Housing Privatization as Intergenerational Redistribution*

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Abstract

In economies with temporary fast-growing wage incomes, housing purchase subsidy to initial cohorts can serve as an effective tool of redistribution from the future towards current generations. Using China's housing reforms in the 1990s as a policy experiment, we show quantitatively that housing purchase subsidies to cohorts who entered the labor market before 1994 substantially increased their home ownership rates in the initial years. This allowed them to reap substantial capital gains – equivalent to intergenerational transfers – when selling their houses to later generations. Quantitative welfare analysis based on a calibrated model suggests that, relative to alternative social security reforms as an intergenerational transfer scheme to the early cohorts, housing purchase subsidy during the housing privatization caused welfare losses for cohorts born in the 1980s but was more desirable for subsequent generations. Alternative social security reforms would have required high social security tax burdens for those generations that would experience a slowdown in wage growth, which is particularly detrimental to their welfare.

Keywords: Housing; Intergenerational redistribution; Capital gains; Social security;
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1 Introduction

Many emerging economies, such as Japan, Korea, Taiwan, China, and other East Asian Miracle economies, had experienced rapid growth of wage incomes, often for a couple of decades, but the rapid income growth eventually slows down for various reasons. Fast wage growth during a period of economic transition might benefit the young more than the old, since the latter will not be able to enjoy the full course of rising incomes due to the, sometimes mandatory, retirement age. A social planner, therefore, may desire intergenerational redistribution from the richer future generations toward the initial generations at the early phases of the rapid economic growth, so that the latter can also share the benefits of the fast growth of the economy.

A conventional policy instrument for intergenerational transfers is through the social security system, which could potentially transfer resources from future generations to the initial ones by offering a generous pension replacement rate. However, given the inevitable *eventual slowdown* of wage income growth, such a redistributive policy, if not timely reformed, will impose a heavy tax burden on future generations. Population aging will further compound the challenge of a generous pension scheme, as it implies a higher old age dependence ratio facing future taxpayers. Thus, a pension system as an intergenerational transfer scheme is not financially sustainable in the *long* run when income growth slows down, unless of course the pension system is reformed by reducing the pension replacement rate and/or raising the retirement age, to reflect the eventual reality of slower growth in the future. However, reducing the generosity of the social security system has often proved politically difficult, if not infeasible.[1]

In this paper, we argue, in the context of China, that during a transition stage featuring fast wage growth, subsidizing housing purchases by the low-wage initial cohorts as part of a housing market privatization can play an important role of intergenerational transfer from the future higher-income generations to the initial generations. The idea is as follows. The housing purchase subsidy to the initial generation allows them to purchase cheap housing at the beginning of the privatization of the housing market, akin to offering them

[1]National surveys consistently show strong opposition to reducing Social Security benefits. In 2024, 79% of adults said benefits should not be reduced in any way (Pew Research Center, 2024). A 2025 Pew review reiterates that most adults oppose any reductions (Pew Research Center, 2025). A Quinnipiac national poll, reported by Yahoo Finance, found 78% of Americans opposed raising the full retirement age to 70 (Quinnipiac University Poll, 2025). Earlier polling summarized by Gallup also found majorities opposed raising the retirement age for younger cohorts (Gallup, 2016). Internationally, the OECD characterizes normal-retirement-age increases as both "crucial and controversial" in aging economies (OECD, 2023). Public reactions often confirm this: for example, the Associated Press reported over 1.1 million people protested France's proposed age increase on Jan. 19, 2023 (Associated Press, 2023). Similarly, a Tokyo Foundation report (Oshio, 2021) explicitly describes raising Japan's statutory pension/retirement age as a "political third rail," underscoring the reform's sensitivity even amid widespread acknowledgment of pension sustainability challenges.

a subsidy to purchase the "initial public offerings" (IPO) of Chinese housing before it is "publicly listed," i.e. before the housing market becomes fully commercialized with the introduction of residential mortgages. As wage income subsequently grows rapidly, commercialized housing also appreciates rapidly, increasing the value of "housing IPO" the initial generation purchased at the beginning of housing privatization with the help of government subsidies. In addition, the initial cohorts' housing ownership rate, partially accentuated by the government subsidy, also helps drive up the growth in housing prices as they trade up for larger commercial houses later on. When the initial generations retire, they can potentially sell or downsize their houses to partially reap the capital gains to finance retirement consumption; as such, the appreciation of housing prices represents an intergenerational transfer of wealth from the younger cohorts to them.

A particularly attractive feature of using housing subsidy to the initial generation as a way to transfer wealth from future generations is that housing prices incorporate and adjust to the future income growth: when the future income growth is fast, the housing price will rise fast; and when the income growth slows down, the housing price will flatten. That is, housing prices as an instrument of intergenerational transfer have an "*automatic adjustment*" property that adapts the intensity of transfer to the wage growth rates of the transition economy, as well as potential demographic changes. As such, one can avoid the politically difficult reforms, as mentioned in Footnote 1, that are necessary for the pension system as the intergenerational transfer instrument.

We study the role of housing privatization as a mechanism for intergenerational redistribution in the context of China, but the insights are more generally applicable in other emerging economies whose rapid income growth is inevitably followed by an eventual slowdown. China offers an ideal setting to study such a policy experiment. Since the early 1990s, China has experienced fast growth in wage incomes until recent years. Between 1992 and 2012, the urban real wage grew at an average rate of 10.8%; since then, wage growth has started to slow down, averaged around 5.5% from 2013 to 2022, including a growth rate of 4.63% in 2022. The wage growth rate is expected to slow down further to 2% after 2050 (Fang and Meyer, eds, 2025). Accompanied with China's fast wage growth was housing market privatization, featured by two key reforms, which we will refer to as the "94 reform" and the "98 reform," respectively. In 1994, the Chinese government allowed employees in the state-owned sector to purchase full or partial property rights to their current apartment units at highly subsidized prices, the so-called "*reformed housing*." In 1998, the central government further approved the use of mortgage loans for commercial housing purchases and terminated the practice of housing provision by the work units as in-kind benefits to their employees. Instead, employers had to include the provision of all implicit and in-kind housing benefits

in the salaries of their workers. [2] The Chinese housing market thus became completely commercialized, and since then, it has appreciated at an annual rate of 10% or higher until around 2021 (see, e.g., Fang et al., 2016).

To quantify the role of China's privatization of the housing market as an intergenerational transfer, we develop a dynamic model of heterogeneous agents who make consumption, saving, housing and inter vivos bequest decisions, where households are heterogeneous in their birth cohorts and skill levels. The model incorporates the most prominent features of the 1994 and 1998 housing reforms – the "94 reform" and the "98 reform" mentioned above. To capture the key features of the Chinese housing system, we incorporate two types of housing, *government* housing and *commercial* housing, where government housing is smaller in size offerings than commercial housing. In the initial pre-1994 steady state before the start of the housing reform, all households rented government houses at discounted rental rates. When the "94 reform" started, households who entered the labor market before 1994 were eligible to purchase government housing at discounted prices with discounts that vary with their labor market tenure, or to purchase commercial housing at the market prices, while retaining the option to continue to rent government housing at a discount. In contrast, households who entered the labor market between the "94 reform" and the "98 reform" lose the option to purchase government housing at the discounted prices but could still rent the government housing at a discount; Finally, those who entered the labor market after the "98 reform" further lose the options of renting government housing at a discount. The model also features a social security system as in China, which, as we explain below, also has some built-in features of intergenerational transfer.

We calibrate the model to the Chinese economy during the period 1994-2014 to target key aggregate moments, including the growth rates of wages and house prices, and the aggregate ownership rates of different types of housing. Our quantitative model can reasonably replicate the ownership rate of government housing along the transition path for different age groups and skill types. We then performed a counterfactual analysis to quantify the impacts of the "94 reform," i.e., price discounts or subsidies, to purchase government-owned houses, on various cohorts that were present in 1994. Our analysis suggests that the "94 reform" significantly increased the ownership rate for government housing, especially among skilled households, in the 1990s, and that for commercial housing during the first decade of the 2000s. The key mechanism is that subsidies to purchase government housing during 1994 and 1998 to the initial cohorts in the 1994 economy facilitate these cohorts to become homeowners, who later traded up to purchase commercial housing.

To evaluate the cross-cohort redistributive effects of the housing purchase subsidy policy

[2] The two decrees issued by the Chinese State Council are "*A Decision on Deepening the Urban Housing Reform*" (July 1994) and "*Notice on Furthering the Urban Housing System Reform and Accelerating Housing Construction*" (1998).

relative to social security system as an intergenerational transfer mechanism, we construct a counterfactual economy without housing purchase subsidies,y but where the pension replacement rates of the initial cohorts are adjusted upward so that these cohorts are as well off as in our benchmark economy with housing subsidies. To capture the downward rigidity of the pension policy (as noted in Footnote 1), we keep the same replacement rate for all future generations; thus, a higher social security tax rate than the benchmark level is necessary to balance the government's intertemporal budget on the pension system. Of course, in the counterfactual economy without government housing subsidies, the payroll tax could be lowered. Despite the countervailing forces between the two tax rates, we find that newborns are always better off in the housing reform compared to those in the pension reform. This is driven by a higher ownership rate in the housing reform, which pushes up the bequest passed from the parents' generations to the newborns, and thus makes them better off.

We assess the sustainability of the government budget under an economic slowdown by simulating a scenario where wage growth unexpectedly declines in 2010 from 8.1% to 5%. We then compare the resulting government deficits under housing reform and pension reform. Slower wage growth reduces pension contributions, leading to a deficit if the social security tax is not adjusted. This effect is more pronounced under pension reform, which entails a higher replacement rate. Our results show that the government deficit equals 24.% of tax revenue under housing reform and 28.7% under pension reform. Because China's pension payments are linked to the current social average wage, slower wage growth also slows pension payouts, partially offsetting the deficit. In a counterfactual scenario where pension benefits are not tied to current wages, the deficit rises to 21.9% under housing reform and 36.8% under pension reform. These findings further highlight the fiscal advantages of housing reform over pension reform.

Related Literature. Our paper is related to several strands of literature. First, our paper is related to the literature on the role of asset prices in intergenrational redistribution.

Our paper is related to several strands of literature. First, our paper is related to the literature on the role of asset prices in intergenerational redistribution. Glover et al. (2020) show that during the Great Recession, a decrease in asset prices benefits younger generations who do not yet have much existing asset holdings but can now buy these assets at low prices, potentially compensating them for the fall in earnings they experience. In contrast, old households experience a large welfare loss because they rely on sales of risky assets, whose value declined in the Great Recession, to finance consumption. More recently, Fagereng et al. (2025) develop a sufficient statistic for the money-metric welfare gain of deviations in asset valuations, where this welfare gain depends on the present value of an individual's net asset sales rather than asset holdings: higher asset valuations benefit prospective sellers and

harm prospective buyers.

Although our paper also highlights the intergenerational redistributive effects of asset price dynamics, specifically housing price, it differs from Glover et al. (2020) in several dimensions. First, our paper explores the redistributive effects of asset prices in the context of an emerging economy, which features temporarily fast growth in wage incomes and increasing asset prices, while Glover et al. (2020) study the redistributive effects of asset prices during the Great Recession, which features large declines in labor incomes and asset prices. Second, the mechanisms for asset prices to redistribute intergenerationally are different. In our paper, redistribution works through a housing purchase subsidy during a phase of privatization of the housing market.[3] In contrast, in Glover et al. (2020), aggregate risks, which temporarily dampened asset prices, and the differential portfolio holdings of young and old, are the main driver of intertemporal redistribution. Consequently, the direction for intergenerational transfers via asset prices is different. In our paper, intergenerational transfer occurs from young generations to old, while in Glover et al. (2020) intergenerational redistribution goes in opposite directions.

The second strand of literature to which our paper contributes is the emerging literature on China's housing reforms and its macroeconomic impacts. Wang (2011) is the first to study the effects of China's 1994 privatization of state-owned housing and argues that the impacts of such reforms on housing prices depend on the degree of housing misallocation prior to the reform. However, the focus of the article is not the role of housing privatization as an instrument of intergenerational redistribution. Fang et al. (2010) study the winners of privatization of public housing in 1994, with a focus on the distributional effects *among* the initial generations. Existing studies in the literature also explore the impacts of housing prices on different households. For example, Fang et al. (2016) argue that in anticipation of rising housing prices, potential home buyers may be forced to buy housing early in life with a high down payment, which can suppress other consumption. Furthermore, for a mortgage borrower, purchasing housing early in life would increase their monthly mortgage payment as a fraction of income.[4] All these findings are in line with our model's prediction on the welfare effects of rising housing prices on cohorts entering the labor force in the 2000s.

The third strand of literature to which our paper contributes to is the literature on reforming China's social security system.[5] Song et al. (2015) is the first to explore social

[3]Since households are borrowing constrained, we show that without housing purchase subsidies the incentives for the initial cohorts to become homeowners would be much weaker. With a similar logic, Hur (2018) finds that the welfare benefit of declining asset prices during the Great Recession is greatly dampened because young generations are often subject to binding borrowing constraints.

[4]See also Chen and Wen (2017) and Jiang et al. (2022), which study the welfare effects of housing bubble bursts in a general equilibrium model.

[5]For an overview of China's pension system, see Fang and Feng (2020).

security reforms as intergenerational transfer in a fast-growing economy such as China.[6] They find that in such an environment, a delayed reform of the current pension system, which involves paying generous pensions to the generations who are currently working or already retired, and negative pensions to subsequent generations, is a second-best policy. In this paper, we show that the housing purchase subsidy for the initial cohorts can provide an alternative approach to transfer to the initial old from the future cohorts via the dynamics of housing prices. Our paper takes seriously the political constraints faced by the government in reducing the social security replacement rate in the future when income growth slows down and/or when the dependency ratio increases.[7]

The remainder of the paper is structured as follows. In Section 2 we describe the institutional background for the evolution of the Chinese housing system, particularly the 1994 and 1998 housing reforms; in Section 3 we develop a full-blown multi-period OLG framework and incorporate two housing reforms occurring in China in the 1990s; in Sections 4 we calibrate the model to match key aggregate and cross-sectional moments; in Section 5 we use the calibrated model to quantify the distributive effects of housing privatization on different generations of individuals; in Section 6 we compare the welfare effects of housing purchase subsidy to initial cohorts on future generations with those of a counterfactual social security reform that achieves the same extent of intergenerational transfer to the initial cohorts; finally in Section 7, we conclude.

2 Institutional Background

Pre-1994 Housing System. From 1949 to 1978, housing in China was allocated through a work unit-employee link as a form of in-kind compensation. The size and location of the housing were determined by factors such as the size of the household and the length of service of the employee in the work unit (Fang et al., 2016).

Between 1978 and 1987, China conducted limited housing reform experiments in selected cities. These trial reforms included encouraging the sale of newly built housing priced according to construction costs, subsidizing the sales of existing public housing units, and increasing the rents charged for public housing (Garriga et al., 2017). However, during this period, most of the participants in the housing market were employers, not individual households. Employers (the so-called "work units") purchased housing and then provided

[6]Similarly, for the U.S., Peterman and Sommer (2019) quantifies the role of the social security system in the United States as intergenerational transfer. The federal government established the social security system in the 1930s to tax the young and transfer to the old by exempting the initial old from paying payroll taxes.

[7]See also Imrohoroglu and Zhao (2018b) and Deng et al. (2023), among others, for discussions of other proposals for reforming China's pension system.

these units to their employees at rents that were substantially below market-based rental rates.

Housing Reforms in the 1990s. It was not until the mid-1990s that China launched its massive privatization reform of the housing market. In July 1994, the State Council issued "*A Decision on Deepening Urban Housing Reform*," which allowed employees in the state-owned sector to purchase at subsidized prices the full or partial property rights of their current apartment units, which were often referred to as the "*reformed housing*." The same document also required employers to gradually terminate the provision of housing as in-kind benefits to their workers.

During this stage, the price on which the discount was based was called the "cost price," which was lower than the market price applicable to the high-income household who purchased houses from the market.[8, 9]

In July 1998, the State Council issued the "*Notice on Furthering the Urban Housing Reform and Accelerating Housing Construction*," commonly referred to as the "No. 23 Decree". According to this decree, employers were no longer allowed to develop or purchase new housing units for their employees. Instead, employers had to include the provision of all implicit and in-kind housing benefits in the salaries of their workers. The purpose of the notice was to establish a market-based housing sector priced according to what high-, medium-, and low-income households could afford to pay. High-income households were expected to buy commercial housing, whereas households that could not afford commercial housing could either buy economically affordable housing or rent relatively cheap housing from the public housing system. To promote commercial housing, in August 2003, the State Council issued "*Notice on Promoting a Sustainable and Healthy Development of the Real Estate Market*," which decreed that commercial housing units would be established as the primary form of housing provision going forward.

Along with housing privatization came the introduction of mortgages. In April 1998, China's central bank, the People's Bank of China (PBOC), issued "*Notice on Individual Mortgage Loan*," which for the first time allowed commercial banks to offer residential mortgage loans to qualified buyers of commercial residential housing. To encourage house purchases, the PBOC set aside 100 billion RMB mortgage loan quota; the mortgage loan

[8] Another pricing rule was called the "standard price," under which the household only enjoyed partial ownership right. However, according to Wang (2011), only 18% of Chinese households had partial ownership. The Chinese government explicitly mandated that the standard price should only be temporary and should be replaced by the break-even cost price by 2000.

[9] Evidence shows that most households who purchased housing during this period (1994-2000) at the cost prices and received a price subsidy from the government. For example, using 2000 census data, Fang et al. (2010) found that among China's eight most populous cities, the average discounted price of housing relative to the market price was about 38%.

term can be 20 years at maximum; and the maximum loan-to-value ratio was established to be 70%. In February 1999, the PBOC issued the "*Guideline for Developing Individual Consumer Credit*" to further promote house purchases with mortgage borrowing.

Housing Market Boom in the 2000s. China experienced an enormous and sustained housing boom in the decade since 2003. For example, Fang et al. (2016) find an average annual real growth rate of 13.1 percent in the four first-tier cities, 10.5 percent in the second-tier cities, and 7.9 percent in the third-tier cities between 2003 and 2013. Wu et al. (2014) show that the national real housing price indices for the 35 major cities increased 17 percent per year between 2006:Q1 and 2010:Q4. In comparison, annual house price growth in US first-tier cities between 1996 and 2006 was slower than annual house price growth that occurred in third-tier Chinese cities between 2003 and 2013, and was only 40% of annual house price growth of Chinese first-tier cities during this period (Glaeser et al., 2017). More recently, Liu and Xiong (2020) show that following a temporary slowdown between late 2013 and 2014, housing prices in the first and second tier cities experienced a remarkable increase during 2015 and 2016, before prices stabilized in 2017. The persistent increase in housing prices in major Chinese cities between 2003 and 2016 has been referred to as the "Great Chinese Housing Boom" (Chen and Wen (2017)).

