Impact Trickles Down: A General Equilibrium Theory of Stakeholder Exit and Engagement *

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Abstract

How do purpose-driven stakeholders induce reform? Using a multi-sided matching equilibrium, we show that whether they exit or engage depends on whether social harm scales with productivity. When harm is uncorrelated—an implicit assumption in most literature—purpose-driven stakeholders engage and compensating differentials adjust. But when harm scales with production, high-productivity firms outbid others for profit-driven stakeholders to avoid mitigation. Purpose-driven stakeholders exit—a seemingly ineffective action at the firm level but whose impact trickles down the productivity ladder and across stakeholder types. This necessitates a multi-sided framework, rationalizes puzzling findings, and explains why studies underestimate the aggregate impact of exit.

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1 Introduction

Employee walkouts at technology firms for social justice, banks divesting from fossil fuels, and consumer boycotts of controversial brands illustrate how purpose-driven stakeholders increasingly pressure firms to mitigate social harm. When do dissatisfied stakeholders exit and reallocate? And when do they engage and reform the firm from within? These questions originate with Hirschman 1970, whose framework of exit and voice by dissatisfied members of an organization has proven remarkably influential—applied by political scientists to voting and emigration (Hirschman 1978, Clark, Golder, and Golder 2024), labor economists to unions and worker turnover (Freeman and Medoff 1984), public administrators to school choice and healthcare (Chubb and Moe 1990, John 2017), and financial economists to corporate governance and socially responsible investing (Broccardo, Hart, Zingales, et al. 2022, Heinkel, Kraus, and Zechner 2001). Yet despite fifty years of application across the social sciences, answers to these questions remain elusive.

We provide answers by developing a tractable framework where firms match with heterogeneous stakeholders—such as workers, banks, and suppliers—who differ in both productivity and purpose. We define purpose-driven stakeholders as those who derive disutility from firm-level social harm—a preference structure that can encompass both warm-glow motivations and non-consequentialist moral constraints.[1] Because purpose-driven stakeholders experience disutility from firm-generated harm, firms face a competitive incentive to mitigate harm to attract and retain them. This structure allows us to study how an increase in purpose-driven stakeholders leads to either optimally severing relationships (exit) or remaining and sharing mitigation costs (engage), i.e. the prevalence of exit and engagement in the economy. We show that these manifestations of purpose and impact depend fundamentally on whether a firm's productivity and its environmental or social harm are correlated.

When they are uncorrelated—an implicit assumption in much of the existing literature—sorting along the dimensions of productivity and purpose are independent of each other. Stakeholders and firms match based on productivity. Holding fixed productivity, firms are indifferent between stakeholders who are profit-driven or purpose-driven. As a result, more purpose-driven stakeholders do not trigger exit—in the sense that stakeholders are always matched with firms of the same productivity before and after the change in the proportion of purpose-driven stakeholders in the population. Firms and stakeholders of the same productivity remain matched, mitigation costs are shared, and prices (compensating differentials) adjust to clear the market. In this setting, there are also no spillovers across different groups of stakeholders—for instance,

[1] Our model follows a large theory literature on prosocial behavior in using non-consequentialist or warm-glow preferences (Andreoni 1989 and Andreoni 1990), such as Bénabou and Tirole 2006 and Besley and Ghatak 2005. These preferences have strong empirical support (Hong and Kacperczyk 2009, Riedl and Smeets 2017, Bonnefon et al. 2025).

more banks becoming purpose-driven do not affect the welfare of employees. This no-spillover result explains why the existing literature has studied each stakeholder group in isolation: studies of green banking (Kacperczyk and Peydró 2022), ethical investing (Hong and Kacperczyk 2009), climate investing (Gormsen, Huber, and Oh 2024) and mission-driven workers (Besley and Ghatak 2005) can each proceed without modeling the other groups, precisely because under independence the groups do not interact.

However, when harm scales with productivity—as is natural for pollution and many other forms of social harm—this isolation breaks down: a preference shock, i.e. a change to one group's proportion, generates spillovers onto all others, making the simultaneous modeling of multiple stakeholder types essential. In this general setting, firms are no longer indifferent between profit-driven versus purpose-driven stakeholders. Because higher output generates greater environmental or social harm, productivity is endogenously less valuable in firms with purpose-driven stakeholders, where additional output requires costly offsetting mitigation. Sorting along the dimensions of productivity and purpose becomes much more complex to characterize. Our contribution is to develop a sequential algorithm where we are able to characterize the equilibrium for an arbitrary number of stakeholder types.

Above a critical productivity threshold, big firms outbid for profit-driven stakeholders to avoid mitigation costs and remain entirely profit-driven. As a result, large purpose-driven stakeholders are forced to sort together since mitigation is non-rival within the firm: its cost is shared among all purpose-driven stakeholders in a team. Hence, stakeholders sort into distinct types: "clean" firms that hire exclusively purpose-driven stakeholders and undertake extensive mitigation, and "dirty" firms that hire profit-driven stakeholders and do not mitigate. Below this threshold, full separation is often infeasible, leading some firms to strategically adopt moderate mitigation to attract productive purpose-driven stakeholders. These "mixed" teams are essential to understanding the aggregate impact of exit versus engagement in the model.

These equilibrium properties dictate the prevalence of exit and engagement following an increase in the proportion of purpose-driven stakeholders. In contrast to the independence benchmark, more purpose-driven stakeholders now lead to both exit and engagement as opposed to just engagement alone. For highly productive stakeholders, inducing large, high-output firms to mitigate is prohibitively expensive; any attempt at engagement would require too large compensation transfers to the firm. Consequently, these stakeholders optimally exit, reallocating toward smaller, purpose-aligned firms. Conversely, lower-productivity stakeholders face a different trade-off. Since their firms are indifferent between stakeholder (purpose) types, having more purpose-driven stakeholders does not destabilize their relationship. Instead, these stakeholders optimally engage, remaining in place and sharing the costs of increased mitigation.

While the exit of highly productive stakeholders often appears ineffective at the firm level—as they move to firms that already mitigate—this narrow view misses the aggregate "trickle-

down" effect. When high-productivity stakeholders exit, they displace less productive purpose-driven stakeholders at their destination firms. These displaced stakeholders then reallocate to other firms, where their presence induces mitigation that would not otherwise occur. Through this chain of displacement, exit generates reallocation spillovers that propagate through the economy, shifting the behavior of far-removed firms.

As a result, exit and engagement differ fundamentally in their aggregate impact. While engagement delivers direct, localized change, exit reshapes mitigation incentives across the broader market via equilibrium matching. In other words, the perceived weakness of exit often reflects a narrow, firm-level perspective, its true impact emerges in general equilibrium. Unlike engagement, which operates locally within a relationship, exit triggers market-wide reallocation spillovers.[2] Through these competitive channels, exit can influence firms that are not directly affected by the initial purpose-driven shock.

Importantly, when harm scales with production, there are spillovers not only down the productivity ladder but also across stakeholder groups. For instance, an increase in purpose-driven banks affects the welfare of workers as well: purpose-driven workers benefit because they can now match with more productive purpose-driven banks, while profit-driven banks earn rents as productive firms bid for their services. These cross-group spillovers are absent under independence and are a key reason why the simultaneous modeling of multiple stakeholder types matters.

Finally, we apply our model to the wave of bank decarbonization commitments following the 2015 Paris Agreement. We show that the model rationalizes several empirical findings that are difficult to explain under independence: the prevalence of exit over engagement by large banks, and the limited firm-level impact documented in empirical studies. Our numerical illustration suggests that a significant fraction of the aggregate impact operates through spillovers that existing firm-level empirical strategies miss.

Related literature. Our general equilibrium theory can be applied to many types of stakeholders and shows when modeling multi-party relationships is important in fully understanding stakeholder exits and engagements. As we alluded to in the Introduction, purpose-driven stakeholders are far-ranging, from workers and suppliers to financial institutions.

Socially responsible finance. In recent years, large financial institutions—institutional investors such as Norges Bank Investment Management (NBIM) and CalPERS, as well as major

[2]Spillovers appear in a number of areas in economics including agglomeration effects of cities (Ellison and Glaeser 1997), spillovers of firm entry (Greenstone, Hornbeck, and Moretti 2010), the spread of technological shocks through production networks (Acemoglu, Akcigit, and Kerr 2016), and macro-financial shocks (Huber 2023). These studies share our focus on how small local changes can have aggregate consequences, though our contribution is to show that purpose-driven preference shocks operate through novel channels in stakeholder–firm matching.

global banks—have been among the most prominent proponents of divestment from firms that generate social harm. Yet most theoretical models of divestment in finance study how portfolio restrictions affect small, dispersed, purpose-driven shareholders (Heinkel, Kraus, and Zechner 2001, Hong, Wang, and Yang 2021, Pástor, Stambaugh, and Taylor 2021, Pedersen, Fitzgibbons, and Pomorski 2021). Investors' portfolio restrictions, which map into non-consequentialist preferences, lead firms to mitigate in exchange for a lower cost of capital, i.e. compensating differentials. But these models do not capture why a large stakeholder like NBIM ends up divesting. Our model, featuring stakeholders with bilateral relationships, arguably better captures the behavior of large financial institutions, since these institutions typically hold enough of a stake to be considered key stakeholders that firms want to maintain a relationship with.

There are two important exceptions. First, Broccardo, Hart, Zingales, et al. 2022 argue that divestment is inferior to engagement (voting) when shareholders have consequentialist preferences—that is, when shareholders care about the social outcomes their votes produce, not merely about their own exposure to harm—and when the cost of voting is zero. Under these two assumptions, even a small probability of being pivotal makes voice worthwhile relative to exit. Second, Oehmke and Opp 2023 consider how funds with consequentialist mandates can influence reform even in a perfectly elastic capital supply setting in the presence of firm financial frictions.

Our model complements their work by analyzing stakeholder exit and engagement using the more standard and empirically grounded non-consequentialist preferences in a multiparty matching equilibrium. As established in our footnote 1, non-consequentialist preferences are the workhorse assumption in the broader literature on prosocial behavior and have strong empirical support. Compared to Broccardo, Hart, Zingales, et al. 2022, our model better rationalizes why we observe such a high propensity for exit among large stakeholders, while nonetheless also seeing engagement by smaller stakeholders in the same equilibrium.

By endogenizing the choice between exit and engagement in a general equilibrium environment with multiple stakeholder types, our analysis offers two primary insights. First, we characterize the sorting patterns of stakeholders, rationalizing why large, highly productive stakeholders—such as major banks or institutional investors—often favor exit over engagement in practice. Second, we show that evaluating exit solely through a firm-level lens—as do virtually all empirical studies (see Section 6)—can be misleading. While exit may appear ineffective for directly targeted firms, it operates through equilibrium reallocation and generates "trickle-down" spillovers that shift mitigation incentives across the entire economy.

We further show that the cost of engagement—modeled as an endogenous subsidy provided by purpose-driven stakeholders to induce mitigation—depends on equilibrium matching and market competition. This cost varies substantially across stakeholder types, helping explain why empirical wage premia for purpose-alignment often exceed green financing premia. Finally,

although we do not model voting explicitly, the effectiveness of voice in our framework depends on the endogenous concentration of purpose-driven stakeholders within the firm. Our sorting results thus complement models of corporate governance by endogenizing the composition—and hence the collective power—of purpose-driven voice.

Matching and sorting on multidimensional characteristics. Methodologically, our environment is a multi-sided matching model with transferable utility and multidimensional heterogeneity, as stakeholders differ in both productivity and purpose.

Most closely related is Boerma, Tsyvinski, and Zimin 2025, who study multi-sided matching where firms choose among worker types under a submodular output function with one-dimensional characteristics. In contrast, we maintain a standard supermodular production technology but introduce two-dimensional heterogeneity. This allows us to study how productivity and purpose interact endogenously to determine surplus and sorting—a feature that is absent in models where multidimensional traits are collapsed into a single index (e.g., Sattinger 1979, Tervio 2008, Gabaix and Landier 2008).

Characterizing sorting patterns in multidimensional environments is notoriously complex, as simple supermodularity no longer uniquely governs assortative matching. Several prior works have characterized equilibrium in bilateral settings under specific conditions (e.g., Dupuy and Galichon 2014, Lindenlaub 2017, Chiappori, McCann, and Pass 2016, Chiappori, Oreffice, and Quintana-Domeque 2018). Methodologically, we contribute a tractable iterative solution method for multi-sided matching problems featuring multidimensional heterogeneity, beyond bilateral matching. This approach allows us to characterize the general equilibrium interactions among diverse stakeholder groups, providing a framework for analyzing spillover effects arising from interdependence between productivity and preferences.

2 Model

2.1 Environment

Production and harm. There are N types of stakeholders and one firm, indexed by $\ell \in L \equiv \{1, 2, \ldots, N+1\}$. Each stakeholder type $\ell \leq N$ has skill $x_\ell \in X_\ell$, while the firm has productivity $x_{N+1} \in X_{N+1}$. All types have unit mass, with smoothly distributed skills on compact supports.

Output depends multiplicatively on all agents' characteristics:

$$y(\mathbf{x}) = \prod_{\ell=1}^{N+1} x_\ell,$$

where $\mathbf{x} = (x_1, \ldots, x_{N+1})$. As discussed in Tervio (2008), the linear assumption on the arguments does not preclude different stakeholders having different skills contributing to their ability to affect output.[3]

Relationship between production and harm. The relationship between harm and production is denoted by

$$\phi(y) = \phi_0 + \sigma y,$$

where $\sigma \geq 0$. When $\sigma = 0$, harm does not increase with production. When $\sigma > 0$, more production generates more environmental or social harm. For example, when harm takes the form of emissions, σ thus represents the emission rate of the firm per unit of production.

Stakeholder preferences. Each stakeholder matches to exactly one firm, capturing the notion of a bilateral relationship (e.g. a bank lending to one firm, or a worker employed at one firm). Stakeholders' utilities differ in whether they are purpose-driven. Purpose-driven stakeholders are modeled as having disutility over the harm, denoted by e, generated by the firm with which they have a relationship.

This warm-glow preference creates incentives for the stakeholder's matched firm to mitigate.[4] Let $\theta_\ell \in \{0, 1\}$ denote type, and the utility is given by

$$u(p, e \mid \theta_\ell) = p - \theta_\ell \psi(e), \tag{1}$$

where p is the transfer received, e is harm created by the hiring firm, and disutility $\psi(\cdot)$ is increasing and convex in harm, $\psi'(e) > 0$ and $\psi''(e) > 0$. That is, profit-driven stakeholders ($\theta_\ell = 0$) care only about compensation. purpose-driven stakeholders ($\theta_\ell = 1$) also dislike the harm produced by their firm. Thus stakeholder types are $a_\ell = (x_\ell, \theta_\ell)$, distributed with measure μ_ℓ on $X_\ell \times \{0, 1\}$. The share of purpose-driven stakeholders of type ℓ is denoted by λ_ℓ.

Firms themselves are profit-maximizing and do not have non-profit-driven preferences, i.e. $a_{N+1} = (x_{N+1}, 0)$ and $\lambda_{N+1} = 0$. That is, firms are special in the sense that their characteristics are always one-dimensional ($\lambda_{N+1} = 0$).[5] Throughout, skills and purpose-driven preferences are assumed to be independently and identically distributed. We use this assumption to highlight that, if there is any correlation between productivity and mitigation, it will arise endogenously through the equilibrium matching.

