K spread increases from 0.443 in the benchmark to 0.812 under the higher-volatility calibration. This response accounts for roughly 60% of the observed post-2000 H/K spread of 1.374 in the data (2001–2023). Importantly, the rise in the intangible gap is not driven by an increase in θ. Instead, the results imply that higher quality-shock volatility can account for a large fraction of the observed rise in intangible investment concentration, even without additional easing of financial constraints.

5 Conclusions

We document a persistently rising intangible-investment gap between small and large firms: small firms allocate a substantially higher share of resources to intangible relative to physical capital, even though they are typically more financially constrained. To understand the mechanisms consistent with these heterogeneous patterns, we develop a dynamic industry-equilibrium model featuring both intangible and physical capital. The key mechanism is the option-like nature of intangible investment, which, combined with the rising uncertainty of intangible outcomes, generates the observed divergence. Our quantitative analysis shows that the model accounts for approximately 60% of the post-2000 increase in the intangible gap. This highlights that the rise in intangible intensity cannot be explained solely by changes in the aggregate production function.

Our findings underscore a broader transformation in the U.S. corporate sector: intangible investment is increasingly driven by smaller, younger, and lower-valuation firms. These firms carry much of the economy's innovation risk, conduct more intangible investment, and disproportionately shape the cross-sectional dispersion in investment behavior. Understanding the dynamics of intangible accumulation—its risks, financing constraints, and heterogeneous adoption across firm types—is therefore essential for interpreting modern corporate investment patterns, evaluating innovation policy, and assessing the long-run implications of the intangible economy.

References

Akcigit, Ufuk, and William R Kerr, 2018, Growth through heterogeneous innovations, *Journal of Political Economy* 126, 1374–1443.

Alexander, Lewis, and Janice Eberly, 2018, Investment hollowing out, *IMF Economic Review* 66, 5–30.

Ayyagari, Meghana, Asli Demirgüç-Kunt, and Vojislav Maksimovic, 2024, The rise of star firms: Intangible capital and competition, *The Review of Financial Studies* 37, 882–949.

Brynjolfsson, Erik, Lorin M Hitt, and Shinkyu Yang, 2002, Intangible assets: Computers and organizational capital, *Brookings papers on economic activity* 2002, 137–181.

Clementi, Gian Luca, and Berardino Palazzo, 2016, Entry, exit, firm dynamics, and aggregate fluctuations, *American Economic Journal: Macroeconomics* 8, 1–41.

Cooley, Thomas F., and Vincenzo Quadrini, 2001, Financial markets and firm dynamics, *American Economic Review* 91, 1286–1310.

Corrado, Carol, Charles Hulten, and Daniel Sichel, 2009, Intangible capital and us economic growth, *Review of income and wealth* 55, 661–685.

Corrado, Carol A, and Charles R Hulten, 2010, How do you measure a "technological revolution"?, *American Economic Review* 100, 99–104.

Corrado, Carol A., Daniel E. Sichel, and Charles R. Hulten, 2004, Measuring capital and technology: An expanded framework.

Covas, Francisco, and Wouter J. Den Haan, 2011, The cyclical behavior of debt and equity finance, *American Economic Review* 101, 877–99.

Crouzet, Nicolas, and Janice Eberly, 2023, Rents and intangible capital: A q+ framework, *The Journal of Finance* 78, 1873–1916.

Crouzet, Nicolas, and Janice C Eberly, 2019, Understanding weak capital investment: the role of market concentration and intangibles, Working Paper 25869, National Bureau of Economic Research.

Eisfeldt, Andrea L, Antonio Falato, and Mindy Z Xiaolan, 2023, Human capitalists, *NBER Macroeconomics Annual* 37, 1–61.

Eisfeldt, Andrea L., and Dimitris Papanikolaou, 2013, Organization capital and the cross-section of expected returns, *The Journal of Finance* 68, 1365–1406.

Falato, Antonio, Dalida Kadyrzhanova, Jae Sim, and Roberto Steri, 2022, Rising intangible capital, shrinking debt capacity, and the us corporate savings glut, *The Journal of Finance* 77, 2799–2852.

Gourio, François, and Leena Rudanko, 2014, Customer capital, *The Review of Economic Studies* 81, 1102–1136.

Guiso, Luigi, Luigi Pistaferri, and Fabiano Schivardi, 2012, Credit within the firm, *The Review of Economic Studies* 80, 211–247.

Gutiérrez, Germán, and Thomas Philippon, 2017, Declining competition and investment in the u.s., Working Paper 23583, National Bureau of Economic Research.

Hall, Robert E, 2015, Quantifying the lasting harm to the us economy from the financial crisis, *NBER Macroeconomics Annual* 29, 71–128.

Hennessy, Christopher A, and Toni M Whited, 2005, Debt dynamics, *The journal of finance* 60, 1129–1165.

Hopenhayn, Hugo, and Richard Rogerson, 1993, Job turnover and policy evaluation: A general equilibrium analysis, *Journal of Political Economy* 101, 915–38.

Hopenhayn, Hugo A., 1992, Entry, exit, and firm dynamics in long run equilibrium, *Econometrica* 60, 1127–1150.

Hulten, Charles R, and Xiaohui Hao, 2008, What is a company really worth? intangible capital and the" market to book value" puzzle, Technical report, National Bureau of Economic Research.

Kogan, Leonid, Dimitris Papanikolaou, Amit Seru, and Noah Stoffman, 2017, Technological innovation, resource allocation, and growth, *The quarterly journal of economics* 132, 665–712.

Lev, Baruch, and Suresh Radhakrishnan, 2005, The valuation of organization capital, in *Measuring capital in the new economy*, 73–110 (University of Chicago Press).

Lucas, Robert E., 1978, On the size distribution of business firms, *The Bell Journal of Economics* 9, 508–523.

Lucas, Robert E., and Edward C. Prescott, 1971, Investment under uncertainty, *Econometrica* 39, 659–681.

Ottonello, Pablo, and Thomas Winberry, 2024, Capital, ideas, and the costs of financial frictions, NBER Working Paper w32056, National Bureau of Economic Research.

Peters, Ryan H, and Lucian A Taylor, 2017, Intangible capital and the investment-q relation, *Journal of Financial Economics* 123, 251–272.

Sun, Qi, and Mindy Z. Xiaolan, 2019, Financing intangible capital, *Journal of Financial Economics* 133, 564 – 588, JFE Special Issue on Labor and Finance.