The nationwide housing market privatization that began in 1994 has resulted in China having one of the highest home ownership rates in the world, rising from 42 percent in 1995 to 78% in 2002 and further increasing to 88.7% by 2007. Since then, China's home ownership rate has remained above 85%.

At least in the early part of the Chinese housing reform, reformed housing played an important role in raising China's home ownership rate before the commercial housing development significantly increased from the 2000s. Sato et al. (2013), using data from the Chinese Household Income Project ("CHIP" henceforth), report that the fraction of households that own privatized public housing increased from 27 percent in 1995 to 61 percent (out of a total national home ownership rate of 78%) in 2002, while during the same seven-year period, the fraction of households that own commercial housing increased only modestly, from 1.3 percent to 7.4 percent. Figure 1 plots the share of various types of housing from 2002-2009, calculated using data from the Chinese Urban Household Survey (UHS). In 2002, reformed houses made up more than 60% of all types of housing, in contrast to a share less than 10% for commercial housing. During 2002-2009, the share of reformed housing steadily decreased, to about 43% in 2009, while the share of commercial houses increased to 36% in 2009. This suggests that households who initially purchased reformed housing later traded up their reformed housing for larger commercial housing. However, reformed housing remained a significant component of China's overall housing stock.

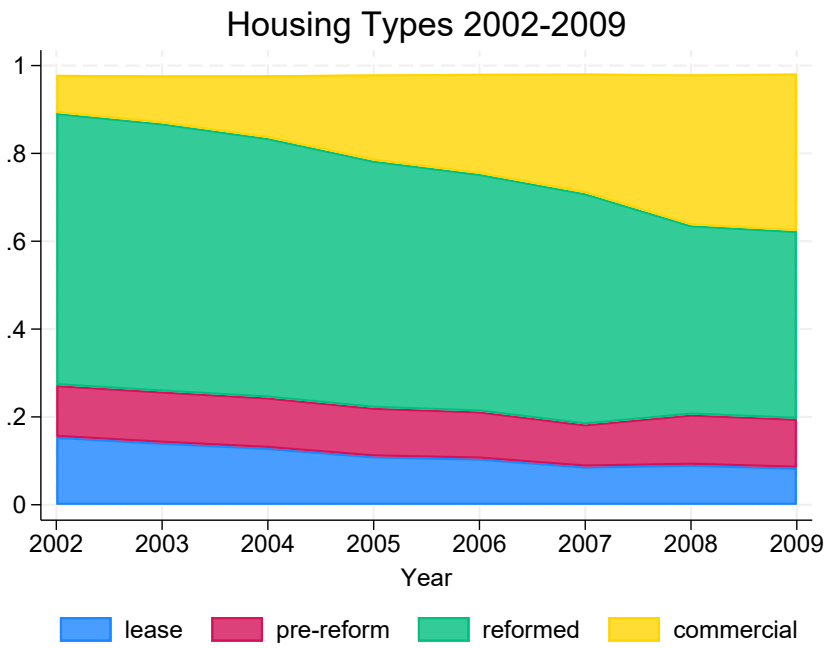


Figure 1: Housing Types: 2002-2009

Notes: The data source is China Urban Household Survey in Various Years.

The trade-up from reformed housing to commercial housing by the initial cohorts who purchased such government housing with subsidies increased not only housing consumption but also nonhousing consumption. For example, using panel data from the China Health and Nutrition Survey ("CHNS" henceforth), Wang (2011) finds that households living in state-owned housing units prior to the reform were consuming approximately 15 percent less housing services than they would have chosen in the private market. Using CHNS data, Yin and Gan (2009) find that after the reforms, households that had purchased public housing had a significantly higher rate of consumption of durable goods a few years later.

3 The Model

In this section, we specify a multi-period overlapping-generations economy, which features economic transition following housing reforms that mimic those in China during the 1990s. For simplicity, we assume that all agents have perfect foresight when the transition started.

3.1 Households

Demographic Structure. In each period, a continuum of households is born and the newborn population grows at an exogenous rate $n_t \geq 0$.[10, 11] and households live a maximum of J periods, where each period represents approximately five years in our quantitative exercises. They enter the economy as adults and work as workers until retirement age J_w. All households have a probability S_j of survival up to age j, and die at age J with certainty, where $S_j = \prod_{k=1}^{j} \psi_k$, and ψ_k is the conditional survival probability from age $k-1$ to age k. The fraction of households of age j at the calendar time t, indicated by $\mu_{t,j}$ is

$$\mu_{t,j} = \frac{\psi_j}{1+n_t}\mu_{t-1,j-1} \text{ for } j \in \{2, ..., J\}; \quad \mu_{t,1} = 1 - \sum_{j=2}^{J} \mu_{t,j}. \tag{1}$$

Preferences. All agents have identical preferences over consumption goods, denoted by c, housing services, denoted by s, and *inter vivos* bequest to children. For a household born in period $\tilde{t}$, his/her lifetime utility is

$$U_{\tilde{t}} = \sum_{j=1}^{J} \beta^{j-1} S_j \left\{ \log(c_{\tilde{t}+j-1,j}) + \phi \log(s_{\tilde{t}+j-1,j} - \underline{s}) + \iota \log(\mathbf{b}_{\tilde{t}+j-1,j} + \underline{\mathbf{b}}) \right\}. \tag{2}$$

In Eq. (2), $\underline{s} > 0$ can be interpreted as the subsistence level of housing services; and $\iota \log(\mathbf{b}_{\tilde{t}+j-1,j} + \underline{\mathbf{b}})$ denotes the *warm-glow* utility of giving *inter vivos* bequest to children.[12] We follow De Nardi (2004) and De Nardi et al. (2010) in our specification of the warm-glow utility function and let $\underline{\mathbf{b}} > 0$ indicate that households will leave a positive inter vivos bequest only if their consumption level exceeds some threshold, suggesting that the bequest is a luxury good. Note that agents can also leave *accidental* bequests at the time of their stochastic deaths; to simplify the analysis, we assume that parents do not obtain utility from accidental bequests. The government will aggregate all inter vivos *and* accidental bequests in the period t and evenly distribute them to newborns who enter the market in period t. Figure 2 depicts our modeling choice for bequests.

The total collectible bequests that will be transferred to the newborn by the government

[10] "Newborns" in our model will correspond to twenty-year-olds who enter the labor market for the first time.

[11] Endogenizing fertility decisions is important, but will significantly complicate the analysis, and we leave it for future research.

[12] Assuming warm-glow utility from inter vivos bequests avoids the complications of having to solve the household's problem as a dynastic problem linking all generations of the households.

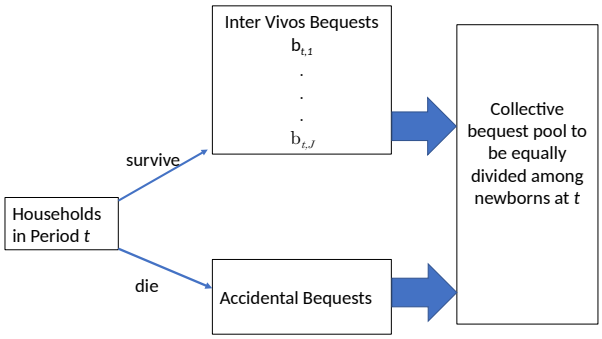


Figure 2: Inter Vivos and Accidental Bequests

can be expressed as

$$\sum_{j=1}^{J} \mathbf{b}_{t+1-j,j} S_j N_{t,j} + (1 - S_j) N_{t,j} NW_{t,j},$$

where $NW_{t,j}$ is the *average net worth* among those who die in period t at age j, among whom the net worth varies endogenously by their choices of housing and asset positions and will be specified later.

Income Processes. Households in our model differ *ex ante* in their skills, proxied by education, and birth cohorts. We assume that the after-tax income of household i in period t at age j, denoted by $y^i_{t,j}$, is given by:

$$y^i_{t,j} = \begin{cases} (1 - \tau^{ss} - \tau) w_t e^i_{t,j} & \text{for } j \leq J_w \\ b^i_{t,j} & \text{for } j > J_w \end{cases} \tag{3}$$

where w_t is the wage rate per unit of efficiency labor supply; $e^i_{t,j}$ is i's efficiency units in period t at age j which is described below in Eq. (4) below; $b^i_{t,j}$ is the social security benefit for a household i that is of age j in period t, which we describe in Eq. (6) below; τ and τ^{ss} are, respectively, the constant labor income tax rate and the social security tax rate. τ^{ss} is determined to balance the government's intertemporal budget on social security, which we describe in Eq. (8) below; and τ is determined to balance the government's intertemporal budget on housing purchase and rental subsidies, which we describe in Eq. (17).

The efficiency unit of household i's labor supply in period t at age j is specified as:

$$e^i_{t,j} = \lambda^i \varepsilon_j z^i_t \epsilon^i_t, \tag{4}$$

where λ^i is a skill-specific component to capture the skill premium; ε_j is a life-cycle income profile that reflects, e.g., the experience effect; z_t^i is a persistent shock that follows an AR(1) process

$$\log(z_t^i) = \rho \log(z_{t-1}^i) + \nu_t^i, \tag{5}$$

where ν_t^i follows a standard normal distribution $N(0, \sigma_\nu)$; finally, $\epsilon_{t,j}$ is a transitory shock drawn randomly from a log-normal distribution in each period.

The stochastic income process as specified by Eq. (3), together with the borrowing constraint, which we specify below, allows us to generate heterogeneity among households in the timing and the sizes of housing purchases.

Social Security Pension Benefits. The government pays social security benefits to retirees. For an individual household i that is of age j in period t, the social security benefit in that period is determined as

$$b_{t,j}^i = \theta[(1-\nu) y_{t-j+J_w, J_w}^i + \nu \bar{y}_{t-1}], \tag{6}$$

where $\theta > 0$ is the *pension replacement rate*, J_w is the maximum working age. The social security pension benefits of a retiree, to a first-order approximation, are affected by the pensioner's own contribution to the system before he/she retires — as proxied by the term y_{t-j+J_w, J_w}^i – the before-tax wage earnings of the household i at its retirement age J_w,[13] and the previous year's "social average wage" of all workers as proxied by $\bar{y}_{t-1}$– the average yearly earnings of all workers in period $t-1$ – which is given by:

$$\bar{y}_{t-1} = \frac{\sum_{j=1}^{J_w} w_{t-1} \mu_{t-1,j} \sum_i \mathbb{E} e_{t-1,j}^i}{\sum_{j=1}^{J_w} \mu_{t-1,j}}. \tag{7}$$

In Eq. (6), $\nu \in (0,1)$ captures the weight between the social average wage and the pre-retirement wage.

Remark 1. *Note that the fact that a retiree's pension benefit depends not only on his/her own earnings history prior to retirement, but also on the social average wage of active workers is a rather unique feature of the Chinese pension system, and it permits a partial indexation of retirees' pension to the growing average income of the economy. Thus, the Chinese pension system in itself permits some degree of intergenerational redistribution. This is a fact that is often not fully appreciated by existing research on the Chinese pension system.*

[13] Note that the subscript of y_{t-j+J_w, J_w}^i, i.e. $t-j+J_w$, is the last working period of retiree i.

Finally, to pin down the social security payroll tax rate τ_t^{ss} in Eq. (3), we assume that τ_t^{ss} is constant over time and endogenously determined to balance the government's intertemporal budget on social security, thus

$$\tau^{ss} \sum_{t=1}^{\infty} \frac{\sum_{j=1}^{J_w} w_t \mu_{t,j} \sum_i \mathbb{E} e_{t,j}^i}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{\sum_{j=J_w+1}^{J} \mu_{t,j} b_{t,j}}{(1+r)^t}, \tag{8}$$

where the left hand side is the discounted sum of the total social security payroll tax revenue from all workers, and the right hand side is the total discounted sum of the pension payments to all retirees, assuming a constant discount rate of r.

3.2 Housing Reforms

Pre-reform Steady State. Before housing reforms, the government owned all rental apartments. As a first-order approximation, during this era, the government assigns to each household an apartment whose size is determined according to the household's skill type λ^i, which proxies educational attainment: households of skill type λ^i are assigned an apartment of size h_g^i by the government where the subscript g indicates that this is a government-owned apartment; households pay a rental rate determined by the government. Again, as a first-order approximation, we assume that the more skilled households are assigned larger apartments, that is, h_g^i increases weakly in λ^i.

We assume that before period 1 (which corresponds to the year 1994 in the data when the Chinese government initiated the housing reform), the economy was in the pre-reform steady state and the government budget was in balance. Also, reflecting the nature of a socialist economy, the households' idiosyncratic shocks are assumed to be perfectly insured in the pre-reform era.

We assume that the government uses land and labor to produce housing and then rents it out to households at a given rental rate. The equilibrium outcomes in the pre-reform steady state determine the initial distribution that we observe in the data when housing reform started in period 1. Appendix B specifies the households' optimization problem in the pre-reform steady state.

Housing Privatization. Because private home ownership in the urban area was prohibited before the housing reform, no households born before period 1 owned any house. Housing privatization began at the beginning of period 1, when the government first sold all the rental apartments it owned to absentee rental companies. From period 1 onward, all households can also choose to rent a commercial apartment $h_R \in \mathcal{H}_R = \{h_R^1, h_R^2, ...h_R^{N_R}\}$, where $h_R^1 < h_R^2 < ... < h_R^{N_R}$ that provides housing service $s = h_R$ at the market rental rate ρ_t, where $\mathcal{H}_R$

is the set of size options among the *commercial rental* units; or purchase an owner-occupied apartment $h_O \in \mathcal{H}_{\mathcal{O}} = \{h_O^1, h_O^2, ...h_O^{N_O}\}$, where $h_O^1 < h_O^2 < ... < h_O^{N_O}$, at market price p_t, where $\mathcal{H}_O$ is the set of size options among the *commercial owner-occupied* units.

We account for several important differences between renting and owing a home. First, the set of size options for commercial rental units ($\mathcal{H}_R$) and owner-occupied units ($\mathcal{H}_O$) may be different, e.g., commercial rental units tend to be smaller on average than owner-occupied units. Second, to capture the fact that there are additional utility from owning than renting a home, we assume that an owner-occupied apartment provides housing services $s = \zeta h$, with $\zeta \geq 1$ reflecting the idea that owning a home allows one to access the local amenities, e.g. attending public schools, or to customize the apartment to one's own liking, while renting does not. Third, from 1998 forwards, home buyers can borrow via mortgages subject to down payment requirements, while renters do not have this option; we will describe the details of the mortgages below. Fourth, owner-occupied houses carry a maintenance cost per period, $\delta_h p_t h_t$, expressed in units of the numeraire good, and we assume that the maintenance fully compensates for any physical depreciation of the dwelling. Fifth, when a household sells its home of size h_t in period t, it incurs a variable transaction cost $\tau_s p_t h_t$ that is proportional to the value of the house transaction.

Subsidized Purchase of Reformed Houses. Households that were already present before period 1 (corresponding to year 1994), who we refer to as the "initial cohorts" throughout this paper, were eligible to purchase the government-assigned apartment they rented in the pre-reform period – the so-called "*reformed houses*" –at the subsidized house price $p_{g,t}(j)$. For a household aged j at period t, the discounted price to purchase government housing, to a first-order approximation, is given by:

$$p_{g,t}(j) = p_t \left[1 - \frac{\min\{\max\{0, 1+j-t\}, J_w\}}{J_w} \times 65 \times 0.9\%\right], \tag{9}$$

where p_t is the market price of housing at period t, and $1 + j - t$ is simply the household's age in 1994 (recall that year 1994 corresponds to the first period, i.e. $t = 1$). According to this formula, the discount on house purchase is cohort-specific: if a household was retired before 1994, then he can enjoy a 58.5% (65*0.9%) price discount.[14] The longer a household worked before 1994, the more discount he/she can enjoy. This reflects the government's desire to transfer resources from future cohorts to these cohorts who entered the labor force before 1994. Note that the term $\{\max\{0, 1+j-t\}$ ensures that only households who were already present in the economy in 1994, i.e., the initial cohorts, were eligible to receive

[14]In reality, the total effective maximum working years of the household in the computation of the house price discount is 65 (35 years for the husband, plus 30 years for the wife, to be consistent with the fact that male retire at age 60 and females retire at age 50.)

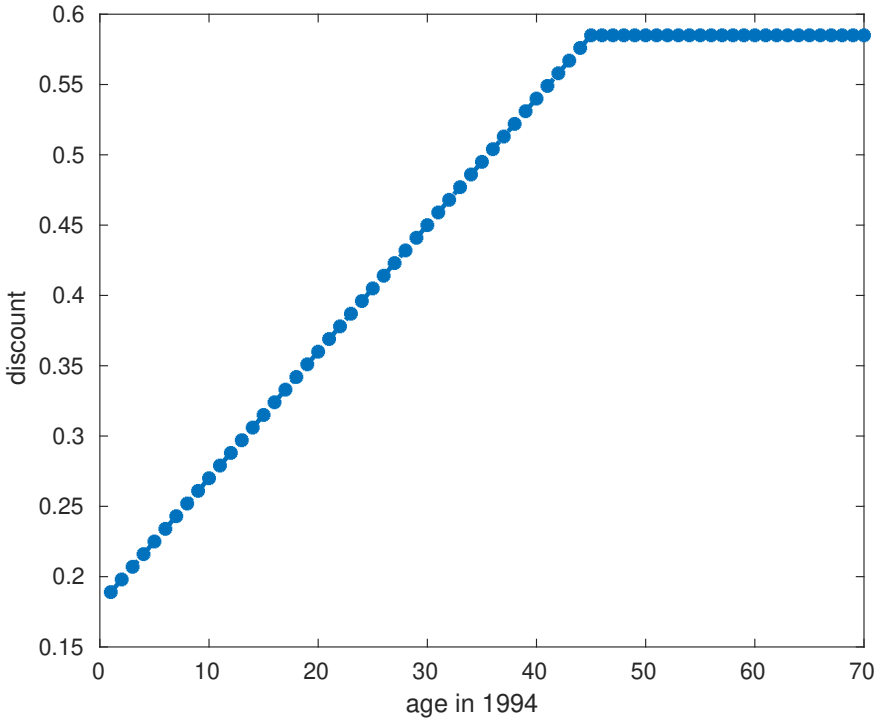


Figure 3: Housing Purchase Discounts for the Initial Cohorts

housing purchase subsidies when they purchase the government-owned apartments. Figure 3 illustrates the cohort specific price discounts.

