A Lost Decade of Fiscal Misallocation

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Abstract

This paper examines how fiscal windfalls from interest rate cuts influenced resource allocation during Japan's Lost Decade (1993-2002). We document two key empirical patterns: a shift of resources from private to government sectors and a deteriorating spatial allocation of government investments. We develop a politico-economic theory where fiscal windfalls induce allocation to be more politically driven than efficiency-oriented. This theoretical mechanism is incorporated into a quantitative model calibrated to the Japanese economy. We conduct counterfactuals to assess the quantitative effects of fiscal windfalls. If the government transfers these windfalls to households while maintaining the same debt trajectory, we find lower government consumption and investment levels, as well as a much improved spatial allocation of government capital, which would have increased aggregate TFP by 0.53% and welfare by nearly 1% during the Lost Decade.

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1 Introduction

Governments worldwide have accumulated substantial public debt, making interest rate adjustments increasingly consequential for fiscal budgets. These budgetary impacts can significantly influence fiscal resource allocation, potentially affecting aggregate productivity and economic welfare. Japan's experience during the "Lost Decade", which refers to the period 1993-2002 in this paper, is particularly revealing for two reasons.[1] First, the real long-term Japanese government bond interest rate declined from an average of 3.8% during 1987-1992 to 2.4% in the Lost Decade. This substantial reduction created considerable fiscal windfalls, even with Japan maintaining its government debt-to-GDP ratio of approximately 80% in the early 1990s. Second, Japan operates under a highly centralized fiscal system (e.g., Sato (2002); Ihori et al. (2009)). The central government actively influenced resource allocation through large government investments around the Lost Decade. This centralization minimizes the confounding influences of local government policies, thereby providing analytical clarity that sharpens our understanding of the underlying politico-economic mechanisms.

While the Lost Decade has been widely regarded as a period characterized by low productivity growth since Hayashi and Prescott (2002), the fiscal dimensions of this era remain understudied. This paper therefore begins with a comprehensive documentation of Japan's fiscal resource allocation during this period. We find that the Lost Decade featured higher government-to-private consumption and investment ratios than the previous decade, indicating a resource shift toward the government sector. Importantly, this shift is associated with a significant deterioration in the spatial allocation of government investments. Prefecture-level government investments exhibited strong negative correlations with their economic returns – a pattern unique to the Lost Decade and never documented before. These empirical observations suggest that fiscal windfalls not only distorted resource allocation between government and private sectors, but also worsened allocation efficiency within the government sector itself.

We next propose a politico-economic theory of fiscal windfall-induced misallocation. The government balances the tradeoff between maximizing household welfare and pursuing political objectives by responding to local demands for government investment. Political incentives are modeled through local lobbying capacity.[2] Compared to the first-best allocation, this political economy exhibits government capital wedges that distort both the consumption-investment decision and the

[1] Our definition of the Lost Decade aligns with the conventional timeframe (early 1990s to early 2000s) commonly adopted in the literature. Hayashi and Prescott (2002), for instance, focuses on 1991-2000 as the Lost Decade, while Peek and Rosengren (2005) and Caballero, Hoshi and Kashyap (2008) examine the periods 1993-1999 and 1990-2002, respectively. We deliberately exclude the initial years of the 1990s to create a clear distinction between the Lost Decade and the preceding asset price bubble burst in 1990-1992.

[2] Local-interest groups exert considerable influence in obtaining substantial transfers from the central government, predominantly in the form of public works (Ihori et al. (2009)). Moreover, these funds are often directed to politically favoured but unproductive uses (Sato (2002)).

spatial investment allocation in the government sector. These distorted allocations represent a compromise between economic efficiency and political influence. In this political economy framework, a fiscal windfall reduces the shadow price of the fiscal budget, inducing allocations to be more politically driven than efficiency-oriented. Moreover, if the government cannot optimize transfers to households when receiving fiscal windfalls, a consumption wedge emerges that increases government consumption relative to the private sector, distorting efficient resource allocation between the two sectors. These model predictions align consistently with the empirical patterns observed during Japan's Lost Decade.

We generalize the simple theory to a small open economy with overlapping generations of households. While interest rate cuts generated fiscal windfalls during Japan's Lost Decade, our model treats these changes as exogenous since the mechanism through which fiscal windfalls induce misallocation does not depend on any particular source of such windfalls. Unlike households, the government administration is long-lived, though it operates with a lower discount factor, reflecting the probability of being replaced by another administration in subsequent periods. The government optimally chooses sequences of consumption, investments, and borrowing, subject to its budget constraint, private-sector decisions, and general equilibrium conditions.[3]

The model incorporates multiple regions, each characterized by a time-varying institutional parameter that captures local lobbying capacity to influence government capital allocation, as well as time-varying private-sector capital and labor wedges that distort local factor prices despite perfect factor mobility across regions. We calibrate these wedges to match private-sector capital and labor distributions in each prefecture. Following the "institutional accounting" framework developed by Song and Xiong (2024), we recover local institutional parameters to match cross-prefecture government capital allocation. Despite the model's rich variation in institutions and frictions across prefectures and over time, our counterfactual analysis shows that these factors play a surprisingly negligible role in explaining both the deteriorating spatial allocation of government capital and the resource shift toward the government sector during the Lost Decade.

We design a specific counterfactual to assess the quantitative effects of fiscal windfalls from interest rate cuts. First, we estimate the magnitude of these windfalls in each period based on changes in expected yield-to-maturity on newly issued government bonds. In our counterfactual, the government transfers these windfalls to households while maintaining the same debt trajectory as in the calibrated economy. This approach ensures that interest rate cuts do not affect the fiscal budget available for government consumption and investments. By deliberately isolating the effect of interest rate cuts on the fiscal budget, we avoid confounding our analysis with the broader impact of interest rate cuts on private consumption and investment – an important but contentious issue

[3] The welfare weights assigned to different household generations in the government objective function ensure time consistency in the dynamic optimization problem. Our main findings remain robust when we assume commitment technology and solve the corresponding Ramsey problem under alternative welfare weights.

on which the literature has yet to reach consensus.

The counterfactual transfers tighten the government budget, representing a reverse of fiscal windfall-induced misallocation. The strong negative cross-prefecture correlation between government investment and its returns disappears. The improved spatial allocation increases the aggregate TFP in the Lost Decade by 0.53%. The welfare also increases by nearly 1% using the equivalent consumption variation. Unlike the counterfactuals examining local institutional parameters and frictions, this counterfactual demonstrates that fiscal windfalls played a decisive role in shaping the allocation of resources within the government sector and between government and private sectors throughout the Lost Decade.

We also consider a scenario where the government can optimize the magnitude of the transfers in response to fiscal windfalls.

The misallocation of fiscal windfalls has been studied in economic literature. Caselli and Michaels (2013) demonstrate how fiscal windfalls from oil output affect disproportionately government expenditures in Brazil. Similarly, Brollo et al. (2013) find that transfers from central to local governments in the same country significantly increase corruption levels. Our study extends this literature by analyzing the production allocation efficiency of fiscal windfalls and quantifying their effects on aggregate TFP and welfare.

Our work is also complementary to the literature on low interest rate and stagnation: Caballero, Hoshi and Kashyap (2008) on zombie lending, Kiyotaki, Moore and Zhang (2021) on overvalued (intangible) assets; and Asriyan et al. (2024) on misallocation via financial frictions.

The rest of the paper is organized as follows. Section 2 presents the basic facts. We construct a simple model in Section 3 to illustrate the politico-economic mechanism behind fiscal windfall-induced misallocation. We extend the simple model into a dynamic framework and calibrate it in Section 4. Section 5 conducts the counterfactual analysis. Section 6 concludes.

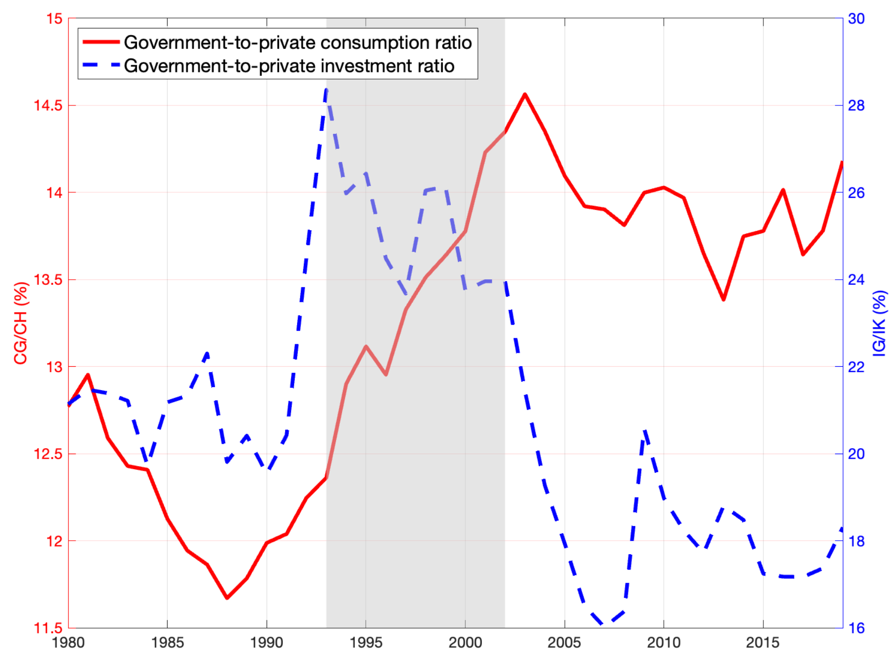
2 Facts

In this section, we present a set of facts regarding the government-private allocation of aggregate consumption and investment, as well as the spatial allocation of government consumption and investment. These empirical observations are complemented by well-documented dynamics of interest rates from the literature. Interest rate data are sourced from Japan's Ministry of Finance, while all other national and prefecture-level data come from Japan's National Accounts and Cabinet Office. Appendix A provides detailed information on data construction procedures.

Government-Private Allocation of Consumption and Investment We first document the consumption allocation between the government and private sectors. Government consumption, denoted by C_t^G, is measured by "government actual final consumption" provided by the central and local governments.[4] Private consumption, denoted by C_t^H, is directly measured by private final consumption expenditure in the National Accounts.

The solid red line in Figure 1 plots the government-to-private consumption ratio since 1980, when the data became available. The shaded area represents the Lost Decade. It is evident that government consumption outgrew private consumption during the Lost Decade. The government-to-private consumption ratio increased from 12.36% in 1993 to a peak of 14.56% in 2003, before declining to an average of 13.82% in the 2010s.