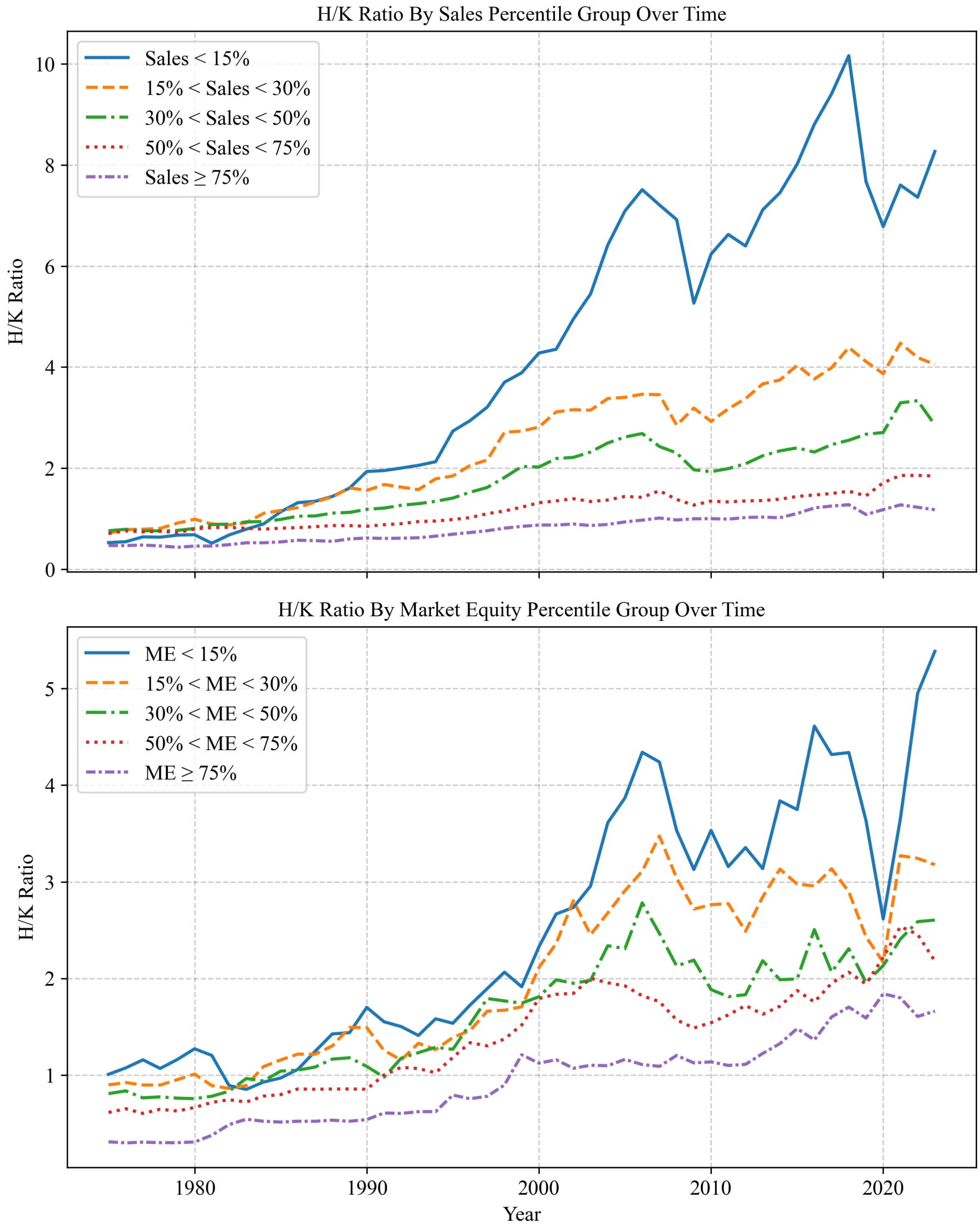

Figure 2: Intangible-Physical Capital Ratio by Firm Size. This figure plots the intangible (H) to physical (K) capital ratio (hkratio) by size over time. Firm size is measured by either sales or market equity, and the hkratio is measured as accumulated intangible investment following Peters and Taylor (2017), H, divided by PPEGT. For each year, firms are sorted into five groups based on their sales or market equity, and the median hkratio is computed within each group to mitigate the impact of outliers.

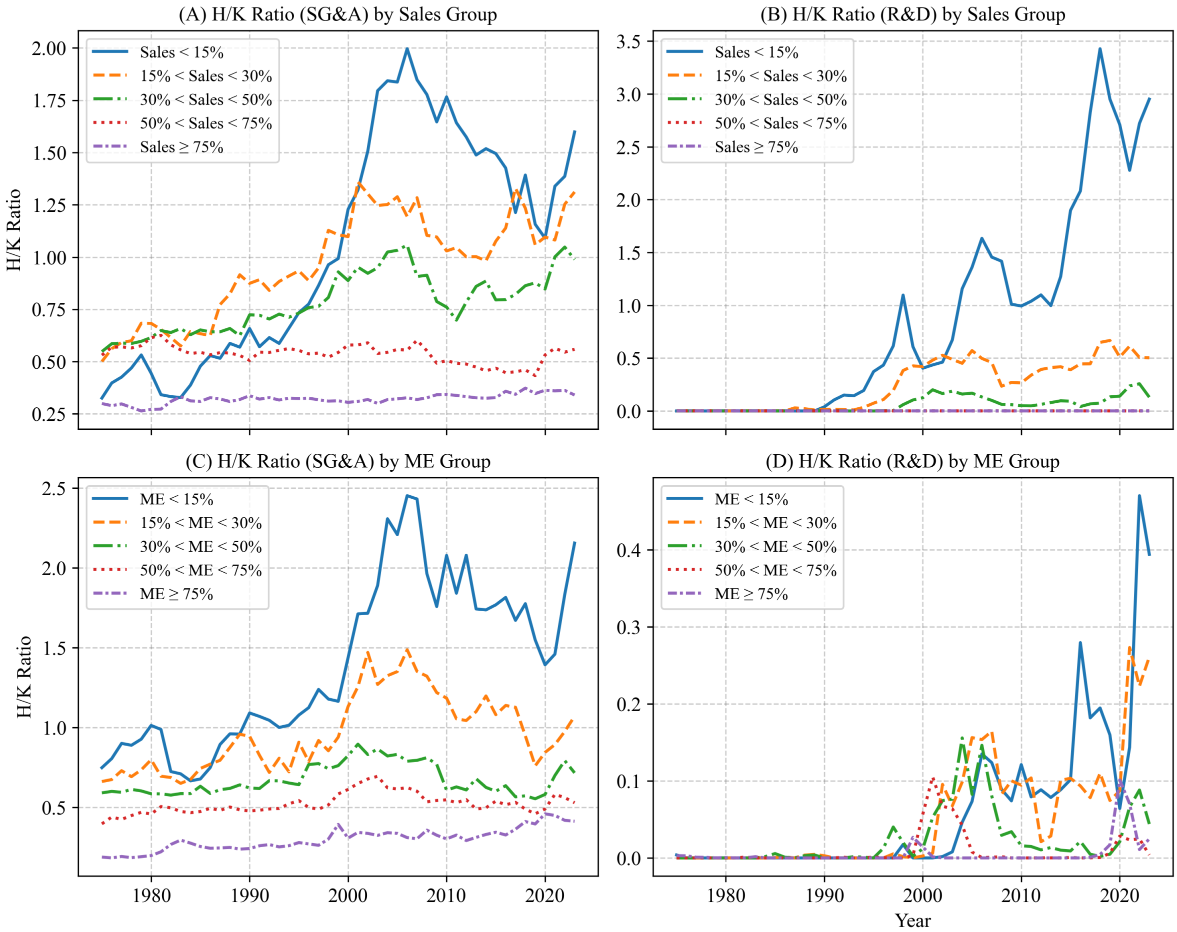

Figure 3: Intangible-Physical Capital Ratio by Firm Size (Robustness). This figure plots additional two types of intangible-to-physical capital ratios. The top panels show ratios constructed from SG&A-based organizational capital (Panel A and Panel C) and R&D-based innovation capital (Panel B and Panel D) across sales percentile groups. The bottom panels present the corresponding ratios across market equity percentile groups. For each year, firms are sorted into five groups based on their sales or market equity, and the median ratio of intangible capital (H) to tangible capital (K, measured by PPEGT) is computed within each group to mitigate the impact of outliers.