[3]Specifically, one could interpret $x_\ell = b_\ell(\hat{x}_\ell)$, which represents the effective ability of some underlying skill $\hat{x}_\ell$, where b_ℓ is an increasing transformation of the scale of measurement for a factor quality. For example, a Cobb-Douglas production function $x_0 \hat{x}_1^{\alpha} \hat{x}_2^{(1-\alpha)}$ can be nested as $x_1 = \hat{x}_1^{\alpha}$ and $x_2 = \hat{x}_2^{1-\alpha}$.

[4]Given the preferences are on firm harm within the team, the matching problem is not subject to externalities.

[5]We use this assumption to highlight that firms only mitigate when matching with purpose-driven stakeholders, and it must be profitable for them to do so. Our setup can easily be extended to allow firms to also care about harm.

Assumption 1. *Skill and purpose are independently distributed. Conditional on skill x_ℓ, a stakeholder of type ℓ is purpose-driven with probability λ_ℓ.*

Lastly, we label stakeholder types such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$. That is, types are ordered by the prevalence of purpose-driven stakeholders.

Mitigation. While firms produce harm, they can reduce harm by $m \geq 0$ at linear cost cm. Hence, the harm generated by a firm in our model is given by $\phi(y) - m$, which depends on two endogenous objects: his productivity (which depends on which stakeholders he employed) and his abatement policy.

Given any match, let $\mathbf{a} = (a_1, \ldots, a_{N+1})$ represent the characteristic of all agents within the team, and $\Lambda(\mathbf{a})$ represent the total surplus of the team. Since we have transferrable utilities, the mitigation within any match must be chosen to maximize the total surplus, which yields

$$\Lambda(\mathbf{a}) = \max_{m \geq 0} \; y(\mathbf{x}) - cm - n(\mathbf{a})\psi(\phi(y(\mathbf{x})) - m), \tag{2}$$

where $n(\mathbf{a}) \equiv \sum_{\ell=1}^{N+1} \theta_\ell$ denotes the *stakeholder-purpose index* of the team, i.e. the number of purpose-driven members within the team.

Stakeholder payoffs. For any stakeholder a_ℓ, let the utility be

$$U_\ell(a_\ell) = \max_{\{a_{\ell'}\}_{\ell' \in L \setminus \{\ell\}}} \Lambda(\{a_{\ell'}\}_{\ell' \in L \setminus \{\ell\}}, a_\ell) - \sum_{\ell' \in L \setminus \{\ell\}} U(a_{\ell'}). \tag{3}$$

That is, the payoff to stakeholder a_ℓ is the team surplus minus the equilibrium utilities of all other members. The firm's problem is a special case of (3) with $\ell = N+1$. Figure 1 summarizes the flow of our model.

Competitive equilibrium. A competitive equilibrium consists of: (1) an allocation function γ that represents the matching between firm and all types of stakeholders, which is defined as a probability measure on the set of $(X_\ell \times \{0, 1\})^{N+1}$, (2) a mitigation policy $m^*(\mathbf{a})$ for each team, and (3) equilibrium utilities $\{U_\ell(a_\ell)\}$ for all types $\forall \ell \in L$. These must satisfy:

- (Optimal mitigation) The mitigation policy $m^*(\mathbf{a})$ solves Equation (2).
- (Stable match) No matched agent can increase their utilities by forming a new team: for any $\mathbf{a} = (a_1, a_2 ... a_{N+1})$, $\Sigma_{\ell \in L} U_\ell(a_\ell) \geq \Lambda(\mathbf{a})$, where an equality obtains on the support of γ.
- (Market clearing) The marginal of γ should equal the marginal distribution of F_ℓ.

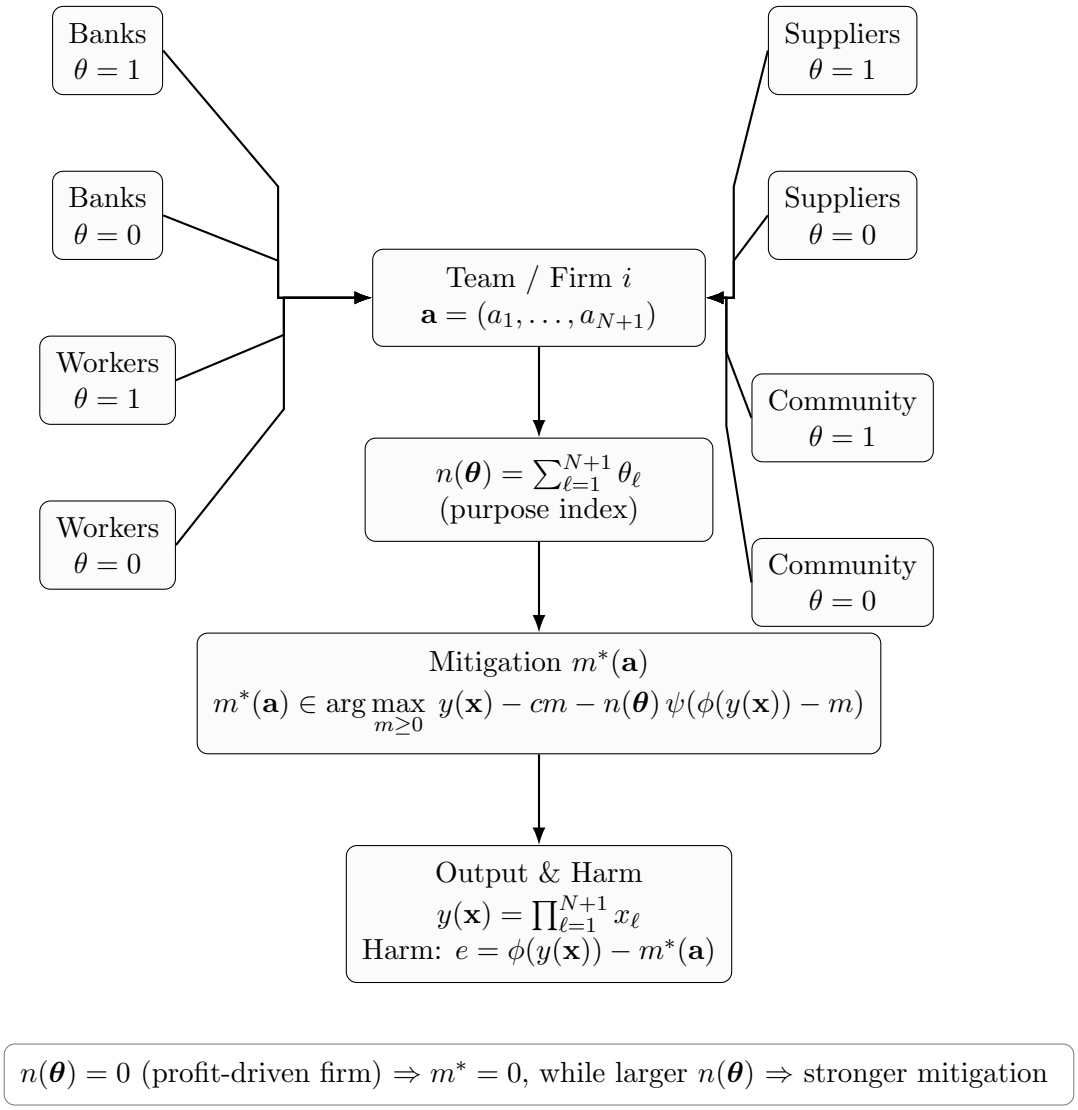


Figure 1: Extended schematic: banks and workers (left), suppliers and community (right) match with a firm, which determines the purpose index, mitigation, and resulting output and harm.

Matching is stable if no coalition can profitably deviate to form a new team. Accordingly, for any assignment $\mathbf{a}'$ outside the support of γ, the joint surplus generated by such a deviation, $\Lambda(\mathbf{a}')$, must be weakly lower than the sum of the members' equilibrium utilities. Following the matching literature with transferable utility, any stable outcome solves the planner's problem. [6] Therefore, the equilibrium maximizes aggregate surplus. Finally, given equilibrium utilities and the mitigation policy, transfers are uniquely pinned down for each stakeholder by Equation 1.

Remarks on disutility specification. *Efficiency and Externalities.* Our setting does not explicitly feature externalities, as stakeholders experience disutility only from the harm gen-

[6] Specifically, Boerma, Tsyvinski, and Zimin 2025 has established the equivalence in the setting with multi-sided matching, based on the optimal transport literature (see Galichon 2016 for a survey).

erated by their matched firm. One could alternatively assume that stakeholders also suffer disutility from aggregate emissions. This extension would not affect the equilibrium allocation, since stakeholders are infinitesimal and take aggregate emissions as given. However, it would generally imply that the decentralized equilibrium is no longer efficient.

Disutility over Level vs. Rate. We specify stakeholder disutility over the level of harm. An alternative formulation would assume that stakeholders experience disutility over the *rate* of harm—for instance, emissions per unit of output. In that case, $\psi(e)$ would be replaced by $\psi(e/y)$, and the relevant condition for our results would be whether more productive firms must incur greater mitigation effort to achieve the same harm rate. Our main economic results hold under either specification. For tractability, we adopt the level specification.

2.2 General Properties

Observe that a team can be characterized by two sufficient statistics: its productivity and its stakeholder-purpose index. Specifically, from Equation 2, the surplus of any team can be written as $\Lambda(\mathbf{a}) = \Omega\left(\prod_{\ell=1}^{N+1} x_\ell, \sum_{\ell=1}^{N+1} \theta_\ell\right)$,where

$$\Omega(y, n) = \max_{m \geq 0} \Big\{ y - cm - n\psi(\phi(y) - m) \Big\}. \tag{4}$$

We impose the following assumptions to guarantee that mitigation is interior whenever $n \geq 1$, and that production is always valuable despite mitigation costs. Let $y_{min} \equiv y(\underline{\mathbf{x}})$ denote the lowest production, which is composed of the lowest skill of type ℓ,denoted by $\underline{x_\ell}$.

Assumption 2. *(i) (Interior mitigation)* $\psi'(\phi(y_{min})) > c$. *(ii) (Production valuable)* $1 - c\sigma > 0 \, \forall y$ *and* $\Omega(y_{min}, N) > 0$.

The first condition represents that, at the lowest productivity (and thus lowest possible harm), it is nevertheless optimal to have positive mitigation as long as there is one purpose-driven stakeholder. The second condition guarantees that $\Omega_y(y, n) > 0$, as for any $n \geq 1$,

$$\Omega_y(y, n) = 1 - n\psi'(\phi(y) - m^*(y, n))\phi'(y) = 1 - c\sigma, \tag{5}$$

where we use the fact the optimal mitigation $m^*(y, n)$ must satisfy the FOC condition, $n\psi'(\phi(y) - m^*(y, n)) = c$. Hence, taking into account the additional abatement cost generated by the production, production is always valuable.

Sorting on purpose. Observe that adding a purpose-driven stakeholder is costly ($\Omega_n(y, n) < 0$), but the marginal cost is decreasing ($\Omega(y, n)$ is strictly convex in n). This is because mitigation is non-rival and has an economic scale.

This implies clustering: purpose-driven agents prefer to join each other. Let $n^*_{-\ell}(x_\ell, \theta)$ be the index of the matching team of stakeholder (x_ℓ, θ), which consists of other types of stakeholders.

Lemma 1. *(Concentration of purpose-driven stakeholders) Consider two stakeholders of type ℓ with the same ability $(x_\ell = x_\ell)$, then*

$$n^*_{-\ell}(x_\ell, 1) \;\geq\; n^*_{-\ell}(x_\ell, 0).$$

That is, conditional on the skill, purpose-driven stakeholders must join a team with a higher purpose-index than the equivalent profit-driven stakeholders. Otherwise, one can switch these two agents, which results in a more extreme distribution of n but does not affect the output in each team, and such a deviation generates higher payoff as $\Omega(y, n)$ is convex in n.

Productivity–purpose interdependence. Next we study how productivity and purpose interact. Output always raises surplus ($\Omega_y(y, n) > 0$), but the marginal value of productivity can depend on the team's purpose index. Specifically, from Equation 5, for any $n \geq 1$,

$$\Omega_y(y, n) = 1 - c\sigma \geq \Omega_y(y, 0) = 1,$$

where the equality holds only when $\sigma = 0$. Intuitively, as long as higher output creates more harm ($\sigma > 0$), productivity becomes less valuable to teams with purpose-driven members than to purely profit-driven teams.

Let $(y_{-\ell}, n_{-\ell})$ denote the productivity and purpose-driven index of the team that excludes agent of type ℓ, and $V(y_{-\ell}, n_{-\ell})$ represent the total utility accruing to all members in the team. The matching between the team $(y_{-\ell}, n_{-\ell})$ and the stakeholder (x_ℓ, θ_ℓ) can thus be rewritten as

$$U(x_\ell, \theta_\ell) = \max_{(y_{-\ell}, n_{-\ell})} \; \Omega(x_\ell y_{-\ell},\, n_{-\ell} + \theta_\ell) - V(y_{-\ell}, n_{-\ell}), \tag{6}$$

As standard, all else equal, a more productive stakeholder always generates higher team surplus and thus must obtain a higher market utility (i.e., $U(x_\ell, \theta)$ increases in x_ℓ). Under the market competition, it is thus the team that is most willing to pay for higher skill will end up hiring a higher skilled stakeholder. Specifically, the marginal contribution of a more productive profit-driven stakeholder $(x_\ell, 0)$ to the team $(y_{-\ell}, n_{-\ell})$ is given by

$$\frac{\partial}{\partial x_\ell}\Omega(y_{-\ell}x_\ell,\, n_{-\ell}) = \begin{cases} (1 - c\sigma)\, y_{-\ell} & n_{-\ell} \geq 1 \\ y_{-\ell} & n_{-\ell} = 0. \end{cases} \tag{7}$$

First of all, due to the standard complementary in production, fixing $n_{-\ell}$, a more productiv-

ity team $y_{-\ell}$ must value a high skilled worker more. Moreover, note that for any $\sigma > 0$, fixing the productivity $y_{-\ell}$, a profit-driven stakeholder's skill is relatively more valuable to a purely profit-driven team ($n_{-\ell} = 0$) than to the team that consists of purpose-driven stakeholders. This is because a higher productivity also means more abatement to the team with $n_{-\ell} \geq 1$.

The joint effect of productivity and purpose composition on the marginal return to profit-driven skill can thus be summarized by the index $z(y_{-\ell}, n_{\ell})$ below,

$$z(y_{-\ell}, n_{-\ell}) \equiv \begin{cases} (1 - c\sigma)y_{-\ell} & n_{-\ell} \geq 1 \\ y_{-\ell} & n_{-\ell} = 0. \end{cases} \tag{8}$$

The z-index captures the effective productivity of a team from the perspective of a profit-driven stakeholder: it equals the team's raw productivity when the team is entirely profit-driven, but is discounted by the factor $(1 - c\sigma)$ when the team contains purpose-driven members, reflecting the additional mitigation cost that higher output entails.