The initial cohorts can also continue to rent government-assigned apartments at discounted rental rates, which we assume to be a fraction ω of the market rental rate ρ_t.

In addition to buying or renting government-assigned apartments at discounted price or discounted rental rate, after the housing reform began, initial cohorts could also buy or rent housing from the commercial market, giving them the option to choose the size of the house they desire from the set $\mathcal{H}_O$ and $\mathcal{H}_R$, respectively. However, once the initial cohorts chose to move out of government-assigned rental units and move into commercial housing, they became ineligible to purchase the reformed house throughout the rest of their life.

Elimination of Government Rental Houses in the 1998 Reform. The 1998 housing reform eliminated, for cohorts who entered the economy after 1998, the option to rent government houses at discounted rental rates. However, all cohorts who entered the economy on or before 1998 could continue renting government-assigned apartments at discounted rent until they moved to commercial housing. The 1998 reform also clarified that the initial cohorts, i.e. those in the economy before 1994, could continue to have the option of purchasing the rental housing at a discounted price until the year 2004.

Mortgage Market. Between 1994 and 1998, home buyers in China did not have access to mortgages. The 1998 reform allowed banks to issue mortgages to home buyers using housing as collateral starting from 1998 (corresponding to period 5 in our model). All mortgages are long-term, subject to a fixed mortgage origination cost $\bar{m}$, and amortized over the remaining life of the buyer at a constant real interest rate r_m. At the time of mortgage origination, households are subject to the loan-to-value ratio limits. The initial mortgage balance d must be less than a fraction of $1-\gamma$ of their housing purchase prices; thus, γ is the downpayment requirement, which was 30%.

After the origination of the mortgage loan, for a household of age j with an outstanding mortgage balance d, the *minimum mortgage payment* per-period, $\underline{m}$ is determined by the constant amortization formula as follows:

$$m_j \geq \underline{m} \equiv \frac{r_m(1+r_m)^{J+1-j}}{(1+r_m)^{J+1-j}-1}d \tag{10}$$

Accordingly, the principal evolves according to:

$$d_{j+1} = d_j(1+r_m) - m_j. \tag{11}$$

3.3 Household Decision

We denote the potential status of a household's housing at the beginning of the period t as $I_t \in \{R_g, R_c, O\}$, where R_g stands for the rental of an apartment owned by the government; R_c for the rental of commercial housing; and O for the ownership of a home. Depending on the year they entered the economy, households of different cohorts have different housing choice sets. In this section, we describe the household decisions for three groups of cohorts: the initial cohorts who were in the economy prior to 1994; the cohorts who entered the economy between 1994 and 1998; and finally, the cohorts who entered the economy on or after 1998. A full description of the household problem in recursive form and a formal definition of equilibrium are provided in Appendices B and 3.6, respectively.

Initial Cohorts. Figure 4(a) summarizes the housing options for the initial cohorts who were already in the economy before 1994. If the household rented a government-assigned apartment at the beginning of a period, i.e., if $I_t = R_g$, he/she has four options: i) keep renting the government-assigned apartment of size h_g^i at a discount; ii) purchase the government-assigned apartment of size h_g^i that he/she previously rented at a subsidized price; iii) rent a house from the commercial market with the size option set $\mathcal{H}_R$; iv) purchase a commercial house, which is not eligible for government subsidy, from the size option set $\mathcal{H}_O$. Those who remain as renters or owners of government housing – recall that its size is fixed

by the government – choose the quantity of non-durable goods to consume and how much to save in the liquid asset.[15] Before 1998, home buyers could not borrow to buy houses; but from 1998 onward, home buyers also chose the value of the mortgage after meeting the down payment requirement and paying the mortgage initiation fees.

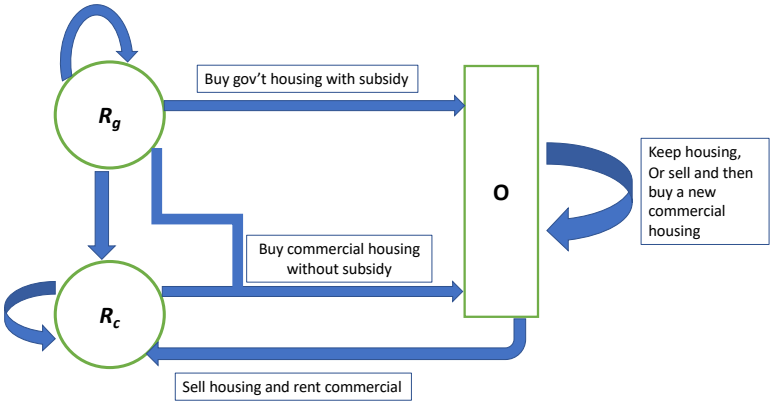
For households that rent commercial houses (i.e., $I_t = R_c$), they can only rent or purchase commercial houses. For a homeowner (i.e. $I_t = O$), whether he/she owns a reformed house or a commercial house, he/she has three options: i) keep the current house; ii) sell the current house, after which the household can buy a commercial house; iii) sell the current house, after which the household can rent a commercial house. Once the household sells a house, he/she needs to pay down all existing mortgage balances, if any, in addition to any housing transaction costs.

The decision on whether to purchase a reformed house or a commercial house is based on the following trade-off. Reformed housing is cheaper than commercial housing. However, purchase of a commercial house does not have a size restriction, while the household can only purchase the reformed house he/she previously rented. Therefore, high-income households may choose to purchase commercial housing for a more desirable home size and for more capital gain. A middle-income household, whose optimal housing size is small, may choose to purchase the reformed housing at the initial stage; after they accumulate enough savings for a down payment for commercial houses, they may sell their reformed houses and trade up for commercial houses. Low-income households may choose to continue to rent the government house at the subsidized rental rates and save for the down payment for the reformed houses.

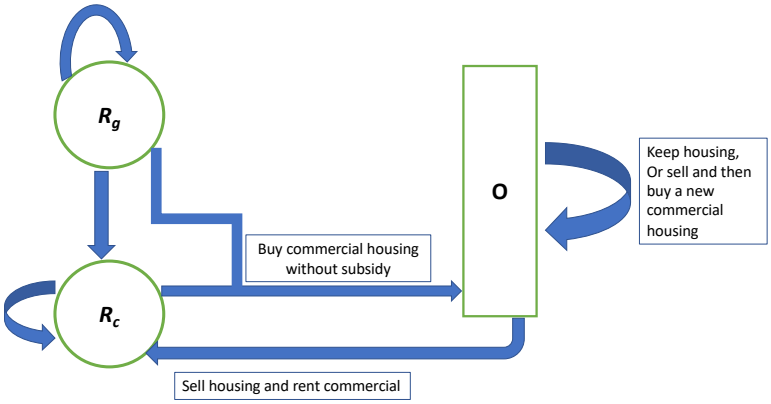
Cohorts Who Enter the Economy between 1994 and 1998. Figure 4(b) summarizes the housing options for cohorts who entered the economy between periods 1 and 4 (between 1994 and 1997). Their problem is similar to the initial cohorts, with only one difference: even if they are renters of government-assigned apartments (i.e., $I_t = R_g$) , they do not have the option of purchasing the unit as reformed houses.

Similarly to initial cohorts, low-income households born during these years tend to choose to rent government-assigned apartments rather than rent commercial houses, tolerating the distortion on housing sizes in exchange for the discounted rental rate of government-assigned apartments. After they have accumulated more wealth, they may switch to rent commercial houses, followed by purchasing commercial houses, when he/she has accumulated enough savings for a down payment.

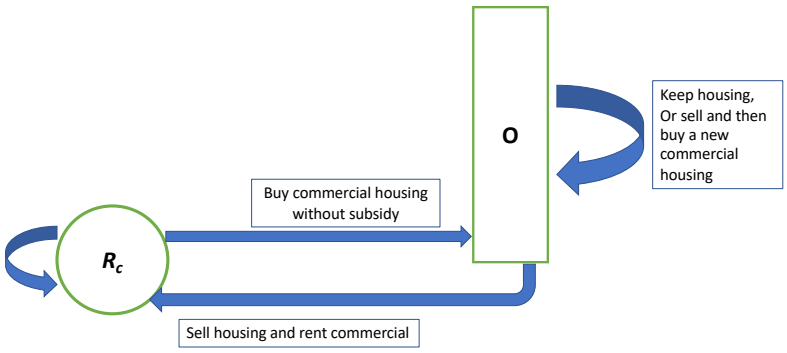
[15] We assume free conversion between consumption goods and a rental apartment. Therefore, if households choose to move out of the reformed house they were initially assigned to rent, these houses are then converted to consumption goods at no cost.



(a) Initial Cohorts (Before 1994)



(b) Cohorts Between 1994 and 1997



(c) Cohorts from 1998

Figure 4: Housing Options for Various Cohorts

Cohorts Who Enter the Economy On or After Year 1998. Figure 4(c) summarizes the housing options for cohorts who entered the economy on or after period 5 (year 1998). As mentioned previously, These cohorts no longer have the option to rent government houses. Consequently, they face the standard trade-off between buying and renting a commercial house. The costs of buying include the initial down payment and the mortgage initiation fees, and the benefits of owning a house are that it yields an extra housing service flow, as well as the capital gains from potential house price appreciation.

3.4 Production

Following Kaplan et al. (2020), we assume that there are two production sectors in the economy: a final goods sector, which produces nondurable consumption (the numeraire good of the economy); and a residential construction sector, which produces new houses. We assume that labor is perfectly mobile across the two sectors.

The consumption goods sector is competitive and operates a constant return-to-scale production technology as follows:

$$Y_{ct} = A_t N_{ct}, \tag{12}$$

where A_t denotes labor productivity and N_{ct} is the total units of labor services in terms of *efficiency units* used to produce consumption goods in period t. Therefore, the wage rate per efficiency unit of labor supply is given by

$$w_t = A_t. \tag{13}$$

We assume that A_t evolves according to an exogenous process which we will calibrate to match observed wages.

The residential construction sector sells newly built houses to both absentee rental companies and buyers of owner-occupied houses at a price p_t. The construction of residential housing requires new land L_{ht} issued by the government and urban labor N_{ht} using constant return-to-scale production technology:

$$Y_{ht} = (A_t N_{ht})^{\alpha} L_{ht}^{1-\alpha}, \text{ where } \alpha \in (0,1)\,. \tag{14}$$

Construction firms choose N_{ht} to maximize profits:

$$\max_{\{N_{ht}, L_{h,t}\}} p_t Y_{ht} - w_t N_{ht} - p_{Lt} L_{ht}, \tag{15}$$

where p_{Lt} is the land price in period t.

Absentee rental companies lease apartments to urban residents for rent ρ_t. Rental

companies must be indifferent between two options: selling an apartment in this period at unit price p_t; and renting it out this period for ρ_t and selling next period at unit price p_{t+1}, which implies the following relationship between apartment prices and rents.

$$p_t = \rho_t + \frac{1-\delta_h}{1+r} p_{t+1}, \tag{16}$$

where δ_h is the depreciation rate of the rental housing units.

3.5 Government Budget

The choice of the earmarked social security tax rate τ^{ss}, as shown in Eq. (8), guarantees that the intertemporal budget constraint on the social security program is guaranteed. In addition, the government also needs to impose an income tax to ensure that it balances the other government outlays, including both housing purchase subsidies and rental subsidies to those who either buy or rent the "reformed houses." Specifically, we assume that the government maintains a balanced intertemporal budget constraint by imposing an income tax rate τ so that:

$$\begin{aligned}
&\sum_{t=1} \frac{1}{(1+r)^t} \sum_{j=1}^{J} \sum_{i} \left[\int (p_t - p_{g,t}(j)) h_g^i \mathbf{I}^{B_g}(\mathbf{x}_t^R) d\mu_{t,j}^{R_g}(\mathbf{x}_t^R) + \int (1-\omega)\rho_t h_g^i \mathbf{I}^{R_g}(\mathbf{x}_t^R) d\mu_{t,j}^{R_g}(\mathbf{x}_t^R) \right] \\
&= B_0 + \sum_{t=1} \frac{1}{(1+r)^t} \sum_{j=1}^{J_W} \tau w_t \mu_{t,j} \sum_i \mathbb{E} e_{t,j}^i,
\end{aligned} \tag{17}$$

where the left-hand side of (17) is the present discounted value of government expenditures including the purchase and rental subsidies of government-owned apartments for eligible households, captured by the first and second terms, respectively; and the right-hand side is the present value of income tax revenues and initial holdings of government assets.

In Eq. (17), B_0 denotes the asset position of the government at the beginning of period 1.[16] $\mathbf{I}^{B_g}(\mathbf{x}_t^R)$ is an indication function that takes the value 1 if renters of government-owned apartments with state variables $\mathbf{x}_t^R \equiv (\lambda^i, h_g^i, j, y_t, z_t^i, t)$ decides to purchase their government assigned apartment in the housing reform (see Figure 4(a)); and $\mu_{t,j}^{R_g}(\mathbf{x}_t^R)$ is the mass of renters of reformed houses with state variables $\mathbf{x}_t^R$ at the beginning of period t before they make their purchase/rental decisions. Similarly, $\mathbf{I}^{R_g}(\mathbf{x}_t^R)$ is an indication function that takes the value 1 if renters of government-owned apartments with state variables $\mathbf{x}_t^R$ decide to continue renting government-assigned housing (see Figures 4(a) and 4(b)). This intertemporal government budget constraint implies that all transfers to early cohorts in

[16] We assume that, when the housing reform began, the government transferred the ownership of all reformed houses to absentee rental companies; Revenue from such transfers is absorbed in B_0.

terms of housing purchase and rent subsidies need to be financed by future generations.[17]

3.6 Equilibrium

Before we define the equilibrium of our model, let us recap the key aspects of our model environment. The vector of government policies include $\langle r_m, \theta, \nu, \gamma, \omega, p_{g,t}(j)\rangle$ where r_m is the constant mortgage interest rate [see (10)], θ and ν are respectively the pension replacement rate and the weight on the social average wage in the pension benefit formula [see (6)]; γ is the mortgage down payment requirement; ω is the discount on rent for government-assigned apartments for eligible households, and finally $p_{g,t}(j)$ is the discount price function (9) for the initial cohorts when they purchase the reformed houses. The sequence of aggregate variables, whose evolution we take as exogenous, includes $\langle A_t, L_t, n_t\rangle_{t=1}^{\infty}$, where $\langle A_t\rangle_{t=1}^{\infty}$ is the sequence of aggregate productivity shocks [see Eqs. (12) and (14)], $\langle L_t\rangle_{t=1}^{\infty}$ is the sequences of land supplies in the construction of residential housing [See Eq. (14)], and $\langle n_t\rangle_{t=1}^{\infty}$ is the sequence of population growth rates.

To simplify the notation, in the definition of equilibrium, we denote the vector of individual states for renters and homeowners as $\mathbf{x}_t^R := (\lambda^i, h_t, j, y_t, z_t^i, t)$ and $\mathbf{x}_t^O := (\lambda^i, h_t, j, y_t, d_t, z_t^i, t)$, respectively. $\mathbf{x}_t^R$ was already introduced in the text below Eq. (17); in $\mathbf{x}_t^O$, h_t is the house owned by the household in period t, and d_t is the remaining balance of the mortgage principal. We use $\mathbf{I}^{B_g}(\mathbf{x}_t^R)$ and $\mathbf{I}^{B_c}(\mathbf{x}_t^R)$ to denote the decision of *renters* of government-owned housing and commercial housing to buy a house, respectively. We use $\mathbf{I}_{R_g}^{R_g}(\mathbf{x}_t^R)$ and $\mathbf{I}_{R_g}^{R_c}(\mathbf{x}_t^R)$ to denote the decision of a current renter of government housing to continue the government rental lease, and to switch to renting a commercial house, respectively. Similarly, we use $\mathbf{I}^{S_g}(\mathbf{x}_t^O)$ and $\mathbf{I}^{S_c}(\mathbf{x}_t^O)$ to denote the decision of *owners* of reformed houses and commercial houses to sell, respectively. We use $\mathbf{I}_O^{R_c}(\mathbf{x}_t^O)$ to denote the decisions of current homeowners (regardless of whether they own reformed or commercial houses) to switch to rent a commercial property. Finally , we use $\mathbf{I}^{T_c}(\mathbf{x}_t^O)$ to denote the decisions of current homeowners (regardless of whether they own reformed or commercial houses) to trade up/down their housing in the commercial market.[18]

Now we are ready to define the equilibrium of our model. Given government policies $\langle r_m, \theta, \nu, \gamma, \omega, p_{g,t}(j)\rangle$, and a sequence of aggregate variables $\langle A_t, L_t, n_t\rangle_{t=1}^{\infty}$, a dynamic equilibrium of our model consists of quantities $\langle N_{ct}^*, N_{ht}^*, L_{h,t}^*\rangle$, prices $\langle p_t^*, \rho_t^*, w_t^*, p_{Lt}^*\rangle$, taxation policies $\langle \tau, \tau^{ss}\rangle$, household value functions $\mathbb{V} \equiv \langle V_t^{R_g}, V_t^{R_c}, V_t^{B_g}, V_t^{B_c}, V_t^{O_g}, V_t^{O_c}\rangle$ and associated policy

[17] Local governments in China are monopoly land owners; and land sales revenues become important components of *local* government finance (Fang et al., 2016). In this paper, we focus on the central government's problem and assume for simplicity that local government revenues from land sales are beyond the control of the central government.

[18] Note that there are some natural restrictions on these indicator functions to ensure that every household lives in one and only one house, which can either be a rental property or an owner-occupied property.

functions $\mathbb{P}^* \equiv \langle \mathbf{I}^{*B_g}(\cdot), \mathbf{I}^{*R_g}_{R_g}, \mathbf{I}^{*R_c}_{R_g}, \mathbf{I}^{*B_c}(\cdot), \mathbf{I}^{*S_g}(\cdot), \mathbf{I}^{*S_c}(\cdot), \mathbf{I}^{*R_c}_{O}(\cdot), \mathbf{I}^{*T_c}(\cdot), c^*_t(\cdot), a^*_t(\cdot), \mathbf{b}^*_t(\cdot), h^*_t(\cdot), m^*_t(\cdot)\rangle$, and distributions of the state variables among renters and owners of government-owned and commercial houses $\boldsymbol{\mu}_t \equiv \langle \mu_t^{R_g}, \mu_t^{R_c}, \mu_t^{O_g}, \mu_t^{O_c}\rangle$, measured at the beginning of period t, that satisfy the following conditions:

1. Given the prices $\langle p^*_t, \rho^*_t, w^*_t\rangle$ and the household value functions $\mathbb{V}$, the household policy functions $\mathbb{P}^*$ solve the households' recursive problems.[19]

2. The household value functions $\mathbb{V}$ are consistent with the households' policy functions $\mathbb{P}^*$ and the evolution of the state variables.