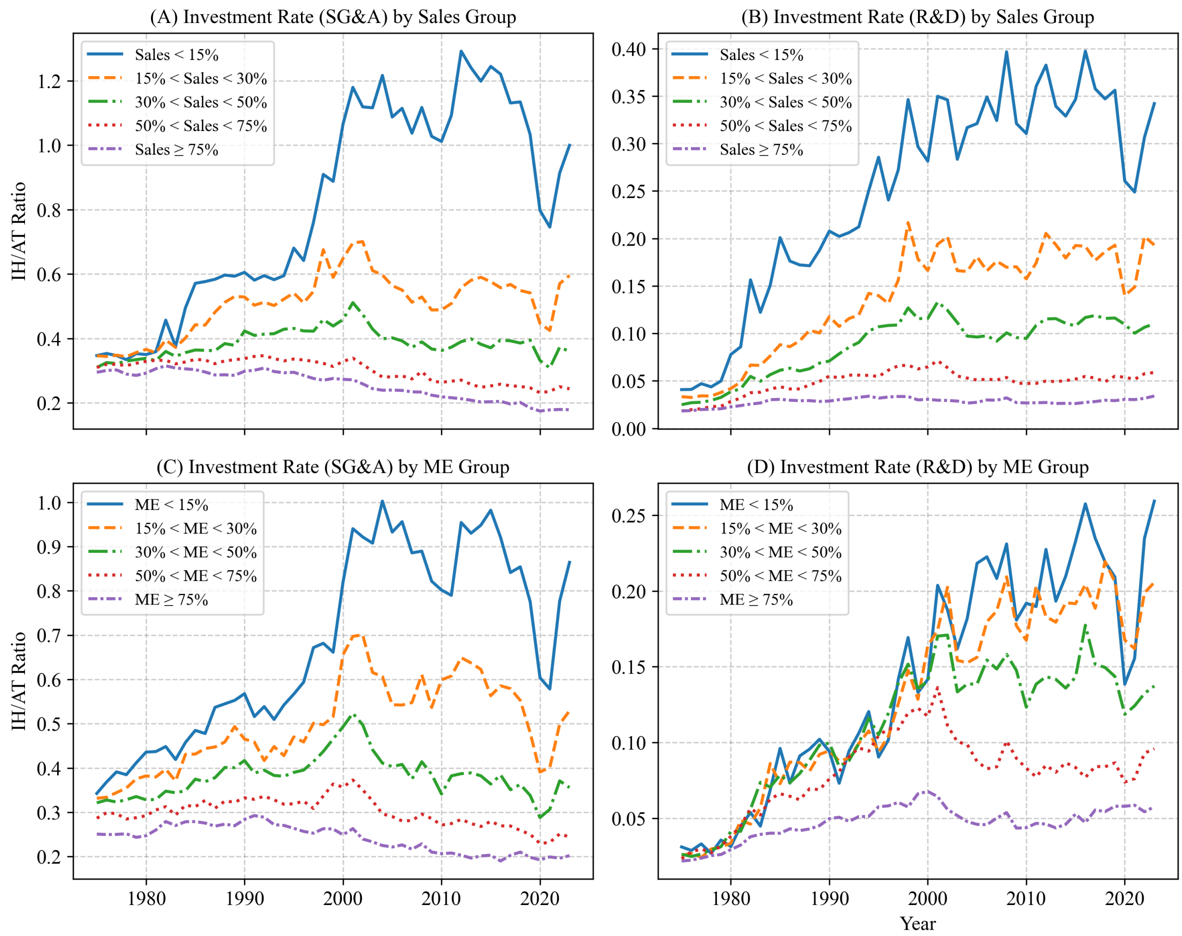

Figure 4: Investment Rate of Intangible Capital by Firm Size. This figure presents the investment in intangible capital (ih) to total assets ratio (AT) by size over time. Size is measured by sales or market equity, and the investment in intangible capital is measured as XRD or XSGA in Compustat. For each year, firms are categorized into five groups based on their sizes, and we estimate the average ik ratio within each group for a given year. Source: Compustat Fundamentals Annual for 1975-2023.