Lemma 2. *(Positive assortative matching) (i) Among profit-driven stakeholders ($\theta_\ell = 0$), higher skill x_ℓ implies matching with a team with a higher effective productivity (z-index). (ii) Among purpose-driven stakeholders ($\theta_\ell = 1$), higher skill x_ℓ implies matching with a team with higher productivity $y_{-\ell}$.*

By monotone comparative statics, a higher-skill profit-driven stakeholders must be matched with the team with z-index (i.e., with higher willingness to pay for profit-driven stakeholder's skill), which thus explains result (i).

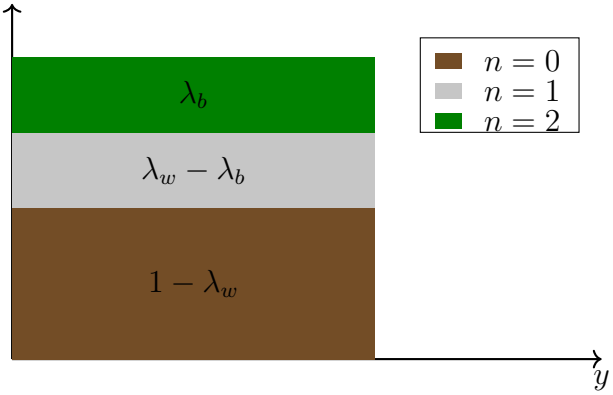
On the other hand, for a purpose-driven stakeholder $(x_\ell, 1)$, productivity is always discounted regardless of purpose-driven index of the team $n_{-\ell}$, as

$$\frac{\partial}{\partial x_\ell}\Omega(y_{-\ell}x_\ell,\, n_{-\ell} + 1) = (1 - c\sigma)y_{-\ell}. \tag{9}$$

As a result, a team's willingness to pay for purpose-driven stakeholder's skill can be reduced to the ranking of their productivity $y_{-\ell}$. Thus, higher-skill purpose-driven stakeholders therefore must match with more productive teams.

Lemma above thus suggests that, for any $\sigma > 0$, the more productive teams will not necessarily match with more productive stakeholders. Intuitively, since teams without purpose-driven members value profit-driven stakeholders relatively more, as they can avoid mitigation, equilibrium matching generally distorts productivity. Hence, equilibrium sorting must jointly account for productivity and purpose.

Figure 2: Distribution of purpose-driven index across y, where $\lambda_w \geq \lambda_b$



3 Independence Benchmark

We begin with the benchmark case in which harm is independent of firm productivity—that is, $\sigma = 0$. As we argued in the Introduction, this assumption is implicit in much of the existing literature on exit and engagement. Under independence, the model delivers two sharp results: purpose shocks operate exclusively through engagement, and there are no spillovers across stakeholder groups. These results establish a clean benchmark against which the general case ($\sigma > 0$) can be understood.

3.1 Sorting and Compensating Differentials under Independence

When $\sigma = 0$, surplus becomes separable in y and n, so sorting on ability and sorting on purpose can be solved independently. From Equation (5), the reason is that the marginal value of productivity does not depend on the team's purpose-driven index, since $\Omega_y(y, n) = 1 \, \forall n \geq 0$ when $\sigma = 0$. There is therefore positive assortative sorting on x_ℓ for all ℓ. More productive stakeholders are matched with one another, and consequently, with more productive firms. Hence, the productivity of a team with ranking i is thus given by $y[i] = \prod_{\ell=1}^{N+1} x_\ell[i]$. Given productivity, how then are the distribution of the index n and compensation determined?

The Case of $N = 2$. For expository purposes, let us first consider the case of $N = 2$. Figure 2 illustrates the simple case in which firms form relationships with two stakeholder types ($N = 2$, with $\ell \in \{w, b\}$), interpreted as workers and banks, and where $\lambda_w \geq \lambda_b$. Because purpose-driven banks are relatively scarce, all such banks can be matched with purpose-driven workers (according to Lemma 1). Hence, the measure of firms with exactly two purpose-driven stakeholders, $n = 2$, is given by λ_b. By market clearing, some purpose-driven workers must then be matched with profit-driven banks, thereby forming firms with $n = 1$. The measure of

firms with exactly one purpose-driven stakeholder is therefore $\lambda_w - \lambda_b$. The remaining firms are purely brown, with measure $1 - \lambda_w$.

Compensating differentials (due to equilibrium variation in mitigation across firms) are pinned down by the following indifference conditions. First, a firm must be indifferent between hiring a $n = 0$ team (profit-driven worker and bank) and hiring a $n = 1$ team (purpose-driven worker and profit-driven bank since $\lambda_w > \lambda_b$). Hence, the difference in compensation between profit-driven versus purpose-driven workers must reflect the difference in the firm's mitigation cost:

$$p_w(x_w, 0) - p_w(x_w, 1 \mid 1) = c\left(\xi_0^* - \xi_1^*\right).$$

Second, the difference in compensation between profit-driven and purpose-driven banks is pinned down by the condition that a firm is indifferent between hiring a $(n = 1)$ team (purpose-driven worker and profit-driven bank) and hiring a $(n = 2)$ team (purpose-driven worker and bank). This yields

$$p_b(x_b, 0) - p_b(x_b, 1 \mid 2) = c\left(\xi_1^* - \xi_2^*\right) - \left[\psi(\xi_1^*) - \psi(\xi_2^*)\right].$$

The first term represents the reduction in mitigation cost. The second term captures the fact that the firm must also pay more to its purpose-driven worker when harm is higher, because that worker's disutility rises.

These results generalize immediately to any N, where types are ranked based on λ_ℓ.

Proposition 1. *When $\sigma = 0$:*

1. *(Sorting on productivity) Two stakeholders with the same productivity but different preferences work for teams with the same productivity, and more productive stakeholders work for more productive teams;*

2. *(Sorting on preferences) For any given y, the measure of firms with stakeholder-purpose index ℓ is $\lambda_\ell - \lambda_{\ell+1}$, while the measure of purely profit-driven firms is $1 - \lambda_1$*

3. *(Compensating differentials) The higher compensation for profit-driven compared to purpose-driven stakeholders is due to the savings in the cost of mitigation:*

$$\Delta_\ell(x_\ell) \equiv p_\ell(x, 0) - p_\ell(x, 1 \mid \ell) = c\left(\xi_{\ell-1}^* - \xi_\ell^*\right) - (\ell - 1)\left[\psi(\xi_{\ell-1}^*) - \psi(\xi_\ell^*)\right] \qquad \forall \ell \geq 1,$$

where $\xi_0^* = \phi_0$.

Note that the compensating differential again has two terms. The first term captures the firm's benefit from a lower mitigation cost. At the same time, greater harm imposes additional

disutility on existing purpose-driven stakeholders and therefore requires higher compensation to those stakeholders.

3.2 Engagement Only and No Spillovers

We now consider the effect of a change in λ_ℓ, an increase in the measure of purpose-driven stakeholders of type ℓ. Our goal is to determine whether the effect operates through engagement or through exit. Formally, we say that a stakeholder *engages* if his equilibrium team, summarized by $z^*(x_\ell, \theta)$, remains unchanged, and *exits* if it changes. [7]

Because sorting on productivity is independent of preferences or purpose, any change in λ_ℓ affects only the distribution of the purpose-driven index n, holding fixed productivity. For example, consider again the $N = 2$ case of banks and workers with $\lambda_w > \lambda_b$. Increasing the measure of purpose-driven banks, λ_b, by δ_b raises the measure of firms with two purpose-driven stakeholders by δ_b and lowers the measure of firms with one purpose-driven stakeholder by the same amount. In this sense, when banks become purpose-driven, they simply induce their firms to adopt more mitigation. These treated banks therefore effectively subsidize their firms by giving up their premium, since the difference in their compensation is exactly $p_b(x_b, 0) - p_b(x_b, 1|2)$. Intuitively, equilibrium prices ensure that firms are indifferent between profit-driven and purpose-driven stakeholders. When a stakeholder becomes purpose-driven, the firm is therefore willing to engage and saves the premium it had previously been paying for profit-driven stakeholders. Therefore, treated stakeholders will not exit.

Corollary 1 generalizes this intuition to any N.

Corollary 1. *(Impact Via Engagement Only) Suppose the share of purpose-driven type-ℓ stakeholders increases by a small amount $\delta_\ell > 0$, with $\lambda_\ell + \delta_\ell < \lambda_{\ell-1}$. When $\sigma = 0$, all treated stakeholders work for firms of the same size as before the shock, compensating differentials adjust, and for any y, a measure δ_ℓ of firms reduces harm by $\xi^*_{\ell-1} - \xi^*_\ell > 0$.*

The intuition is straightforward: when harm is independent of firm size, a purpose-driven stakeholder's disutility does not depend on which firm she works for—only on whether her firm mitigates. There is no incentive to leave a large firm for a smaller, cleaner one: the stakeholder can induce mitigation wherever she is. Engagement weakly dominates exit.

No Spillover Effect on Stakeholders' Utilities We now analyze how such shocks affect stakeholders' utilities. According to Equation (6), the return to skill x_ℓ is determined by the

[7] Conditional on $z^*(x_\ell, \theta)$, agents may be indifferent across purpose-driven indices n, so such variation is payoff-irrelevant. For this reason, we define engagement as the case in which $z^*(x_\ell, \theta)$ remains unchanged. Equivalently, one may interpret the preference shock as operating at the marginal firm, namely the firm with index $n = \ell - 1$, since the aggregate effect must occur there.

stakeholder's equilibrium team, $z^*(x_\ell, \theta)$. By the envelope theorem, we have

$$\frac{\partial U_\ell(x_\ell, \theta)}{\partial x_\ell} = z^*(x_\ell, \theta), \tag{10}$$

where $z^*(x_\ell, \theta)$ denotes the stakeholder's equilibrium matching team. Hence, the utility of stakeholder (x_ℓ, θ) is given by

$$U_\ell(x_\ell, \theta) = \int_{\underline{x}_\ell}^{x_\ell} z^*(\tilde{x}_\ell, \theta)\, d\tilde{x}_\ell + U_\ell(\underline{x}_\ell, \theta). \tag{11}$$

Note that, for any ℓ, the least valuable stakeholder is the least productive purpose-driven stakeholder. This stakeholder must therefore obtain his outside option, which we normalize to zero. Hence, $U_\ell(\underline{x}_\ell, 1) = 0$. In contrast, the least productive pecuniary stakeholder earns an additional premium due to compensating differentials.

Therefore, given the matching outcome $z^*(x_\ell, \theta)$, equilibrium utilities for all stakeholders are uniquely pinned down. It follows immediately that if such shocks do not affect the matching outcome, then stakeholders' utilities remain unchanged.

Corollary 2. *(No Reallocation and No Spillover) A preference or purpose shock of size δ_ℓ does not affect the equilibrium allocation or the utilities of any agents. That is, $z^*(x_{\ell'}, \theta)$ and $U_{\ell'}(x, \theta)$ remain unchanged for all ℓ', x, and θ. Conditional on skills, the utility gain of a profit-driven relative to a purpose-driven stakeholder remains the same, which is given by*

$$U_\ell(x_\ell, 0) - U_\ell(x_\ell, 1) = \Delta_\ell + \psi(\xi_\ell^*), \tag{12}$$

and decreases in ℓ.

In this sense, the payoffs of different stakeholder types are isolated: an increase in the prevalence of purpose-driven preferences among type-ℓ stakeholders does not affect the utilities of other agents. This no-spillover result has a direct empirical interpretation: a commitment by banks to become purpose-driven does not affect the welfare of workers at those same firms, or at any other firm. The commitment is absorbed locally through compensating differentials without propagating across stakeholder types or across firms.

Note that while utilities remain the same, the payment to existing purpose-driven stakeholders would decrease if the firm were to reduce the harm so that the purpose-driven stakeholder remains indifferent. Thus, the compensation of a purpose-driven stakeholder of type ℓ in a firm with index $n \geq \ell$ is

$$p_\ell(x, 1 \mid n) = U_\ell(x, 1) + \psi(\xi_n^*) \qquad \forall n \geq \ell, \tag{13}$$

where ξ_n^* denotes equilibrium harm under optimal mitigation, characterized by $n\psi'(\xi) = c$,

which is strictly decreasing in n.

4 General Characterization

We now consider the general setting where environmental or social harm scales with production, i.e. $\sigma > 0$. Intuitively, larger firms now have stronger incentives to avoid mitigation and are willing to match with less productive profit-driven stakeholders to do so. Hence, unlike the independence benchmark of Section 3, the most productive firms will compete for relatively less productive profit-driven stakeholders. Sorting on the dimensions of productivity and purpose are no longer independent. For this reason, the multi-dimensional matching problem is much more complex.

We first analyze a simple environment, where firms form relationships with two types of stakeholders. We then extend the construction to general N via a sequential algorithm.

4.1 Illustrative example with $N = 2$

To fix ideas, consider again that firms form relationships with two types of stakeholders (say workers and banks), with purpose-driven probability λ_w and λ_b.

We first focus on the matching between workers and banks. Since firms are all profit-driven, the match between the firm and the team consisting of workers and banks can then be characterized by PAM on the firm's productivity x_{N+1} and the team's z-index. Thus, workers and banks understand that if their team has a higher z-index, they will be able to match with a larger firm.

Balanced supply: full segmentation. When the shares of purpose-driven workers and banks are identical, $\lambda_w = \lambda_b$, equilibrium matching between workers and banks is again very simple, as it must be fully segmented. Because of clustering (Lemma 1), purpose-driven workers match exclusively with purpose-driven banks, and profit-driven workers match exclusively with profit-driven banks. Within the purpose-driven or profit-driven segment, matching is positive assortative in skills by Lemma 2.

Lemma 3. *(Balanced Supply) When $\lambda_w = \lambda_b$, equilibrium matching is fully segmented, with positive assortative matching within each segment. Moreover, there will be a positive sorting between firms and the z-index of the team.*

We can visualize the equilibrium matching outcomes in Figure 3. Consider two banks of the same talent x_b — one profit-driven $(x_b, 0)$ and one purpose-driven $(x_b, 1)$. The profit-driven bank's matching worker is given by the dashed brown line, while the purpose-driven bank's is

given by the dashed green line. They both match with a worker of the same quality x_w but the profit-driven bank matches with the worker with a higher z-index, which in this case is just x_w while the purpose-driven bank matches with a worker with a z-index of $(1 - c\sigma)x_w$ (following Equation 8). The positive slope of the two lines reflects positive assortative matching of bank and worker quality.

Unbalanced supply: segmentation at the top, mixing at the bottom. When $\lambda_w \neq \lambda_b$, full segmentation is no longer feasible due to the need for market clearing. Suppose $\lambda_w > \lambda_b$, so that workers have relatively more purpose-driven stakeholders. Market clearing then requires some purpose-driven workers to match with profit-driven banks. We show that such mixing occurs only at the bottom but not at the top of the skill distribution.

Lemma 4. *(Full Separation At The Top) When $\lambda_w \geq \lambda_b$, there exists a cutoff $\hat{x}_b$ such that, for all $x_b \geq \hat{x}_b$, profit-driven (purpose-driven) banks match only with profit-driven (purpose-driven) workers.*

Relative to the balanced benchmark, profit-driven workers are scarce, so maintaining pure profit-driven teams requires sacrificing productivity. At high skill levels, profit-driven banks continue to prefer pure profit-driven matches, while at lower skill levels they optimally accept purpose-driven workers, generating mixing.