3. Given $\langle A_t, L_t\rangle$ and prices $\langle p^*_t, \rho^*_t, w^*_t, p^*_{Lt}\rangle$, the demand for labor N^*_{ht} and the demand for land $L^*_{h,t}$ solves the problem of profit maximization (15) of construction firms.

4. The labor market clears at the wage rate $w^*_t = A_t$, i.e., $N^*_{ct} + N^*_{h,t} = N_t$.

5. The land market clears in each period, i.e., $L^*_{ht} = L_t$, for all t where L_t is the exogenous land supply.

6. Given the households' policy functions and state distributions, and the new housing supply from construction firms as given by (14), housing price p_t clears the housing market in each period, i.e.:

$$
\begin{aligned}
&\overbrace{\left[\int h^i_g[1-\mathbf{I}^{*B_g}(\mathbf{x}^R_t)-\mathbf{I}^{*B_c}(\mathbf{x}^R_t)]\mathbf{I}^{*R_g}_{R_g}(\mathbf{x}^R_t)d\mu^{R_g}_t + \int h^*_t[1-\mathbf{I}^{*B_g}(\mathbf{x}^R_t)-\mathbf{I}^{*B_c}(\mathbf{x}^R_t)]\mathbf{I}^{*R_c}_{R_g}(\mathbf{x}^R_t)d\mu^{R_g}_t\right]}^{\text{Rental Demand from Previous Renters of Reformed Houses}} \\
+ &\overbrace{\int h^*_t[1-\mathbf{I}^{*B_c}(\mathbf{x}^R_t)]d\mu^{R_c}_t}^{\text{Rental Demand from Previous Commercial Renters}} + \overbrace{\int h^*_t\mathbf{I}^{*R_c}_{O}(\mathbf{x}^O_t)d[\mu^{O_g}_t+\mu^{O_c}_t]}^{\text{Rental Demand from Previous Homeowners}} \\
+ &\overbrace{\left[\int h^*_t\mathbf{I}^{*B_c}(\mathbf{x}^R_t)d\mu^{R_c}_t + \int h^*_t\mathbf{I}^{*B_c}(\mathbf{x}^R_t)d\mu^{R_g}_t + \int h^i_g\mathbf{I}^{*B_g}(\mathbf{x}^R_t)d\mu^{R_g}_t + \int h^*_t\mathbf{I}^{*T_c}(\mathbf{x}^O_t)d[\mu^{O_g}_t+\mu^{O_c}_t]\right]}^{\text{Ownership Demand}} \\
+ &\overbrace{\delta_h\left[\int h^i_g[1-\mathbf{I}^{*S_g}(\mathbf{x}^O_t)]d\mu^{O_g}_{t,j} + \int h^*_{t-1}[1-\mathbf{I}^{*S_c}(\mathbf{x}^O_t)]d\mu^{O_c}_{t,j}\right]}^{\text{Maintenance Demand}} \\
= &\overbrace{H_{t-1}(1-\delta_h)}^{\text{Depreciated Existing Housing}} + \overbrace{Y^*_{ht}}^{\text{New Construction}}, \qquad (18)
\end{aligned}
$$

where $Y^*_{ht} = (A_t N^*_{ht})^\alpha L_t^{1-\alpha}$ denotes the new construction in period t.[20]

[19] Details of the recursive problems are contained in the Appendix **??**.

[20] The housing demand consists of (i) rental demand from previous renters of reformed houses (the first row), including those from renters who continue to rent from the government (the first terms) and those who switch to renting commercial property (the second term); (ii) rental demand from previous commercial renters (the first term in the second row); (iii) rental demand from previous homeowners who decide to switch to rent commercial housing (the second term in the second row); (iv) ownership demand (the third

7. The no-arbitrage condition between housing price p_t^* and rental rates ρ_t^*, as described by Eq. (16), is satisfied all all period t.

8. The social security tax τ^{ss} balances the intertemporal budget of the social security system (8).

9. The evolution of state variables, encapsulated by $\boldsymbol{\mu}_t$, is consistent with exogenous stochastic processes including income shocks and aging/death processes and all endogenous household decision rules $\mathbb{P}^*$.

Remark 2. *Our model does not have capital goods and households are allowed to save. As a result, the consumption goods market does not always clear in our model. We assume that excess supply or excess demand is met by exports or imports, which is consistent with the role of international trade in the Chinese economy*

Remark 3. *In our benchmark analysis, we assume that the vector of government policy* $\langle r_m, \theta, \nu, \gamma, \omega, p_{g,t}(j)\rangle$ *is fixed at the actually implemented policies. We evaluate the effects of alternative policies in Section 6.*

4 Calibration

4.1 Model Parameters

We calibrate the benchmark economy to match stylized facts in the Chinese economy. Each period corresponds to five years in the data. The pre-reform period refers to the period before 1995, and the economic transition began in 1995, when housing reform began in practice. The main data source we rely on are the population census conducted in 2000 and 2005. In both censuses, the questionnaire specifically asks households whether they currently rent or own a reformed house.

Demography. In our model, $j = 1$ corresponds to age 21 in real life, and $J_w = 8$ so that agents retire at age 60 in real life. We set $J = 14$, corresponding to a maximum lifetime of 90 years. The age-specific conditional survival probabilities $\{\psi_j\}_{j=1}^J$ are taken from Imrohoroglu and Zhao (2018a).

row), including those from previous renters who decide to switch from renting to owning commercial houses (first two terms), from previous renters of subsidized government housing who decide to purchase them (third term), and from previous homeowners who decide to sell their previous homes and trade up/down for another commercial housing (the fourth term); (v) maintenance demand (the fourth row), from previous homeowners (reformed or commercial housing) so that they can compensate for the depreciation and continue to enjoy the same housing service.

The top panel of Figure 5 plots the dynamics of population growth rates n_t.[21] In the preformed periods, the population growth rate is set to be 2.4%, which was the average growth rate of the working-age population growth rate until 1995; between 1995 and 2020, the annual population growth rate is taken from the data on working-age population growth rate by the United Nations; between 2021 and 2045, the projected growth rate is as low as 0.088 percent; after 2045, the projected annual population growth rate is set to be zero.

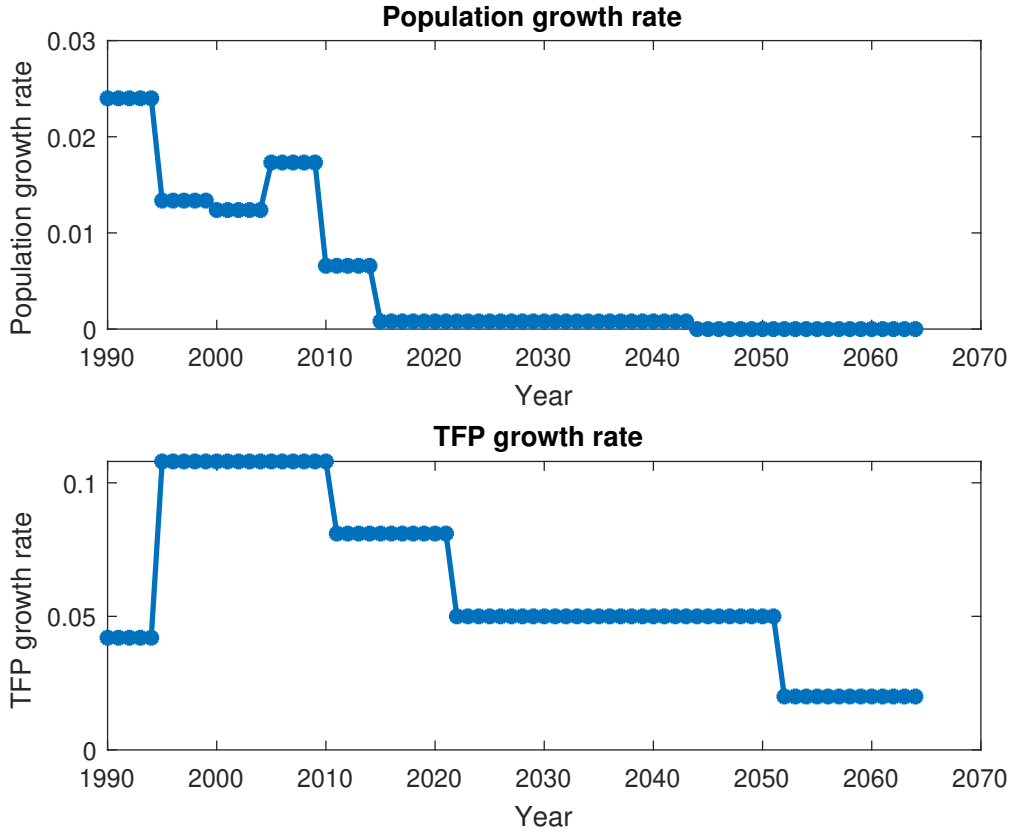


Figure 5: Dynamics of Wage and Population during Economic Transition

Process for Labor Supply Efficiency Units. The components in the process for labor supply efficiency units (4) are calibrated as follows. For the skill-specific component, we normalize $\lambda^L = 1$. λ^H is calibrated to match the ownership rate of 75-percent in the year 2000 among skilled households, who are defined as those who have at least completed high school. The life-cycle income profile, ϵ_j is taken from He et al. (2017) who use the CHNS data to estimate them. The persistent shock to labor supply (until the retirement), z follows an AR(1) process shown in equation 5; we follow He et al. (2017), and take $\rho = 0.84$ and

[21]Note that since 2010, China has started to experience population aging. The growth rate of the working-age population decreased from 1.73% on average between 2005-2010 to 0.66% on average between 2010-2015.

$\sigma_v^2 = 0.055$ for a period of five years. We then discretize this process into a seven-state Markov chain using the Tauchen method.

Preference. The discount factor, β, is set to be 0.59 corresponding to an annual discount factor of 90-percent. Utility parameter for housing service ϕ is calibrated to match an average 20 percent of expenditure share on rents among renters. The subsistence housing service $\underline{s}$ is chosen so that the price elasticity of rental demand is 0.8 according to Stephen et al. (1981). ι captures the intensity of the bequest motive. $\underline{b}$ determines the curvature of the bequest function. They are calibrated to match the ratio of wealth at age 75 to age 50 (0.82), and the median wealth-to-income distribution (0.63) from the China Household Finance Survey. We calibrate the home service flow premium for owning a house relative to renting, ζ, to match a home-ownership rate of 71 percent in the year 2000.

Housing. Housing depreciation rate, δ_h, is set to be 22.6%, which corresponds to a 5-percent annual depreciation rate. The downpayment requirement was 100 percent during 1995-1999, and 30 percent afterward to be consistent with the timing of the establishment of the mortgage market in China. The discounts offered to the renters of reformed houses (ω) is set to be 20% of the market rental rate. The annual return of risk-free bonds is $r = 0.02$ to match the real interest rate. The mortgage interest rate r_m is set to be 5 percent.

We choose the size of smaller reformed house (h_g^1) to match an average of housing price to income ratio at 9.1 in 2005 according to Fang et al. (2016). The size of the larger reformed house (h_g^2) offered to skilled households is chosen so that the population share of reformed homeowners in 2000 is 51 percent according to the population census. We allow owner-occupied commercial houses take six different sizes. The two smallest sizes coincide with the size of the two types of reformed houses. The largest commercial house is set to be five times the smallest housing size. Rental apartments have four different sizes, and they take the smallest four sizes of the owner-occupied houses.

Production Sector. The construction technology parameter α is set to be 0.8, so that price elasticity of new housing supply equals 4, about the average value by Wang et al. (2012) for the 35 major cities in China. The TFP process for consumption goods, $\{A_t\}$is set to match the urban hourly wage growth. In particular, in the bottom panel of Figure 5, the annual growth rate of wages is 4.2 percent in the preformed periods, which is the average growth rate of the urban wage rate between 1980 and 1993. The growth rate sharply increased to 10.8 percent during 1995-2010 to mimic the rapid economic growth in China during the first decade of the 21st century. The annual wage growth then slightly declines to 8.1 percent during 2010-2020. During 2020-2050, the wages grow at a modest speed of 5 percent. We

assume a balanced growth path is reached after 2050, in which the annual wage grows at a constant rate of 2 percent. We normalize the series so that the average wage in 2005 is 1. Housing prices are normalized in the same way as the TFP series. We calibrate the series of land supply to match the evolution of housing prices. Both series of land supply and matched housing prices are plotted in Figure 6.

Figure 6: Calibrated Land Supply and Housing Price Dynamics

Government sector. The pension replacement rate θ is calibrated to be 0.6, which is the replacement rate for the retirees between 1997 and 2011 covered by the system according to Song et al. (2015). We calibrate the government's initial asset holding to match an average payroll tax rate of 10% in the data. The weight between the social average wage and the pre-retirement wage, ν, is set to be 0.4.

Table 1 summarizes the parameters determined without calibration and Table 2 presents the calibrated parameter values and compare the model prediction with their targeted moments.

In the calibration, we only target ownership rate and the share of reformed homeowners in 2000. In Table 3, we also compare the model prediction with the respective share in 2005. The model is still able to match well with the overall ownership rate, but underpredicts the share of reformed homeowners by about 9 percentage point. This is likely because in the model the purchase of reformed houses at a discounted price is only allowed till year 2005, but in practice some state-owned enterprises or non-profit institutions still distribute housing subsidies to their employees after 2005. We have also compared the model-predicted

Table 1: Summary of Pre-determined Model Parameters

Description	Parameter	Value	Sources
Life time	J	14	Age 21-90
Working time	J_w	7	Age 21-60
Depreciation rate	δ_h	0.02	OECD estimates
Replacement rate	θ	0.6	Song et al. (2015)
Discount Factor	β	0.90	standard
auto-corr.of persistent shock	ρ	0.84	He et al. (2017)
std. dev.of persistent shock	σ_v	0.055	He et al. (2017)
std. dev.of transitory shock	σ_ϵ	0.055	Fan et al. (2010)
Minimum Down Payment Ratio	γ	0.3	government policy
Interest Rate for Savings	r	0.02	Government policy
Mortgage Interest Rate	r_m	0.06	Government policy
Land share	α	0.7	Wang et al. (2012)
Seller Transaction Cost	τ_s	0.12	Guren et al.(2020)
TFP growth rate	g_w	Figure 5	urban wage growth 1994-2050
Population growth rate	n_t	Figure 5	population growth 1994-2050
Conditional survival prob	$\{S_j\}$		Imrohoroglu and Zhao (2018a)
Life-cycle income profile	$\{\varepsilon_j\}$		He et al. (2017)
Initial TFP level	A_1	1.3	normalize average wage in 2005 to be 1

Table 2: Summary of Jointly-Calibrated Model Parameters

Para	Description	Value	Target	Data	Model
ζ	util. premium of owning	1.65	own rate in 2000	0.697	0.713
ι	Intensity of bequest	0.85	ratio of wealth at age 75 to age 50	0.82	0.81
$\underline{b}$	Curvature of bequest	0.12	median wealth-to-income distribution	0.634	0.60
ϕ	coef. on housing service	5.6	expenditure share on rents	0.20	0.20
$\underline{h}$	subsistence housing service	0.17	price elas. of rental demand	-0.8	-0.8
h_1^g	size of small reformed house	9.1	price to income ratio	9.0	9.0
h_2^g	size of large reformed house	12.7	share of reformed homeowners in 2000	0.513	0.486
λ^H	Skill premium	1.49	Own rate of skilled in 2000	0.749	0.738
B_0	Initial gov assets	5.3	payroll tax	0.1	0.1
L_{ht}	Land supply	Figure 6	Housing price dynamics		

ownership rate by skills in year 2005 with the data in Table 4. The model has done a decent job in matching those untargeted moments. The ownership rate by age groups are not targets of our calibration, Table 5 shows that the model is able to mimic the ownership pattern over life cycle. This suggests that both the inter-vivo and accidental bequests work well in promoting ownership rate among the young and dampen the ownership rate among the old.

Table 3: Ownership rate by type of houses: Model vs. Data

	2000		2005	
	Data	Model	Data	Model
Ownership	0.697	0.713	0.753	0.741
Pop share own ref. house	0.513	0.486	0.431	0.344
Pop share own com. house	0.184	0.227	0.322	0.397

Table 4: Ownership rate by type of skills: Model vs. Data

	2000		2005	
	Data	Model	Data	Model
Overall	0.697	0.713	0.753	0.741
Unskilled	0.672	0.692	0.738	0.691
Skilled	0.749	0.738	0.793	0.800

Table 5: Ownership rate by age: Model vs. Data

	2000		2005	
	Data	Model	Data	Model
21-30	0.570	0.603	0.545	0.630
31-60	0.705	0.728	0.777	0.738
61-90	0.634	0.600	0.736	0.658

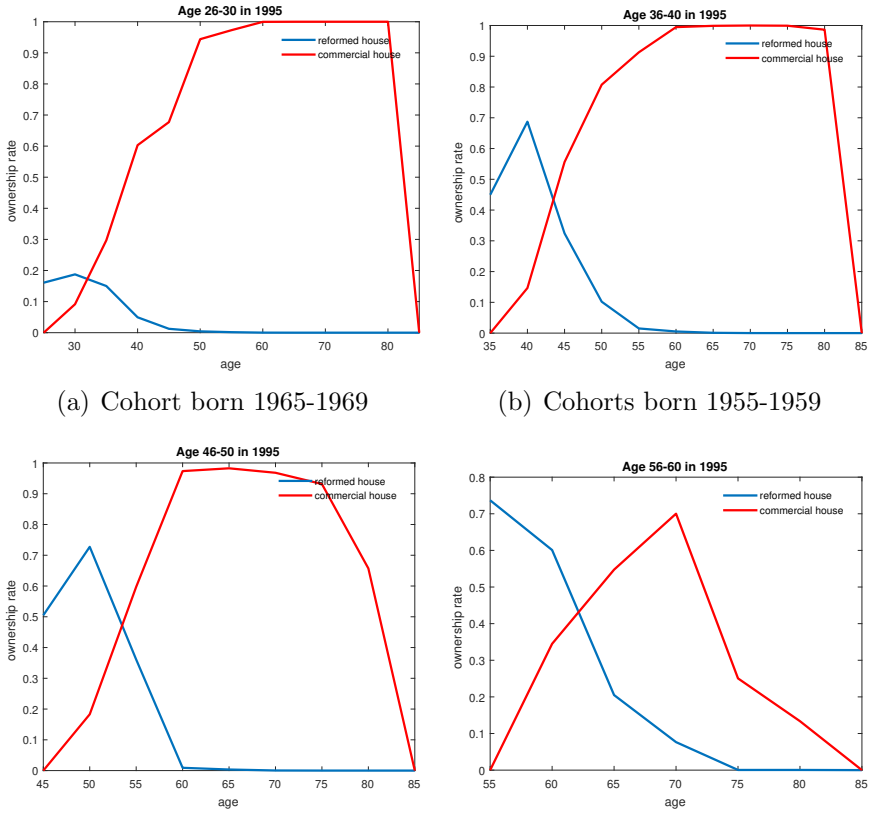
5 Benchmark Results

Figure 7 plots the age profile of ownership rate of reformed houses and commercial houses by four different initial cohorts, those born in the 1930s, 1940s, 1950s, and 1960s.[22] Despite the 100% down payment requirement (because mortgage loans were not yet introduced in this period), a significant fraction of the initial cohorts of different ages bought their reformed houses during 1995-1999. Among those born between 1940s and 1960s we see that the ownership rate of reformed houses exhibits a hump-shaped pattern that peaks at 1999 when mortgages became available. The ownership rate of commercial houses is steadily increasing and becomes stable afterwards before the dramatic decline towards the end of the life-cycle. For example, for those born during the 1950s, the ownership rate of reformed houses peaks at 70% in their 40s and declines to almost zero when they reach the retirement age. By contrast, the ownership rate of commercial houses increases from 0% to almost 100% over the life cycle.