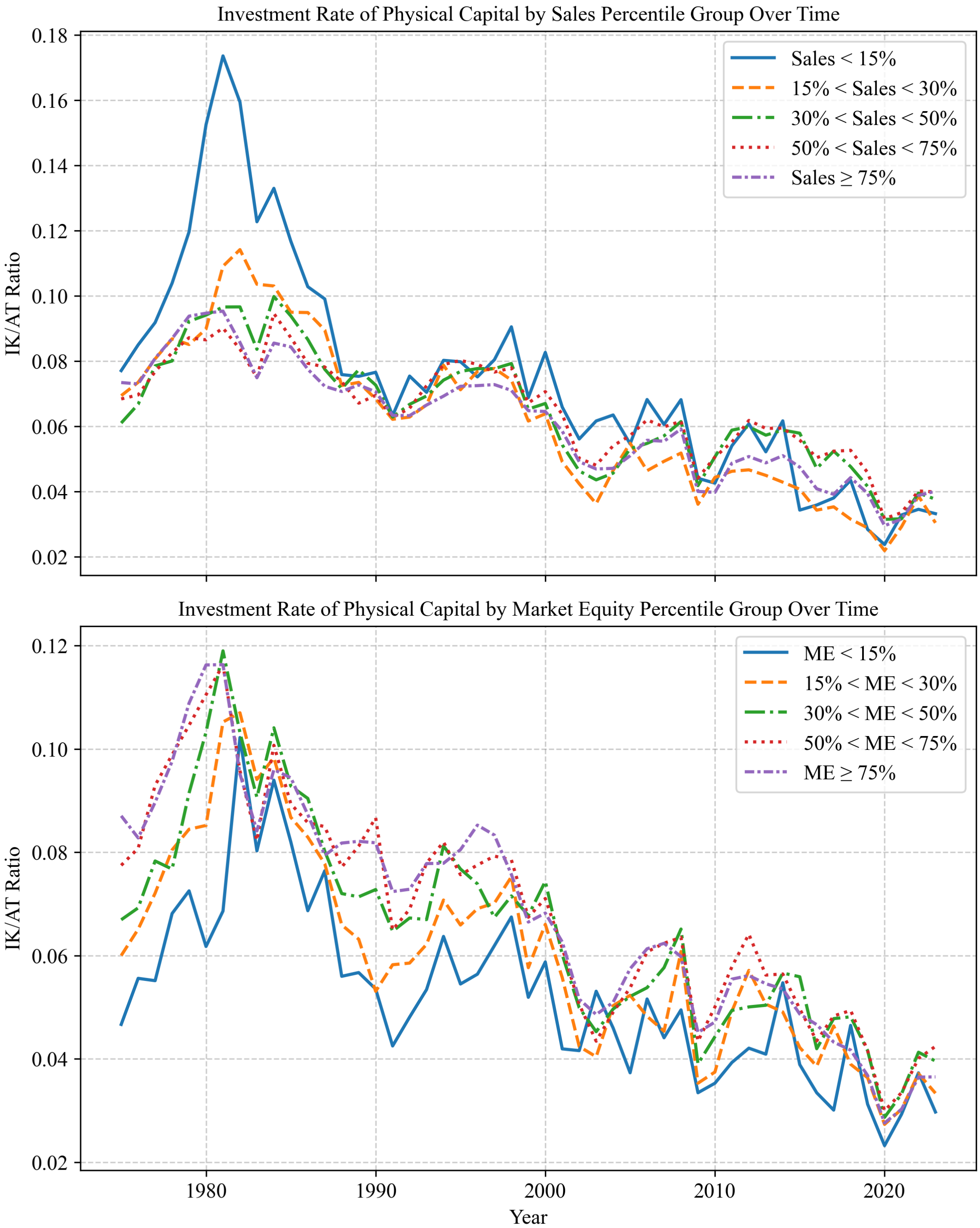

Figure 5: Investment Rate of Physical Capital by Firm Size. This figure presents the investment in physical capital (K) to total assets ratio (ik) by size over time. Size is measured by sales or market equity, and the investment in physical capital is measured as CAPX in Compustat. For each year, firms are categorized into five groups based on their sizes, and we estimate the average ik ratio within each group for a given year. Source: Compustat Fundamentals Annual for 1975-2023.

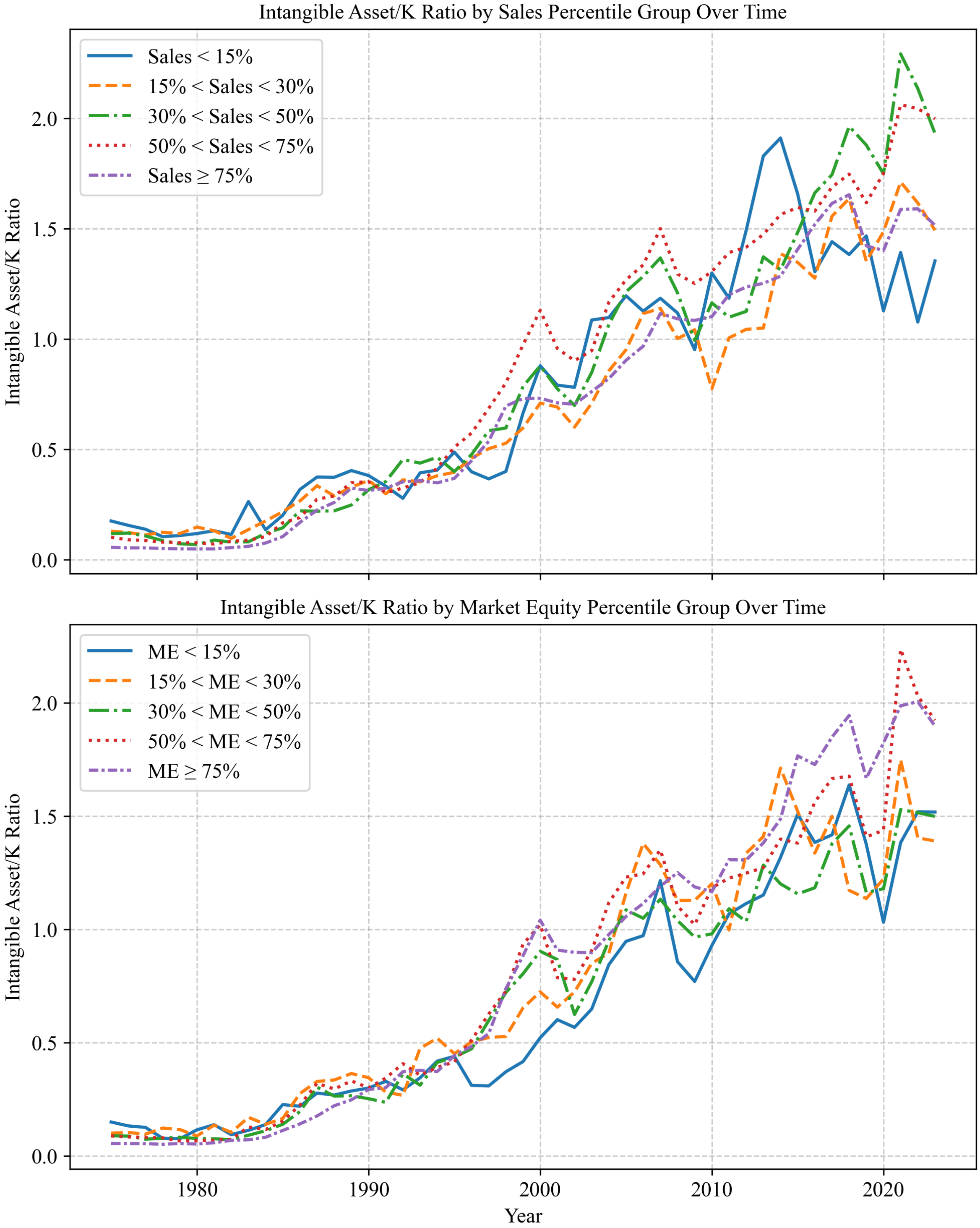

Figure 6: Intangible Assets by Firm Size. This figure presents the aggregate intangible asset (Intang) to physical (K) capital ratio by size over time. Size is measured by sales or market equity, and the ratio is measured as the total intangible assets (INTAN) divided by PPEGT. For each year, firms are categorized into five groups based on their total assets, and we estimate the intangible assets to physical capital ratio within each group for a given year. Source: Compustat Fundamentals Annual for 1975-2023.