Productivity loss versus mitigation cost. A profit-driven bank faces a fundamental trade-off: it can match with a lower-skill profit-driven worker to avoid mitigation costs, or match with a higher-skill purpose-driven worker and incur those costs. Figure 3 illustrates how this trade-off resolves under unbalanced supply ($\lambda_w > \lambda_b$).

The dashed lines represent the balanced supply benchmark ($\lambda_w = \lambda_b$). The solid lines represent the imbalanced case. To understand the shift, consider two banks of identical skill x_b, one profit-driven $(x_b, 0)$ and one purpose-driven $(x_b, 1)$. Because the cost of mitigation is proportional to total output, high-talent profit-driven banks have the strongest incentive to avoid purpose-driven partners. Consequently, they aggressively out-bid other stakeholders for the now-scarce pool of high-talent profit-driven workers.

However, because these profit-driven workers are scarce relative to the bank population, these high-talent banks must "settle" for workers of lower skill than they would in a balanced market. This is why the solid brown line lies below the dashed brown line—it represents a downward displacement in the quality of profit-driven matches. Conversely, since there is a surfeit of talented purpose-driven workers, the solid green line sits above the dashed green line, as purpose-driven banks find it easier to secure top-tier talent.

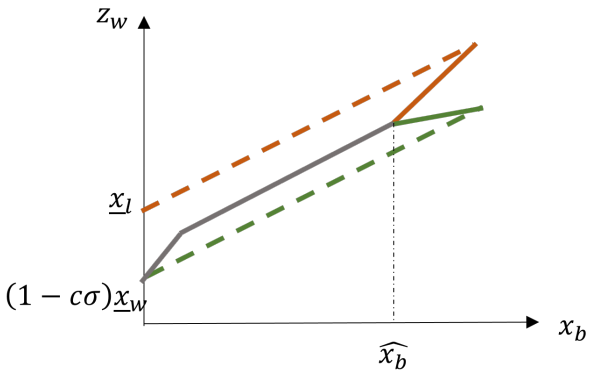


Figure 3: Dashed lines represent balanced supply, where the z-index of purpose-driven matches is lower due to the discount factor $(1 - c\sigma)$. The solid lines represent unbalanced supply, where full separation occurs the top. The dashed purpose-driven (profit-driven) line represents the z-index of the matching workers under the balanced supply for purpose-driven (profit-driven) bank.

For "middling" profit-driven banks (those below the cutoff $\hat{x}_b$), the scarcity of profit-driven workers becomes prohibitive. Having been out-competed for the remaining pool of reasonable profit-driven talent, these banks find that the productivity loss of matching with an untalented profit-driven worker exceeds the cost of mitigation. At this threshold, they are willing to match with high-talent purpose-driven workers. This transition is represented by the solid gray line in the mixing region. Here, the productivity of the purpose-driven match is high enough that the bank is willing to internalize the mitigation tax rather than match with the low-skill profit-driven workers left at the bottom of the distribution. The profit-driven bank is indifferent between profit-driven or purpose-driven workers now as long as these workers have the same effective productivity z-index.

PAM between banks (x_b) and workers' effective productivity (z_w) in the mixing region. In the mixing region, we show that matching can be constructed based on the positive sorting between the effective productivity of workers z_w and banks ability x_b

Because profit-driven banks rank potential workers according to their effective productivity, we define a cumulative measure of "effective supply" among workers. Let $g_w^1(y)$ and $g_w^0(y)$ denote the probability density functions of the productivity distribution for teams with purpose-driven members ($n \geq 1$) and purely profit-driven members ($n = 0$), respectively. We define $\Psi_w(z)$ as the total mass of worker teams whose effective productivity is less than or equal to z:

$$\Psi_w(z) \equiv \int_{\underline{y}}^{z/(1-c\sigma)} g_w^1(\tilde{y})\, d\tilde{y} + \int_{\underline{y}}^{z} g_w^0(\tilde{y})\, d\tilde{y}. \tag{14}$$

In this expression, the first integral accounts for purpose-driven teams. Because their productivity is discounted by $(1 - c\sigma)$, a purpose-driven team must have a higher raw productivity

($\tilde{y} = z/(1 - c\sigma)$) to provide the same effective z as a profit-driven team.

Market clearing then requires that the rank of a bank in their skill distribution $F_b(x_b)$ matches the rank of their partner in the effective productivity distribution $\Psi_w(z)$. Let $\eta_m(x_b)$ denote the matching function that assigns a stakeholder of skill x_b to a team with a specific z-index in the mixing region. The equilibrium matching is determined by:

$$\Psi_w(\eta_m(x_b)) = F_b(x_b), \tag{15}$$

as illustrated by the solid gray line in Figure 3. This construction implies that two banks with the same rank but different preferences $(x_b, 0)$ and $(x_b, 1)$ will be matched with workers with the same z-index of $\eta_m(x_b)$.

Clustering within the mixing region. While banks with a given x_b are matched with workers of the same z-index, according to Lemma 1, stakeholders with the same preference are matched with each other as much as they can. Specifically, conditional on x_b and z, all profit-driven workers with index z and $n = 0$, which are relatively scarce, will be matched with profit-driven banks, forming a team index with $n = 0$. On the other hand, since profit-driven banks are relatively abundant, some of them will then be matched with purpose-driven workers with the same index z, forming teams with $n = 1$. Purpose-driven banks, which again are relatively scarce, will all match with purpose-driven workers, yielding teams with $n = 2$.

Summary. High-skill stakeholders remain segmented by their purpose, as the cost of mixing is prohibitive at the top of the productivity distribution. Mixing occurs at the bottom, where productivity losses from matching with a less-skilled partner are small. This is the origin of the trickle-down effect that we will analyze in Section 5: exit occurs at the top, engagement at the bottom. Competition endogenously determines which stakeholders bear the cost of mitigation.

4.2 General case: sequential algorithm for $N \geq 2$

We now construct the equilibrium with N types of stakeholders via a recursive sequence of bilateral matches. Types are ordered by the prevalence of purpose-driven stakeholders. Label stakeholder types such that

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N,$$

where λ_ℓ denotes the share of purpose-driven stakeholders of type ℓ. Type 1 is thus the most purpose-driven, while later types are increasingly profit-driven. For instance, workers might be λ_1, suppliers might be λ_2, while banks might be λ_N, and so on.

While the multi-sided matching problem with N types is high-dimensional, the following sequential algorithm simplifies it into a series of nested bilateral decisions.

1. **Initial Match:** Match type 1 and type 2 stakeholders using the bilateral results from Section 3.2. This generates a distribution of teams with type 1 and 2 stakeholders with productivity $y_2 = x_1 x_2$ and a purpose-count $n_2 \in \{0, 1, 2\}$.

2. **Iteration:** For each subsequent stage $\tau \geq 3$, the team formed at stage $\tau - 1$, is treated as a single partner with characteristics $(y_{\tau-1}, n_{\tau-1})$ and is matched with stakeholders of type τ.

   The matching construction follows the same patterns shown in Figure 2: full separation holds at the top of the distribution, while mixing arises at the bottom. The x-axis now represents the ability of type τ, x_τ, and the y-axis is the effective z-index of the team, given by $z_{\tau-1}(y_{\tau-1}, n_{\tau-1})$.

   Let $x^*_\tau(y_{\tau-1}, n_{\tau-1})$ and $\theta^*_\tau(y_{\tau-1}, n_{\tau-1})$ denote the characteristics of the stakeholder of type τ matched to a team $(y_{\tau-1}, n_{\tau-1})$. After matching with type τ, team characteristics then evolve according to

   $$y_\tau = y_{\tau-1} \cdot x^*_\tau(y_{\tau-1}, n_{\tau-1}), \qquad n_\tau = n_{\tau-1} + \theta^*_\tau(y_{\tau-1}, n_{\tau-1}). \tag{16}$$

   After the matching at stage τ, the team now comprises types $1, \ldots, \tau$, with characteristics (y_τ, n_τ). Let $g^1_\tau(y_\tau)$ and $g^0_\tau(y_\tau)$ denote the densities of teams with $n_\tau \geq 1$ and $n_\tau = 0$, respectively.

3. **Final Firm Allocation:** The completed N-stakeholder teams, characterized by their final effective productivity z_N, are matched with firms (type $N + 1$) based on the firm's productivity x_{N+1}.

Monotonicity of scarcity condition. Note that one key assumption for constructing PAM based on x_τ (e.g. banks) and the effective productivity index of the team, $z_{\tau-1}$, is that type τ has relatively abundant profit-driven stakeholders and they are the ones mixing under our construction. The measure of the profit-driven stakeholder conditional on x_τ is constant under the i.i.d assumption, which is given by $(1 - \lambda_\tau)$. On the other hand, the measure of the pure profit-driven stakeholder conditional on z_τ is given by $\Pr(n_\tau = 0|z_\tau) = \frac{g^0_\tau(z)}{g^0_\tau(z)+g^1_\tau(z/(1-c\sigma))}$, which is generally not constant. Hence, the following condition below is needed to ensure that the measure of the profit-driven stakeholder of type τ is higher than the measure of pure profit-driven team everywhere under the mixing region.

Assumption 3. $(1 - \lambda_\tau) \geq \Pr(n_{\tau-1} = 0|z_{\tau-1})$ *for* $\forall z_{\tau-1} \leq (1 - c\sigma)\bar{z}$.

Proposition 2. *(Sequential Construction of Equilibrium) Suppose Assumption 3 holds at every stage τ. Then the equilibrium matching outcome can be constructed sequentially as follows. At each stage τ, the matching between teams $(y_{\tau-1}, n_{\tau-1})$ and stakeholders (x_τ, θ_τ) is characterized by a cutoff $\hat{x}_\tau$ such that:*

1. Full separation at the top: *profit-driven (purpose-driven) stakeholders above the cutoff match exclusively with teams with $n_\tau = 0$ ($n_\tau \geq 1$).*

2. Mixing at the bottom: *below the cutoff, matching is characterized by positive assortative matching between stakeholder skill x_τ and the team z-index $z_{\tau-1} = z(y_{\tau-1}, n_{\tau-1})$. Conditional on $(z_{\tau-1}, x_\tau)$, purpose-driven stakeholders are matched first with teams with higher purpose-driven indices.*

Team characteristics evolve according to (16).

Evolution of purpose-driven teams. The sequential structure has two implications. First, a purely profit-driven team ($n_\tau = 0$) at period τ remains profit-driven in all subsequent stages, since later stakeholder types are weakly more profit-driven. As a result, the total mass of profit-driven teams is $(1 - \lambda_1)$, the mass of profit-driven type-1 stakeholders. Second, in later stages, purpose-driven stakeholders are increasingly scarce. Consequently, only teams with higher purpose-driven indices continue to match with purpose-driven stakeholders, so that purpose-driven intensity becomes progressively concentrated among a shrinking subset of teams.

4.3 Rents

Under the independence benchmark of Section 3, a treated stakeholder can simply subsidize their original firm so that the firm is willing to pay for additional mitigation costs. When $\sigma > 0$, differences in compensation between profit-driven and purpose-driven stakeholders are no longer due solely to compensating differentials. An additional component emerges: scarce profit-driven stakeholders at the top of productivity distribution earn rents because productive firms compete aggressively to avoid mitigation. This avoidance rent drives a wedge beyond compensating differentials and explains why treated stakeholders who are highly productive cannot afford to engage—no transfer is large enough to compensate the firm.

Specifically, for type 1 stakeholders (the scarcest profit-driven type), our equilibrium construction implies that they remain in purely profit-driven teams and facilitate the avoidance of mitigation costs. They therefore match with more productive firms and earn higher wages relative to their purpose-driven counterparts. For type $\ell \geq 2$ stakeholders, the picture is more nuanced. In the separation region ($x_\ell > \hat{x}_\ell$), profit-driven stakeholders match with teams of

higher z-index than their purpose-driven counterparts and earn rents beyond compensating differentials. In the mixing region ($x_\ell \leq \hat{x}_\ell$), profit-driven and purpose-driven stakeholders of the same skill match with teams of the same z-index, so the premium reflects only compensating differentials.

Proposition 3 formalizes this intuition.

Proposition 3. *(Rent for Scarce Profit-Driven Stakeholders) Profit-driven stakeholders of type* 1 *and for* $\forall \ell \geq 2$ *earn rents (beyond compensating differentials), where* $z^*(x_\ell, 0) > z^*(x_\ell, 1)$*, and* $p_\ell(x, 0) - p_\ell(x, 1|\ell) > \Delta_\ell(x_\ell)$ *for* $x_\ell > \hat{x}_\ell$*. In contrast, profit-driven stakeholders in the mixing region earn no additional rent. That is, for* $\forall \ell \geq 2$*,* $p_\ell(x, 0) - p_\ell(x, 1|\ell) = \Delta_\ell(x_\ell)$ $\forall x_\ell \leq \hat{x}_\ell$*.*

5 Exit vs. Engagement in General Equilibrium

We now use the equilibrium characterization to study how an exogenous increase in the share of purpose-driven stakeholders affects firm-stakeholder relationships and firm harm. Consider a shock that increases the measure of purpose-driven stakeholders of type ℓ (i.e., λ_ℓ). Following the shock, do affected stakeholders optimally remain with their current firms (*engagement*) or reallocate to new firms (*exit*)? And what happens to aggregate mitigation?

In contrast to the independence benchmark from Section 3, we show that aggregate outcomes now manifests very differently. First, both exit and engagement can arise. Second, despite that the preference shocks are i.i.d across the productivity distribution, little direct impact or mitigation materializes at the top of this distribution. Instead, aggregate impact is concentrated in the middle region of firm productivity due to equilibrium spillovers or a trickle-down effect. Lastly, having more type ℓ stakeholders now affect the utilities of other stakeholders.

5.1 Prevalence of Exit versus Engagement

Illustration: $N = 2$ case. To illustrate the intuition, consider our $N = 2$ case with banks (λ_b) and workers (λ_w) from Section 4.1. In Figure 4, we show what happens to the equilibrium matching outcomes when λ_b increases by δ, i.e., the $t = 0$ equilibrium versus the $t = 1$ equilibrium. The solid lines represent the $t = 0$ equilibrium as illustrated in Figure 3. The dotted lines represent the $t = 1$ equilibrium. More purpose-driven banks reduce competition for profit-driven workers. Hence, the remaining profit-driven banks can now match with a more productive profit-driven worker, which is reflected in the dotted brown line being above the solid brown line. Similarly, purpose-driven banks are now matched with less productive purpose-driven workers, which is reflected in the dotted green line being below the solid green line.

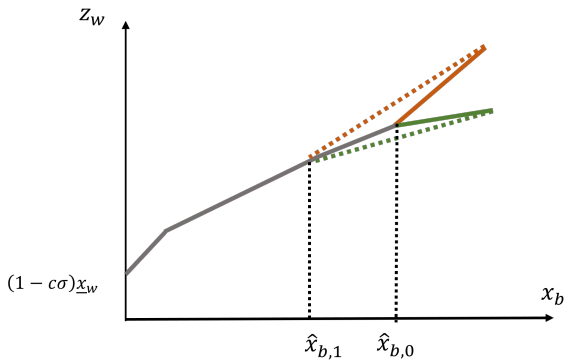


Figure 4: The solid (dotted) line represents the matching before (after) the preference shock that increases the measure of purpose-driven banks.