Among all cohorts, we observe that the ownership rate of reformed houses is initially higher than and eventually overtaken by that of commercial houses. These patterns suggest a pattern of trading up over the life cycle by various initial cohorts from reformed to commercial houses. This trade-up pattern is evident in Figure 8. It plots the share of trade-up demand in the total demand for commercial houses and the share of trade-up sellers in all sellers in selective years.[23] In 2000, trade-up contributes to about 60% of demand for commercial

[22]For each cohort, the x-axis starts from their specific age in 1995 to age 85.

[23]We only keep track of those who sell their reformed houses and purchase commercial houses within the same period; that is, we do not record those who may purchase commercial houses a few periods later than selling the reformed houses.



(a) Cohort born 1965-1969

(b) Cohorts born 1955-1959

(c) Cohort born 1945-1949

(d) Cohort born 1935-1939

Figure 7: Ownership over life cycle among initial cohorts

houses, and this number rises to 75% in 2005. From the supply side, almost all supply of houses (other than new construction) is from trade-up sellers. The contribution of trade-up to the demand for commercial houses steadily declined to about 40% in 2010. In 2015, less than 10% of the demand for commercial houses was attributable to housing trade-up, suggesting that first-time homebuyers have become the dominant drivers of the demand for commercial houses over time. During these periods, trade-up sellers still dominate the supply side. This suggests that most owners of commercial houses decide to keep their current houses.

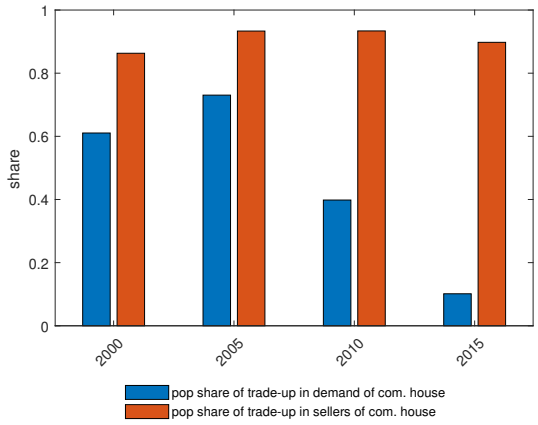


Figure 8: Importance of Trade-up for Commercial Housing Demand

Figure 9 reports the loan-to-value (LTV) ratio at origination among those initial cohorts who upgrade from reformed to commercial houses and those first-time homebuyers, respectively. The results show that all the first-time homebuyers take a mortgage equivalent to 50-70 percent of the value of the houses. This also holds among a dominant proportion of upgraders. But still more than 30 percent of upgraders choose mortgages equivalent to 10-50 percent of the housing value, and about 5 percent of upgraders choose to take a very small amount of mortgages. This suggests that the potential capital gain from selling their original reformed houses serves as a down payment for upgrading to commercial houses.

5.1 The Role of Housing Reforms

To evaluate the quantitative importance of housing reforms, we conduct two counterfactual exercises. In the first one, we remove the housing purchase subsidy to those initial cohorts, but they are still allowed to purchase the government house that they were initially assigned to at the market price. We call such a counterfactual economy the economy without housing purchase subsidy. In the second exercise, we further eliminate the option for those initial cohorts to purchase the reformed houses in which they previously resided. As a result, if they plan to own a house, they have to purchase a commercial house. We call this counterfactual

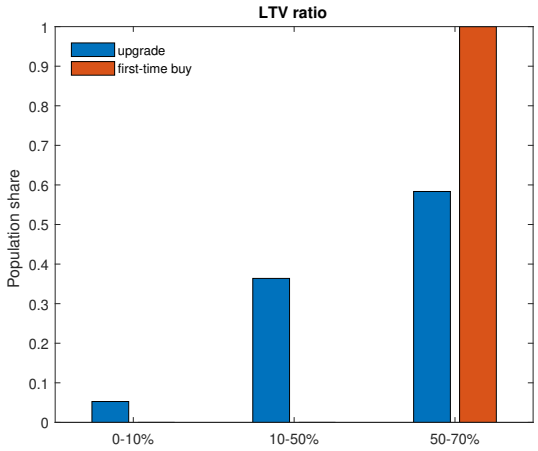


Figure 9: LTV Distribution by Trade-up and First-time Homebuyers

economy the economy without the 1994 reform.[24] The difference in home ownership rate between our benchmark economy and the economy without purchase subsidy captures the effects of purchase subsidy, while the difference in home ownership rate between the economy without purchase subsidy and the one without the 1994 reform captures the effects of the availability of reformed houses.

Figure 10 presents the evolution of housing prices among different scenarios. To understand the housing price dynamics, first note that the ownership rate of reformed houses will not affect housing prices. Consider two extremes in which everyone either rents or owns a reformed house. Housing prices will be the same in the two extremes. This is because those who own reformed houses they just purchased the houses that they initially rented. Therefore, the only housing demand in these two extreme economies is the demand led by housing depreciation, and thus housing prices are the same in both scenarios. This also implies that the initial housing price in both counterfactual scenarios will be higher than the benchmark counterpart, led by additional demand for commercial houses, as shown in Figure 10. Without a housing purchase subsidy or a reformed housing market, the government is able to levy a lower payroll tax to balance the budget. This positive income effect tends to trigger a stronger demand for commercial houses among younger cohorts, regardless of renting or owning, who tend to be borrowing-constrained, than those in the benchmark economy. In addition, the bequest left among newborns might be less than the benchmark scenario since the ownership rate in both counterfactual scenarios might be lower due to weaker affordability. On the other hand, the trade-up motive among early cohorts is weaker or absent in both counterfactual scenarios, and this puts downward pressure on housing demand. Our results show that the first two channels dominate the last one during early

[24]We maintain housing market reform in 1998, so eligible households can still rent their reformed houses at a discounted price.

periods and vice versa in later periods.

When housing purchase subsidy or housing reform are removed, between 1995-2005, benchmark housing prices are on average 19% lower, they become 4.8% percent higher during 2015-2025. It might be surprising to see that housing prices are higher in the counterfactual economy despite the lower ownership rate, as shown later in Figure 11. This is because those initial cohorts switch to renting a bigger apartment instead of buying their current reformed houses when they are no longer given discounts or the option to buy. The larger demand for rental apartments also works to push up housing prices despite the low ownership rate. Later on, as a result of a lower population share of reformed homeowners, less trade-up housing demand is expected, and this explains why housing prices in the baseline economy overtake those in the counterfactual economy during 2015-2025. In the long run, as shown in the Figure 10, housing prices converge to the same level among all scenarios after all early cohorts exit the economy.

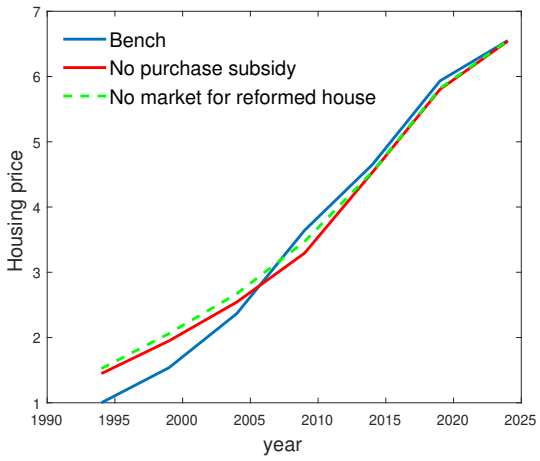


Figure 10: Housing prices among different scenarios

In Figure 11, we further compare the evolution of home ownership rate among different groups of households and different types of houses. The top two panels show the dynamics of home ownership rates for the unskilled and the skilled households. For both skill types, the home ownership rate in the benchmark economy is the highest during the first three decades since the reform started. In particular, the ownership rate in the benchmark economy starts from 50 percent and steadily increases over time. In contrast, the ownership rate in both counterfactual economies starts at a much lower level, at about 10 percent, and increases dramatically over time till they catch up with the benchmark levels. The ownership rate of skilled is higher than that of unskilled in all scenarios. The ownership rates without housing purchase subsidy track closely with those without housing reform. This suggests that when the housing purchase subsidy is removed, most demand is directed into commercial houses just as if there were no reformed housing market.

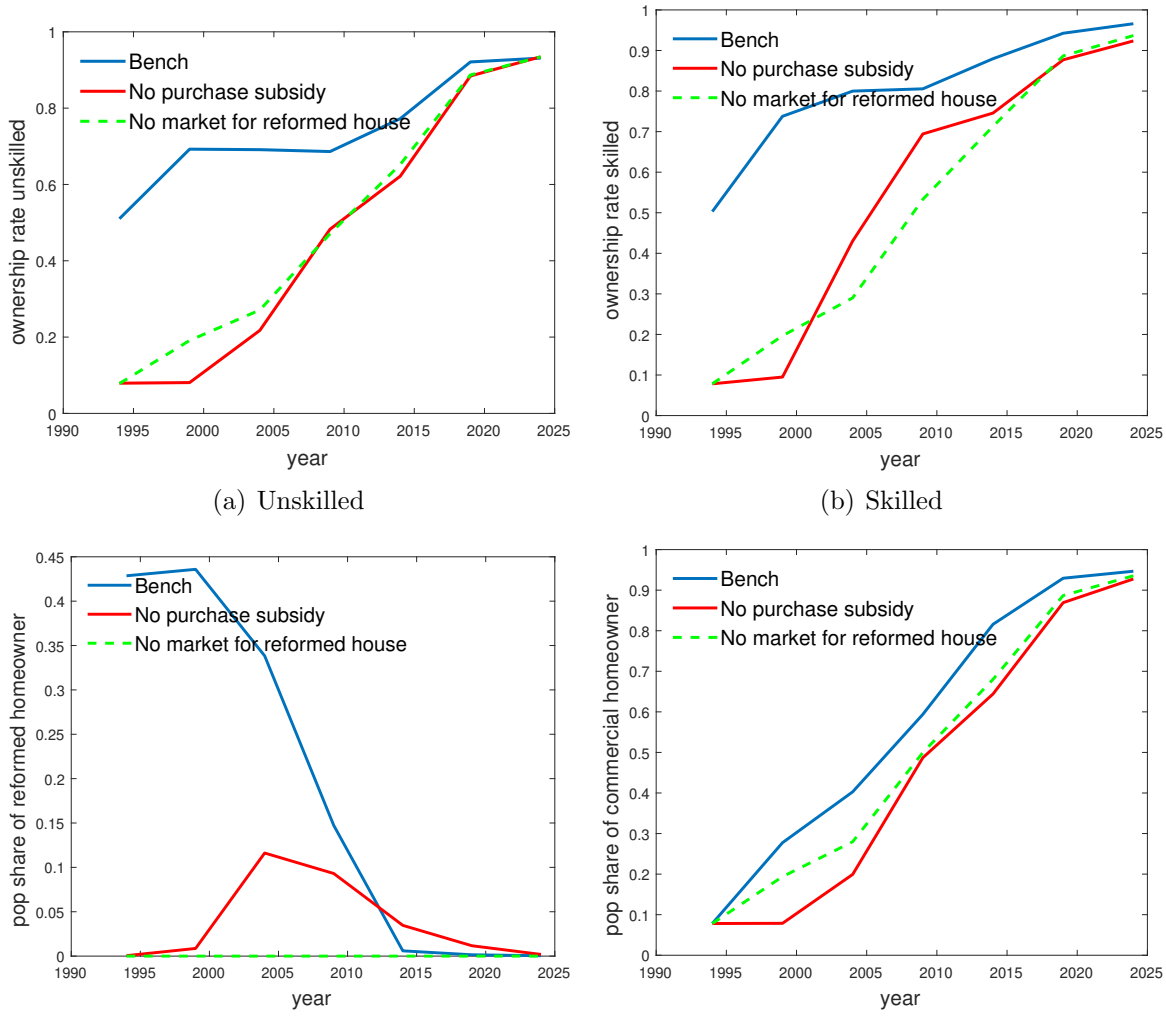
The bottom two panels capture the role of the 1994 housing reform on the ownership rate of two types of houses. It is clear that before 2000, the ownership rate of reformed houses was significantly higher in the benchmark economy than in the economy without a housing purchase subsidy.[25] Without a housing purchase subsidy, the population share of reformed homeowners is much lower than the benchmark counterpart. As discussed before, those initial cohorts may switch to renting a larger apartment when a purchase discount is not given. As a result, the population share of commercial homeowners is also significantly lower than the benchmark counterpart without either a housing purchase subsidy or housing market reform. This is partially due to a lack of trade-up demand from those reformed homeowners. In addition, a lower ownership rate also implies fewer accidental bequests offered to the newborn, and thus fewer newborns are able to afford commercial houses in the counterfactual economy.

We also explore the welfare implications of the housing market reforms. We use consumption equivalent variation (CEV) to measure the welfare effects of a particular redistributive policy. Specifically, CEV is defined as the uniform percentage change in expected consumption in each period over the remainder of an individual's lifetime that makes the individual from cohort s, i.e., born in period s, indifferent between the benchmark and the counterfactual scenario. A positive number implies welfare gain in the counterfactual economy equivalent to the percentage increase in the benchmark consumption.

$$E\Big[u(c_s^B, h_s^B) + \sum_{j=1}^{J-s}\big[u(c_{s+j}^B, h_{s+j}^B)\big]\Big] = E\Big[u\big(c_s^C(1+\frac{CEV}{100}), h_s^C\big) + \sum_{j=1}^{J-s}\big[u\big(c_{s+j}^C(1+\frac{CEV}{100}), h_{s+j}^C\big)\big]\Big]$$

We focus on the welfare of unskilled and skilled newborns separately. Our results suggest that removing both the housing purchase subsidy and the housing market reform results in welfare loss among newborns. This is due to higher housing prices and lower bequests, as discussed before. Figure 13 presents the level of bequests among different scenarios. On average, the bequests in the benchmark economy are 1.43 times those in the other two counterfactual scenarios during 2000-2025. Comparing the two counterfactual economies, removing housing market reform seems to result in higher welfare loss than removing the housing purchase subsidy. Across skilled and unskilled households, skill households initially in 1995 suffer a much bigger welfare loss of 60 percent from removing housing market reform than their unskilled counterparts. This is likely because the lower payroll tax in the counterfactual economy may push some households to rent bigger commercial houses, and the resulting higher housing price may make some newborn skilled individuals who were

[25] By construction, the ownership rate of the reformed houses is zero in the economy without the 1994 reform.



(a) Unskilled

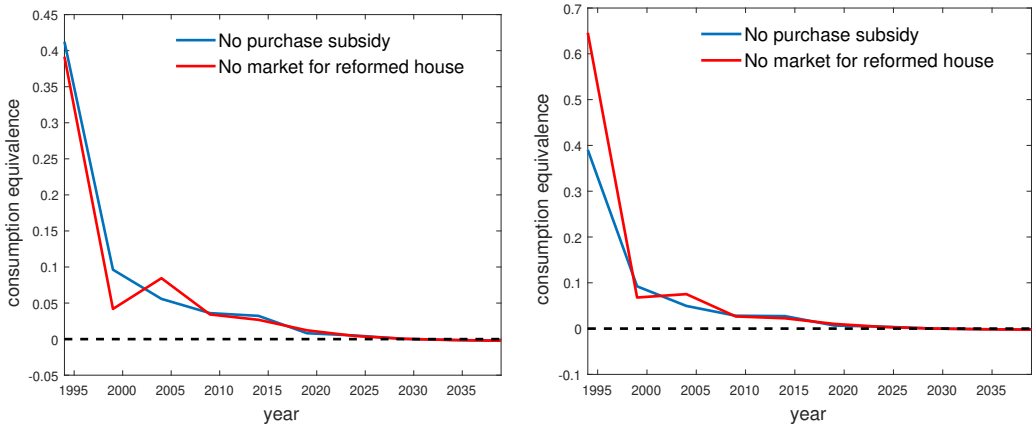
(b) Skilled

(c) Reformed House

(d) Commercial House

Figure 11: Ownership rate among different scenarios

able to afford a house in the benchmark economy unable to afford a house anymore. To sum up, housing reform is found to benefit more than pension reform. Skilled households benefit proportionally more than their unskilled counterparts.



(a) unskilled (b) skilled

Figure 12: Welfare relative to benchmark among newborn

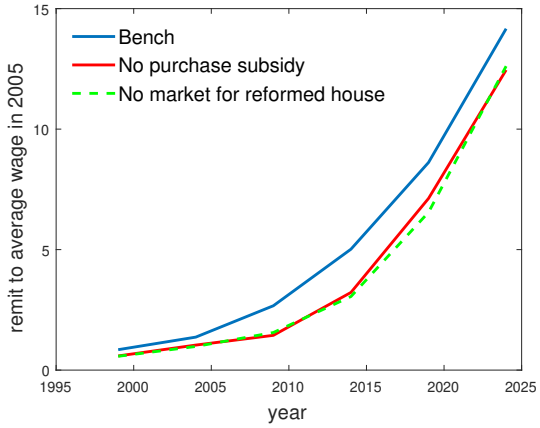


Figure 13: Bequests among different scenarios

6 Welfare Effects of Housing Reforms vs. Pension Reforms

In this section, we compare the roles of intergenerational transfer between housing purchase subsidies and social security subsidies. To this end, we construct a counterfactual economy without housing purchase subsidies for initial cohorts, but with an alternative social security system that serves as a tool of intergenerational transfer from the later generations to earlier

generations. We then explore the welfare impacts of such an alternative social security system, in comparison with housing reforms, on younger cohorts.