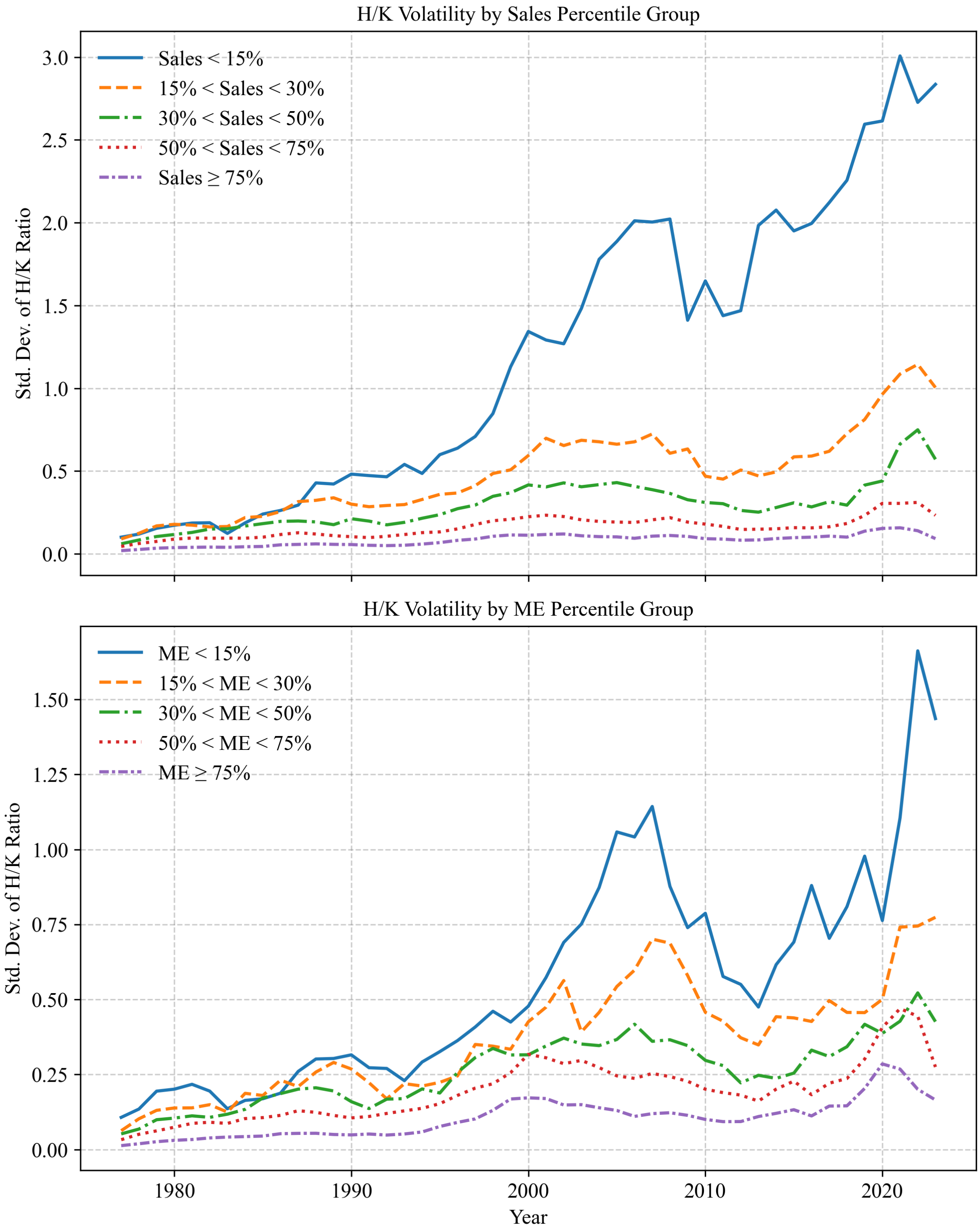

Figure 7: Intangible Volatility by Firm Size. This figure presents the firm-level volatility of intangible (H) to physical (K) capital ratio (hkratio) by size over time. Size is measured by sales and market equity. The volatility of hkratio is measured at the firm level. For each firm j year t, we estimate the standard deviation of the $hkratio_{i,t}$ over the last 5 years. The estimation is done over a rolling window. For each year, firms are categorized into five groups based on their total assets, and we estimate the median hkratio volatility within each group for a given year. Source: Compustat Fundamentals Annual for 1975-2023.

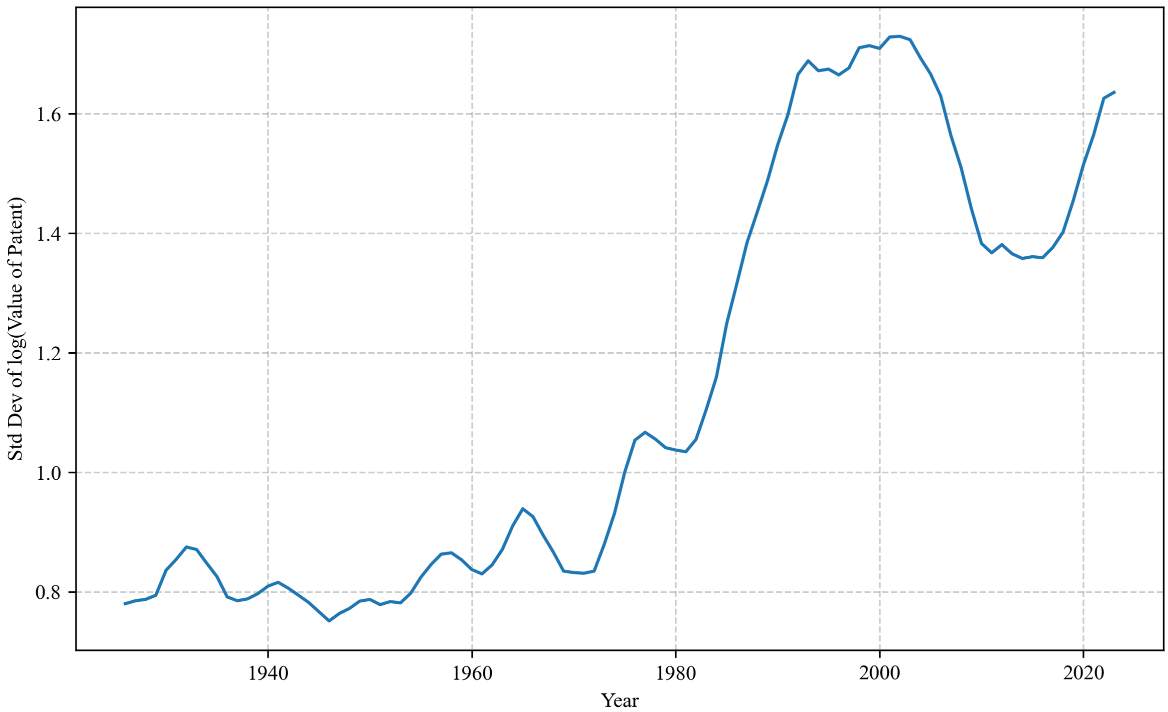

Figure 8: Standard Deviation of Log(Value of Patent) by Year. This figure plots the time series of the cross-sectional dispersion in patent value. The vertical axis shows the standard deviation of the logarithm of normalized patent value, computed within a rolling three-year window. For each year t, the dispersion is calculated using all patents granted in years $t-2$ through t. The patent value measure is obtained directly from Kogan et al. (2017), who construct market-based valuations of U.S. patents using stock market reactions to patent grant announcements. Each patent's value is normalized by the median within the same CPC subclass and year to account for technological field heterogeneity. The 1975 standard deviation is normalized to one.