As a result, there is a lower separation threshold (i.e., $\hat{x}_{b,1} < \hat{x}_{b,0}$) or a larger separation region, which can be interpreted as profit-driven banks are less willing to mix. This means that for treated banks below the new cut-off $\hat{x}_{b,1}$, they remain with their previous matches, i.e. engage. For treated banks above this new cut-off, there is in general reallocation or exit.

We now show that this illustration holds for general N.

Proposition 4. *(Exit vs. Engagement) Suppose $\ell \geq 2$ and the share of purpose-driven type-ℓ stakeholders increases by a small $\delta > 0$, with $\lambda_\ell + \delta < \lambda_{\ell-1}$. Then there exists a cutoff firm size such that: (i) treated stakeholders at firms below the cutoff engage; (ii) treated stakeholders at firms above the cutoff exit to smaller firms with higher stakeholder-purpose indices.*

Engagement at the bottom. Consider a treated stakeholder with ability $x_\ell < \hat{x}_{\ell,1}$, which means that he is below the cutoff before and after the shock. Recall that our equilibrium construction implies that stakeholders $(x_\ell, 0)$ and $(x_\ell, 1)$ are matched with teams of the same z-index in this region.[8] Therefore, when a low-ability stakeholder becomes purpose-driven, the ranking of their equilibrium match does not change: $z^*(x_\ell, 1) = z^*(x_\ell, 0)$. They can thus remain with a firm of the same size, which we interpret as engagement.[9] Specifically, let $\hat{x}_{N+1,t}$ be the firm that is matched to the cutoff stakeholder $\hat{x}_{\ell,t}$ at period t, which is given by $F_{N+1}(\hat{x}_{N+1,t}) = F_\ell(\hat{x}_{\ell,t})$. Then, all treated stakeholders at firms below the cutoff $\hat{x}_{N+1,1}$ will engage.

Exit at the top. For stakeholders that are at the top of the distribution (i.e., $x_\ell > \hat{x}_{\ell,1}$), on the other hand, the equilibrium matching implies that $z^*(x_\ell, 1) < z^*(x_\ell, 0)$. Hence, once treated stakeholders become purpose-driven, they are no longer valuable to the large, purely

[8] In the mixing region, the sorting function $\eta_m(x_\ell)$ that solves (15) is independent of λ_ℓ.

[9] For profit-driven stakeholders $(x_\ell, 0)$, the equilibrium z-index is uniquely pinned down, although they may be indifferent across teams with different n. Conditional on z, all such allocations are payoff equivalent. We therefore define engagement as remaining with a firm of the same size.

profit-driven firms. As a result, they optimally exit to smaller firms after the shock. Moreover, due to the concentration, they must join firms with higher stakeholder-purpose indices.

5.2 Impact Trickles Down

We now study how preference shocks affect firm harm. Recall that firm harm depends on productivity y and the stakeholder-purpose index n:

$$e^*(y,n) = \begin{cases} \phi(y), & n = 0 \quad \text{(purely profit-driven team)}, \\ \xi_n^*, & n \geq 1 \quad \text{(purpose-driven team)}. \end{cases}$$

When $n \geq 1$, harm depends only on n and not on productivity. Hence, the first-order impact of preference shocks operates through changes in the distribution of the stakeholder-purpose index across firms. For purely profit-driven firms, harm also depends on productivity, but this channel is quantitatively minor. We therefore focus on the distribution of n.

Aggregate impact. As is in the independent case, because purpose-driven stakeholders cluster, the distribution of firms by stakeholder-purpose index is determined by $(\lambda_1, \ldots, \lambda_N)$, where the measure of firms with stakeholder-purpose index ℓ is $\lambda_\ell - \lambda_{\ell+1}$, and the measure of purely profit-driven firms is $1 - \lambda_1$.[10] Hence, conditional on the ordering of λ_ℓ remaining unchanged, increasing λ_ℓ by δ raises the share of firms with index ℓ by δ and lowers the share with index $\ell - 1$ by δ.

Micro-level impact. We next study the impact at the stakeholder level. Consider a mass δ of treated type-ℓ stakeholders who switch from profit-driven to purpose-driven preferences. Their effect depends on whether they engage or exit, as characterized in Proposition 4.

When a low-ability stakeholder becomes purpose-driven, his equilibrium match does not change. He therefore remains with the same firm, which now has one additional purpose-driven member. This lowers firm harm, which we refer to as the *engagement effect*. By contrast, treated high-ability stakeholders must exit to firms with higher purpose-driven indices. This may have no direct effect if the receiving firm already has purpose-driven stakeholders, but it displaces untreated purpose-driven stakeholders downward in the distribution.

Trickle-down effect in the middle. This displacement generates additional harm reduction in the middle of the distribution. Treated high-ability stakeholders crowd out lower-ability untreated ones, who then join smaller firms and make those firms more purpose-driven. As

[10] The key difference is that the distribution of y is no longer independent of the distribution of n.

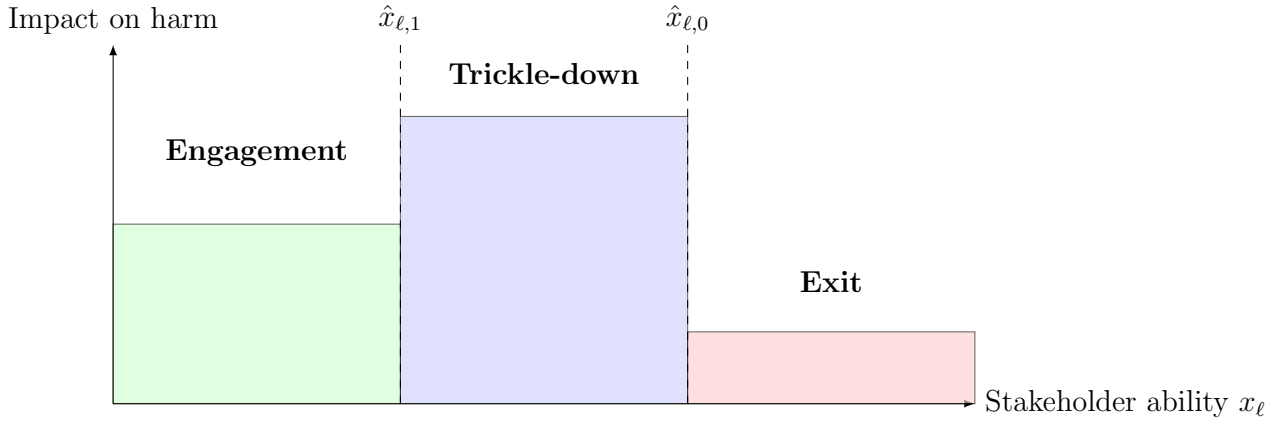


Figure 5: Schematic of micro-level impact of a purpose-driven preference shock. At the bottom ($x_\ell \leq \hat{x}_{\ell,1}$), treated stakeholders stay and reduce harm directly (engagement). In the middle ($\hat{x}_{\ell,1} < x_\ell < \hat{x}_{\ell,0}$), displacement of untreated stakeholders generates additional harm reduction (trickle-down). At the top ($x_\ell \geq \hat{x}_{\ell,0}$), treated stakeholders reallocate but often have little direct impact (exit).

a result, harm reduction in the middle can exceed the measure of treated stakeholders. In particular, firms with $n = 1$ in the region $x_\ell \in [\hat{x}_{\ell,1}, \hat{x}_{\ell,0}]$ before the shock must have $n = 0$ or $n = 2$ afterward, since they lie above the new cutoff $\hat{x}_{\ell,1}$.

Recall that aggregate harm reduction equals $\delta(\xi^*_\ell - \xi^*_{\ell-1})$. Since engagement at the bottom accounts exactly for $\delta(\xi^*_\ell - \xi^*_{\ell-1})$, and not all treated stakeholders at the top have direct effects, harm reduction in the middle must exceed this amount. Thus, the aggregate impact is realized disproportionately in the middle of the distribution, even though the preference shock is uniformly distributed.

Proposition 5. *(Trickle-Down Effect) (i) At the bottom ($x_\ell \leq \hat{x}_{\ell,1}$), harm reduction equals the measure of treated stakeholders. (ii) In the middle region ($x_\ell \in [\hat{x}_{\ell,1}, \hat{x}_{\ell,0}]$), harm reduction exceeds the measure of treated stakeholders due to displacement of untreated ones. (iii) At the top ($x_\ell \geq \hat{x}_{\ell,0}$), some treated stakeholders have no direct effect on harm.*

This is one of the paper's central results: while exit appears ineffective at the firm level—treated stakeholders at the top reallocate to firms that already mitigate—its true impact is realized in the middle of the distribution through the displacement of untreated stakeholders. The trickle-down effect means that the aggregate impact of exit is systematically larger than what any firm-level comparison of treated and untreated firms can detect.

5.3 Spillover Effects on Payoffs

We finally examine how preference shocks affect equilibrium payoffs. Recall that in Section 3 where $\sigma = 0$ there are no spillovers across types of stakeholders. When $\sigma > 0$, this is no longer

true. Although only type-ℓ stakeholders are directly treated, the induced reallocation alters competition and matching economy-wide.

An increase in λ_ℓ intensifies competition among purpose-driven type-ℓ stakeholders, lowering the z-index of their equilibrium matches at the top of the distribution (the green line in Figure 3 shifts downward). Conversely, profit-driven type-ℓ stakeholders become scarce and match with higher-ranked teams. Thus, purpose-driven (profit-driven) type-ℓ stakeholders at the top are worse (better) off.

For stakeholders of other types $\ell' \neq \ell$, the effect is reversed. Due to complementarities in production, improvements in the skill distribution of purpose-driven teams raise productivity and benefit purpose-driven stakeholders of other types. In contrast, profit-driven stakeholders of other types, however, face less productive matches.

Proposition 6. *(Spillover Effects on Payoffs) An increase in the share of purpose-driven type-ℓ stakeholders results in a lower (higher) payoff for purpose-driven (profit-driven) stakeholders of type ℓ that are relatively productive. On the other hand, it results in a higher (lower) payoff for purpose-driven (profit-driven) stakeholders of any other type $\ell' \neq \ell$ that are relatively productive. Payoffs are unchanged for all low-ability stakeholders.*

Engagement does not generate payoff spillovers, since matching at the bottom remains unchanged. However, because engagement reduces firm harm, the monetary payments to other purpose-driven stakeholders must decline.

6 Discussions

We use the calibrated model to discuss several stylized facts surrounding the wave of bank decarbonization commitments in the mid-2010s. Following the 2015 Paris Agreement, a significant fraction of large global banks pledged to reduce their exposure to fossil fuels (Kacperczyk and Peydró 2022). The resulting patterns have been difficult to reconcile under the standard independence assumption. We highlight three:

(i) *Large stakeholders exited rather than engaged.* Major global banks overwhelmingly responded to their green commitments by divesting—trimming participation in fossil fuel loans at the extensive margin—rather than by staying in relationships and imposing conditions on borrowers.[11]

[11]The same pattern holds for large institutional investors: Norges Bank divested from coal companies in 2015, and CalPERS has pursued similar exclusion policies. Despite holding large positions that give them real relationships with portfolio firms—and thus genuine leverage for engagement—these large stakeholders chose exit.

This prevalence of exit among the largest, most productive stakeholders is a natural prediction of the model under non-independence: high-productivity green stakeholders find engagement prohibitively costly because mitigation scales with firm output, and instead exit toward already-clean firms.

(ii) *Firm-level impact appears limited.* Kacperczyk and Peydró 2022 find that while committed banks reduce lending to high-emission firms, there is no measurable improvement in environmental outcomes at treated firms. Brown regional banks—such as Citizens Financial, Truist, Fifth Third, and BOK Financial—sharply expanded fossil fuel lending to fill the gap vacated by exiting global banks, with combined lending jumping over 70% on an annualized basis (according to industry report (Mathews 2024, DiFeliciantonio 2024).[12][13] This substitution by smaller brown lenders is consistent with the model: when large green stakeholders exit, brown stakeholders sort into the vacated relationships at the top of the firm distribution, leaving firm-level outcomes apparently unchanged.

(iii) *Aggregate effects are underestimated.* Standard diff-in-diff designs compare treated firms (those losing a committed bank) to untreated (already green) firms, attributing impact only to direct changes in bank-firm relationships. But the model predicts that impact operates primarily through spillovers: the exit of large green banks displaces smaller green banks, who reallocate to mid-tier firms and induce mitigation there. These indirect effects are absorbed into the control group in firm-level regressions, biasing estimated impact toward zero.[14] The calibration illustrates this quantitatively.

6.1 Calibration

The model accommodates general N stakeholder types, but a disciplined calibration requires observable data on green shares, compensating differentials, and a well-identified purpose shock for each group. We focus on two stakeholder types—banks and workers—for which such data are available. Direct evidence on green shares comes from bank commitment data (Kacperczyk and Peydró 2022) and worker surveys (Krueger, Metzger, and Wu 2021), while compensating differentials are observable in both interest-rate spreads and wage discounts. Data on

[12]Duchin, Gao, and Xu 2022 document similar null results in a broader ESG context, attributing them to greenwashing. Green and Vallee 2025 provide an important counterpoint: specific, verifiable bank coal exit policies do reduce coal firm debt issuance, plant retirements, and emissions. Their setting—coal, where emissions scale tightly with output and bank relationships are non-substitutable—corresponds to the conditions under which our model predicts exit is most effective.

[13]Hartzmark and Shue 2022 document a related pattern in equity markets: ESG-motivated divestment can be counterproductive when capital flows to less elastic brown buyers. Their setting—liquid equity markets with dispersed ownership—lacks the bilateral relationship structure of our model, but the prevalence of exit by large investors is consistent with our prediction.

[14]This bias is analogous to concerns in the spillovers literature more broadly Greenstone, Hornbeck, and Moretti 2010: treatment effects estimated in partial equilibrium miss general equilibrium reallocation.

other stakeholder groups—such as suppliers and customers—are currently lacking in comparable form. Our $N = 2$ calibration should therefore be interpreted as conservative: the model predicts that trickle-down spillovers grow with the number of stakeholder types, since a purpose shock to one group propagates onto all others through equilibrium reallocation. With only two groups, we capture within-group spillovers (bank-to-bank displacement) and cross-group effects onto workers, but miss the additional amplification that would arise from further stakeholder types.

We calibrate to the 500 most carbon-intensive publicly listed firms, primarily in the power sector.

Green shares. Green preferences are substantially more prevalent among workers than banks. Survey evidence indicates that 33% of workers accept green jobs at an average wage discount of 28% (Krueger, Metzger, and Wu 2021). By contrast, Kacperczyk and Peydró 2022 report that only 7% of bank lending flows to green firms prior to the shock. This asymmetry in green shares is reflected in compensating differentials: workers accept large wage discounts while banks show only small interest-rate differentials—as the model predicts when green workers are relatively more abundant than green banks.

We set $\lambda_1 = 33\%$ (workers) and $\lambda_2 = 7\%$ (banks) at $t = 0$. Bank talent parameters are calibrated on $[1.1, 1.4]$ to match asset and debt distributions from bank loan data. From Branikas et al. 2022, worker talent is distributed with $\gamma_1 = -0.4$ on $[0.08, 0.3]$ and firm productivity follows $\gamma_3 = 5$ on support $[5{,}000, 100{,}000]$.