Table 6: Payroll and Social Security tax under Different scenarios

	Payroll tax	Social Security tax
Baseline	0.100	0.231
No purchase subsidy	0.065	0.231
No 94 reform	0.064	0.231
Pension reform	0.066	0.270
Baseline slow down	0.100	0.231
Pension reform slow down	0.100	0.270

In the counterfactual economy, we increase the replacement rate so that the average welfare among those initial cohorts born before 1994 is kept at the same level as in the benchmark economy with housing reform. Due to the downward rigidity of the replacement rate, the same (or higher) replacement rate will be implemented across all cohorts. Table 6 compares the payroll tax and the social security tax in both the benchmark and the counterfactual economy with alternative pension reform. Due to lower housing subsidies, the payroll tax becomes lower and closer to the economy without housing purchase subsidies or a reformed housing market. The social security is higher than the benchmark economy as a result of a higher replacement rate. The replacement among initial cohorts is found to increase by 12 percentage points to $\theta = 0.72$. The social security tax increases from 0.23 to 0.27 percent. Figure 14 shows the evolution of housing prices in both scenarios. Housing prices in the housing reform were initially lower than those in the pension reform and then overtook them. In the long run, housing prices converge to almost identical levels. When those early cohorts are subsidized with a higher replacement rate, different from the benchmark economy, not all of them can immediately realize these benefits. They will have to wait for their retirement. As a result, the ownership rate was initially much lower than the benchmark counterpart, as shown in Figure 15. They do not catch up till 2020, similar to the period when housing prices overtake. There is a larger share of the population that owns commercial houses than the economy can support with pension reform, suggesting the prevalence of the trade-up mechanism in housing reform.

Figure 16 compares the welfare among newborns under both reforms. The results suggest the newborns benefit more from housing reform than pension reform till 2030. Afterwards, the welfare difference is negligible due to the exits of early cohorts. In the pension reform, the newborns are levied a higher social security tax, and they also receive fewer bequests due to the lower ownership rate in early periods. Both channels work to dampen the welfare of newborns in the economy through pension reform, despite the lower payroll tax.

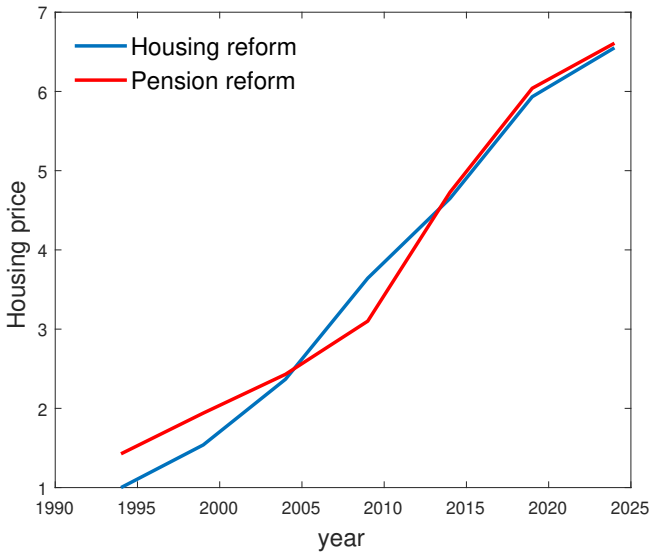
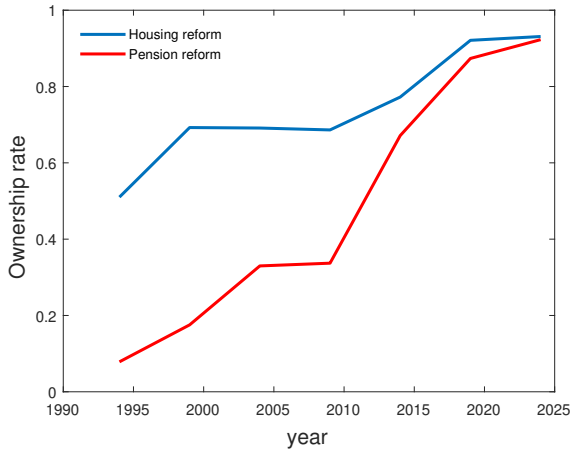


Figure 14: Housing Prices: housing reform vs. pension reform

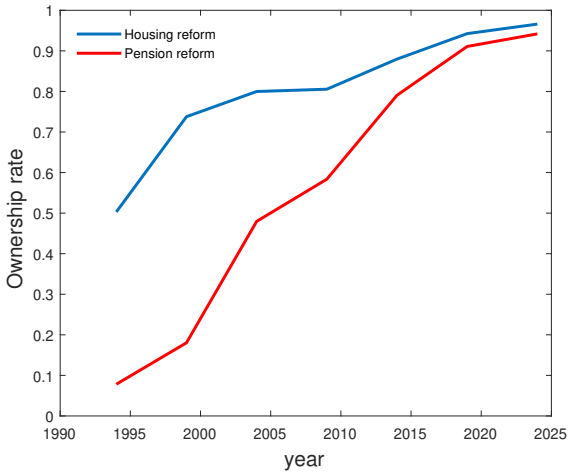
6.1 Economy Slowdown under Both Reforms

In this subsection, we further assess the sustainability of the government budget in the event of an unexpected slowdown in wages. Since a large proportion of the households that bought their house during the first two decades after the housing market reform, we assume that wage growth started to slow down after 2010. Specifically, in the benchmark economy, wage growth during 2010-2020 is 8.1 percent, and becomes 5 percent afterwards till 2050. In this counterfactual exercise, we let wage growth be 5 percent between 2010 and 2050. Figure 17 plots the evolution of TFP and average wages during both the benchmark and counterfactual economies.

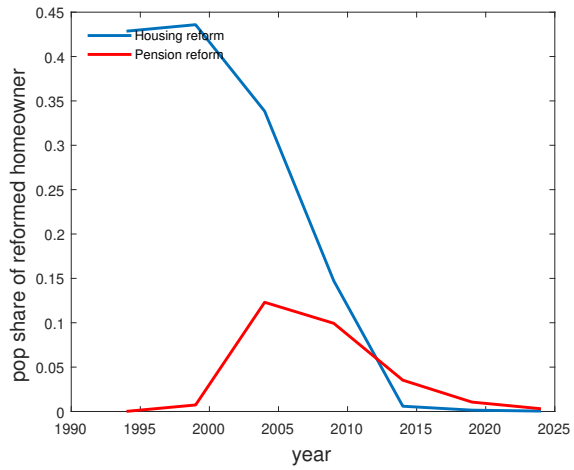
When wage growth slows down, the contribution to the pension system among working cohorts will be less, so the government's budget on the social security tax is expected to run a deficit in both reforms, given the original social security tax rate. In addition, the government is also expected to collect less payroll tax due to lower wages, although the government's expenditure on pension payments will also be less since pension is indexed to current wages. Overall, the government is expected to run a deficit in both the pension and payroll systems. However, in the case of pension reform, to keep the average welfare among those initial cohorts the same as in the housing reform, the replacement rate is set to be higher than that in the housing reform (0.72 vs. 0.6). Therefore, the government may run a larger deficit from the pension system than in the housing reform. This is shown in Table 7. Total subsidy refers to the housing purchase and rental subsidies offered to the initial cohorts. Therefore, they are higher in the housing reform. Tax income is also higher in the housing reform since the payroll tax is 0.1 and 0.066 percent in the housing and pension reform, respectively, as shown in Table 6. The pension deficit, on the other hand,



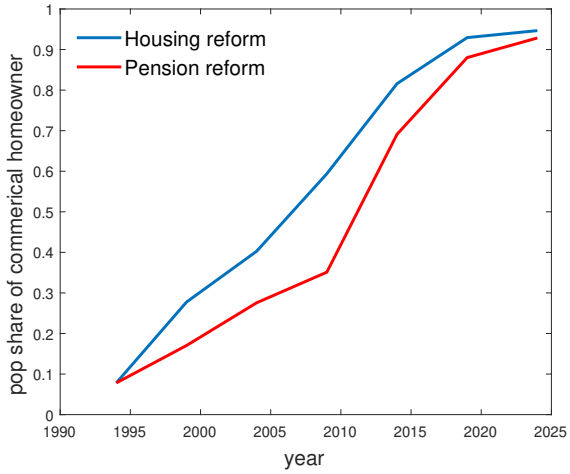
(a) unskilled



(b) skilled

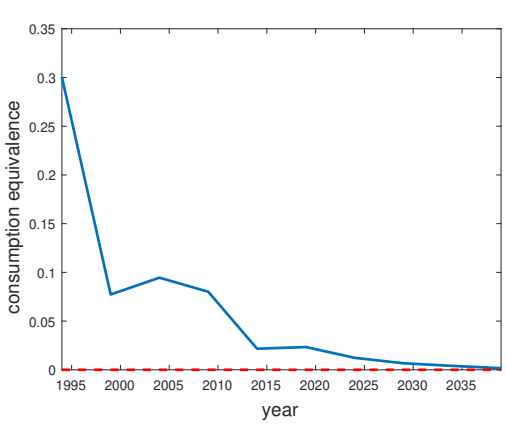


(c) reformed house

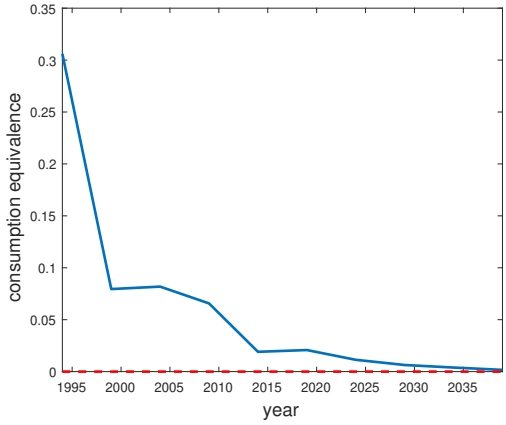


(d) commercial house

Figure 15: Ownership: housing reform vs. pension Reform

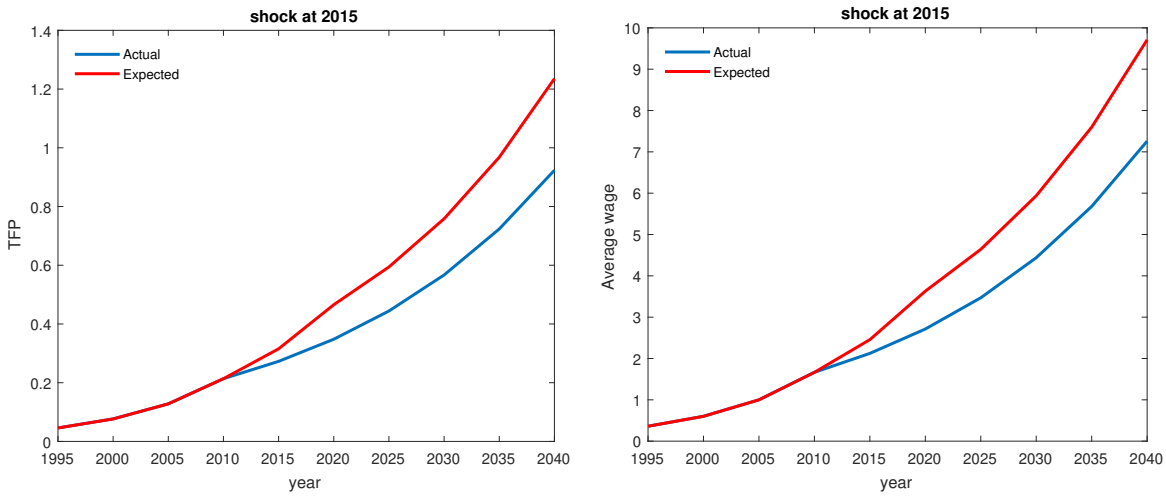


(a) unskilled



(b) skilled

Figure 16: Welfare relative to benchmark among newborn



(a) TFP (b) Average wage

Figure 17: Evolution of TFP and average wages

is higher in the pension reform due to the higher replacement rate. As a result, the fraction of the government's total deficit, which is the sum of total subsidy and pension deficit net of tax income, in tax income is 24.8% in housing reform and 28.7% in the pension reform. This suggests that, in the event of an economic slowdown, proportionally, the government runs a lower deficit in the housing reform than in the pension reform. This highlights the housing prices' automatic adjustment property that adapts the intensity of transfer to the wage growth rates in the transition economy.

Table 7: Government Budget under Housing or Pension Reforms

	Housing Reform	Pension Reform
tot subsidy	6.753	4.430
per-capita subsidy	0.834	0.547
tax income	5.662	3.727
per-capita tax income	0.699	0.460
pension deficit	0.313	0.365
per-capita pension deficit	0.039	0.045
total deficit	1.404	1.069
per-capita deficit	0.173	0.132
tot deficit/tax income	0.248	0.287

In Appendix A, we present results when the pension payment is no longer indexed to the current wage. In this case, when wage growth slows down, pension payments do not decrease in tandem with pension contributions, and thus we expect a larger deficit in the pension system, as shown in Table A.1. Pension deficit increases from 0.313 to 0.489 in

the housing reform and from 0.365 to 0.570 in the pension reform. The gap in the pension deficit between the two reforms has widened. As a result, the fraction of total deficit in the taxation income is also significantly higher in the pension reform than in the housing reform. This further strengthens the benefits of housing reform in enhancing the sustainability of the government budget.

7 Conclusion

Many emerging economies experienced fast growing but eventually declining wage income growth, which makes intergenerational transfer from future generations to the initial generations challenging via conventional redistributive policies such as social security. This paper uses China's housing reforms in the 1990s as a policy experiment to show that in such economies housing purchase subsidy to initial cohorts when housing privatization starts can serve as an alternative intergenerational redistributive scheme. By developing a quantitative general equilibrium model and calibrating it to Chinese economy, we find that housing purchase subsidy to these initial cohorts facilitates them, especially the skilled workers, to become homeowners, who later on reap the capital gain from selling their houses to future generations. Our welfare analysis suggests that for a majority of future generations, housing purchase subsidy is more desirable than an alternative social security reform that redistribute from future generations to the initial ones, as the latter involves high social security tax burdens for those generations experiencing slowing down in wage growth. This is despite the fact that cohorts born in the 1980s would prefer social security as intergenerational transfer to avoid higher housing prices caused by housing purchase subsidy to initial cohorts. Our findings provides a concrete step towards future research on how a combination of housing purchase subsidy to earlier generations with social security reforms can jointly redistribute from future towards current generations for economies that experience fast growing wage incomes during their economic transition.

References

Associated Press, "France: Over 1 million march against raising retirement age," 2023.

Chen, Kaiji and Yi Wen, "The Great Housing Boom of China," *American Economic Journal: Macroeconomics*, April 2017, *9* (2), 73–114.

Deng, Yuanyuan, Hanming Fang, Katja Hanewald, and Shang Wu, "Delay the Pension Age or Adjust the Pension Benefit? Implications for Labor Supply and Individual Welfare in China," *Journal of Economic Behavior and Organization*, 2023, *212*, 1192–1215.

Fagereng, Andreas, Matthieu Gomez, Émilien Gouin-Bonenfant, Martin Holm, Benjamin Moll, and Gisle Natvik, "Asset-Price Redistribution," *Journal of Political Economy*, 2025, *133* (11), 3494–3549.

Fan, Xiaoyan, Zheng Song, and Yikai Wang, "Estimating Income Processes in China," 2010. Working Paper.

Fang, Hanming and Jin Feng, "The Chinese Pension System," in Marlene Amstad, Guofeng Sun, and Wei Xiong, eds., *Handbook of China's Financial System*, Princeton, New Jersey: Princeton University Press, 2020.

_ **and Marshall Meyer, eds**, *The Arc of the Chinese Economy*, Cambridge University Press, 2025.

_ **, Quanlin Gu, Wei Xiong, and Li-An Zhou**, "Demystifying the Chinese Housing Boom," *NBER Macroeconomics Annual 2015*, 2016, *30.*

Fang, Yiping, John Logan, and Zhanxin Zhang, "The Winners in China's Urban Housing Reform," *Housing Studies*, 2010, *25*, 101–117.

Gallup, "Social Security: Historical Trends (as summarized by Social Security Works)," 2016.

Garriga, Carlos, Aaron Hedlund, Ping Wang, and Yang Tang, "Rural-Urban Migration, Structural Transformation, and Housing Markets in China," September 2017. NBER Working Paper 23819.

Glaeser, Edward, Wei Huang, Yueran Ma, and Andrei Shleifer, "A Real Estate Boom with Chinese Characteristics," *Journal of Economic Perspectives*, 2017, *31*, 93–116.

Glover, Andrew, Jonathan Heathcote, Dirk Krueger, and Jose Victor Rios-Rull, "Inter-generational Redistribution in the Great Recession," *Journal of Political Economy*, 2020, *128*, 3730–3778.

He, Hui, Lei Ning, and Dongming Zhu, "The Impact of Rapid Aging and Pension Reform on Savings and the Labor Supply: The Case of China," *Working Paper*, 2017.

Hur, Sewon, "The lost generation of the Great Recession," *Review of Economic Dynamics*, 2018, *30*, 179–202.

Imrohoroglu, Ayse and Kai Zhao, "The Chinese Saving Rate: Long-term Care Risks, Family Insurance, and Demographics," *Journal of Monetary Economics*, 2018, *96*, 33–52.

_ **and** _ , "Intergenerational transfers and China's social security reform," *Journal of Economics of Aging*, 2018, *11*, 62–70.

Jiang, Shenzhe, Jianjun Miao, and Yuzhe Zhang, "China's Housing Bubble, Infrastructure Investment, and Economic Growth," *International Economic Review*, 2022, *63*, 1189–1237.

Kaplan, Greg, Kurt Mitman, and Giovanni L Violante, "The Housing Boom and Bust: Model Meets Evidence," *Journal of Political Economy*, 2020, *128* (9), 3285–3345.

Liu, Chang and Wei Xiong, "China's Real Estate Market," in Marlene Amstad, Guofeng Sun, and Wei Xiong, eds., *China's Financial System*, Princeton, New Jersey: Princeton University Press, 2020.