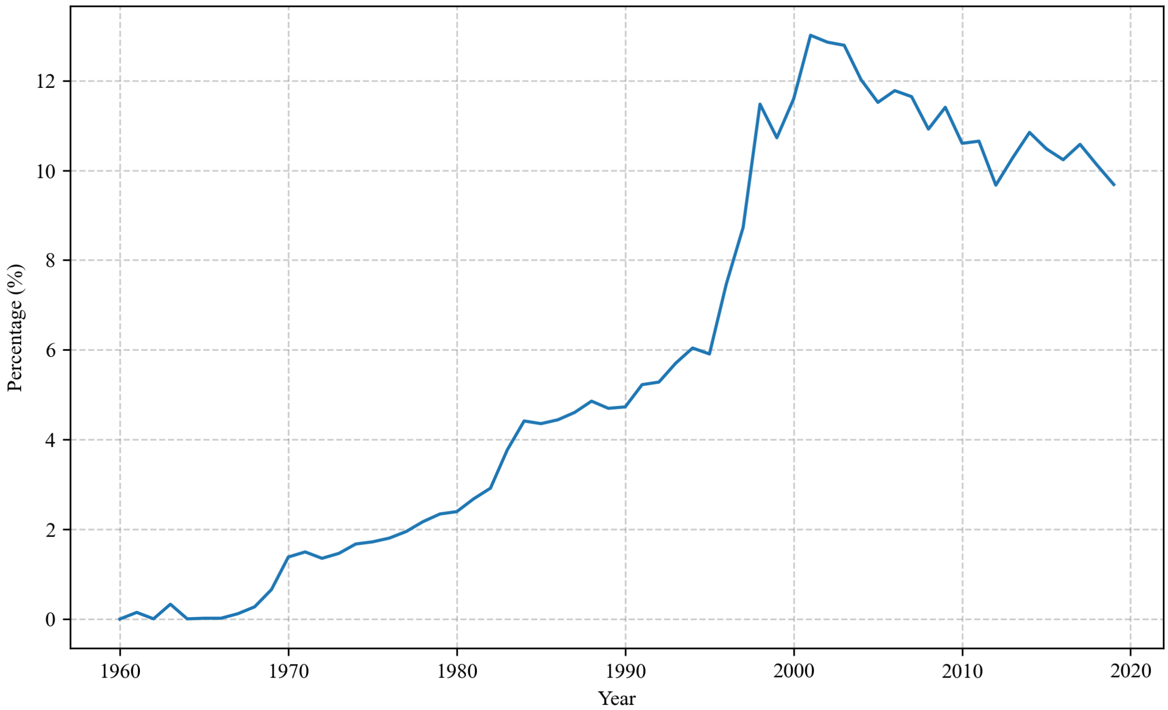

Figure 9: Share of Equity Allocated to Employee Incentives by Year. This figure shows the annual percentage of total corporate equity allocated to employee ownership and stock-based compensation. The firm-level employee stock-based compensation data are obtained from Eisfeldt, Falato, and Xiaolan (2023).

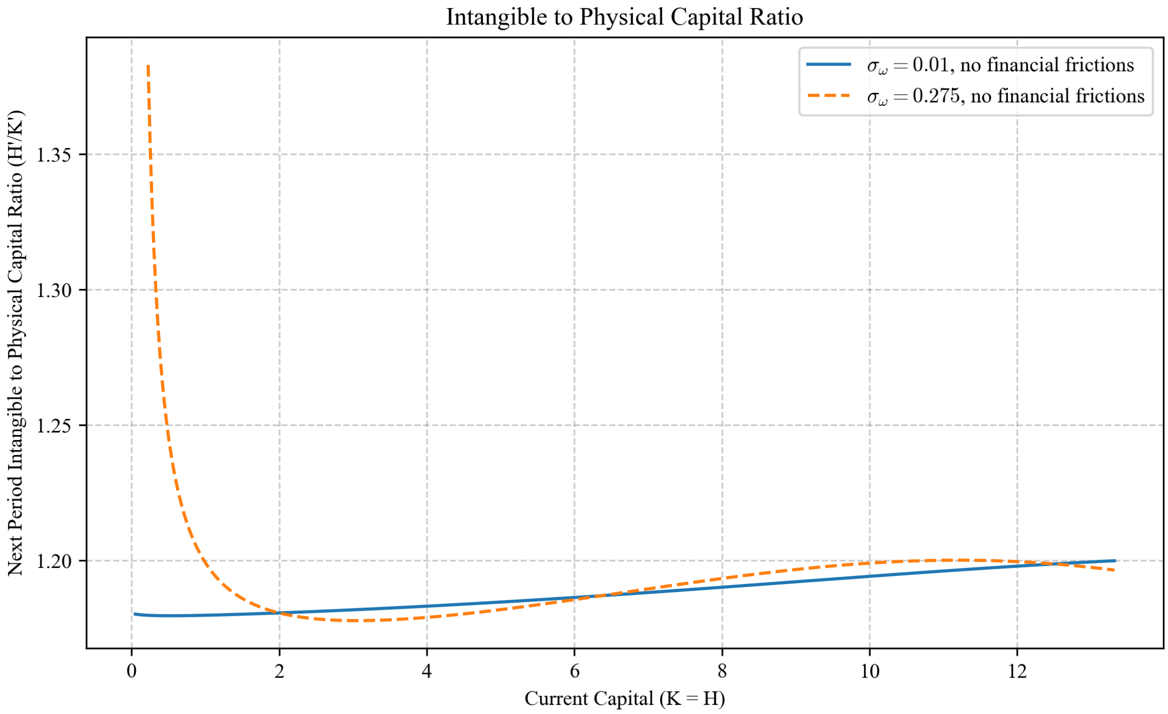

Figure 10: Policy Functions of Intangible to Physical Capital Ratio. This figure plots the ratio of next-period intangible to physical capital H'/K' as a function of current capital ($K = H$) under two levels of quality-shock volatility, $\sigma_\omega = 0.01$ and $\sigma_\omega = 0.275$, in an environment without financial frictions.

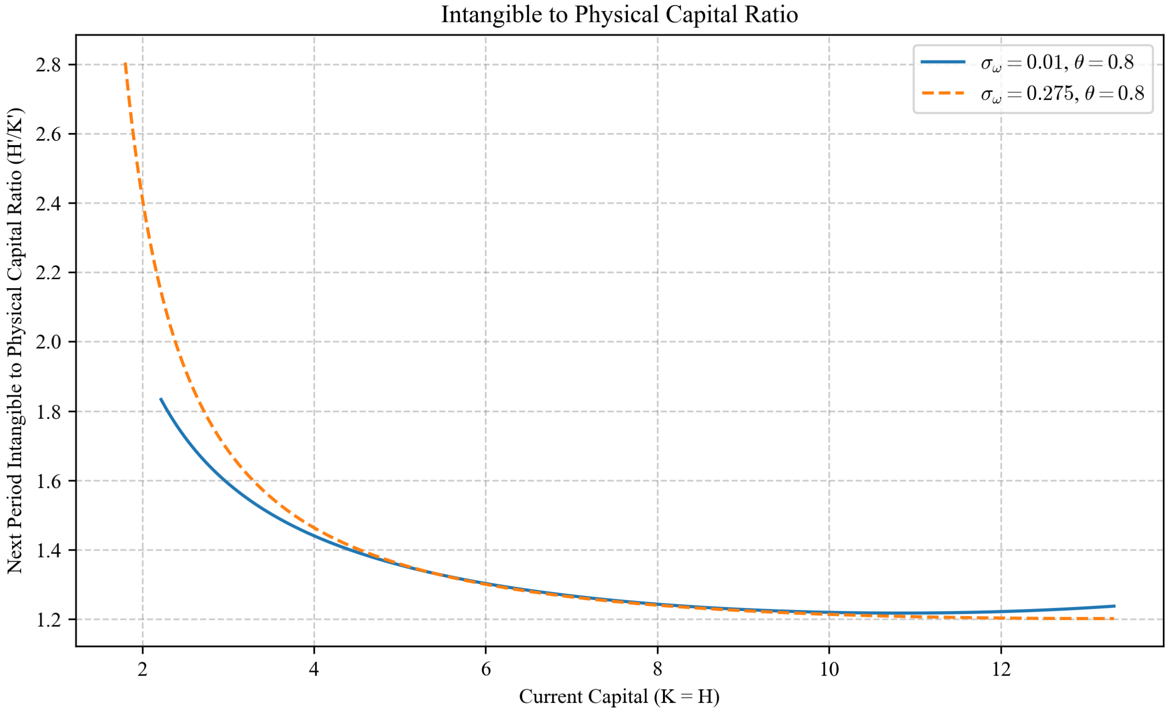

Figure 11: Policy Functions of Intangible to Physical Capital Ratio. This figure plots the optimal ratio of next-period intangible to physical capital H'/K' as a function of current capital ($K = H$) under two levels of quality-shock volatility, $\sigma_\omega = 0.01$ and $\sigma_\omega = 0.275$, when financial frictions are present ($\theta = 0.8$).