Figure 1: Government-to-Private Consumption and Investment Ratios (%)



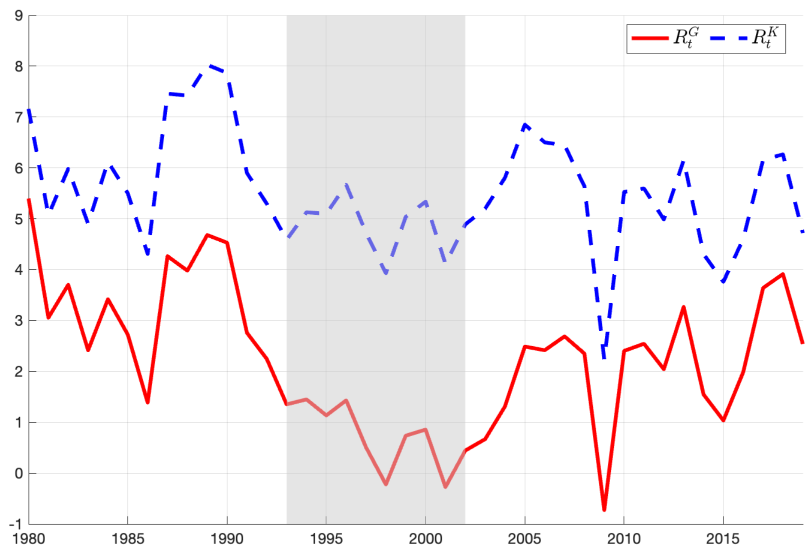
The dashed blue line in the figure plots the government-to-private investment ratio. Government investment (I_t^G) is measured by gross fixed capital formation (GFCF) by the central and local governments.[5] Private investment (I_t^K) is calculated as total GFCF minus government investment (I_t^G). The increase in government investment relative to private investment was more dramatic than the consumption ratio at the beginning of the Lost Decade. The government-to-private investment ratio remained at a high level throughout most of the Lost Decade and then rapidly declined afterward, stabilizing at an average of 17.95% in the 2010s.

[4]Our measure represents a narrowly defined government consumption. We exclude "social transfers in kind" from government consumption, as well as "government actual final consumption" provided by social security funds. However, using more inclusive definitions of government consumption would lead to the same finding or even strengthen it.

[5]Consistent with our definition of government consumption, we exclude GFCF provided by social security funds and public enterprises. Our finding is, again, robust to the inclusion of these entities.

We next calculate the aggregate returns to government capital (R_t^G) and private-sector capital (R_t^K), which will be specified in our quantitative model in Section 4. Figure 2 illustrates these returns, with R_t^G represented by the solid red line and R_t^K by the dashed blue line. While both returns reached their lowest levels during the Lost Decades, the gap between them widened to its maximum. The average difference ($R_t^K - R_t^G$) measured 4.1% during the Lost Decades, exceeding the averages of 3.0% and 3.7% observed in the preceding and subsequent decades, respectively.

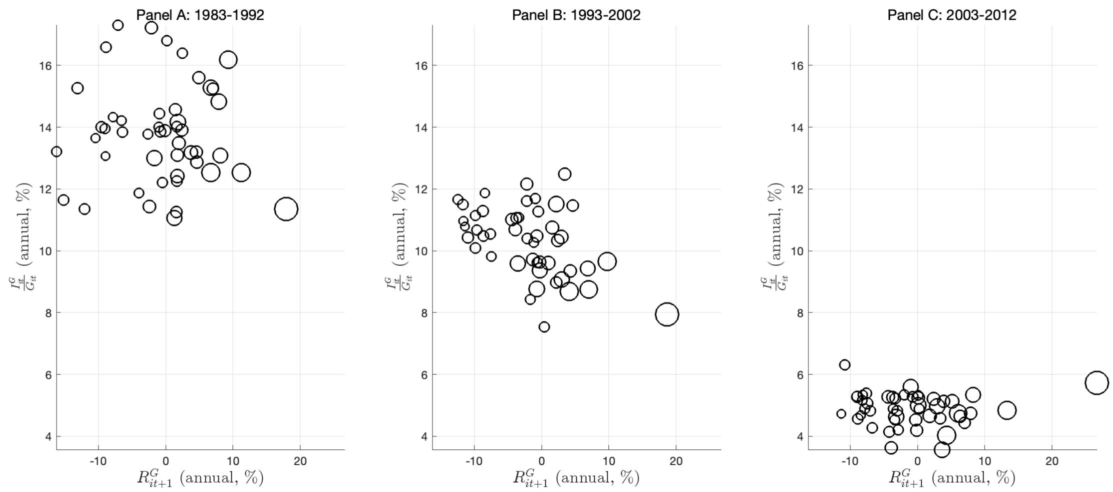
Figure 2: Aggregate Returns to Government and Private-sector Capital (%)



In summary, the Lost Decade features a higher government-to-private consumption ratio than the previous decade, and a higher government-to-private investment ratio than both the preceding and subsequent decades. The dispersion of the aggregate returns to government and private-sector capital indicates a worsening of government and private-sector capital allocation efficiency in the Lost Decade.

Spatial Allocation of Government Consumption and Investment We compile prefecture-level data on government investment and capital stock, denoted by I_{it}^G and G_{it}, respectively. We next calculate the aggregate returns to prefecture government capital, R_{it}^G, which will be defined by equation (26) in our quantitative model. Figure 3 plots the relationship between R_{it+1}^G and I_{it}^G/G_{it} across the three time periods.

Figure 3: $\frac{I_{it}^G}{G_{it}} \sim R_{it+1}^G$



The first notable observation is the large but relatively stable cross-prefecture dispersion of R_{it+1}^G, with standard deviations of 7.2%, 6.4%, and 7.1% in the respective time periods. However, the correlation between R_{it+1}^G and I_{it}^G/G_{it} varies substantially across periods. The local government investment rate is uncorrelated with returns both before and after the Lost Decade. This pattern contradicts the prediction of standard investment models, which suggest that investment rates should be higher when returns are higher, thus indicating spatial misallocation of government capital during these periods. More strikingly, there is a strong negative correlation during the Lost Decade, signaling a significant deterioration in spatial allocation efficiency during that period. These observations are confirmed by simple OLS regression results presented in Table 1.

Table 1: $\frac{I_{it}^G}{G_{it}} \sim R_{it+1}^G$

$\frac{I_{it}^G}{G_{it}}$	1983-1992	1993-2002	2003-2012
R_{it+1}^G	-0.008	-0.084***	-0.002
	(0.033)	(0.023)	(0.011)
Observations	47	47	47
R^2	0.001	0.223	0.001

These observations indicate substantial variation in prefecture-level government investments over time. The average government investment rate $\frac{I_{it}^G}{G_{it}}$ during the Lost Decade exhibits zero correlation with corresponding averages in either the preceding or subsequent decades (correlation of 0.04 and 0.06).[6] The time-varying spatial allocation of government investment cannot be explained

[6]Government investment growth $\frac{I_{it}^G}{I_{it-1}^G}$ during the Lost Decade shows no significant correlation with growth in

by changes in local fiscal conditions. Rather, changes in prefecture-level government investment are highly correlated with fluctuations in fiscal transfers from the central government to prefecture governments. This finding aligns with the highly centralized structure of the Japanese fiscal system, where central authorities maintain substantial control over regional investment patterns.

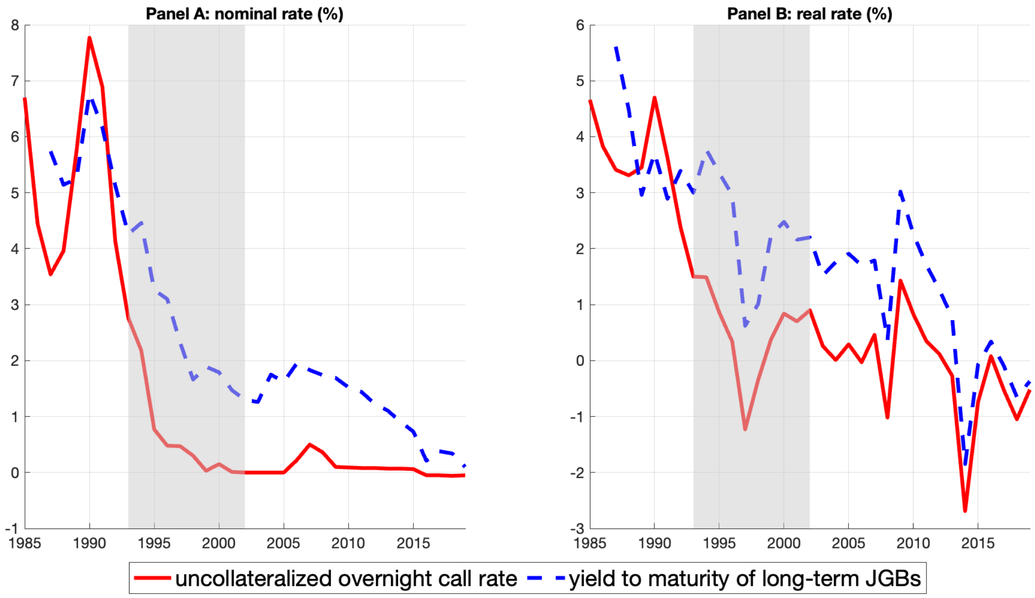
We also compile prefecture-level data on government consumption, denoted by C_{it}^G, where i represents each prefecture, from the national accounts (Appendix A.2.2). The spatial allocation of government consumption demonstrates considerably greater persistence. The correlation of government consumption growth $\frac{C_{it}^G}{C_{it-1}^G}$ between the Lost Decade and the subsequent decade reaches 0.61, which stands in stark contrast to the zero correlation observed in government investment growth. In the following analysis, we will abstract away from the spatial allocation of government consumption and focus on the spatial allocation of government investment.

Interest Rate Cuts Japan's monetary policy during the Lost Decade has been intensively studied in the literature. Following Hayashi and Koeda (2013), we plot the uncollateralized overnight call rate in Figure 4 (solid line) to illustrate the rapid adoption of the near-zero interest rate policy – a pivotal policy shift that underlies the mechanism central to our study. A similarly pronounced decline is observable in Japanese Government Bond (JGB) interest rates, as depicted by the dashed line representing the yield to maturity of long-term JGBs.[7]

the following decade (correlation of -0.18).

[7] These interest rate cuts were largely unanticipated and initially perceived as temporary, primarily due to the BOJ's optimistic assessments of economic conditions (as evidenced in the *Monthly Bulletin* publications of 1991-92 analyzed by Jinushi, Kuroki and Miyao (2000)). Similarly, the implementation of the Zero Interest Rate Policy (ZIRP) in 1999 and subsequent Quantitative Easing Policy (QEP) in 2001 represented unexpected monetary interventions that significantly influenced market expectations. As Nakazawa and Osada (2024) demonstrates, these policies had significant effects on the expected short-term rate component of long-term JGB yields.