Nardi, Mariacristina De, "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, 2004, *71* (3), 743–768.

_ **, Eric French, and John B. Jones**, "Why Do the Elderly Save? The Role of Medical Expenses," *Journal of Political Economy*, 2010, *118* (1), 39–75.

OECD, "Is it worth raising the normal retirement age?," 2023.

Oshio, Takashi, "Is Working Good for Our Seniors? Mulling a Change in Japan's Retirement Age," 2021. Tax & Social Security Reform.

Peterman, William and Kamila Sommer, "A historical welfare analysis of Social Security: Whom did the program benefit?," *Quantitative Economics*, 2019, *10*, 1357–1399.

Pew Research Center, "Americans' views of government aid to the poor, role in health care, and Social Security," 2024.

_ , "What the data says about Social Security," 2025.

Quinnipiac University Poll, "Nearly 80% Oppose Raising Social Security's Full Retirement Age," 2025. via Yahoo Finance.

Sato, Hiroshi, Terry Sicular, and Ximing Yue, "Housing Ownership, Incomes, and Inequality in China, 2002–2007," in Shi Li, Hiroshi Sato, and Terry Sicular, eds., *Rising Inequality in China: Challenges to a Harmonious Society*, Cambridge, UK: Cambridge University Press, 2013, chapter 3.

Song, Zheng, Kjetil Storesletten, Yikai Wang, and Fabrizio Zilibotti, "Sharing High Growth across Generations: Pensions and Demographic Transition in China," *American Economic Journal: Macroeconomics*, 2015, *7*, 1–39.

Stephen, Mayo et al., "Theory and estimation in the economics of housing demand," *Journal of Urban Economics*, 1981, *10*, 95–116.

Wang, Shing-Yi, "State Misallocation and Housing Prices: Theory and Evidence from China," *The American Economic Review*, 2011, *101*, 2081–2157.

Wang, Songtao, Su Han Chan, and Bohua Xu, "The estimation and determinants of the price elasticity of housing supply: Evidence from China," *Journal of Real Estate Research*, 2012, *34* (3), 311–344.

Wu, Jing, Yongheng Deng, and Hongyu Liu, "House Price Index Construction in the Nascent Housing Market: The Case of China," *Journal of Real Estate Finance and Economics*, 2014, *48*, 522–545.

Yin, Zhicao and Li Gan, "The effect of housing reforms on durable consumption in China," *China Economic Quarterly*, 2009, *9*, 53–72.

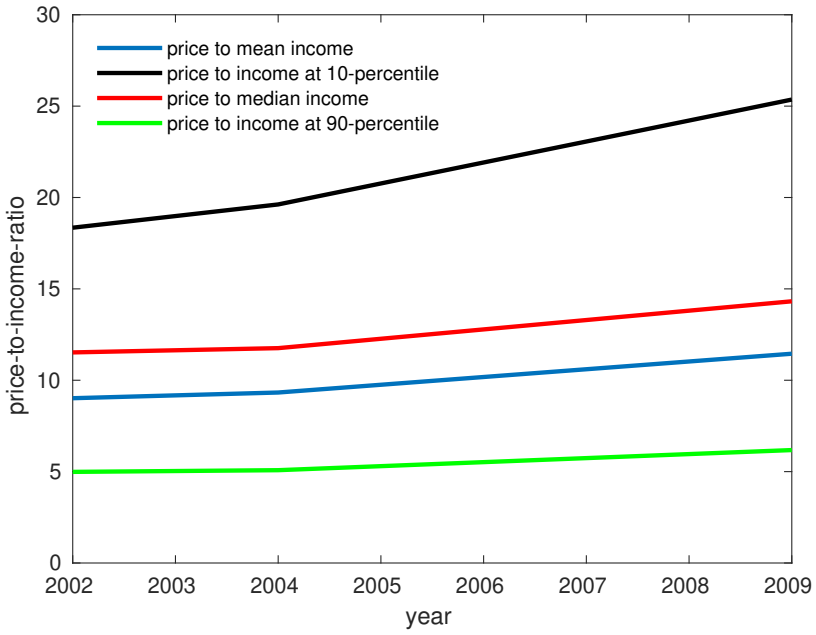


Figure 18: Simulated House Price to Income Ratios for Households of Different Income Levels in the Benchmark Economy

Online Appendix (Not For Publication)

A Quantitative results with pension system that does not index current wage

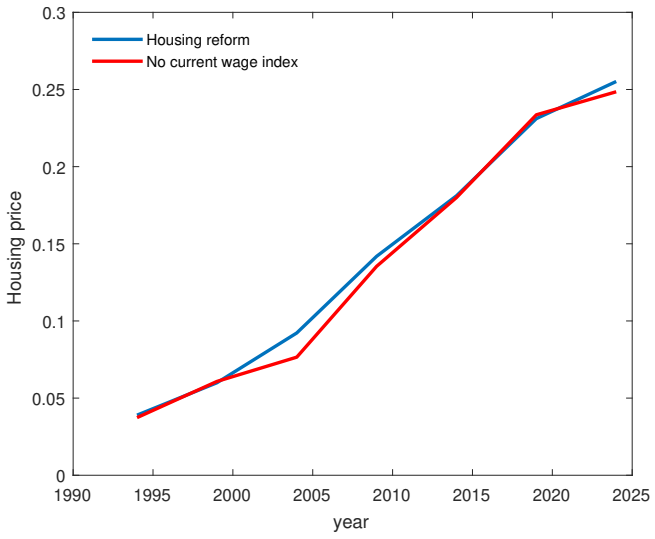
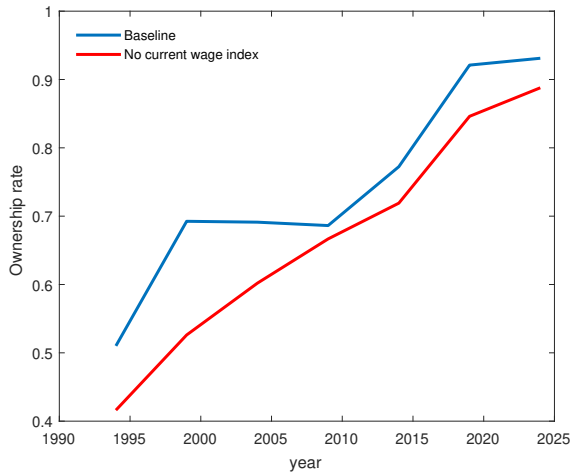
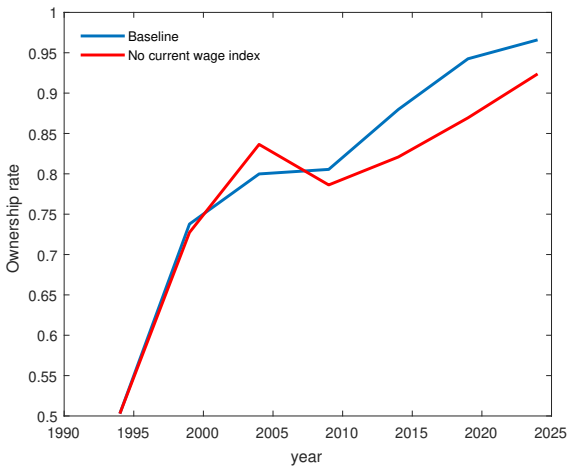


Table A.1: Government Budget under Housing or Pension Reforms when pension does not index current wage

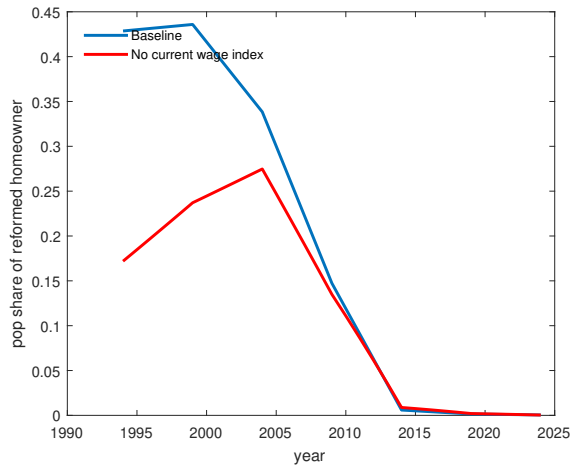
	Housing Reform	Pension Reform
tot subsidy	6.413	4.526
per-capita subsidy	0.792	0.559
tax income	5.662	3.727
per-capita tax income	0.699	0.460
pension deficit	0.489	0.570
per-capita pension deficit	0.060	0.070
total deficit	1.240	1.370
per-capita deficit	0.153	0.169
tot deficit/tax income	0.219	0.368



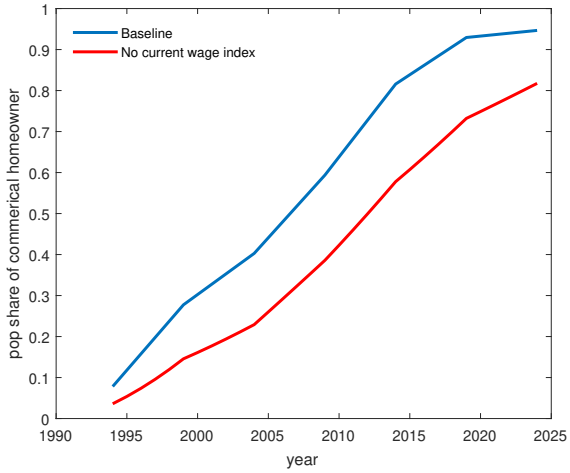
(a) unskilled



(b) skilled

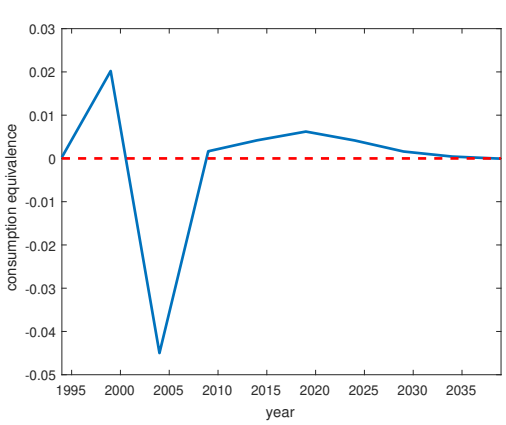


(c) reformed house

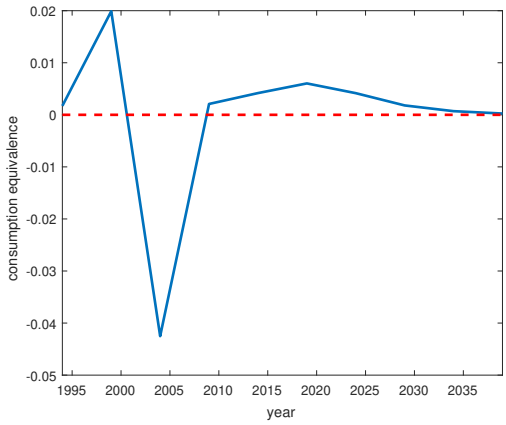


(d) commercial house

Figure A.1: Ownership: Benchmark v.s. Pension system that does not index current wage



(a) unskilled



(b) skilled

Figure A.2: Welfare relative to benchmark among newborn

B Households' Optimization Problem

B.1 Pre-reform Periods

In the pre-reform periods, every household lived in government-assigned apartments whose area depended on their skill type λ^i. Therefore, all households were renters of government housing, i.e., R_g, in the pre-reform periods. We also assume that in the pre-reform era households did not experience idiosyncratic shocks in the earnings equation (3), which approximates a socialist economy. Thus, all households of the same age and skill type carry the same income y. Since their housing consumption h_g^i, the skill-type-specific size of government-assigned housing specified in the main text, was not a choice in the pre-reform period, they chose only the consumption c, the inter vivos bequest $\mathbf{b}$, and saving in the form of risk-free bonds a', after paying subsidized rents for the government-assigned housing.

For a household of skill type i and age j in the pre-reform period t, their problem is:

$$V_0^{R_g}\left(\lambda^i, j, y, t\right) = \max_{\{c,a',\mathbf{b}\}} u\left(c, h_g^i, \mathbf{b}\right) + \beta\psi_{j+1}\mathbb{E}\left[V_0^{R_g}\left(\lambda^i, j+1, y', t+1\right)\right], \tag{A1}$$

$$\text{s.t.} \quad c + \mathbf{b} + a' + \omega\rho_t h_g^i = y, \tag{A2}$$

$$y' = \begin{cases} (1-\tau_0^{ss})\lambda^i w_{t+1}\varepsilon_{j+1} + a'(1+r) & \text{for } j \leq J_w \\ b_{t+1,j+1}^i + a'(1+r) & \text{for } j > J_w \end{cases} \tag{A3}$$

The steady state in the pre-reform periods is defined as the state where all exogenous series are constant over time, and the distributions of households' state variables by skill types are time-invariant.

B.2 Post-Reform Periods

In post-reform periods, households can be in four groups: renters of reformed (R_g) or commercial houses (R_c), and owners of reformed (O_g) or commercial (O_c) houses. We now characterize the optimization problem in recursive forms for households of different groups.

As we described in Section 3.6, the vector of individual states for renters and homeowners is denoted by $\mathbf{x}_t^R := (\lambda^i, h_t, j, y_t, z_t^i, t)$ and $\mathbf{x}_t^O := (\lambda^i, h_t, j, y_t, d_t, z_t^i, t)$, respectively.

As we described in Section 3.3, only households who entered the labor market before 1998 were eligible to rent the government-assigned houses with subsidized rents, and $\omega\rho_t$ is the subsidized rental rate. Other than rental payments, households make standard consumption and saving decisions a' given after-tax income.

At the beginning of the next period, households choose whether or not to continue renting the current government-assigned apartment and remain type-R_g; they can choose to switch to rent a commercial house from the set $\mathcal{H}_R$, and become type-R_c, or buy a commercial

property from the set $\mathcal{H}_O$ and obtain value $V_{t+1}^{B_c}$, or buy the government-assigned house as a reformed house and obtain value $V_{t+1}^{B_g}$. $\mathcal{T}_{t+1}$ denotes the transfer made by the government to ensure a subsistence level of consumption ($\underline{c}$) and housing services among renters:

$$\mathcal{T}_t(y) = \max\{0, \rho_t H_R^1 + \underline{c} - y\},$$

where H_R^1 is the smallest rental option in the commercial market.

Specifically, the value function of a type-R_g renter of state variable $\mathbf{x}_t^R = (\lambda^i, h_g^i, j, y_t, z_t^i, t)$ at the beginning of period t can be expressed as

$$V_t^{R_g}(\lambda^i, j, h_g^i, y, z) = \max_{\{c,a',\mathbf{b}\}} u(c, h_g^i, \mathbf{b}) + \beta\psi_{j+1}\mathbb{E}\Bigg[\max\Bigg\{V_{t+1}^{R_g}\left(\lambda^i, j+1, h_g^i, y', z'\right), V_{t+1}^{B_g}\left(\lambda^i, j+1, h_g^i, y', z'\right), \tag{A4}$$

$$V_{t+1}^{R_c}\left(\lambda^i, j+1, y', z'\right), V_{t+1}^{B_c}\left(\lambda^i, j+1, y', z'\right)\Bigg\}\Bigg] \tag{A5}$$

$$\text{s.t.} \quad c + \mathbf{b} + a' + \omega\rho_t h_g^i = y \tag{A6}$$

$$y' = \begin{cases} (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z' + a'(1+r) + \mathcal{T}_{t+1} & \text{for } j \le J_w \\ b_{t+1,j+1}^i + a'(1+r) + \mathcal{T}_{t+1} & \text{for } j > J_w \end{cases} \tag{A7}$$

$$a' \ge 0 \tag{A8}$$

Unlike renters of government-assigned houses, renters of commercial houses can choose housing sizes from the option set $\mathcal{H}_R$. However, they do not enjoy rent subsidies and once they become commercial renter, they would no longer be able to choose to rent a reformed house in the future even though they were born before 1998. Thus, at the beginning of the next period, they choose between continuing to be a commercial house renter, that is, staying as type-R_c, and buying a commercial house and obtaining value $V_{t+1}^{B_c}$. The value function of households who are commercial renters, i.e., the type-R_c households, can be expressed as

$$V_t^{R_c}(i, j, y, z) = \max_{\{c,a',h_R\in\mathcal{H}_R,\mathbf{b}\}} u(c, h_R, \mathbf{b}) + \beta\psi_{j+1}\mathbb{E}\left[\max\left\{V_{t+1}^{R_c}\left(y', z', j+1\right), V_{t+1}^{B_c}\left(i, j+1, y', z'\right)\right\}\right] \tag{A9}$$

$$\text{s.t.} \quad c + \mathbf{b} + a' + \rho_t h_R = y \tag{A10}$$

$$y' = \begin{cases} (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z' + a(1+r) + \mathcal{T}_{t+1} & \text{for } j \le J_w \\ b_{t+1,j+1}^i + a'(1+r) + \mathcal{T}_{t+1} & \text{for } j > J_w \end{cases} \tag{A11}$$

$$a' \ge 0 \tag{A12}$$

We next switch to characterize households who are buyers of either reformed or commercial houses at the beginning of the period. For a household that chooses to buy a reformed house, its optimization problem is

$$V_t^{B_g}(i,j,y,z) = \max_{c,a',d',\mathbf{b}} u(c,\zeta h_g^i,\mathbf{b}) + \beta\psi_{j+1}\mathbb{E}\Bigg[\max\Bigg\{V_{t+1}^{O_g}(i,j+1,y',z',\zeta h_g^i,d',), \tag{A13}$$

$$V_{t+1}^{R_c}(i,j+1,y'_{\text{sell}},z'), V_{t+1}^{B_c}(i,j+1,y'_{\text{sell}},z')\Bigg\}\Bigg] \tag{A14}$$

$$\text{s.t.} \quad c + \mathbf{b} + a' + p_{gt}(j)h_g^i = y + d' \tag{A15}$$

$$y' = \begin{cases} (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z' + a'(1+r) & \text{for } j \le J_w \\ b_{t+1,j+1}^i + a'(1+r) & \text{for } j > J_w \end{cases} \tag{A16}$$

$$y'_{\text{sell}} = y' + (1-\tau_s)p_{t+1}h_g^i - d'(1+r_m) + \mathcal{T}_{t+1} \tag{A17}$$

$$d' \le (1-\gamma)p_{gt}(j)h_g^i \tag{A18}$$

$$a' \ge 0 \tag{A19}$$

If the household is a buyer of the reformed house, then the discounted house price he paid is $p_{g,t}(j)$ as given by Eq. 9. Households can take mortgages and borrow up to a fraction $(1-\gamma)$ of the purchase price. At the beginning of the next period, a household may choose to remain the owner of the house (O_g), or sell the house and become either a renter (R_c) or a buyer (O_c) of a commercial house. Note that once a household sells the reformed house, it fetches the market price, which gives rise to a potential capital gain from the difference between the subsidized purchase price and the sales price.