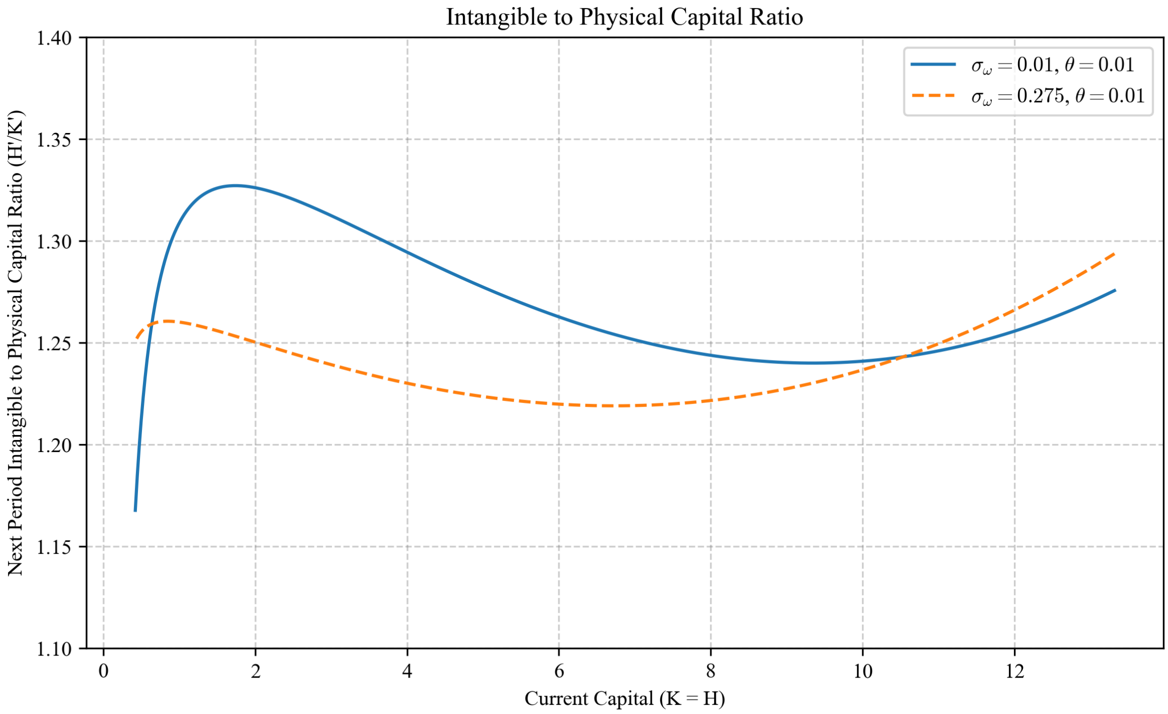

Figure 12: Policy Functions of Intangible to Physical Capital Ratio. This figure plots the optimal ratio of next-period intangible to physical capital H'/K' as a function of current capital $(K = H)$ under two levels of quality-shock volatility, $\sigma_\omega = 0.01$ and $\sigma_\omega = 0.275$, when financial frictions are substantial $(\theta = 0.01)$.

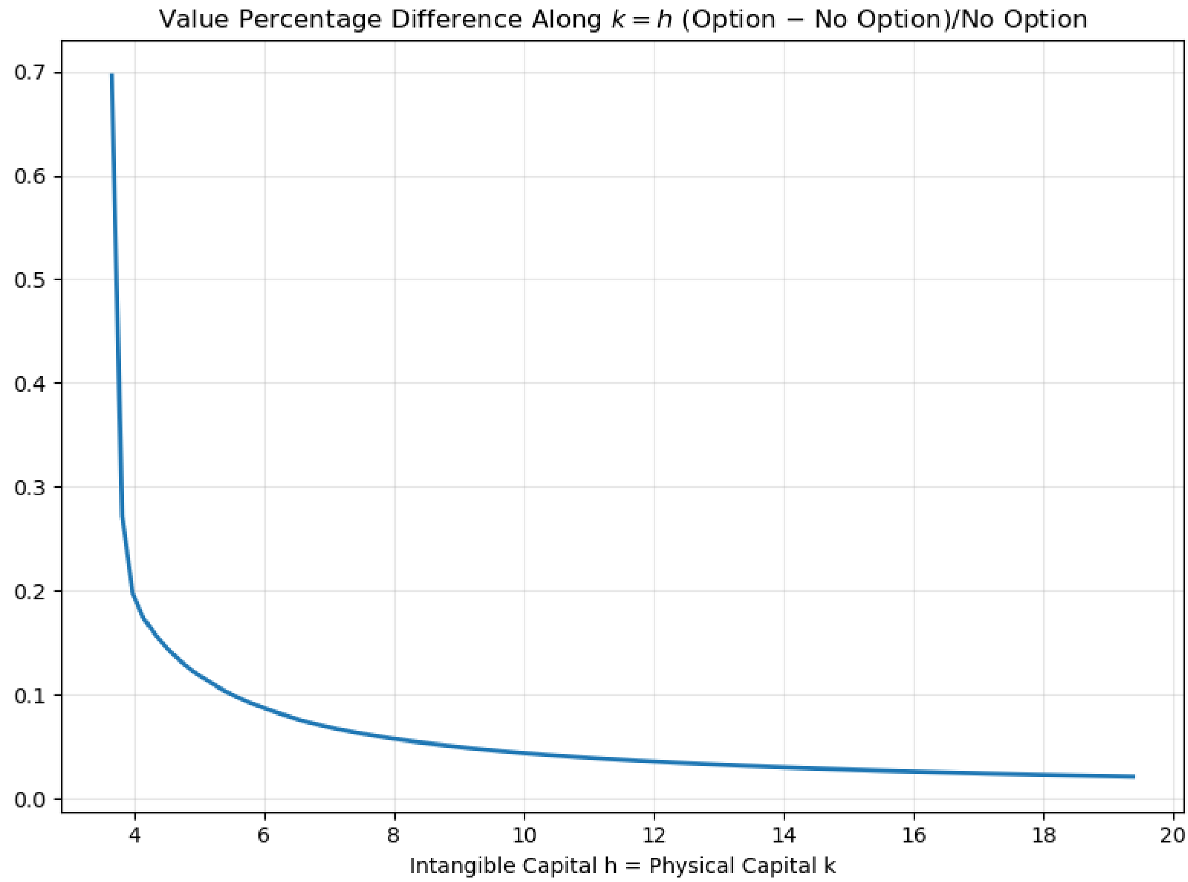

Figure 13: Value of Exit Option.

Table 1: Summary Statistics

This table reports summary statistics for key investment, and financial condition variables across firm size groups. Firm size is defined based on the distribution of market equity (ME). All variables are expressed in real terms and deflated using the Consumer Price Index (CPI). The sample includes U.S. public firms from Compustat Annual over the period 1975–2023.