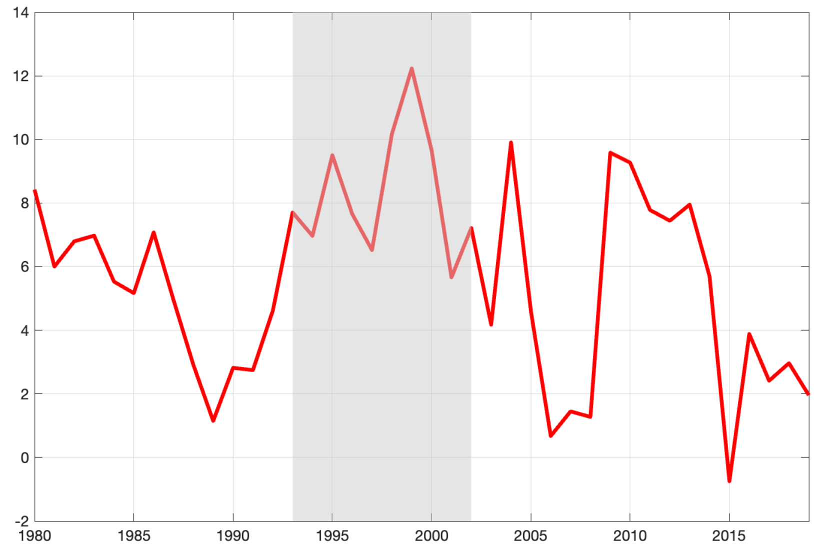
Figure 4: Interest Rate



The right panel presents real interest rates – nominal rates adjusted for inflation using the CPI. While the decline in real interest rates appears less dramatic, it remains quantitatively significant. The real yield to maturity of long-term JGBs fell from an average of 3.84% during 1987-1992 to 2.38% during the Lost Decade. As demonstrated in Section 4, this substantial reduction in government borrowing costs significantly expanded fiscal capacity and can account for all the empirical patterns documented above.

Government Borrowing We denote D_t total government debt, which includes both central and local government debt but excludes Fiscal Investment and Loan Program (FILP) bonds from central government debt (see Appendix A.3). The dramatic interest rate cut in the Lost Decade was associated with a substantial increase in government borrowing $B_t \equiv D_{t+1} - D_t$ (Figure 5). The government borrowing GDP ratio increased from an average of 4.4% during 1983-1992 to 8.3% during the Lost Decade.

Figure 5: Government Borrowing (in percent of GDP)



Fiscal Windfalls We write the government budget constraint as follows.

$$C_t^G + I_t^G = NB_t + B_t - r_t^D D_t, \tag{1}$$

The left-hand side of equation (1) represents total government expenditure, including aggregate government consumption and investment. The right-hand side captures government net revenue, including non-borrowing income NB_t and borrowing revenue B_t net of interest payment $r_t^D D_t$, where r_t^D denotes the interest rate for government debt. We define fiscal windfalls as the unanticipated change in government net revenue. Section 4.6 will quantify both the magnitude and composition of the fiscal windfalls. The interest rate reduction depicted in Figure 4 directly contributes to fiscal windfalls through the decline in $r_t^D D_t$ during the Lost Decades and subsequent periods. The expanded government borrowing illustrated in Figure 5 also generates fiscal windfalls, although this effect diminishes over the long run.

3 A Simple Model

This section develops a one-period model that identifies efficiency wedges by contrasting a multi-region political economy with the first-best allocation. Government capital wedges emerge as the central mechanism through which more government wealth may lead to overinvestment and spatial misallocation of government capital.

3.1 The Economy

We consider a simple economy consisting of N regions indexed by i. Each region i has a production function given by:

$$Y_i = A_i G_i^{\alpha}, \tag{2}$$

where Y_i denotes regional output, A_i represents regional total factor productivity, G_i is regional government capital, and $\alpha \in (0,1)$ is the output elasticity of government capital. Aggregate capital and output are $G \equiv \sum_i G_i$ and $Y \equiv \sum_i Y_i$, respectively.

The representitative household derives utility from a Cobb-Douglas aggregation of household consumption C^H and government consumption C^G: $U^H = u(C)$, where $u' > 0$, $u'' < 0$, $\lim_{C\to 0} u'(C) = \infty$, $\lim_{C\to\infty} u'(C) = 0$, and

$$C = \left(C^H\right)^{\rho} \left(C^G\right)^{1-\rho}. \tag{3}$$

$\rho \in [0,1]$ captures the intensity of preference for household consumption. Unlike G_i, which varies across region i, we do not distinguish region-specific government consumption. This simplification reflects the empirical finding that cross-prefecture allocation of government consumption has exhibited remarkable persistence over time (see Section 2).

The resource constraint is

$$C^G + C^H + G = W^G + W^H + Y, \tag{4}$$

where W^H and W^G represent household and government wealth.

3.2 First-Best Allocation

A benevolent planner maximizes household utility U^H by optimally allocating resources among government consumption, household consumption, and regional government capital, subject to the resource constraint (4).

The first-order with respect to G_i yields:

$$R_i^G = 1, \tag{5}$$

where we define the aggregate returns to G_i as $R_i^G \equiv \frac{\partial Y}{\partial G_i} = \alpha \frac{Y_i}{G_i}$. R_i^G is equalized across regions. Denote by A the aggregate TFP: $A \equiv \frac{Y}{G^{\alpha}}$. A is maximized by equalizing R_i^G.

$$\bar{A} = \left(\sum_i A_i^{\frac{1}{1-\alpha}}\right)^{1-\alpha}, \tag{6}$$

where $\bar{A}$ denotes the maximum aggregate TFP.

The first-order conditions with respect to C^G and C^H yield the optimal consumption allocation rule:

$$\frac{C^G}{C^H} = \frac{1-\rho}{\rho}. \tag{7}$$

3.3 Political Economy

We now introduce a political economy framework wherein resource allocation decisions are made by the central government, which balances the tradeoff between maximizing household welfare and pursuing political objectives. Specifically, we assume the following government objective function:

$$U^G = u(C) + \sum_i \kappa_i v\left(G_i\right), \tag{8}$$

where $\kappa_i > 0$ is an institutional parameter that captures local lobbying capacity and $\kappa \equiv \sum_i \kappa_i$; $v' > 0$, and $v'' < 0$. The central government collects tax revenue together with its wealth W^G to finance government consumption, regional government capital, and lump-sum transfers T to households. The government budget constraint is

$$C^G + \sum_i G_i + T = W^G + \tau Y, \tag{9}$$

where τ is a flat-rate tax on output.

Household consumption is financed by their wealth W^H, after-tax output, and lump-sum transfers from the government:

$$C^H = W^H + (1-\tau)\, Y + T \tag{10}$$

We examine two cases. In our first scenario, the central government optimally selects aggregate government consumption C^G, region-specific government capital allocations $\{G_i\}$, and lump-sum transfers T to maximize equation (8), subject to its budget constraint (9). We assume τ is exogenous and outside the government's decision framework.[8] In our second scenario, we constrain the government's fiscal instruments to C^G and G_i while treating transfers T as exogenously determined.

To guarantee the non-emptyness of the feasibility set, we assume $W^G > \underline{W}^G$ and $W^G > \underline{W}^G(T)$ for the two scenarios, respectively, where $\underline{W}^G \equiv -W^H - (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}\left(\bar{A}\right)^{\frac{1}{1-\alpha}}$ and $\underline{W}^G(T) \equiv T - (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}\left(\tau\bar{A}\right)^{\frac{1}{1-\alpha}}$ are the lower bounds of government wealth.

In the first scenario with optimized transfers T, the optimal allocation between household consumption C^H and government consumption C^G replicates the first-best condition given in equation (7). The allocation of regional government capital follows

[8]The alternative case – optimizing τ while keeping T exogenous – would yield similar insights in this simple model. However, in our full-fledged dynamic framework, the implications would differ due to the distortionary nature of τ.

$$\left[1+\underbrace{\frac{\kappa_i C^G}{(1-\rho)\,\alpha Y_i}\frac{v'(G_i)\,G_i}{u'(C)C}}_{\text{Government Capital Wedge}}\right]R_i^G=1 \tag{11}$$

Comparing equation (11) with equation (5) reveals capital wedges arising from lobbying incentives, which direct greater government capital toward regions with higher political weights κ_i.

The capital wedges lead to two sources of misallocation. First, it distorts the allocation between C^G and $\{G_i\}$ by overinvesting $\{G_i\}$ relative to the first-best allocation. Moreover, the capital wedge creates misallocation of government capital across regions. To see this, we rewrite equation (11) as

$$\kappa_i v'(G_i)=\underbrace{\frac{(1-\rho)\,C}{C^G}u'(C)}_{\text{Shadow Price}}\times\left(1-R_i^G\right) \tag{12}$$

The left-hand side of equation (12) represents the marginal benefit the government derives from increasing G_i. On the right-hand side, the first component captures the shadow price of G_i, while the second component reflects the marginal cost of providing G_i through its effect on aggregate output Y in the budget constraint (9). Unless local lobby capacity precisely aligns with local productivity – where $\kappa_i v'(G_i)$ is function of R_i^G – government capital returns R_i^G will not be equalized across regions, resulting in aggregate TFP losses.

In the second scenario with exogenous transfers T, the allocation between household consumption C^H and government consumption C^G may deviate from the optimal ratio specified in equation (7), creating an additional consumption wedge.

$$\text{Consumption Wedge}=\frac{\rho}{1-\rho}\frac{C^G}{C^H}-1 \tag{13}$$

This distortion not only affects consumption allocation but also complicates capital allocation. The first-order conditions with respect to C^G and $\{G_i\}$ establish

$$\left[1+\underbrace{\frac{\kappa_i C^G}{(1-\rho)\,\alpha Y_i}\frac{v'(G_i)\,G_i}{u'(C)C}+(1-\tau)\times\text{Consumption Wedge}}_{\text{Government Capital Wedge}}\right]R_i^G=1 \tag{14}$$

Compared to the capital wedge in equation (11), exogenous T introduces an additional component derived from the consumption wedge following equation (13). Since G_i represents the government's sole fiscal instrument for balancing government and household consumption, the consumption wedge inevitably influences the capital wedge. A positive consumption wedge – i.e., the ratio C^G/C^H exceeds its optimal level – the government allocates more resources from C^G to G_i, thereby increasing the government capital wedge. Conversely, when this ratio falls below its optimal level, the government diverts more resources from G_i to C^G, resulting in a reduced government capital wedge.

The consumption wedge also affects misallocation of government capital across regions. When T is exogenous, equation (12) becomes

$$\kappa_i v'(G_i) = \underbrace{\frac{(1-\rho)C}{C^G} u'(C)}_{\text{Shadow Price}} \times \left[1 - R_i^G \left(1 + (1-\tau) \times \text{Consumption Wedge}\right)\right]. \tag{15}$$

Relative to the marginal costs depicted in equation (12), the scenario with exogenous T introduces an additional component. A positive consumption wedge (C^G is too high) lowers the marginal cost of $\{G_i\}$ because more $\{G_i\}$ increases household income and consumption, indirectly correcting the consumption misallocation.

When $\tau = 1$, equation (14) becomes identical to equation (12). In this special case, C^H is entirely determined by W^H, prohibiting the government from influencing C^H through G_i. Consequently, the consumption wedge does not affect capital allocation decisions.

3.4 Government Wealth and Misallocation

In this subsection, we first distinguish two special types of cross-region government capital allocation in our political economy framework, in contrast to the first-best government capital allocation $G_i = (\alpha A_i)^{\frac{1}{1-\alpha}}$ implied by $R_i^G = 1$. The first type, $G_i \propto A_i^{\frac{1}{1-\alpha}}$, features identical marginal returns R_i^G and thereby maximizes aggregate TFP. We term this the efficiency-driven spatial allocation of government capital.