Emissions and mitigation. The harm-productivity link is set at $\sigma = 5{,}000$, calibrated from the ratio of carbon emissions to firm revenues in Trucost data. Mitigation cost $c = 0.00008$ is based on carbon capture cost surveys, implying $c\sigma = 0.04$. Remaining parameters $(\kappa, \rho, \xi_1^*, \xi_2^*)$ are chosen to fit the observed production–emissions relationship (Figure 6).

6.2 Illustrative Results

We model the purpose shock as an increase in the green bank share from $\lambda_2 = 7\%$ to $\lambda_2 = 15\%$, consistent with the wave of bank commitments following the Paris Agreement.

Engagement at the bottom, exit at the top. Figure 7 displays matching outcomes before and after the shock. At the bottom of the firm distribution, exactly 8% of firms transition from brown to green, equal to the treated share. These are small firms where newly green banks engage directly—remaining in their existing relationships and inducing mitigation. At the top, the impact is less than 8%: large firms retain brown banks, and newly green banks reallocate

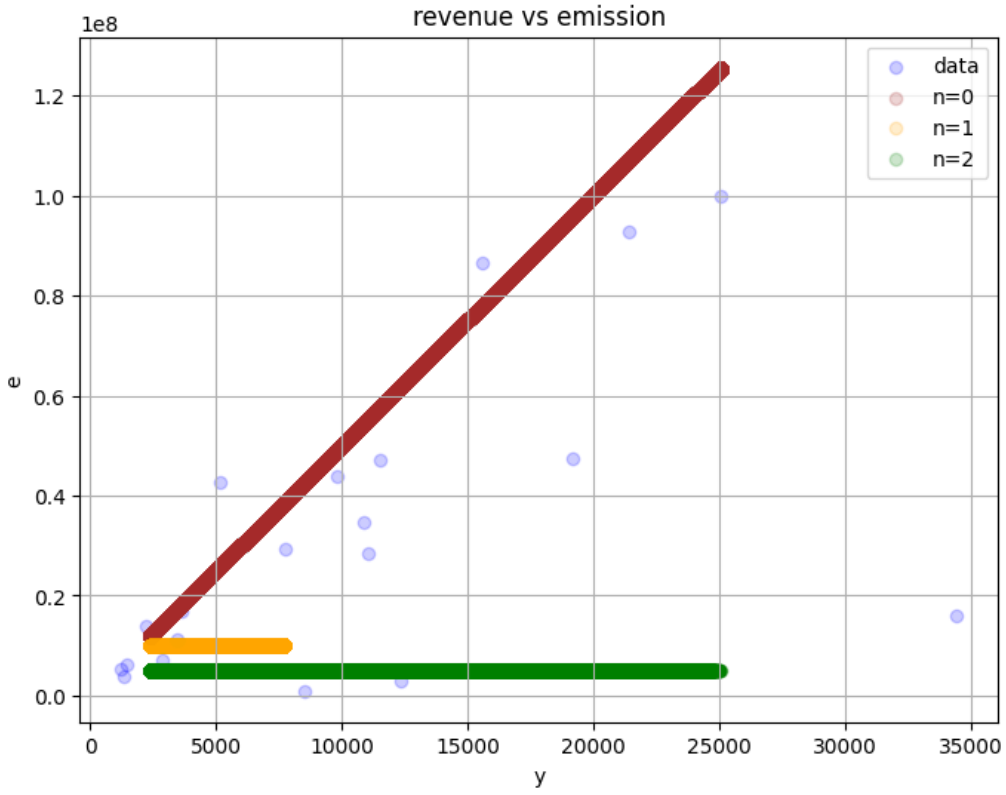


Figure 6: Calibrated production–emission relation.

away. In the middle of the distribution (firms ranked 400–600), the transition rate exceeds 8%, as firms that previously had one green stakeholder acquire a second through the reallocation of displaced green banks. This is the trickle-down mechanism at work.

Displacement and spillovers. Figure 8 decomposes the impact by bank type. If all treated banks simply engaged their current firms, 53% of green banks would have measurable impact. In the calibrated equilibrium, only the bottom treated banks engage directly. At the top, treated banks exit without direct effect—but displace incumbent green banks, who sort into mid-tier firms and induce mitigation that would not otherwise occur. The aggregate emissions reduction therefore exceeds the direct contribution of treated banks alone.

Connecting to the evidence. The calibration illustrates—without claiming to match the data precisely—why the patterns described above emerge naturally under non-independence. Large banks exit because engagement is too costly at high-output firms. Firm-level studies find null results because brown banks substitute into the vacated relationships while spillover-driven mitigation at mid-tier firms is absorbed into the control group. The true aggregate impact of the green shock is substantially larger than what any firm-level regression can recover. Moreover, since the $N = 2$ calibration omits spillovers onto additional stakeholder groups—suppliers,

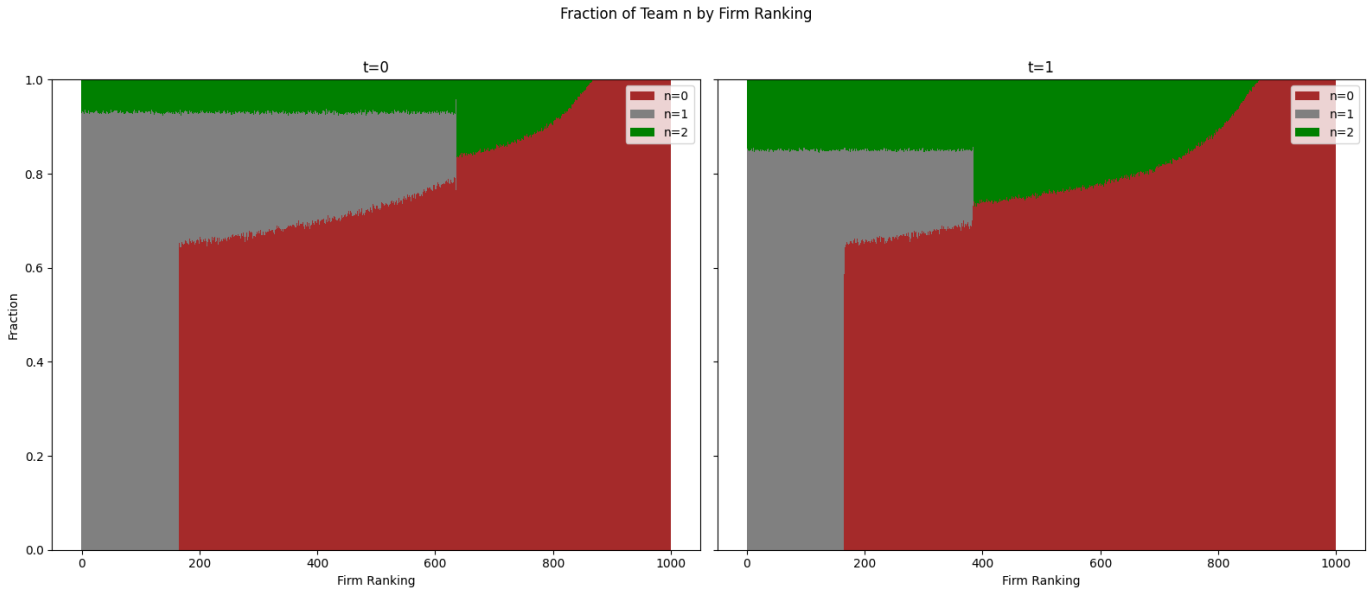


Figure 7: Firm matching outcomes before (left pabel) vs. after the shock (right panel).

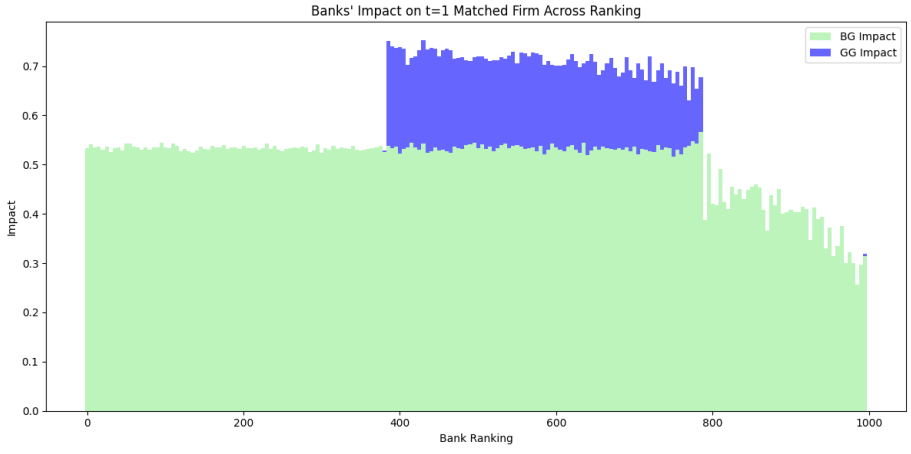


Figure 8: Share of banks with impact at $t = 1$. BG = treated, GG = already green.

customers, institutional investors, and others who would also be affected by the reallocation of green banks—it likely understates the full trickle-down effect.

7 Conclusion

Hirschman (1970) posed the question of when dissatisfied members of an organization exit versus stay and push for reform from within? More than fifty years later, this question remains central — and newly urgent — as purpose-driven stakeholders increasingly pressure firms to mitigate social harm. We show that the answer hinges on a key structural feature: whether harm scales with firm productivity.

Under independence — an assumption implicit in much of the existing literature — purpose-driven stakeholders engage, compensating differentials adjust, and there are no spillovers across stakeholder groups. When harm scales with productivity, the equilibrium is fundamentally different. Large purpose-driven stakeholders exit, and their reallocation triggers trickle-down spillovers: displaced purpose-driven stakeholders sort into mid-tier firms, inducing mitigation that would not otherwise occur. Exit and engagement are complements in aggregate, even as they are substitutes for any individual stakeholder.

An application to the wave of bank decarbonization commitments following the 2015 Paris Agreement illustrates these dynamics, rationalizing the prevalence of exit by large banks, the null firm-level results, and the substitution by regional lenders. Our findings have direct implications for empirical work and policy. Firm-level studies that compare treated and untreated firms absorb trickle-down spillovers into the control group, systematically underestimating the aggregate impact of exit. Policy debates that dismiss exit as ineffective relative to engagement may therefore be drawing the wrong conclusion.

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A Appendix

A.1 Proof of Lemma 1

Proof. Given that $y - cm - n\psi\big(\phi(y) - m\big)$ is affine in n,the surplus

$$\Omega(y, n) = \max_{m \geq 0} y - cm - n\psi\big(\phi(y) - m\big) \tag{A.1}$$

is decreasing and convex in n. Because $\Omega(y, n)$ is convex in n, it satisfies increasing differences in (n, θ) when evaluated at $n + \theta$. That is, for any $n' > n$ and $\theta' > \theta$,

$$\Omega(y, n' + \theta') - \Omega(y, n + \theta') \geq \Omega(y, n' + \theta) - \Omega(y, n + \theta). \tag{A.2}$$

Hence, $\Omega(y, n_{-\ell} + \theta)$ has increasing differences in $(n_{-\ell}, \theta)$. According to Equation 6, by monotone comparative statics (Topkis' theorem), the optimal choice $n^*_{-\ell}(x, \theta)$ must be weakly increasing in θ. Therefore, conditional on productivity x, agents with higher preference type θ are matched with the team with weakly higher n. □

A.2 Proof for Lemma 2

Proof. Since Equation 9 implies complementarity between green agent $(x_\ell, 1)$ and $y_{-\ell}$, hence, by the monotonic comparative statics, a green agent with higher ability must choose a team with a higher productivity than a green agent with lower ability. Similarly, Equation 7 implies complementarity between brown agent $(x_\ell, 0)$ and $z(y_{-\ell}, n_{-\ell})$; hence, a more skilled brown agent must choose a team with a higher z-index. □

A.3 Proof for Proposition 1

Proof. Let γ denote a feasible assignment on A^{N+1}, with $a_\ell = (x_\ell, \theta_\ell)$. Output and the number of purpose-driven stakeholders are given by $y(\boldsymbol{x}) = \prod_{\ell=1}^{N+1} x_\ell$, and $n(\boldsymbol{\theta}) = \sum_{\ell=1}^{N+1} \theta_\ell$.

The planner solves

$$\max_{\gamma} \int \big[y(\boldsymbol{x}) - C(n(\boldsymbol{\theta}))\big] d\gamma, \tag{A.3}$$

where

$$C(n) \equiv \min_{m} \{cm + n\psi(\phi_0 - m)\},$$

which is independent of y when $\sigma = 0$. Under independence, $F(x, \theta) = F_x(x)F_\theta(\theta)$. Hence the feasible set factorizes as

$$\Upsilon(F, \ldots, F) = \Upsilon(F_x, \ldots, F_x) \times \Upsilon(F_\theta, \ldots, F_\theta). \tag{A.4}$$

Because the surplus is additively separable, the problem decomposes:

$$\max_{\gamma} \int y(\boldsymbol{x})\, d\gamma^x - \min_{\gamma^\theta} \int C(n(\boldsymbol{\theta}))\, d\gamma^\theta. \tag{A.5}$$

Since $y(\boldsymbol{x})$ is supermodular in $\boldsymbol{x}$, positive assortative matching in productivity follows from standard matching arguments. Since $-C(n)$ is convex, the matching is positively assortative in θ. Let λ_ℓ denote the mass of stakeholders with type weakly above ℓ. Positive sorting implies that firms with at least ℓ purpose-driven stakeholders have measure λ_ℓ. Therefore firms with index ℓ have measure $\lambda_\ell - \lambda_{\ell+1}$, and firms with index 0 have measure $1 - \lambda_1$.

Compensating differentials. Firms must be indifferent between a team with index ℓ and one with index $\ell - 1$. Recall that type ℓ stakeholders must work for firms with at least $n \geq \ell$ purpose-driven stakeholders. Hence, a team with index ℓ (respectively, $\ell - 1$) consists of purpose-driven stakeholders with types $k \leq \ell$ (respectively, $k \leq \ell - 1$) and profit-driven stakeholders with types $k > \ell$ (respectively, $k > \ell - 1$).
Conditional on y, indifference between a team of index ℓ and $\ell - 1$ is therefore equivalent to the firm being indifferent between matching a purpose-driven and a profit-driven stakeholder of type ℓ. This implies

$$\Omega(y, \ell-1) - \left\{ \sum_{k=1}^{\ell-1} U_k(x,1) + \sum_{k=\ell}^{N} U_k(x,0) \right\} = \Omega(y,\ell) - \left\{ \sum_{k=1}^{\ell} U_k(x,1) + \sum_{k=\ell+1}^{N} U_k(x,0) \right\}. \tag{A.6}$$

Rearranging yields

$$U_\ell(x,0) - U_\ell(x,1) = \Omega(y, \ell-1) - \Omega(y,\ell) \tag{A.7}$$

Since $p_\ell(x,0) = U_\ell(x,0)$ and $p_\ell(x,1 \mid \ell) = U_\ell(x,1) + \psi(\xi^*_\ell)$, it follows that

$$p_\ell(x,0) - p_\ell(x,1 \mid \ell) = c(\xi^*_{\ell-1} - \xi^*_\ell) - (\ell - 1)\left[\psi(\xi^*_{\ell-1}) - \psi(\xi^*_\ell)\right]. \tag{A.8}$$

□

A.4 Proof for Stable Matches at period ℓ

Let $G_\ell(z_\ell, n_\ell)$ denote the cumulative distribution function for teams with characteristics (z_ℓ, n_ℓ) at period ℓ, and let $G^0_\ell(z)$ and $G^1_\ell(z)$ denote the measure of teams with index $n_\ell = 0$ and $n_\ell \geq 1$, respectively, having productivity no larger than z.