	Size				
Market Equity	$<15\%$	15–30%	30–50%	50–75%	$>75\%$
	Panel A: Firm Dynamics				
AT (real)	18.52	56.55	147.36	437.84	17011.93
Age (Found)	20.53	23.67	31.86	44.03	84.63
Labor Growth (%)	3.17	3.48	3.37	2.63	1.06
Asset Growth (%)	2.34	3.62	3.83	3.30	1.92
Std of Asset Growth (%)	7.98	6.59	6.05	5.47	4.81
Std of Labor Growth (%)	11.88	9.80	9.06	7.97	6.32
Tobin's q	2.82	2.48	2.25	2.11	1.94
ROA (%)	-8.27	6.19	10.28	11.96	13.29
	Panel B: Investment				
H(SGA)/K	1.93	1.48	1.21	0.93	0.77
H(RD)/K	1.35	0.95	0.76	0.49	0.37
Stkco / ME (%)	1.71	1.24	1.00	0.78	0.48
Stkco / AT (%)	5.69	3.70	2.22	1.60	0.85
R&D / AT (%)	13.96	7.54	4.82	3.30	3.43
SGA / AT (%)	47.61	37.38	31.92	29.14	23.98
AD / AT (%)	1.70	1.77	1.84	1.71	2.10
Patent Value / AT (%) (Citation Weighted)	66.93	24.21	15.22	13.31	7.18
Patent Value / AT (%) (Equally Weighted)	6.62	4.42	4.94	8.57	16.34
	Panel C: Financial Conditions				
DIV	0.19	0.32	0.45	0.59	0.88
KZ	-59.03	-57.31	-20.30	-7.54	-5.94
SA	-2.44	-2.95	-3.31	-3.70	-4.30
WW	-0.14	-0.20	-0.25	-0.31	-0.46
Book Leverage	0.41	0.39	0.42	0.48	0.56
Market Leverage	0.17	0.20	0.23	0.27	0.32

Table 2: Intangible-Physical Capital Ratio and Firm Size. This table summarizes the results from running the following regression:

$$\frac{H_{i,t}}{K_{i,t}} = \beta_0 + \beta_1 \log(size_{i,t}) + \alpha_i + \delta_t.$$

This table reports firm-level regressions of different measures of the intangible-to-physical capital ratio (H/K) on firm size. The dependent variable represents different measures of intangible capital intensity, including total intangible capital, SG&A-based organizational capital, and R&D-based innovation capital. Firm size is measured alternatively by the logarithm of sales (Columns 1–3) and the logarithm of market equity (Columns 4–6). The sample period is 1975–2023. Standard errors are clustered at firm level and are collected in parentheses. $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

		By Sales			By Market Equity	
	Total	R&D	SG&A	Total	R&D	SG&A
	(1)	(2)	(3)	(4)	(5)	(6)
log(sales)	-2.248***	-1.181***	-1.199***			
	(0.303)	(0.174)	(0.231)			
log(me)				-1.237***	-0.770***	-0.691***
				(0.161)	(0.124)	(0.089)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Adj R^2	0.274	0.252	0.215	0.276	0.260	0.212
Observations	257,238	257,238	257,238	225,426	225,426	225,426

Table 3: Parameter Values: Benchmark

Parameter	Symbol	Value
Discount factor	β	0.96
Physical capital share	α_k	0.3
Intangible capital share	α_h	0.4
Depreciation of physical capital	δ_k	0.1
Depreciation of intangible capital	δ_h	0.1
Intangible capital financing capacity	θ	0.8
Fixed production cost	ξ	2.45
Adjustment cost of physical capital	ρ_k	0.1
Adjustment cost of intangible capital	ρ_h	0.1
Standard deviation of intangible quality shock	σ_ω	0.2
Idiosyncratic productivity shock persistence	ρ_s	0.8
Idiosyncratic productivity shock volatility	σ_s	0.1

This table reports the parameter values of our benchmark calibration.

Table 4: Model Fit: Data and Size Moments. The model is calibrated using U.S. firm-level data over the period 1975–2000 to match moments that capture firms' physical and intangible capital dynamics. Panel A reports moments from the 1975-2000 sample and from the model simulated under parameter values ($\theta = 0.8, \sigma_\omega = 0.2$), including the firm exit rate, the H/K spread, the share of employee equity in firm value, and the average firm age. H/K spread $= (H/K)_{size>75\%} - (H/K)_{size<15\%}$, where firm size is measured by market equity. Panel B evaluates the model's performance in replicating the cross-sectional pattern of intangible intensity across firm size groups, comparing the H/K ratios across size quantiles in the data and in the model.

Panel A: Data and Model Statistics

	Data: 1975-2000	$(\theta, \sigma_\omega) =$ $(0.8, 0.2)$	$(\theta, \sigma_\omega) =$ $(0.8, 0.01)$	$(\theta, \sigma_\omega) =$ $(0.01, 0.2)$	$(\theta, \sigma_\omega) =$ $(0.01, 0.01)$
Exit rate	0.046	0.035	0.022	0.215	0.261
H/K spread	0.47	0.44	0.20	-0.25	-0.29
Investment rate of K	0.10	0.14	0.13	0.01	-0.04
Investment rate of H	0.11	0.14	0.12	0.02	-0.05
Employee equity to value	0.05	0.11	0.07	0.01	0.01
Age	44	58	78	173	129

Panel B: Intangible Investment by Size

	<15%	[15%,30%]	[30%,50%]	[50%,75%]	>75%
H/K Ratio Data	1.676	1.437	1.374	1.336	1.209
H/K Ratio $\theta = 0.8, \sigma_\omega = 0.2$	1.818	1.375	1.348	1.333	1.315
H/K Ratio $\theta = 0.8, \sigma_\omega = 0.01$	1.541	1.340	1.340	1.340	1.340
H/K Ratio $\theta = 0.01, \sigma_\omega = 0.2$	1.123	1.158	1.355	1.327	1.370
H/K Ratio $\theta = 0.01, \sigma_\omega = 0.01$	1.060	1.285	1.201	1.354	1.347

Table 5: Comparative Statics: Effects of Quality Shock Volatility and Financing Capacity. This table examines how quality shock volatility (σ_ω) and financing capacity (θ) affect firms' intangible intensity. Each row represents a different calibration scenario, varying σ_ω and θ to isolate the effects of the real-option and financing channels. The first column reports the model-generated H/K spread, defined as $(H/K)_{>75\%} - (H/K)_{<15\%}$ based on firm size measured by market equity. The second column shows the ratio of employee equity to firm value, and the third column presents the ratio of the model-implied H/K spread to the empirical counterpart. Higher σ_ω increases the dispersion in intangible intensity through the real-option mechanism.

	H/K spread	Employee equity to value	% of data (H/K spread)
Data: 1975-2000	0.467	4.8%	100%
Low $\sigma_\omega = 0.2$, low $\theta = 0.8$	0.443	10.5%	94.5%
Low $\sigma_\omega = 0.2$, high $\theta = 1.2$	0.160	16.2%	34.3%
Data: 2001-2023	1.374	11.1%	100%
High $\sigma_\omega = 0.275$, low $\theta = 0.8$	0.812	18.7%	60.1%