Minimal Government Wealth and Efficiency-Driven Spatial Allocation Regardless of whether T is optimized or exogneous, one can easily see that as government wealth W^G approaches its lower bound $\underline{W}^G$ (for optimized T) or $\underline{W}^G(T)$ (for exogenous T), the allocation G_i always converges to the efficient spatial allocation. An impoverished government faces an infinitely high shadow price for G_i and, consequently, prohibitive marginal costs for any inefficient spatial allocation. This compels the government to prioritize efficiency, which overrides political considerations.

The second type is the politically-driven spatial allocation, where $v'(G_i) \propto \frac{1}{\kappa_i}$ is determined entirely by political considerations captured by the left-hand side of equation (12) or (15).

Abundant Government Wealth and Politically-Driven Spatial Allocation Under optimized transfers T, as $W^G \to \infty$, $\{G_i\}$ converges to the politically-driven spatial distribution. The intuition is straightforward: abundant government wealth reduces the shadow price to zero, allowing allocation decisions to be driven entirely by political considerations.

However, this mechanism operates differently under exogenous transfers T. With $\tau < 1$ and finite household wealth W^H, as $W^G \to \infty$, C^G approaches infinity, generating an infinitely high consumption wedge. Consequently, the marginal cost of G_i on the right-hand side of equation (15) remains non-zero. In this case, $\{G_i\}$ converges to a hybrid formula that balances efficiency and political factors: $\kappa_i v'(G_i) = \left(\frac{1-\rho}{C^G} - \frac{\rho}{C^H}(1-\tau) R_i^G\right) u'(C) C$. Two important observations emerge. First, in the absence of lobbying incentives ($\kappa_i = 0$ for all i), this hybrid formula simplifies to the efficiency-driven allocation rule $G_i \propto A_i^{\frac{1}{1-\alpha}}$. Second, when either $\tau = 1$ or $W^H \to \infty$, the formula reduces to the purely politically-driven allocation $v'(G_i) \propto \frac{1}{\kappa_i}$.

Under some additional assumption, we can prove the following proposition that characterizes resource allocations between the two extreme cases of W^G.

Proposition 1 (Government Wealth and Misallocation) *Assume $u(\cdot) = v(\cdot) = \log(\cdot)$, $\kappa_i \geq 0$ $\forall i$, and sufficiently high τ under exogenous transfers T.*

1. *Level of Government Capital: G_i exceeds its first-best level. Moreover, the discrepancy between G_i and its first-best level increases monotonically with government wealth W^G.*

2. *Spatial Allocation of Government Capital: The spatial allocation efficiency, as measured by aggregate TFP A, decreases monotonically with government wealth W^G.*

3. *Government to Household Consumption Ratio (for exogenous transfers T only): The ratio C^G/C^H increases monotonically with government wealth W^G. Furthermore, there exists a threshold such that C^G/C^H exceeds its first-best value if and only if W^G is above the threshold.*

The first two parts of the proposition show that increased government wealth exacerbates both the oversupply and spatial misallocation of government capital. Under exogenous T, the last part of the proposition establishes that when C^G is already excessive, more government wealth also further distort the allocation between C^G and C^H.

Finally, under optimized transfers T, we can establish that $\frac{dT}{dW^G} > 0$ holds if and only if W^G exceeds a threshold. An increase in W^G simultaneously raises C^G and G_i, with the latter enhancing aggregate output Y and consequently household consumption C^H. When government wealth is sufficiently high, the marginal increase in C^G outpaces the corresponding increase in C^H, pushing up transfers T to restore the optimal government-to-household consumption ratio.

4 A Quantitative Model

4.1 Household

The economy is inhabited by overlapping generations of households. Each generation lives for two periods with logarithm preferences:

$$U_t^H = \log\left(\left(C_{t,t}^H\right)^\rho \left(C_t^G\right)^{1-\rho}\right) + \beta \log\left(\left(C_{t,t+1}^H\right)^\rho \left(C_{t+1}^G\right)^{1-\rho}\right) \tag{16}$$

where U_t^H stands for the utility of the households born at period t; $C_{t,t}^H$ and $C_{t,t+1}^H$ represent their consumption in periods t and $t+1$, respectively; C_t^G and C_{t+1}^G are the government consumption in the two periods. Households take government consumptions as given. $\rho \in [0,1]$ captures the intensity of preference over household consumption. β is household's discount factor.

The household budget constraints are $C_{t,t}^H + W_{t+1}^H = Y_t^H + T_t$ and $C_{t,t+1}^H = (1 + r_{t+1})W_{t+1}^H$, where W_{t+1}^H denotes household savings, Y_t^H represents the total income of the young household (to be specified later), and T_t is a lump-sum government transfer or tax. With log preferences, household saving choices are independent of government consumptions. Household consumptions have the closed-form solutions:

$$C_{t,t}^H = \frac{1}{1+\beta}\left(Y_t^H + T_t\right), \quad C_{t,t+1}^H = \frac{\beta\left(1 + r_{t+1}\right)}{1+\beta}\left(Y_t^H + T_t\right). \tag{17}$$

The aggregate labor supply from young households is L_t, which is assumed to be exogenous in the benchmark specification. Young households competitively allocate their labor L_{it} across regions according to local labor demand that will be specified below.

4.2 Local Production and Aggregate Output

We consider a small open economy that consists of N regions. The regional output in period t is given by:

$$Y_{it} = A_{it} G_{it}^{\alpha_G} K_{it}^{\alpha_K} L_{it}^{\alpha_L}, \tag{18}$$

where A_{it}, G_{it}, K_{it} and L_{it} represent total factor productivity, government capital, private-sector capital, and labor in region i in period t, respectively. We assume $\alpha_G + \alpha_K + \alpha_L < 1$.[9]

Each region has a representative firm. The output of the firm is subject to a proportional tax rate τ_t that is identical across regions. Private capital and labor are mobile across regions but government capital is immobile.

Denote by r_t^K and w_t the rental rate of private capital and wage rate, respectively. In this small open economy, the rental rate of private capital r_t^K is exogenously determined by the world interest

[9] One may interpret $1 - \alpha_G - \alpha_K - \alpha_L$ as the land output elasticity. If we assume fixed land supply in each region, A_{it} will be a composite of total factor productivity and local land supply.

rate, denoted r_t^w. From the bank's zero-profit condition, r_t^K satisfies

$$1 + r_t^K = \frac{(1 + r_t^w)\, q_{t-1} - (1 - \delta)\, q_t}{1 - \xi_t^K} \tag{19}$$

where q_t is the relative price of investment in terms of consumption, δ is the capital depreciation rate, and ξ_t^K is an inverse measure of financial intermediation efficiency in the private sector (e.g., Song, Storesletten and Zilibotti (2011))

In contrast, the wage rate w_t is endogenously derived through labor market clearing: $\sum_{i=1}^N L_{it} = L_t$. Our model incorporates region-specific distortions in factor markets through wedges on private capital rental (τ_{it}^K) and labor hiring (τ_{it}^L). In contrast to private inputs, government capital is provided to firms within each region without direct user charges.

The firm's profit-maximization problem is given by:

$$\max_{K_{it}, L_{it}} \Pi_{it} = (1 - \tau_t)\, Y_{it} - \left(1 + \tau_{it}^K\right)\left(1 + r_t^K\right) K_{it} - \left(1 + \tau_{it}^L\right) w_t L_{it}. \tag{20}$$

The first-order conditions imply the local demand of K_{it} and L_{it}:

$$\frac{K_{it}}{L_{it}} = \frac{\alpha_K}{\alpha_L} \cdot \frac{w_t}{1 + r_t^K} \cdot \frac{1 + \tau_{it}^L}{1 + \tau_{it}^K}, \tag{21}$$

$$L_{it} \propto Z_{it} G_{it}^{\frac{\alpha_G}{1 - \alpha_K - \alpha_L}}, \tag{22}$$

where $Z_{it} \equiv \left(\frac{A_{it}}{(1+\tau_{it}^K)^{\alpha_K}(1+\tau_{it}^L)^{1-\alpha_K}}\right)^{\frac{1}{1-\alpha_K-\alpha_L}}$ is a composite term capturing the region-specific productivity and distortions.

The local output follows

$$Y_{it} \propto Z_{it}\left(1 + \tau_{it}^L\right) G_{it}^{\frac{\alpha_G}{1 - \alpha_K - \alpha_L}}, \tag{23}$$

In a partial equilibrium framework with exogenous wage rate w_t, the elasticity of local output with respect to G_{it} would be $\frac{\alpha_G}{1-\alpha_K-\alpha_L}$. However, in general equilibrium, changes in G_{it} trigger a reallocation of labor L_{it} across regions, which generates additional impacts through the endogenously determined w_t. Denote by $Y_t \equiv \sum_{i=1}^N Y_{it}$ the aggregate output,

$$Y_t = \left(\frac{\alpha_K (1 - \tau_t)}{1 + r_t^K}\right)^{\frac{\alpha_K}{1-\alpha_K}} \left(\frac{L_t}{\sum_{i=1}^N Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}}\right)^{\frac{\alpha_L}{1-\alpha_K}} \sum_{i=1}^N \left(Z_{it}\left(1 + \tau_{it}^L\right) G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}\right), \tag{24}$$

The middle term on the righ-hand-side of equation (24) captures the general equilibrium effect of G_{it}. The marginal product of G_{it} at the aggregate level is

$$\frac{\partial Y_t}{\partial G_{it}} = \frac{\alpha_G}{1 - \alpha_K - \alpha_L} \frac{Y_{it}}{G_{it}} - \frac{\alpha_G \alpha_L}{(1 - \alpha_K - \alpha_L)(1 - \alpha_K)} \frac{L_{it}}{L_t} \frac{Y_t}{G_{it}}, \tag{25}$$

The first term on the right-hand-side of equation (25) is the direct effect of G_{it} on local production in region i, and the second term is its general equilibrium effects through wage.

We generalize the aggregate returns to G_{it} in the simple model to

$$R_{it}^{G} \equiv \frac{\partial Y_t}{q_{t-1}\partial G_{it}} + \frac{q_t}{q_{t-1}}(1-\delta) - 1, \tag{26}$$

and the aggregate TFP to $A_t \equiv \frac{Y_t}{G_t^{\alpha_G} K_t^{\alpha_K} L_t^{\alpha_L}}$.

In a special case where $\tau_{it}^{L} = \tau_t^{L}$, the aggregate output Y_t is given by

$$Y_t = \left(\frac{\alpha_K(1-\tau_t)}{1+r_t^K}\right)^{\frac{\alpha_K}{1-\alpha_K}} L_t^{\frac{\alpha_L}{1-\alpha_K}} \left(\sum_{i=1}^{N} Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}\right)^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K}}, \tag{27}$$

where $Z_{it} \equiv (\frac{A_{it}}{(1+\tau_{it}^K)^{\alpha_K}})^{\frac{1}{1-\alpha_K-\alpha_L}}$. Here, G_{it} contributes to the aggregate output through a CES fashion, with the elasticity of substitution of $\frac{1-\alpha_K-\alpha_L}{1-\alpha_K-\alpha_L-\alpha_G} > 1$. When G_{it} is efficiently allocated – i.e., $G_{it} \propto Z_{it}^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K-\alpha_L-\alpha_G}}$, we obtain the maximized A_t as $\frac{\left(\sum_{i=1}^{N} Z_{it}^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K-\alpha_G-\alpha_L}}\right)^{1-\alpha_G-\alpha_L}}{\left(\sum_{i=1}^{N} \frac{1}{1+\tau_{it}^K} Z_{it}^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K-\alpha_G-\alpha_L}}\right)^{\alpha_K}}$.