Let $\eta^\theta_s(x_{\ell'})$ denote the productivity of the team matched with stakeholder $(x_{\ell'}, \theta)$ in the

full-separation region. The market-clearing conditions yield

$$G_\ell^0(\bar{y}_\ell) - G_\ell^0\big(\eta_s^0(x_{\ell'})\big) = (1-\lambda_{\ell'})(F_{\ell'}(\bar{x}_{\ell'}) - F_{\ell'}(x_{\ell'}))\,, \tag{A.9}$$
$$G_\ell^1(\bar{z}_\ell) - G_\ell^1\big(\eta_s^1(x_{\ell'})\big) = \lambda_{\ell'}(F_{\ell'}(\bar{x}_{\ell'}) - F_{\ell'}(x_{\ell'}))\,.$$

Cutoff characterization. The cutoff $\hat{x}_{\ell'}$ can be determined in two ways. For relatively small $\lambda_{\ell'} < \hat{\lambda}_{\ell'}$, the cutoff is given by

$$\eta_s^0(\hat{x}_{\ell'}) = \eta_s^1(\hat{x}_{\ell'}). \tag{A.10}$$

Otherwise, the cutoff solves

$$G_\ell^0(\bar{z}_\ell) - G_\ell^0(z_{\ell,L}^0) = (1-\lambda_{\ell'})(F_{\ell'}(\bar{x}_{\ell'}) - F_{\ell'}(\hat{x}_{\ell'}))\,, \tag{A.11}$$

where $z_{\ell,L}^0$ denotes the lowest productivity among teams with index $n=0$. Define $z_{\tau+1} = \eta_s^0(\hat{x}_{\tau+1})\cdot\hat{x}_{\tau+1}$.

Lemma 5. *Assume that $G_{\ell-1}(z,n)$ satisfies Assumption 3, and there exists $\hat{z}_{\ell-1}$ such that for all $z \geq \hat{z}_{\ell-1}$, $n \in \{0,\, \ell-1\}$. The stable matching between (z,n) and (x,θ) can be characterized by a cutoff $\hat{x}_\ell$ with the following structure.*

1. ***Full separation above the cutoff.*** *For $x_\ell > \hat{x}_\ell$, teams with $n_\ell = 0$ are matched exclusively with stakeholders of type $\theta = 0$, and teams with $n_\ell = \ell - 1$ are matched exclusively with stakeholders of type $\theta = 1$. Conditional on θ, the matching is positively assortative (PAM) in team productivity $z_{\ell-1}$ and stakeholder ability x_ℓ. Let $\eta_s^\theta(x_\ell)$ denote the z-index of the team matched with stakeholder (x_ℓ,θ) in this region. The market-clearing conditions are*

$$G_{\ell-1}^0(\bar{z}) - G_{\ell-1}^0\big(\eta_s^0(x_\ell)\big) = (1-\lambda_\ell)(F_\ell(\bar{x}_\ell) - F_\ell(x_\ell))\,, \tag{A.12}$$
$$G_{\ell-1}^1((1-c\sigma)\bar{z}) - G_{\ell-1}^1\big(\eta_s^1(x_\ell)\big) = \lambda_\ell(F_\ell(\bar{x}_\ell) - F_\ell(x_\ell))\,. \tag{A.13}$$

2. ***Mixing below the cutoff.*** *For $x_\ell \leq \hat{x}_\ell$, the matching is positively assortative in $z_{\ell-1}$ and x_ℓ. Conditional on $(z_\ell, x_{\ell+1})$, every value-driven stakeholder $(x_\ell, 1)$ is matched with a team of index $n = \ell - 1$. Let $\eta_m(x_\ell)$ denote the assignment function. Then*

$$G_{\ell-1}^0(\eta_m(x_\ell)) + G_{\ell-1}^1(\eta_m(x_\ell)) = F_\ell(x_\ell).$$

Proof. Let ξ_n^* solve $c = n\psi'(\xi_\ell^*)$, and thus $m^*(y,n) = \phi(y) - \xi_\ell^*$. For $n \geq 1$, the surplus can be

rewritten as

$$\begin{aligned}\Omega(y,n) &= y - c(\phi(y) - \xi_\ell^*) - n\psi(\xi_n^*) \\ &= (1-c\sigma)y - \{c(\phi_0 - \xi_\ell^*) + n\psi(\xi_n^*)\} \\ &= (1-c\sigma)y - C(n)\end{aligned} \tag{A.14}$$

The planner's objective can be written as

$$W = \max_{\gamma\in\Gamma} \int [\tilde{z}(y,n,\theta) - C(\theta+n)]\, d\gamma,$$

where Γ represents the permissible set and

$$\tilde{z}(y,n,\theta) = \begin{cases} y & \text{if } n+\theta = 0, \\ (1-c\sigma)y & \text{if } n+\theta \geq 1, \end{cases}$$

The proof proceeds in three steps.

Step 1: Value-driven stakeholders are never matched with $n = 0$ teams. We claim that $\gamma(x, 1, (y, 0)) = 0$ for all x and y. Because profit-driven stakeholders $(x, 0)$ are in excess supply, it is always feasible to match every $n = 0$ team with a $\theta = 0$ stakeholder. Any matching with $\gamma(x, 1, (y, 0)) > 0$ would yield a lower effective productivity $\tilde{z}(y, 0, 1) = (1 - c\sigma)y < y = \tilde{z}(y, 0, 0)$, thereby shifting the distribution of effective productivity downward in the sense of first-order stochastic dominance. This strictly reduces welfare.

Step 2: Reduction to constrained one-dimensional sorting. Since every $n = 0$ team must be matched with a $\theta = 0$ stakeholder, the effective productivity within each match simplifies to $z(y, n)\, x$. Indeed, for $n \geq 1$ we have $\tilde{z}(y, n, \theta) = (1 - c\sigma)y = z(y, n)$ regardless of θ, and for $n = 0$ we have $\tilde{z}(y, 0, 0) = y = z(y, 0)$. The objective thus reduces to

$$W = \max_{\gamma\in\Gamma_C} \int [z(y,n)\, x - C(n+\theta)]\, d\gamma, \tag{A.15}$$

subject to the constraint $\gamma(\{\theta = 1,\, n = 0\}) = 0$.
The constructed solution minimizes $\int C(\theta + n)\, d\gamma$, since all value-driven stakeholders $(x, 1)$ are matched with teams of index $n = \ell - 1$, yielding the most extreme feasible distribution of $n + \theta$. It therefore remains to verify optimality with respect to the first term, $\int z(y, n)\, x\, d\gamma$.

The supermodularity of $z \cdot x$ implies that the admissible optimum preserves PAM wherever the type-exclusion constraint is slack and departs from it minimally where the constraint binds. By the standard theory of optimal transport, the optimal assignment satisfies monotonicity and no-mass splitting conditional on θ. That is, letting $z^*(x, \theta)$ denote the equilibrium z-index

matched with stakeholder (x, θ), the function $z^*(\cdot, \theta)$ is strictly increasing in x, and, conditional on θ, each stakeholder type is matched to a unique z-index with no mass split across multiple partners.

Moreover, for $n \geq 1$ the constraint does not bind, so there exists a unique ability level x matched to each team (y, n). Hence, if $\gamma((x, 0), (y, n)) > 0$ for some $n \geq 1$, then $\gamma((x, 1), (y, n)) > 0$ as well: both stakeholder types at the same ability level x are assigned to the same team whenever $n \geq 1$.

Step 3: Separation at the top. The optimal matching features full separation at the top of the productivity distribution. Specifically, the highest-ability profit-driven stakeholder must be matched with the most productive $n = 0$ team. To see this, note that for $\sigma > 0$ there exists $\epsilon > 0$ such that $z(y, 0) > z(\bar{y}, n')$ for all $y > \bar{y} - \epsilon$ and $n' \geq 1$. Matching these top $n = 0$ teams with top $\theta = 0$ stakeholders preserves PAM without violating the type-exclusion constraint. The highest-ability value-driven stakeholders, on the other hand, are then matched with the most productive teams having $n' \geq 1$.

Since profit-driven stakeholders are in excess supply, some of them must be matched with teams of index $n \geq 1$. The key decision reduces to identifying which profit-driven stakeholders join such teams. If stakeholder $(x, 0)$ is matched with a team (y, n) with $n \geq 1$, then optimality requires that it be paired with the team currently assigned to $(x, 1)$, which has productivity index $z^*(x, 1)$. The payoff difference between matching x to its $n = 0$ team (with index $z^*(x, 0)$) versus the $n \geq 1$ team (with index $z^*(x, 1)$) is $\left[z^*(x, 0) - z^*(x, 1)\right] x$. Under Assumption 3,

$$\frac{(1 - \lambda_\ell)\, f_\ell(x)}{\lambda\, f_\ell(x)} > \frac{g^0_{\ell-1}(z)}{g^1_{\ell-1}(z)},$$

and therefore, at any point where $\eta^0_s(x_\ell)$ and $\eta^1_s(x_\ell)$ coincide at some productivity level z, the derivatives satisfy

$$\frac{d\eta^0_s(x_\ell)}{dx} = \frac{(1 - \lambda_\ell)\, f_\ell(x)}{g^0_{\ell-1}(\eta^0_s(x_\ell))} > \frac{\lambda_\ell\, f_\ell(x)}{g^1_{\ell-1}(\eta^1_s(x_\ell))} = \frac{d\eta^1_s(x_\ell)}{dx}.$$

It follows that the two assignment functions can cross at most once, at a point $\hat{x}_\ell$, and that $\eta^0_s(x_\ell) - \eta^1_s(x_\ell) > 0$ for all $x > \hat{x}_\ell$. In the interior case, the cutoff is determined by

$$z^*(\hat{x}_\ell, 0) = \eta^0_s(\hat{x}_\ell) = \eta^1_s(\hat{x}_\ell) = z^*(\hat{x}_\ell, 1),$$

and full separation is strictly optimal for all $x > \hat{x}_\ell$. If no such interior crossing exists, it is optimal to maintain separation until all $n = 0$ teams are exhausted. □

A.5 Proof for Lemma 3 and 4

Proof. In the case that $N = 2$, then $G^0_\ell(y) = (1-\lambda_\ell)F_\ell(y)$ and $G^1_\ell(y) = \lambda_\ell F_\ell(y)$. Hence, Proposition follows directly from Lemma 5. The balanced supply is a special case where Equation A.11 can be written as

$$(1-\lambda_\ell)(F_\ell(\bar{x}_\ell) - F_\ell(\underline{x_\ell})) = (1-\lambda_\ell) = (1-\lambda_{\ell'})(F_{\ell'}(\bar{x}_{\ell'}) - F_{\ell'}(x^0_{\ell',L})),$$

and thus $x^0_{\ell',L} = \underline{x_{\ell'}}$ when $\lambda_\ell = \lambda_{\ell'}$. Hence, full separation between stakeholders must hold. □

A.6 Properties of Matches

Lemma 6 (Properties of the Evolution)**.** *Let* $\hat{i}_\ell \equiv F_\ell(\hat{x}_\ell)$.

1. *For* $i \le \hat{i}_\ell$*, the ranking of the z-index for* $x_\ell[i]$ *remains unchanged after the match. For* $i > \hat{i}_\ell$*, the ranking of the z-index of* $x_\ell[i]$ *increases for profit-driven stakeholders and decreases for purpose-driven stakeholders.*

2. *The separation region* $(1-\hat{i}_\ell)$ *is decreasing in* ℓ.

3. *For all* $n_{\ell-1} < \ell - 1$*, we have* $n_\ell = n_{\ell-1}$*. The measure of teams with index* $n_\ell = \ell$ *after the match is* λ_ℓ.

Proof. Part (1). For $z_{\ell-1} \le z_{\ell-1}[\hat{i}_\ell]$, positive assortative matching between $(z_{\ell-1}, x_\ell)$ in the mixing region implies that $z_\ell[i] = z_{\ell-1}[i] \cdot x_\ell[i]$. Hence,

$$\Pr(z < z_\ell[i]) = \Pr(x < x_\ell[i]) = i.$$

For those in the separation region, we have $z^*(x,0) > z^*(x,1)$, and thus for a profit-driven stakeholder,

$$z_\ell[i] = z^*(x_\ell[i],0) \cdot x_\ell[i] > z^*(x_\ell[i],1) \cdot x_\ell[i].$$

Hence, for a profit-driven stakeholder with ranking i,

$$\Pr(z < z_\ell[i]) > \lambda_\ell\, F_\ell(x_\ell[i]) + (1-\lambda_\ell)\, F_\ell(x_\ell[i]) = i,$$

and vice versa for purpose-driven stakeholders.

Part (2). We show that $\hat{i}_\ell$ is increasing in ℓ. Assuming that the interior cutoff holds, at the cutoff $\hat{i}_\ell$ we must have

$$\frac{G^0_{\ell+1}(\bar{z}_{\ell+1}) - G^0_{\ell+1}\big(z_{\ell+1}(\hat{i}_\ell)\big)}{G^1_{\ell+1}(\bar{z}_{\ell+1}) - G^1_{\ell+1}\big(z_{\ell+1}(\hat{i}_\ell)\big)} = \frac{G^0_\ell(\bar{z}_\ell) - G^0_\ell\big(z(\hat{i}_\ell)\big)}{G^1_\ell(\bar{z}_\ell) - G^1_\ell\big(z(\hat{i}_\ell)\big)} = \frac{1-\lambda_\ell}{\lambda_\ell} < \frac{1-\lambda_{\ell+1}}{\lambda_{\ell+1}},$$

where the first equality uses the fact that $G^0_{\ell+1}\big(z_{\ell+1}(\hat{i}_\ell)\big) = G^0_\ell\big(z(\hat{i}_\ell)\big) = \hat{i}_\ell$. Since the ratio

$$\frac{G^0_{\ell+1}(\bar{z}_{\ell+1}) - G^0_{\ell+1}(z_{\ell+1})}{G^1_{\ell+1}(\bar{z}_{\ell+1}) - G^1_{\ell+1}(z_{\ell+1})}$$

is increasing in $z_{\ell+1}$, it follows that $\hat{i}_{\ell+1} > \hat{i}_\ell$. If instead the cutoff is given by the corner solution, then $\hat{i}_\ell$ is trivially increasing in ℓ.

Part (3). We show that the measure of teams with index $n_\ell = \ell$ after the match is λ_ℓ. It suffices to show that $(x_\ell, 1)$ joins only teams with $n_{\ell-1} = \ell - 1$. This holds by construction in the separation region, so it remains to verify it in the mixing region. Since our construction implies that $(x_\ell, 1)$ always joins the team with the highest $n_{\ell-1}$, we only need to show that there are always relatively more teams with $n_{\ell-1} = \ell - 1$. This is indeed the case, since

$$\frac{G_{\ell-1}(z,\, \ell-1)}{\sum_{\tilde{n}} G_{\ell-1}(z,\, \tilde{n})} \geq \lambda_{\ell-1} \geq \lambda_\ell,$$

where the first inequality uses the fact that there is at least a $\lambda_{\ell-1}$ measure of teams with index $\ell - 1$, as purpose-driven stakeholders ranking after matches is weakly lower than profit-driven stakeholders, according to property (1). □

A.7 Proof for Proposition 2

Proof. We show that Lemma 5 can be applied at any stage N under the sequential ordering. Since the property (3) of Lemma 6 implies that the distribution of n must maximize the second term, we focus on the first term.