The problem for buyers of commercial houses only differs from that for buyers of reformed houses in two aspects. First, buyers of commercial houses no longer receive purchase subsidies offered by the government, so they buy at the market price. Second, buyers get to choose

among available sizes of houses ($\mathcal{H}_O$).

$$V_t^{B_c}(i,j,y,z) = \max_{c,a',d',h_o \in \mathcal{H}_O, \mathbf{b}} u(c, \zeta h_o, \mathbf{b}) + \beta \psi_{j+1} \mathbb{E}\Bigg[\max\Bigg\{ V_{t+1}^{O_c}(i, j+1, y', z', h_o, d'), \\ V_{t+1}^{R_c}(i, j+1, y'_{\text{sell}}, z'), V_{t+1}^{B_c}(i, j+1, y'_{\text{sell}}, z') \Bigg\} \Bigg]$$

$$\text{s.t.} \quad c + \mathbf{b} + a' + p_t h_o = y + d'$$

$$y' = \begin{cases} (1 - \tau^{ss} - \tau)\lambda^i w_{t+1} \varepsilon_{j+1} z' + a'(1+r) & \text{for } j \le J_w \\ b^i_{t+1,j+1} + a'(1+r) & \text{for } j > J_w \end{cases}$$

$$y_{\text{sell}}{}' = y' + (1 - \tau_s) p_{t+1} h_o - d'(1 + r_m) + \mathcal{T}_{t+1}$$

$$d' \le (1 - \gamma) p_t h_o$$

$$a' \ge 0$$

The optimization problem for an owner of a reformed house can be expressed as follows:

$$V_t^{O_g}(i,j,y,z,h_g^i,d) = \max_{c,a',m,\mathbf{b}} u(c, \zeta h_g^i, \mathbf{b})$$

$$+ \beta \psi_{j+1} \mathbb{E}\Big[\max\left\{ V_{t+1}^{O_g}(i, j+1, y', z', h_g^i, d'), V_{t+1}^{R_c}(i, j+1, y'_{\text{sell}}, z'), V_{t+1}^{B_c}(i, j+1, y'_{\text{sell}}, z') \right\} \Big]$$

$$\text{s.t.} \quad c + \mathbf{b} + a' + \delta_h p_t h_g^i + m = y$$

$$m \ge \frac{r_m (1 + r_m)^{J+1-j}}{(1+r)^{J+1-j} - 1} d$$

$$d' = d(1 + r_m) - m$$

$$y' = \begin{cases} (1 - \tau^{ss} - \tau)\lambda^i w_{t+1} \varepsilon_{j+1} z' + a'(1+r) & \text{for } j \le J_w \\ b^i_{t+1,j+1} + a'(1+r) & \text{for } j > J_w \end{cases}$$

$$y'_{\text{sell}} = y' + (1 - \tau_s) p_{t+1} h_g^i - d'(1 + r_m) + \mathcal{T}_{t+1}.$$

$$a' \ge 0$$

Similarly, the value function for an owner of a commercial house is expressed as follows.

$$V_t^{O_c}(i,j,y,z,h_o,d) = \max_{c,a',m,\mathbf{b}} u(c,\zeta h_o,\mathbf{b})$$
$$+\beta\psi_{j+1}\mathbb{E}\Big[\max\left\{V_{t+1}^{O_c}(i,j+1,y',z',h_o,d'),V_{t+1}^{R_c}(i,j+1,y'_{\text{sell}},z'),V_{t+1}^{B_c}(i,j+1,y'_{\text{sell}},z')\right\}\Big]$$
$$\text{s.t.}\quad c+\mathbf{b}+m+a'+\delta_h p_t h_o = y$$
$$m \geq \frac{r_m(1+r_m)^{J+1-j}}{(1+r)^{J+1-j}-1}d$$
$$d' = d(1+r_m)-m$$
$$y' = \begin{cases} (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z' + a'(1+r) & \text{for } j \leq J_w \\ b^i_{t+1,j+1} + a'(1+r) & \text{for } j > J_w \end{cases}$$
$$y'_{\text{sell}} = y' + (1-\tau_s)p_{t+1}h_o - d'(1+r_m) + \mathcal{T}_{t+1}.$$
$$a' \geq 0$$

C Algorithm

We have two algorithms here. One algorithm is to solve the pre-reform steady state. The other is to solve jointly the steady state after 2013 and the transition path from some initial state of the economy to the steady state after 2013. For example, in the benchmark, we need to use transition path algorithm twice. Firstly, we solve the transition path from the pre-reform to the steady state after 2013. Secondly, we use the economy in year 1998 as the initial state and solve for the transition path again.

Pre-reform steady state:

To solve for the pre-reform steady state, we assume households taking the assigned housing service as given, and government plans for the life cycle consumption profile across permanent efficiency shock groups.

Given government policy, τ, b, and interest rate r and wage rate w in pre-reform steady state.

1. Parameterize the model, and calculate the density of retired workers in the population, μ_t, $t = J_{w+1}, ..., J$.

2. Given the government expenditure, $\bar{g}$, guess the pre-reform rental rate R_c;

3. Given the policy function (analytical) of households,

   (a) guess the initial bequest.

(b) simulate the optimal path for consumption and saving for the new born generation by forward induction given the initial bequest.

(c) aggregate household's decision, and calculate the bequest leftover on the path.

(d) update the guess of the initial bequest until it converges. (Guass-Seidel method)

4. Aggregate government's tax revenue, renting revenue, and pension expenditure. Check whether government's intertemporal budget is balanced, and update the guess of the rental rate R_c.

5. Check whether R_c match the calibration target, if not, update $\bar{g}$. Derive the allocation of $\overline{H}$.

Transition path:

We need to find the equilibrium path of housing price and the tax rate in the final steady state. Assume we know the state of the economy at $t = 1$, and the economy reaches the final steady state after some periods T. (T is larger than the three times of the maximum lifespan.)

To solve the transition path, we have the following steps:

Given government policy, $\{\tau_t, b_t\}_{t=1}^{\infty}$, discount housing policy, land supply, $\{H_t\}_{t=1}^{\infty}$, and interest rate $\{r_t\}_{t=1}^{\infty}$ and wage rate $\{w_t\}_{t=1}^{\infty}$ on the path, and the initial distribution of household on the state space (initial state).

1. Choose the number of transition periods T.

2. De-trend the economy by the time T variables.

3. Provide an initial guess for tax rate in the steady state, τ.

4. Given all policy variables, solve for the final steady state housing price that clear the housing market by bisection method.

5. Provide an initial guess for housing price on the path, $\{p_t\}_{t=0}^{T}$, and solve household's problem backwards:
   At period t, compute the value functions and policy functions for the new born at t, which has a perfect foresight.

6. Compute the transition path: Compute the optimal path for consumption, housing, and saving by forward induction given the initial state in period $t = 1$. In initial state, households receive assignment of public housing from the government, $\bar{H}$. The bequests for period t newborn are collected from the household passing away at period t.

7. Aggregate household's net housing demand each period. Check if housing market in each period is clear. If not, update the guess of $\{p_t\}_{t=1}^{T}$, and go to step 5.

8. Aggregate government tax revenue, housing sale revenue, pension expenditure on the path. Combined with government's deficit/surplus in the steady state, check whether government's intertemporal budget is balanced. If not, update the guess of τ, and go to step 4.

9. Check whether p_T is close enough with the final steady state housing market price. If not, increase T, and go to step 2.

Long run equilibrium:

The price adjustment step. Because it is the long run equilibrium

$$I_t = \delta H$$

$$p_0 \Longrightarrow H(p_0) + \widetilde{H}(p_0) \Longrightarrow I_t(p_0) \Longrightarrow p_1 = 0.2 * \frac{1}{\alpha}\left(\frac{I_t(p_0)}{L}\right)^{\frac{1-\alpha}{\alpha}} + 0.8 * p_0$$

D Detrending

This section describe how we detrend the individual and aggregate variables. Since the initial steady state and final steady state have different trend growth rates for productivity and population, we detrend the initial steady state and transitional paths separately. After we solve for the initial steady state, we back out the original values for the initial steady state and detrend it with the growth rate of the final steady state as the initial values in the transition path.

D.1 Transitional path and final steady state

Denote g as the balance-growth rate of the wage rate, g_p and g_I as the growth rate of housing prices and housing investment at the balance growth rate in the final steady state. Note that according to (**??**) , since there is no secular growth of land at the steady state, the growth rate of housing investment at balance growth path is

$$1 + g_I = [(1+g)(1+n)]^{\alpha}$$

or

$$g_I = \alpha(g+n)$$

Then, according to (**??**) , the growth rate of housing price at balance growth path is

$$1 + g_p = [(1 + g)(1 + n)]^{1-\alpha}$$

or

$$g_p = (1 - \alpha)(g + n)$$

For individual housing growth, notice that at balance growth path

$$\sum_{j=1}^{J} N_j \sum_i h_j^i = H$$

Therefore, the balance growth rate of h_j^i, denoted as g_h is simply as

$$g_h = g_I - n$$

Note that

$$1 + g = (1 + g_h)(1 + g_p)$$

For individual variables, we detrend as follows

$$\begin{aligned}
\widehat{c}_{t,j} &= c_{t,j}(1+g)^{T+j-t-1} \\
\widehat{a}_{t+1,j+1} &= a_{t+1,j+1}(1+g)^{T+j-t-1} \\
\widehat{h}_{t+1,j+1} &= h_{t+1,j+1}(1+g_h)^{T+j-t-1} \\
\widehat{s}_{t,j} &= s_{t,j}(1+g_h)^{T+j-t-1} \\
\widehat{\widetilde{h}}_{t+1,j+1} &= \widetilde{h}_{t+1,j+1}(1+g_h)^{T+j-t-1} \\
\widehat{b}_{t,j} &= b_{t,j}(1+g)^{T+j-t-1} \\
\widehat{y}_{t,j} &= y_{t,j}(1+g)^{T+j-t-1}
\end{aligned}$$

For aggregate variables, we detrend as follows

$$\begin{aligned}
\widehat{w}_t &= w_t(1+g)^{T-t} \\
\widehat{N}_{et} &= N_{et}(1+n)^{T-t} \\
\widehat{N}_{ht} &= N_{ht}(1+n)^{T-t} \\
\widehat{p}_t &= p_t(1+g_p)^{T-t} \\
\widehat{p}^g_{t,j} &= p^g_{t,j}(1+g_p)^{T-t} \\
\widehat{I}_t &= I_t(1+g_I)^{T-t} \\
\widehat{L}_t &= L_t \\
\widehat{H}_t &= H_t\,(1+g_I)^{T-t} \\
\widehat{\rho}_h &= \rho_h(1+g_p)^{T-t} \\
\widehat{R}_{c,t} &= R_{c,t}(1+g_p)^{T-t} \\
\widehat{\tilde{H}} &= \tilde{H}'\,(1+g_I)^{T-t} \\
\widehat{Y}_t &= Y_t\,[(1+g)\,(1+n)]^{T-t} \\
\widehat{z}_t &= z_t(1+g)^{T-t}
\end{aligned}$$

Finally, we assume that the operating cost ψ grows at a constant rate g_I along transition and

at the final steady state. Therefore we have the detrended version of the above equations as

$$
\begin{aligned}
\widehat{c}^i_{t,j} &= \widehat{y}^i_{j,t} + \widehat{a}^i_{t,j} - (q\widehat{a}^i_{t+1,j+1} + \widehat{R}_c\widehat{s}^i_{t,j}(1+g_p)^{j-1}) \\
\widehat{s}^i_{t,j} &= \widehat{\widehat{h}}^i_{1,1+j-t}; \widehat{s}^i_{t,j} = \widehat{h}^i_{t+1,j+1} \\
\widehat{y}^i_{j,t} &= \begin{cases} (1-\tau_t-\tau^{ss}_t)\widehat{w}_t(1+g)^{j-1}e^i_{t,j} \text{ for } j \le J_w \\ \widehat{b}^i_{t,j} \text{ for } j > J_w \end{cases} \\
\widehat{c}^i_{t,j} &= \widehat{y}^i_{j,t} + \widehat{a}^i_{t,j} - (q\widehat{a}^i_{t+1,j+1} + \widehat{\rho}_{h,t}\widehat{\widehat{h}}^{i\prime}_{t+1,j+1}(1+g_p)^{j-1}), \\
\widehat{c}^i_{t,j} &= \widehat{y}^i_{j,t} + \widehat{a}^i_{t,j} - (q\widehat{a}^i_{t+1,j+1} + \widehat{p}^g_{t,j}\widehat{h}^i_{t+1,j+1}(1+g_p)^{j-1}) \\
\widehat{c}^i_{t,j} &= \widehat{y}^i_{j,t} + \widehat{a}^i_{t,j} - (q\widehat{a}^i_{t+1,j+1} + \widehat{p}_t\widehat{h}^i_{t+1,j+1}(1+g_p)^{j-1}), \\
\widehat{a}^i_{t+1,j+1} &\ge -\Upsilon\widehat{p}^g_{t,j}\widehat{h}^i_{t+1,j+1}(1+g_p)^{j-1} \\
\widehat{a}^{i,n}_{t,j} &= \widehat{a}^i_{t,j} + (1-\delta_h-\kappa)\widehat{p}_t\widehat{h}^i_{t,j}(1+g_p)^{j-1} \\
\widehat{\rho}_{h,t}(\Phi) &= \psi + \widehat{p}_t(\Phi) - \frac{(1-\delta_h)(1+g_p)}{1+r}E_{\Phi'}[\widehat{p}_{t+1}(\Phi)|\Phi] \\
\widehat{Y}_t &= \widehat{z}_t\widehat{N}_{ct} \\
\widehat{I}_t &= \left(\widehat{z}_t\widehat{N}_{ht}\right)^{\alpha}\left(\widehat{L}_t\right)^{1-\alpha} \\
\widehat{I}_t &= (\alpha\widehat{p}_t)^{\frac{\alpha}{1-\alpha}}\widehat{L}_t \\
\widehat{N}_{ht} &= (\alpha\widehat{p}_t)^{\frac{1}{1-\alpha}}\frac{L_t}{\widehat{\widehat{z}}_t} \\
(1+g_I)\left(\widehat{\widehat{H}}_{t+1} + \widehat{H}_{t+1}\right) &= (1-\delta_h)\left(\widehat{\widehat{H}}_t + \widehat{H}_t\right) + \widehat{I}_t \\
(1+g_{Ipre})(h_{pre}) &= (1-\delta_h)(h_{pre}) + \widehat{I}_t \\
\widehat{A}_0 &= \sum_{t=1}^{\infty} R^{-t} \left[\begin{array}{l} \sum_{j=t+1}^{J}\mu_{j,t}\int_{I\in bg}\left(\widehat{p}_t - \widehat{p}^g_{t,j}\right)\widehat{h}^i_{1,1+j-t}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t(s)d \\ \sum_{j=t+1}^{J}\mu_{j,t}\int_{I\in rg}\left(\widehat{\rho}_t - \widehat{R}_{c,t}\right)\widehat{h}^i_{1,1+j-t}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t(s) \\ \sum_{j=1}^{t}\mu_{j,t}\int_{I\in rg}\left(\widehat{\rho}_t - \widehat{R}_{c,t}\right)\widehat{h}^i_{1,j}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t(s)ds \\ -\sum_{j=1}^{J_w}\mu_{j,t}\tau_t\widehat{w}_t/(1+g)^{T-t}\int_s e\Phi_t(s)ds \end{array}\right. \\
\widehat{b}^i_{t,j} &= \theta(0.6\widehat{\widehat{y}}^i_{t-j+J_w,J_w} + 0.4\widehat{\bar{y}}_t(1+g)^{j-1}) \\
\widehat{\widehat{y}}^i_{t-j+J_w,J_w} &= \widehat{w}_{t-j+J_w}(1+g)^{J_w-1}e_{t-j+J_w,J_w} \\
\widehat{\bar{y}}_t &= \frac{\sum_{j=1}^{J_w}\sum_i \mu_{j,t}\widehat{w}_t e^i_{t,j}}{\sum_{j=1}^{J_w}\mu_{j,t}} \\
\tau^{ss}_t &= \frac{\sum_{j=1}^{J_w}\sum_i \mu_{j,t}\widehat{b}^i_{t,j}(1+g)^{1-j}}{\sum_{j=1}^{J_w}\sum_i \mu_{j,t}\widehat{w}_t e^i_{t,j}}
\end{aligned}
$$

D.2 Pre-reform Steady State

Denote g_0 as the balance-growth rate of the wage rate, g_{p0} and g_{I_0} as the growth rate of housing prices and housing investment at the balance growth rate in the initial steady state. Similar to the final steady state, we have

$$\begin{aligned} 1+g_{I0} &= [(1+g_0)(1+n_0)]^{\alpha} \\ 1+g_{p0} &= [(1+g_0)(1+n_0)]^{1-\alpha} \\ g_{h0} &= g_{I0}-n_0 \\ 1+g &= (1+g_{h0})(1+g_{p0}) \end{aligned}$$

For individual variables, we detrend as follows

$$\begin{aligned} \widehat{c}_{t,j} &= c_{t,j}(1+g_0)^{-t} \\ \widehat{a}_{t+1,j+1} &= a_{t+1,j+1}(1+g_0)^{-t} \\ \widehat{s}_{t,j} &= s_{t,j}(1+g_{h0})^{-t} \\ \widehat{y}_{t,j} &= y_{t,j}(1+g_0)^{-t} \end{aligned}$$

For aggregate variables, we detrend as follows

$$\begin{aligned} \widehat{w}_t &= w_t(1+g_0)^{-t} \\ \widehat{N}_{ht} &= N_{ht}(1+n_0)^{-t} \\ \widehat{I}_t &= I_t(1+g_{I0})^{-t} \\ \widehat{L}_t &= L_t \\ \widehat{H}_t &= H_t(1+g_{I0})^{-t} \\ \widehat{H}_t^D &= H_t^D(1+g_{I0})^{-t} \\ \widehat{R}_{c,t} &= R_{c,t}(1+g_{p0})^{-t} \\ \widehat{C}_t &= C_t[(1+g_0)(1+n_0)]^{-t} \\ \widehat{Y}_t &= Y_t[(1+g_0)(1+n_0)]^{-t} \\ \widehat{z}_t &= z_t(1+g_0)^{-t} \end{aligned}$$