4.3 Central Government

The economy has a centralized fiscal system, where the central government chooses government consumption, investment, and borrowing at the beginning of period t. We denote local government investment as I_{it}^{G}, with local government capital evolving according to $G_{it+1} = I_{it}^{G} + (1-\delta)G_{it}$. While the central government could allocate government consumption cross regions, Section 2 demonstrates that cross-regional allocation of government consumption has remained remarkably stable over time. Consequently, we abstract from local government consumption and focus exclusively on aggregate government consumption C_t^{G}. Moreover, because local governments must obtain prior permission from the central government to issue bonds (Ihori and Sato (2002)), we likewise abstract from local government borrowing and consider only aggregate government borrowing.

The central government's objective function is

$$U_t^{C} = \omega \log\left(\left(C_{t-1,t}^{H}\right)^{\rho}\left(C_t^{G}\right)^{1-\rho}\right) + \sum_{j=0}^{\infty} \beta_C^{j}\left(U_{t+j}^{H} + \sum_{i=1}^{N} \kappa_{it+j} \log G_{it+j+1}\right), \tag{28}$$

where κ_{it} is the same institutional parameter as in the simple model that indicates the local government's lobbying capacity for government investment in that region. The household utility and consumption follow equations (16) and (17). The central government cares about all present and future generations. It discounts the future generations' utilities geometrically with a discount factor $\beta_C \in (0,1)$. The weight on the current old generation is ω. We assume $\omega = \frac{\beta}{\beta_C}$ such that the central government's preference is time-consistent and can be rewritten as:

$$U_t^C = \sum_{j=0}^{\infty} \beta_C^j \left(\rho \left(\omega \log C_{t+j-1,t+j}^H + \log C_{t+j,t+j}^H \right) + (1-\rho)(1+\omega) \log C_{t+j}^G + \sum_{i=1}^{N} \kappa_{it+j} \log G_{it+j+1} \right).$$

The central government's budget constraint is:

$$C_t^G + \sum_{i=1}^{N} q_t I_{it}^G + T_t + \Phi_t = \tau_t Y_t + D_{t+1} - \left(1 + r_t^D\right) D_t - \frac{\psi_t^D}{2} \left(r_{t+1}^D D_{t+1} - \overline{rD} \right)^2, \tag{29}$$

Here, D_t denotes the total government debt at the beginning of period t. Government borrowing is subject to a quadratic debt adjustment cost $\frac{\psi_t^D}{2}(r_{t+1}^D D_{t+1} - \overline{rD})^2$, and $\overline{rD}$ is an exogenous parameter about the targeted interest payment. r_t^D is the real effective interest rate of government debt. We assume $r_t^D = r_t + \xi_t^D$, where ξ_t^D captures the gap between the interest rate and effective rate of government debt. Φ_t is a residual component, capturing government spendings that cannot be easily classified as government consumption, investment, or transfer. The aggregate output Y_t follows equation (24).

The central government chooses $\{C_{t+j}^G, G_{it+j+1}, D_{t+j+1}\}_{i=1,j=0}^{N,\infty}$ to maximize the objective function (28), subject to the budget constraint (29).

4.4 Equilibrium Conditions

Household's total income include firm profits ($\sum_{i=1}^N \Pi_{it}$) and wages ($\sum_{i=1}^N w_t L_{it}$):

$$\begin{aligned} Y_t^H &= \sum_{i=1}^{N} \left(\Pi_{it} + w_t L_{it} \right) \\ &= (1-\tau_t) \sum_{i=1}^{N} \left(\left(1 - \alpha_K - \frac{\tau_{it}^L}{1+\tau_{it}^L} \alpha_L \right) Y_{it} \right) \end{aligned} \tag{30}$$

Here we use the equilibrium wage.

We derive the optimality conditions for the central government's problem in Appendix B.4.2. The first-order conditions w.r.t. D_{t+j+1} and G_{it+j+1} are

$$\frac{C_{t+j+1}^G}{C_{t+j}^G} = \frac{\beta_C \left(1 + r_{t+j+1}^D\right)}{1 - \psi_{t+j}^D r_{t+j+1}^D \left(r_{t+j+1}^D D_{t+j+1} - \overline{rD} \right)}, \tag{31}$$

$$\begin{aligned}\frac{C^G_{t+j+1}}{C^G_{t+j}} =&\beta_C\left(\tau_{t+j+1}\frac{\partial Y_{t+j+1}}{\partial G_{it+j+1}}+q^G_{t+j+1}(1-\delta)\right)\\&+\frac{1}{(1-\rho)(1+\omega)q^G_{t+j}}\left(\beta_C\rho\frac{C^G_{t+j+1}}{C^H_{t+j+1,t+j+1}}\frac{\partial Y^H_{t+j+1}}{\partial G_{it+j+1}}+\kappa_{it+j}\frac{C^G_{t+j+1}}{G_{it+j+1}}\right),\end{aligned} \tag{32}$$

where $\frac{\partial Y_t}{\partial G_{it}}$ follow equation (25) and

$$\frac{\partial Y^H_t}{\partial G_{it}} = (1-\tau_t)\sum_{j=1}^{N}\left(\left(1-\alpha_K-\frac{\tau^L_{jt}}{1+\tau^L_{jt}}\alpha_L\right)\frac{\partial Y_{jt}}{\partial G_{it}}\right). \tag{33}$$

Equation (31) represents a standard Euler equation, where consumption growth is determined by the interest rate and additional debt adjustment costs. The right-hand side of equation (32) reveals the multifaceted nature of government capital allocation decisions. The first term captures the economic returns to government capital. The term $\beta_C\rho\frac{C^G_{t+j+1}}{C^H_{t+j+1,t+j+1}}\frac{\partial Y^H_{t+j+1}}{\partial G_{it+j+1}}$ represents the marginal benefit of government investment in region i through positive spillover effects on household consumption. The final term $\kappa_{it+j}\frac{C^G_{t+j+1}}{G_{it+j+1}}$ reflects the political gain from allocating government capital to region i.

4.5 Calibration

4.5.1 External Calibration

We set the aggregate capital share parameter $\alpha = 0.362$ to match the Japanese economy over 1984-1989 (Hayashi and Prescott (2002)). The government capital share parameter α_G is set to 0.05, consistent with the infrastructure capital share adopted in Song and Xiong (2024). This calibration implies a private capital share of $\alpha_K = \alpha - \alpha_G = 0.312$. Assuming a land share of 0.10 and constant returns to scale, we derive a labor share of $\alpha_L = 0.538$.

Each period in the model spans ten years. We set $\delta = 1-(1-0.089)^{10} = 0.606$, following the calibrated annual capital depreciation rate of 0.089 in Hayashi and Prescott (2002).

We set the household annual discount factor to 0.98, yielding a decadal discount factor of $\beta = 0.98^{10} = 0.82$. For the central government, we assume a discount factor $\beta_C = \beta p_C$, where $p_C \in (0,1)$ captures political turnover. We calibrate the annualized political survival probability to $p_C = 0.87$, implying that an administration has a 25% probability of remaining in office for a full decade. This parameterization yields a government discount factor of $\beta_C = 0.85^{10} = 0.20$. The preference parameter $\rho = 0.965$ is calibrated so that the observed government-to-household consumption ratio during the 1983-1992 period matches the first-best allocation.

All the variables for output, capital, consumption, and debt are detrended by the long-run growth rate $g = (1+g_A)^{\frac{1}{1-\alpha_G-\alpha_K}} - 1$, where g_A denotes the long-run TFP growth rate, which is set to an annual rate of 0.5%.

We set the initial household wealth W_0^H to match household financial assets at the end of 1982, which is 160% of GDP.

Table 2 summarizes all time-invariant externally calibrated parameter values.

Table 2: Externally Calibrated Time-Invariant Parameters

Parameters	Value	Target
α_K	0.312	$\alpha_G + \alpha_K = \alpha = 0.362$ (Hayashi and Prescott (2002))
α_G	0.050	Song and Xiong (2024)
α_L	0.538	land share = 0.1
annualized δ	0.089	capital depreciate rate in Hayashi and Prescott (2002)
annualized β	0.980	
annualized β_C	0.850	25% probability of staying in office in a full decade
ρ	0.965	government to household consumption ratio in 1983-1992
annualized g_A	0.005	0.5% annual TFP growth

In the calibrated model, $t = 0, 1, 2$ represents the period 1983-1992, 1993-2002, and 2003-2012, respectively. We use the relative price of fixed capital formation to GDP from the national accounts for q_t, and assume both government and private investment share this same price. The parameter τ_t is chosen to match the ratio of total government revenue to GDP. The results are reported in the first two rows of Table 3).

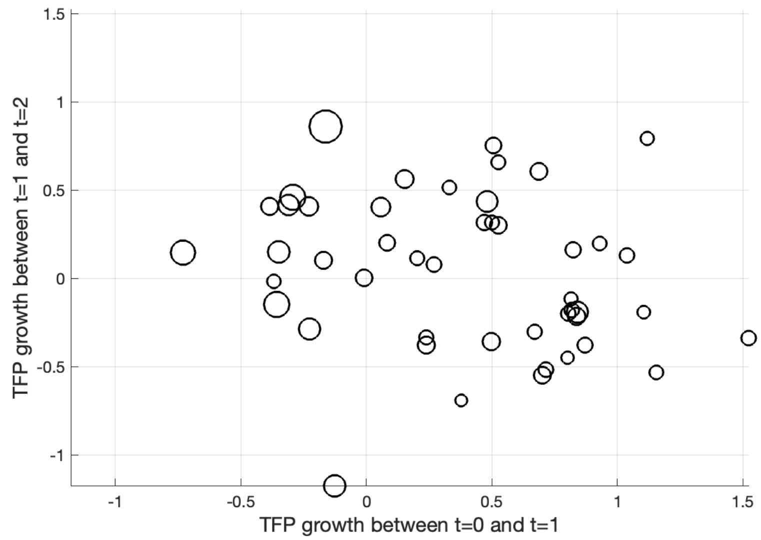
Table 3: Externally Calibrated Time-Varying Parameters

Parameters	$t = 0$ (83-92)	$t = 1$ (93-02)	$t = 2$ (03-12)	Target
q_t	1.40	1.41	1.45	relative price of fixed capital formation
τ_t (%)	23.26	21.26	21.17	total government revenue to GDP ratio
r_t^K(annualized, %)	3.56	1.77	0.61	rental rate of private capital

We derive local productivity, private capital, and labor wedges from the above externally calibrate parameters. Using the perpetual inventory method, we construct prefecture-level government and private capital stocks G_{it} and K_{it} (see Appendix A.2.3 for details). Prefecture TFP is computed as the Solow residual: $\log A_{it} = \log Y_{it} - \alpha_G \log G_{it} - \alpha_K \log K_{it} - \alpha_L \log L_{it}$, where L_{it} denotes employment in the prefecture.