Recall that, conditional on $n_{\tau-1} > 0$ (respectively, $n_{\tau-1} = 0$), the team with a higher z-index must be matched with a more productive stakeholder x_τ. Let $x^1_\tau(z_{\tau-1})$ and $x^0_\tau(z_{\tau-1})$ denote the productivity of the stakeholder matched with the team of index z_τ, conditional on $n_{\tau-1} > 0$ and $n_{\tau-1} = 0$, respectively. The first term of the optimization problem can thus be rewritten as

$$\max_{\gamma \in \Gamma_C(\mu,\nu)} \int \Gamma^\theta(z_{\tau-1}\, x_\tau)\, d\gamma, \tag{A.16}$$

where

$$\Gamma^\theta(z_\tau) = z_\tau \prod_{\ell=1}^{N+1} x^\theta_{\tau+\ell}(z_{\tau+\ell-1}),$$

with $z_{\tau+1} = z_\tau\, x^\theta_{\tau+1}(z_\tau)$ and $\theta \in \{0, 1\}$. Importantly, since all purely profit-driven teams must remain purely profit-driven by construction, the recursive relation $z_{\tau+1} = z_\tau\, x^\theta_{\tau+1}(z_\tau)$ holds throughout.

At period N, $x^\theta_N(z_{N-1})$ is given by Lemma 5 and is strictly increasing in z. Hence, $\Gamma^\theta(z_{N-1})$

is a monotone transformation of z_{N-1} for both values of θ. It follows that, at period $N-1$, any solution γ that maximizes (A.15)—where the product is given by $z_{N-1} = z_{N-2}\, x_{N-1}$—must also maximize (A.16), where $x^{\theta}_{N-1}(z_{N-2})$ is characterized by Lemma 5.

We proceed by induction. Suppose that $x^{\theta}_{t+1}(z_t)$ is characterized by Lemma 5 at period $t+1$. Then the matching at period t must maximize $\Gamma^{\theta}(z_{t-1}\, x_t)$, where $\Gamma^{\theta}_z(z_\tau) > 0$. Hence, the solution at period t is again characterized by Lemma 5. This shows that our sequential algorithm is optimal. □

A.8 Proof for Proposition 4

Proof. Let $\hat{x}_{\ell,0}$ and $\hat{x}_{\ell,1}$ denote the cutoff before and after the shock, respectively. Since the separation region must expand, we have $\hat{x}_{\ell,0} > \hat{x}_{\ell,1}$.

For all stakeholders with ability ranking $x_\ell \leq \hat{x}_{\ell,1}$—that is, those who lie in the mixing region both before and after the shock—we have $z^*_t(x,0) = z^*_t(x,1)$. The z-index of the matched team therefore remains unchanged under our sequential construction. Given positive assortative matching between z_N and x_{N+1}, such a stakeholder continues to work at the firm with ability $x_{N+1}[i]$.

For any $x_\ell > \hat{x}_{\ell,1}$, since profit-driven (purpose-driven) stakeholders must be matched with a team of higher (lower) z-index after the shock, we have $z^*_1(x,0) > z^*_0(x,0)$ and $z^*_1(x,1) < z^*_0(x,1)$. This can be seen from our sequential construction. First of all, since matches before period ℓ depend only on the distribution of stakeholders below type $\ell' < \ell$, the team formation among them remains the same.

We now look at the matching at period ℓ, and such an effect can be seen from Figure 3. More purpose-driven stakeholders of type ℓ must result in a downward shift of the green line in Figure 3. This is because that, in the separation region, $\frac{d\eta^1_s(x_\ell)}{dx} = \frac{\lambda_\ell\, f_\ell(x)}{g^0_{\ell-1}\left(\eta^1_s(x_\ell)\right)}$, and thus

$$\eta^1_s(x_\ell) = (1 - c\sigma)\,\bar{z} - \int_x^{\bar{x}} \frac{\lambda_\ell\, f_\ell(x)}{g^0_{\ell-1}\left(\eta^1_s(x_\ell)\right)}\, dx.$$

Hence, a higher λ_ℓ results in lower $\eta^1_s(x_\ell)$. That is, $(x_\ell, 1)$ will now match with a team with lower productivity. The other way around for $(x_\ell, 0)$.

So, an increase in λ_ℓ must increase the z-ranking of purpose-driven (profit-driven) stakeholders after the matches in period ℓ, which is given by $x_\theta\, \eta^\theta_s(x_\ell)$. Thus, the purpose-driven (profit-driven) stakeholders of type ℓ end up in a less (more) productive team after period ℓ. Given that PAM between z and x must hold for any period τ onward, it thus means that $z^*_0(x_\ell, 1) > z^*_1(x_\ell, 1)$ and $z^*_0(x_\ell, 0) < z^*_1(x_\ell, 0)$.

The z-ranking of a treated stakeholder, who moves from $\theta = 0$ to $\theta = 1$, must drop by $x(z^*_0(x,0) - z^*_1(x,1)) > 0$, where the positivity follows from $z^*_0(x,0) \geq z^*_0(x,1) > z^*_1(x,1)$. A

lower ranking thus means that he must therefore work at a smaller firm. Lastly, since a purpose-driven stakeholder of type ℓ must work at a firm with index ℓ, this implies that the treated stakeholder moves to a smaller firm with values-driven index $n = \ell$. □

A.9 Proof for Proposition 3

Proof. Profit-driven stakeholders earn additional rent if and only if they stay in a team $n = 0$, which includes all type 1 $(x_1, 0)$ and for the retaliative productive stakeholders (where $x_\ell > \hat{x}_\ell$) for type $\ell \geq 2$, according to the first property of Lemma. Using the fact that for the firm to be indifferent between ℓ and $\ell - 1$, he must be indifferent between hiring purpose-driven stakeholder of type ℓ,for any $\ell \geq 2$, we thus have

$$U_\ell(x, 0) - U_\ell(x, 1) = \Omega(yx, \ell - 1) - \Omega(yx, \ell) = C(\ell - 1) - C(\ell)$$

and hence $p_\ell(x, 0) - p_\ell(x, 1|\ell) = C(\ell - 1) - C(\ell) - \psi(\xi_\ell^*) = \Delta_\ell(x_\ell)$. For $x > \hat{x}_\ell$,according to the first property of Lemma 6, since $(x_\ell, 0)$ will have higher ranking than $(x_\ell, 1)$,then $z^*(x_\ell, 0) > z^*(x_\ell, 1)$ for $x > \hat{x}_\ell$. This thus means that for any $(x_\ell, 1)$, there exists $x'_\ell < x_\ell$ such that $z^*(x'_\ell, 0) = z^*(x_\ell, 1)$ in equilbiurm. Hence, we have

$$\begin{aligned} U_\ell(x_\ell, 0) - U_\ell(x_\ell, 1) &= p_\ell(x, 0) - (p_\ell(x, 1|\ell) - \psi(\xi_\ell^*)) \\ &= \int \{z^*(x, 0) - z^*(x, 1)\}\, dx + U_\ell(\underline{x}_\ell, 0) - U_\ell(\underline{x}_\ell, 1) > C(\ell - 1) - C(\ell), \end{aligned}$$

and thus $p_\ell(x, 0) - p_\ell(x, 1|\ell) > \Delta_\ell(x_\ell)$. □

A.10 Proof of Proposition 5

Proof. Let $x_\ell^\theta(x_{N+1})$ denote the productivity of firm x_{N+1}'s matched stakeholder of type ℓ with preference $\theta \in \{0, 1\}$. We first establish that the share of firms in the mid-range matched with a values-driven stakeholder of type ℓ is strictly larger than λ_ℓ.

Define $I_\ell(x) \equiv \dfrac{\displaystyle\int_{\underline{x}_\ell}^{x_\ell^1(x)} \lambda_\ell \, dF_\ell(\tilde{x})}{\displaystyle\int_{x_L}^{x} dF_{N+1}(\tilde{x})}$ which represents the proportion of firms below x that are matched with a purpose-driven type-ℓ stakeholder. By the market-clearing condition,

$$\int_{x_L}^{x} dF_{N+1}(\tilde{x}) = \int_{\underline{x}_\ell}^{x_\ell^1(x)} \lambda_\ell \, dF_\ell(\tilde{x}) + \int_{\underline{x}_\ell}^{x_\ell^0(x)} (1 - \lambda_\ell)\, dF_\ell(\tilde{x}),$$

so we may write

$$I_\ell(x) = \frac{\lambda_\ell \int_{\underline{x}_\ell}^{x_\ell^1(x)} dF_\ell(\tilde{x})}{\int_{\underline{x}_\ell}^{x_\ell^1(x)} \lambda_\ell \, dF_\ell(\tilde{x}) + \int_{\underline{x}_\ell}^{x_\ell^0(x)} (1-\lambda_\ell) \, dF_\ell(\tilde{x})} \;\geq\; \frac{\lambda_\ell \int_{\underline{x}_\ell}^{x_\ell^1(x)} dF_\ell(\tilde{x})}{\int_{\underline{x}_\ell}^{x_\ell^1(x)} dF_\ell(\tilde{x})} = \lambda_\ell \, .$$

The inequality uses the fact that when a firm is indifferent between two stakeholders with different preferences, the purpose-driven stakeholder must be weakly more productive, i.e., $x_\ell^1(x_{N+1}) \geq x_\ell^0(x_{N+1})$, which implies $x_\ell^0(x) \leq x_\ell^1(x)$ and hence the denominator on the left is weakly larger. Equality holds if and only if $x_\ell^1(x) = x_\ell^0(x)$, which occurs only when the stakeholder lies in the mixing region (i.e., $x_\ell < \hat{x}_\ell$).

Part (i). Given that $\hat{x}_{\ell,1} < \hat{x}_{\ell,0}$, consider a firm x_{N+1} matched with a stakeholder satisfying $x_\ell < \hat{x}_{\ell,1}$. Such a stakeholder lies in the mixing region both before and after the shock. Hence, $I_{\ell,1}(x) - I_{\ell,0}(x)$, representing the change in the measure of firms affected by stakeholder $(x_\ell, 1)$, satisfies

$$I_{\ell,1}(x) - I_{\ell,0}(x) = \lambda_{\ell,1} - \lambda_{\ell,0} = \delta.$$

Part (ii). For firms that were in the mixing region before the shock but lie in the separation region after the shock (i.e., $x_{N+1} \in [x_{N+1}^*(\hat{x}_{\ell,1}),\, x_{N+1}^*(\hat{x}_{\ell,0})]$), we have

$$I_{\ell,1}(x) - I_{\ell,0}(x) > \lambda_{\ell,1} - \lambda_{\ell,0} = \delta,$$

where we use the fact that $I_{\ell,0}(x) = \lambda_{\ell,0}$, since $x_{N+1} < x_{N+1}^*(\hat{x}_{\ell,0})$ implies that these firms were in the mixing region before the shock.

Part (iii). The total change in share over the top region $[x_{N+1}^*(\hat{x}_{\ell,0}),\, \bar{x}_{N+1}]$ is

$$\begin{aligned}
&\big\{I_{\ell,1}(\bar{x}_{N+1}) - I_{\ell,1}\big(x_{N+1}^*(\hat{x}_{\ell,0})\big)\big\} - \big\{I_{\ell,0}(\bar{x}_{N+1}) - I_{\ell,0}\big(x_{N+1}^*(\hat{x}_{\ell,0})\big)\big\} \\
&\quad = \delta + \big\{I_{\ell,0}\big(x_{N+1}^*(\hat{x}_{\ell,0})\big) - I_{\ell,1}\big(x_{N+1}^*(\hat{x}_{\ell,0})\big)\big\} \\
&\quad < \delta + \lambda_{\ell,0} - \lambda_{\ell,1} < \delta,
\end{aligned}$$

where the first equality uses the fact that $I_{\ell,t}(\bar{x}_{N+1}) = \lambda_{\ell,t}$ for each t. The second inequality uses $I_{\ell,0}\big(x_{N+1}^*(\hat{x}_{\ell,0})\big) = \lambda_{\ell,0}$ together with $I_{\ell,1}\big(x_{N+1}^*(\hat{x}_{\ell,0})\big) > \lambda_{\ell,1}$, the latter holding because $x_{N+1}^*(\hat{x}_{\ell,0}) > x_{N+1}^*(\hat{x}_{\ell,1})$. Hence, the impact on firms at the top is strictly less than δ.

The impact can equivalently be defined from the viewpoint of stakeholders. Let

$$\tilde{I}_\ell(x) \equiv I_{\ell,1}\big(x_{N+1,1}^*(x)\big) - I_{\ell,0}\big(x_{N+1,0}^*(x)\big).$$

Since $x^*_{N+1}(x)$ is monotone in x, the same conclusions apply to the stakeholder-side formulation.

□

A.11 Proof of Proposition 6

Proof. According to Proposition 4, the allocation remains the same for stakeholders at the bottom. Hence, their utility remains the same, according to Equation 11.

For stakeholders at the top, consider first type ℓ. As shown in Proposition 4, the z-index $z^*(x_\ell, \theta)$ must decrease (increase) for purpose-driven (profit-driven) stakeholders because of increased competition on the same side of the market. Hence, according to Equation 11, $U_\ell(x_\ell, 1)$ decreases, whereas $U_\ell(x_\ell, 0)$ increases at the top.

The opposite effect arises for stakeholders of any other type $\ell' \neq \ell$. Due to complementarity, purpose-driven stakeholders of type $\ell' \neq \ell$ benefit from the presence of more productive purpose-driven stakeholders of type ℓ, which results in a higher $z^*(x_{\ell'}, 1)$ and thus a higher $U_{\ell'}(x_{\ell'}, 1)$. Conversely, profit-driven stakeholders of type ℓ' are worse off.

To see this, consider stakeholders of type $\ell' < \ell$, whose matches may change from period ℓ onward. Suppose such a stakeholder is in team $z_{\ell-1}(y_{\ell-1}, n_{\ell-1})$. Given that $(x_\ell, 1)$ is now matched with a less productive team, it means that, fixing the productivity, the team can instead match with a more productive stakeholder of type ℓ in period ℓ, which increases post-match team productivity, as $z_{\ell-1}x^*_\ell(z_{\ell-1})$ must increase. Conversely, teams with $n_{\ell-1} = 0$ must be matched with less productive profit-driven stakeholders of type ℓ, since there are now fewer of them. Hence, the distribution of productivity for teams with index $n \geq 1$ ($n = 0$) improves (deteriorates) in the first-order stochastic dominance sense.

This also implies that, for any stakeholder of type $\ell' > \ell$, stakeholder $(x_{\ell'}, 1)$ will be able to match with a better team, since such stakeholders face an improved distribution of teams with index $n \geq 1$. By contrast, matches worsen for stakeholders of type $(x_{\ell'}, 0)$.

□