The aggregate TFP growth is 0.10% between 1983-1992 and 1993-2002, and recovers to 0.19% in the subsequent decade. Figure 6 illustrates prefecture-level TFP growth across consecutive decades. The horizontal axis shows growth between 1983-1992 and 1993-2002, while the vertical axis represents growth between 1993-2002 and 2003-2012. The correlation is almost zero, suggesting no persistence in prefecture-level TFP growth.

Figure 6: Prefecture-level TFP Growth (annual, %)

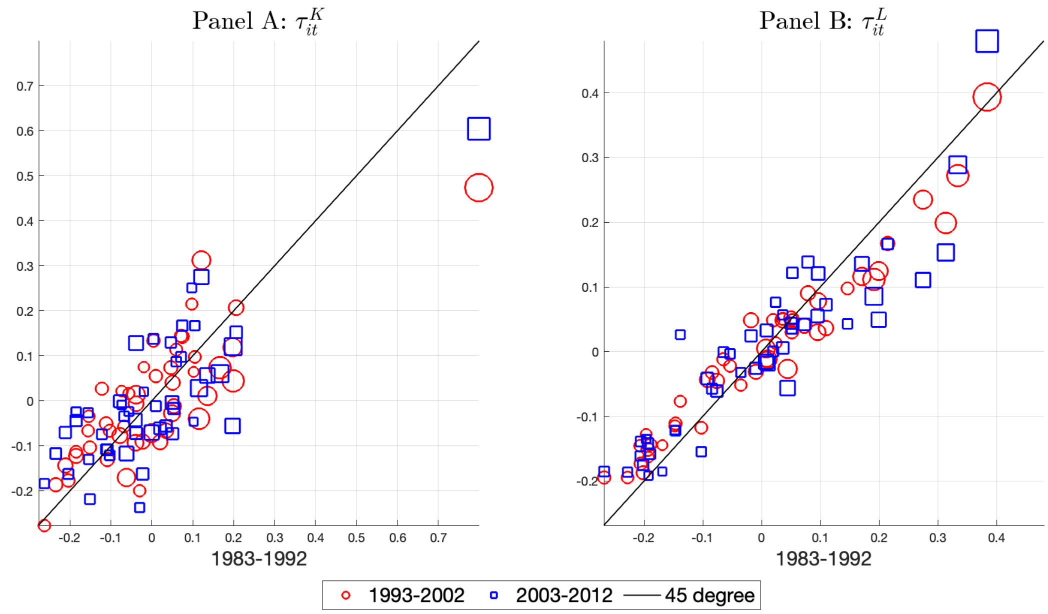


Firms' first-order conditions imply the following prefecture-specific private capital and labor wedges: $1+\tau_{it}^K \propto \frac{Y_{it}}{K_{it}}$ and $1+\tau_{it}^L \propto \frac{Y_{it}}{L_{it}}$. We pin down the level of wedges by imposing $\sum_{i=1}^N \tau_{it}^K K_{it} = \sum_{i=1}^N \tau_{it}^L L_{it} = 0$ – i.e., these wedges do not affect aggregate private capital and labor income. The exogenous rental rate of private capital follows $r_t^K = \frac{\alpha_K(1-\tau)}{N}\frac{Y_t}{K_t}$.[10]

The values of r_t^K are reported in the last row of Table 3. The left panel of Figure 7 plots τ_{it}^K for three time periods: 1983-1992 (x-axis) against 1993-2002 (y-axis, red circles) and 2003-2012 (y-axis, blue squares). The right panel follows the same structure for τ_{it}^L. The visualization shows that prefecture-level wedges exhibit strong persistence across time periods.

[10]Since r_t^K is the only price that influences the demand for private capital, we need not separately specify the world interest rate r_t^w and the capital friction parameter ξ_t^K in equation (19).

Figure 7: Prefecture-level Wedges



4.5.2 Internal Calibration

The remaining parameters are interally calibrated to match the observed data: Φ_t to government consumption C_t^G, T_t to household consumption C_t^H, $\{\kappa_{it}\}_{i=1}^N$ to prefecture-level government capital $\{G_{it+1}\}_{i=1}^N$, and ψ_t^D to government debt D_{t+1}. We calibrate $\overline{rD}$ to match an average adjustment cost that accounts for half of interest payments and management fees.

To simulate the dynamic model, we make two key assumptions. First, for all time-varying parameters (including those in Table 3 and the internally calibrated Φ_t, T_t, κ_{it}, and ψ_t^D) with $t > 2$, we set their values equal to their respective values in period $t = 2$. Additionally, we assume prefecture-level TFP grows at the constant rate g_A for $t > 2$.

Second, regarding expectations, we assume perfect foresight for TFP growth but MIT-shock expectation for all the other parameters. Specifically, for the set $X_t = \{i_{t+1}, \pi_t, \psi_t^D, \Phi_t, T_t, \tau_t, \tau_{it}^K, \tau_{it}^L, \kappa_{it}\}$, we assume

$$\mathbb{E}_t\left[X_{t+j+1}\right] = X_t, \quad \forall j \geq 0. \tag{34}$$

We use the uncollateralized overnight call rate for i_t and the CPI for π_t. The expected real interest rate follows $\mathbb{E}_t[r_{t+j+1}] = \mathbb{E}_t[i_{t+j+1} - \pi_{t+j+1}] = i_{t+1} - \pi_t$, $\forall j \geq 0$. The results are reported in Table (4).

Determining expected real effective rate of government debt, $\mathbb{E}_t[r_{t+j+1}^D]$ $\forall j \geq 0$, is more complex, as it depends on both the historical issuance rates and the maturity composition of outstanding debt. Let $i_t(m)$ denote the yield-to-maturity on newly issued m-year bonds at year t, and let $D_{s,t+1}(m)$ represent the outstanding stock of m-year debt issued at year $t - s$. The nominal effective rate of government debt, i^D_{t+1}, is determined by

$$i^D_{t+1} = \frac{1}{D_{t+1}} \left(\sum_{s=0}^{m-1} i_{t-s}(m) D_{s,t+1}(m) \right). \tag{35}$$

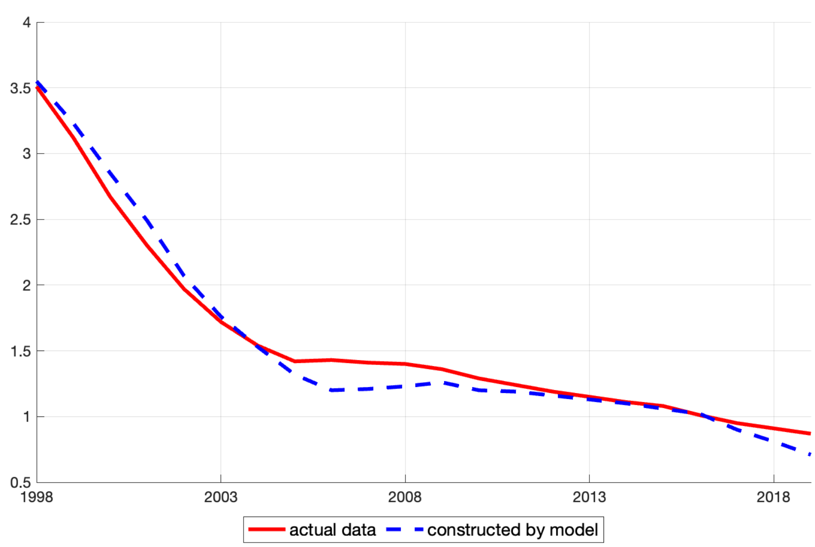
Due to data constraints, we group maturities into three categories (shorter than or equal to one year; between one and ten years; and longer than or equal to ten years) and assume that $D_{t+1}(m) \equiv \sum_{s=0}^{m-1} D_{s,t+1}(m)$ was issued uniformly over the past m years (see Appendix A.3.2 for detailed data descriptions). Despite these limitations, the constructed i^D_{t+1} closely tracks the effective interest rate in the data, as illustrated in Figure 8.

To form expectation for $\mathbb{E}_t[i^D_{t+j+1}]$ for $j \geq 0$, we similarly adopt MIT-shock expectations for future issuance rates and maturity shares:

$$\mathbb{E}_t\left[i_{t+j+1}(m)\right] = i_t(m), \quad \mathbb{E}_t\left[\frac{D_{t+j+1}(m)}{D_{t+j+1}}\right] = \frac{D_t(m)}{D_t}, \quad \forall j \geq 0,$$

We then apply equation (35) to generate $\mathbb{E}_t[i^D_{t+j+1}]$, and obtain $\mathbb{E}_t[r^D_{t+j+1}] = \mathbb{E}_t[i^D_{t+j+1} - \pi_{t+1}]$.[11] Both $\mathbb{E}_t[i^D_{t+j+1}]$ and $\mathbb{E}_t[r^D_{t+j+1}]$ are reported in Table (4).

Figure 8: Effective interest rate i^D_t (%)



[11] We first construct annual expectations using yearly data, and then convert these into ten-year expectations, which are computed as the product of the annual forecasts for $j = 1, \ldots, 10$.

Table 4: Expected Interest Rates (%)

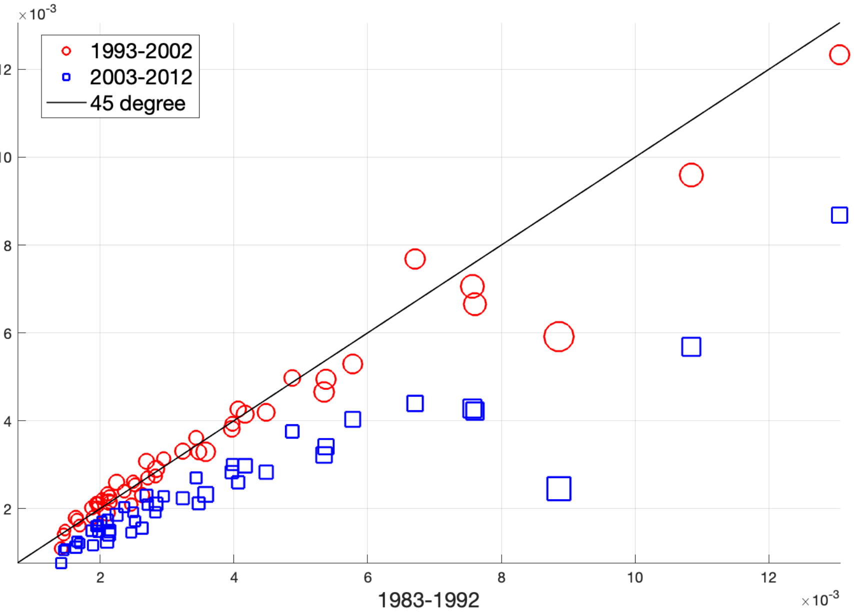
	$t = 1$ (93-02)	$t = 2$ (03-12)	$t = 3$ (13-22)
r_t	0.96	0.26	-0.70
$\mathbb{E}_0[r_t]$	-0.66	-0.66	-0.66
$\mathbb{E}_1[r_t]$	-	-0.04	-0.04
$\mathbb{E}_2[r_t]$	-	-	0.13
i_t^D	3.71	1.40	0.95
$\mathbb{E}_0[i_t^D]$	5.03	4.60	4.60
$\mathbb{E}_1[i_t^D]$	-	1.12	0.85
$\mathbb{E}_2[i_t^D]$	-	-	0.96
r_t^D	3.53	1.53	0.94
$\mathbb{E}_0[r_t^D]$	3.24	2.80	2.80
$\mathbb{E}_1[r_t^D]$	-	0.95	0.67
$\mathbb{E}_2[r_t^D]$	-	-	1.09

The calibration solves for 151 unknows: 4 aggregate variables (Φ_t, T_t, ψ_t^D, $\overline{rD}$), and 47 prefecture-level variables (κ_{it}) across three periods. We obtain $\frac{\overline{rD}}{Y_0} = 1.2\%$, and other results are summarized in Table 5. Figure 9 plots κ_{it} for three time periods: 1983-1992 (x-axis) against 1993-2002 (y-axis, red circles) and 2003-2012 (y-axis, blue squares). Like prefecture-level wedges, κ_{it} is also highly persist across time periods.

Table 5: Internally Calibrated Parameters

Parameters	$t = 0$ (83-92)	$t = 1$ (93-02)	$t = 2$ (03-12)	Target
$\frac{\Phi_t}{Y_t}(\%)$	5.69	8.44	6.28	government consumption
$\frac{T_t}{Y_t}(\%)$	3.76	3.56	4.06	household consumption
ψ_t^D	14.6	292	59.4	government debt
mean of κ_{it} (10^{-3})	3.63	3.50	2.38	prefecture government capital
std of κ_{it} (10^{-3})	2.51	2.22	1.41	prefecture government capital

Figure 9: Prefecture-level κ_{it}



4.6 Fiscal Windfall

We now quantify the magnitude of the fiscal windfall during the Lost Decade. For notational ease, we define changes in expectations as $\Delta\mathbb{E}_t[X_{t+j}] \equiv \mathbb{E}_t[X_{t+j}] - \mathbb{E}_{t-1}[X_{t+j}]$. We can now formally write down the definition of fiscal windfalls in line with equation (1). The government in period t perceives the following fiscal windfall in period $t+j$:

$$\mathbb{E}_t\left[\text{Windfall}_{t+j}\right] = -\Delta\mathbb{E}_t\left[r^D_{t+j}\right] \times D_{t+j} + \Delta\mathbb{E}_t\left[B_{t+j}\right] + \Delta\mathbb{E}_t\left[NB_{t+j}.\right] \tag{36}$$

Here, the government non-borrowing income NB_t is

$$NB_t = \tau_t Y_t - \Phi_t - T_t.$$

The borrowing income B_t is net of debt adjustment costs.

$$B_t = D_{t+1} - D_t - \frac{\psi^D_t}{2}(r^D_{t+1} D_{t+1} - \overline{rD})^2.$$

In the Lost Decade, the expected real interest rate declined from $\mathbb{E}_0[r^D_2] = 2.80\%$ to $\mathbb{E}_1[r^D_2] = 0.95\%$, which saved interest payments. A lower expected real rate also increases borrowing by $\Delta\mathbb{E}_t[B_t]$. Finally, there are other forces that change government non-borrowing net income, $\Delta\mathbb{E}_t[NB_t]$. We report each component as a percentage of GDP in Table 6.

Table 6: Fiscal windfalls perceived at 1993-2002 ($t = 1$, in percent of GDP)

	$t+j=1$ (93-02)	$t+j=2$ (03-12)	$t+j=3$ (13-22)	steady state
$-\Delta\mathbb{E}_t[r^D_{t+j}] \times D_{t+j}$	2.94	5.04	5.08	5.43
$\Delta\mathbb{E}_t[B_{t+j}]$	5.55	7.45	1.35	1.03
$\Delta\mathbb{E}_t[NB_{t+j}]$	-3.39	-3.81	-3.84	-4.70
$\mathbb{E}_t[\text{Windfall}_{t+j}]$	5.11	8.68	2.60	1.77

5 Counterfactuals

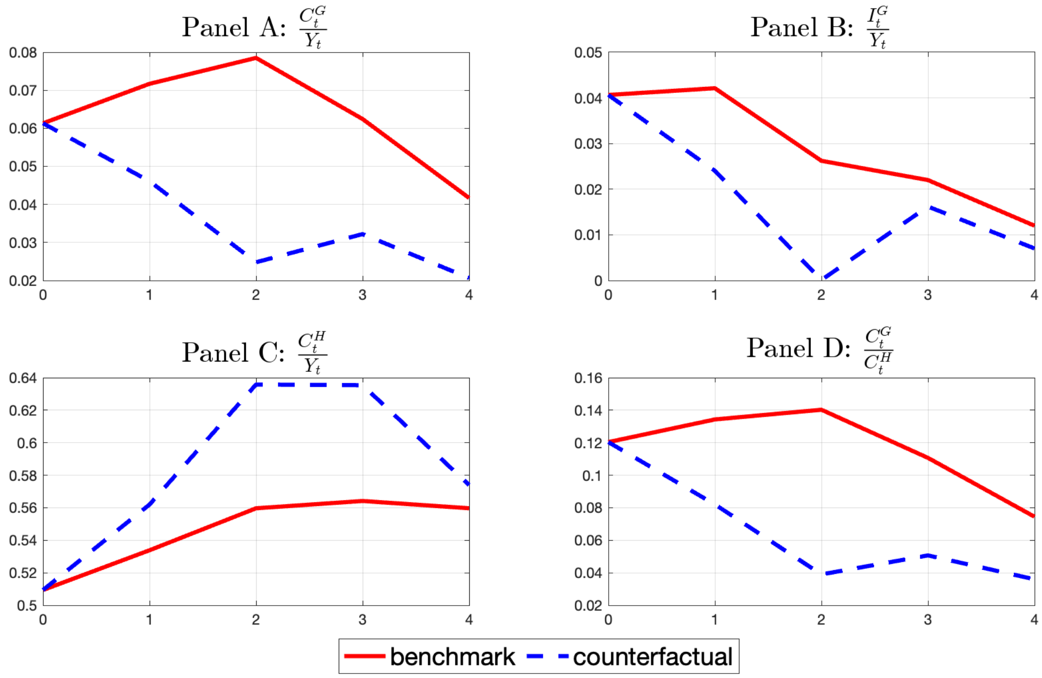
As demonstrated in the simple model, fiscal windfalls may generate several inefficiencies in the economy. In the data, we observe a higher government investment-to-GDP ratio and a worsened spatial allocation of government investment, both consistent with our model predictions. Moreover, we observe a higher government-to-household consumption ratio, aligning with insufficient adjustments in household transfers in response to fiscal windfalls.

To quantitatively examine the effects of these fiscal windfalls, we conduct a counterfactual analysis in which the central government commits to transferring the fiscal windfall $\mathbb{E}_t[\text{Windfall}_{t+j}]$ perceived during the 1993-2002 period ($t = 1$). Consistent with our model, $\mathbb{E}_t[\text{Windfall}_{t+j}]$ is directed exclusively to the young generation in period $t + j$ and is financed from the government budget.

We constrain the government from adjusting its borrowing, ensuring that the debt trajectory remains identical to that in the calibrated model. Consequently, the government can only optimize its choices of C^G_{t+j} and $\{G_{it+1+j}\}$ for $j \geq 0$ in each period $t \geq 1$.

The transitional dynamics of aggregate variables are shown in Figure 10. In the counterfactual, government consumption and investment GDP ratios decrease substantially in the two periods when the government hands out the additional transfers (Panel A and B). In contrast, the household consumption GDP ratio increases (Panel C) and the government to household consumption ratio becomes much smoother (Panel D).

Figure 10: Transitional Dynamics



The additional transfers tighten the government budget. Proposition 1 predicts an increase in the spatial allocation efficiency of government capital in the one-period model. This also happens in the quantitative model as shown by Figure 11. The strong negative correlation between I_{it}^G/G_{it} and R_{it+1}^G in the data is now replaced with zero correlation in the counterfactual (see also the first two columns of Table 7. As a result, the counterfactual aggregate TFP at $t = 2$ is 0.53% higher than the calibrated economy. Aggregate output declines by 2.08%, primarily due to substantially reduced government investment. However, household income increases by 8.65%, resulting in nearly a 1% welfare gain.

Figure 11: $\frac{I^G_{it}}{G_{it}} \sim r^G_{it+1}$ at 1993-2002

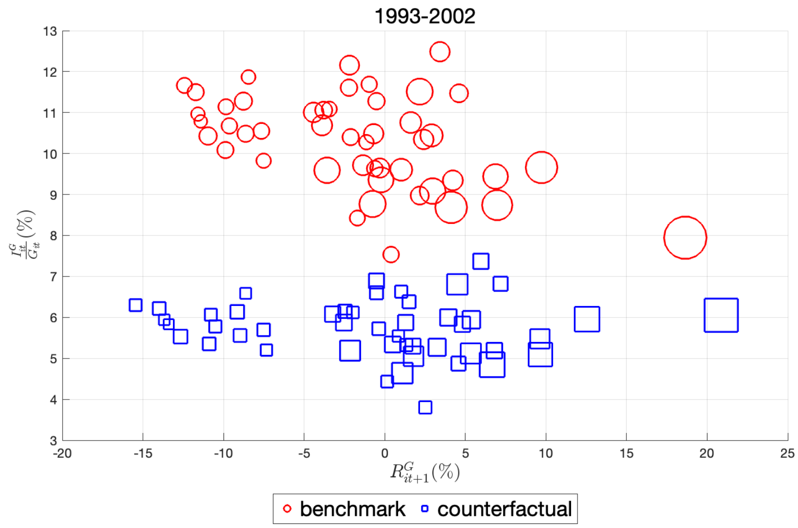


Table 7: $\frac{I^G_{it}}{G_{it}} \sim R^G_{it+1}$ at 1993-2002

$\frac{I^G_{it}}{G_{it}}$	Benchmark	Counterfactuals			
		transfers	$\kappa_{it} = \kappa_i$	$\tau^K_{it} = \tau^K_i$	$\tau^L_{it} = \tau^L_i$
R^G_{it+1}	-0.084***	-0.009	-0.061***	-0.080***	-0.091***
	(0.023)	(0.013)	(0.015)	(0.024)	(0.023)
Obs.	47	47	47	47	47
R^2	0.223	0.010	0.271	0.200	0.251

Table 8: Aggregate TFP, output and welfare changes

	Counterfactuals			
	transfers	$\kappa_{it} = \kappa_i$	$\tau^K_{it} = \tau^K_i$	$\tau^L_{it} = \tau^L_i$
Aggregate TFP change at 2003-2012 (%)	0.53	0.19	0.10	-0.05
Aggregate output change at 2003-2012 (%)	-2.08	-0.10	0.10	-0.06
Household income change in 1993-2002 (%)	8.65	0	0	0
Welfare change (φ, %)	0.99	0.11	-0.07	0.00

Specifically, we assume a social planner maximizing household utility, with the discount factor for future generations' utilities (β_C) the same as the central government. Therefore, the social planner's objective function follows equation (28), but with $\kappa_{it} = 0$. The counterfactual welfare gain or loss is measured using the equivalent consumption variation (φ) relative to the calibrated economy. Specifically, φ is calculated to make the social planner indifferent between two scenarios: a lifetime consumption stream in the calibrated economy, $\{C_t^G, C_{t-1,t}^H, C_{t,t}^H\}_{t=t_0}^{\infty}$, each scaled by $(1+\varphi)$, and a lifetime consumption stream in the counterfactual allocation. t_0 is the period when we start to conduct the counterfactual exercise. The welfare gain in the counterfactual is $\varphi = 0.99\%$. These results are summarized in the first column of Table 8.

5.1 Lobby Incentives and Prefecture-level Wedges

We next conduct the following counterfactuals to examine the effects of changes in κ_{it}, τ_{it}^K, τ_{it}^L in the lost decade.

1. Set the local lobby capacity parameter in the lost decade to its average level: $\kappa_{i1} = \frac{1}{3}\sum_{t=0}^{2}\kappa_{it}$.
2. Set the local private capital or labor wedge in the lost decade to its average level: $\tau_{i1}^K = \frac{1}{3}\sum_{t=0}^{2}\tau_{it}^K$ or $\tau_{i1}^L = \frac{1}{3}\sum_{t=0}^{2}\tau_{it}^L$.

The κ counterfactual modestly weakens the negative correlation between I_{it}^G/G_{it} and R_{it+1}^G (the third column of Table 7). The resulting counterfactual aggregate TFP in 1993-2002 is only 0.19% higher than in the calibrated economy, substantially smaller than the efficiency gain observed in the counterfactual where fiscal windfalls are transferred to households. This limited improvement is expected given the high persistence in κ_{it} (Figure 9). The welfare gain is $\varphi = 0.11\%$.

The counterfactuals for prefecture-level wedges prove inconsequential for the spatial allocation of government capital (as shown in the last two columns of Table 7). This outcome is unsurprising given the high persistence of these wedges demonstrated in Figure 7. Consequently, the counterfactual aggregate TFP and welfare remain virtually identical to those in the calibrated economy. These results are summarized in the last three columns of Table 8.

These results confirm that insufficient adjustments to transfers in response to low-interest-rate-generated fiscal windfalls can largely account for the deterioration in spatial allocation efficiency of government capital. Transferring these fiscal windfalls to households can significantly increase aggregate TFP and welfare. In contrast, changes in local lobbying capacity and local private capital and labor wedges have, at best, marginal effects on aggregate TFP and welfare.

5.2 Optimal Transfers

We solve the optimal transfer problem, where a benevolent planner maximizes the same objective function (28) as the central government. The planner operates under the same budget constraint (29) and possesses the same policy instruments as the central government, with one critical addition:

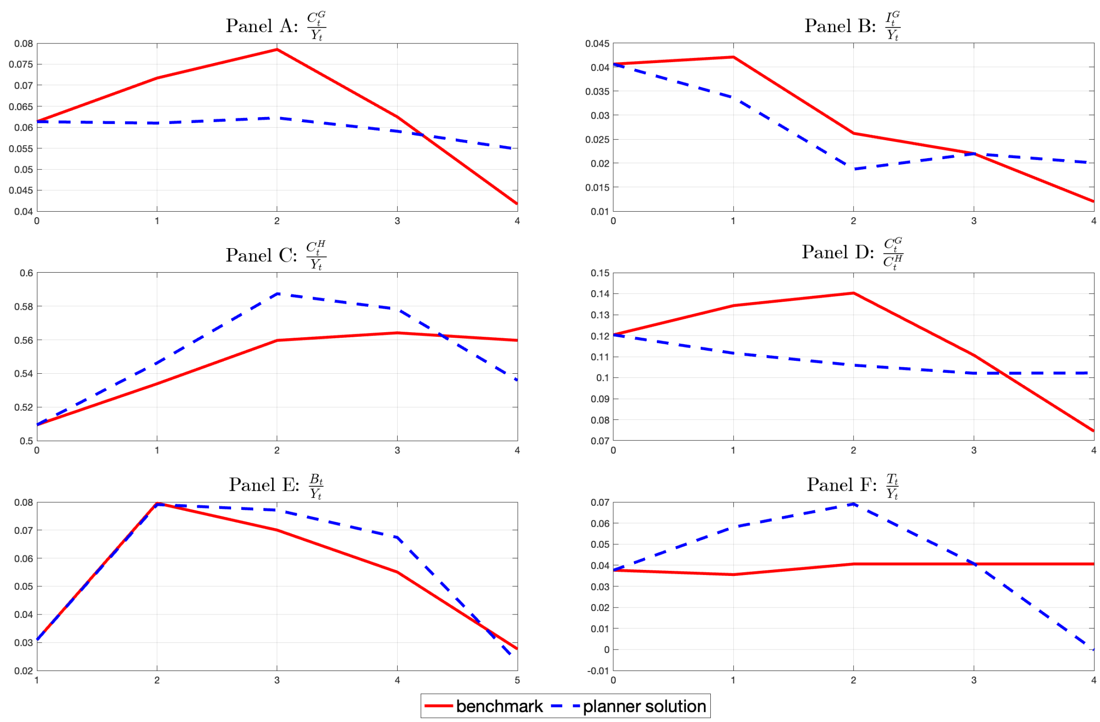
the ability to determine the sequence of lump-sum transfers to young households ($\{T_{t+j}\}_{j\geq 0}$). The first-order condition w.r.t T_{t+j} is:

$$\mathbb{E}_t\left[\frac{C^H_{t+j,t+j}}{C^G_{t+j}}\right] = \frac{\rho}{(1-\rho)\left(1+\frac{\beta}{\beta_C}\right)}, \tag{37}$$

and the first-order conditions with respect to D_{t+j+1} (31) and G_{it+j+1} (32) remain applicable.

Figure 12 plots the transitional dynamics. The bottom right panel shows an increase in the optimal transfers. In this case, the aggregate TFP increases by 0.07% in 2003-2012, and the aggregate output declines by -1.04% in 2003-2012 due to decreased government investments (the top right panel). However, the welfare increases by 0.63%.

Figure 12: Transitional Dynamics



5.3 Institutional Perspective

Through the lens of our model, the Lost Decade of fiscal misallocation concluded with the stabilization of interest rates near zero, which terminated the fiscal windfall created by unanticipated interest rate cuts. However, fiscal restructuring typically necessitates substantial institutional reforms. From an institutional perspective, the end of this decade-long period of fiscal misallocation was achieved through a series of comprehensive fiscal policy reforms. These institutional changes complemented the macroeconomic stabilization and were instrumental in redirecting Japan's fiscal trajectory.

- FILP Reform
- “Trinity Reforms” on central government transfers to local governments
  - Sticks: Fiscal restructuring (Yubari city)
  - Carrots: ”Great Heisei Mergers”

6 Conclusion

To be written

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A Data

A.1 National-level Data from National Accounts

A.1.1 Output and Employment

We collect data on nominal and real gross domestic product (GDP) for the period 1955-2022, and employment data for period 1970-2022.

Japan's national accounts data employ different accounting standards over time. For GDP data, the Cabinet Office publishes 2008SNA standard data covering 1980-2022 and 1968SNA standard data covering 1955-1998. To construct a consistent time series using the 2008SNA standard for 1955-1979, we first calculate growth rates for each GDP component (private final consumption, government final consumption, gross domestic capital formation, exports, and imports) between 1955 and 1980 using the 1968SNA standard. We then apply these growth rates to the 1980 values from the 2008SNA data to reconstruct the historical series. This method yields nominal and real GDP for the entire 1955-2022 period, from which we derive the GDP deflator.

For employment data, the Cabinet Office publishes three series: 2008SNA standard data for 1994-2022, 1993SNA standard data for 1980-2009, and 1968SNA standard data for 1970-1998. We employ a similar methodology, using growth rate information from the 1993SNA and 1968SNA series to construct a consistent time series spanning 1970-2022.

The consumption and investment data discussed in subsequent sections face similar challenges with changing accounting standards over time. We apply analogous methods to construct consistent time series for these variables, though we omit detailed descriptions of this process in the following sections.

A.1.2 Consumption

We collect private final consumption expenditure and government final consumption expenditure for 1955-2022. Private final consumption expenditure serves as our measure of household consumption C_t^H.

Government final consumption expenditure in the National Accounts comprises two components: "government actual final consumption" and "social transfers in kind" provided by the central government, local governments, and social security funds. For our measure of government consumption C_t^G, we use government actual final consumption by central and local governments only. This decomposed data is available for 1980-2022.

A.1.3 Investment

We collect nominal and real gross fixed capital formation (GFCF) for both private and public sectors covering 1955-2022. Public sector GFCF can be decomposed into investments by the central government, local governments, social security funds, and public enterprises. For our measure of government investment I_t^G, we use GFCF by central and local governments only. This decomposed data is available for 1970-2022.

To extend the series back to 1955, we assume that the share of central and local government GFCF relative to total public sector GFCF during 1955-1969 equals the 1970 level. This assumption allows us to construct a complete government investment series for 1955-2022. Total GFCF excluding government investment serves as our measure of private investment I_t^K. The investment deflator is calculated by dividing nominal GFCF by real GFCF.

A.2 Prefecture-level Data from National Accounts

A.2.1 Output and Employment

We collect nominal GDP for each prefecture for the period 1955-2020 and employment for each prefecture for the period 1975-2020. Since aggregated prefecture-level data do not perfectly match national totals, we use national-level data for aggregate figures and allocate these to each prefecture proportionally based on the corresponding prefecture-level values.

A.2.2 Consumption

We obtain private and government final consumption expenditures for each prefecture over 1955-2020. We derive household consumption C_{it}^H using the same allocation method as for output and employment.

From Section A.1.2, we have central government consumption C_{0t}^G and the aggregate local government consumption $\sum_{i=1}^N C_{it}^G$. Unlike national-level data, government final consumption expenditure data at the prefecture level do not provide further decomposed information, leaving us without a direct measure for C_{it}^G. We therefore assume that each prefecture's share of C_{it}^G in aggregate local government consumption $\sum_{i=1}^N C_{it}^G$ equals its share of government final consumption in aggregate local government final consumption. This assumption yields our measure of prefecture-level local government consumption C_{it}^G.

A.2.3 Investment and Capital

We collect nominal GFCF for both private and public sectors for each prefecture for 1955-2020. Denote by $I_{it}^{G,\text{ local}}$ the investment provided by the local government in prefecture i. Since decomposed data within public sector GFCF are not available at the prefecture level, we assume that each prefecture's share of $I_{it}^{G,\text{ local}}$ in $I_t^{G,\text{ local}}$ equals its share of public sector GFCF in aggregate public sector GFCF. This yields prefecture-level local government investment $I_{it}^{G,\text{ local}}$. We further assume that central government investment $I_t^{G,\text{ central}}$ is allocated across prefectures in proportion to their local government investment. Thus, total government investment in prefecture i is

$$I_{it}^{G} = I_{it}^{G,\text{ local}} + I_{t}^{G,\text{ central}} \times \frac{I_{it}^{G,\text{ local}}}{\sum_{i=1}^{N} I_{it}^{G,\text{ local}}}. \tag{38}$$

Total GFCF data for each prefecture provide I_{it}, from which we derive private investment as $I_{it}^{K} = I_{it} - I_{it}^{G}$.

We estimate initial government and private capital stocks at the beginning of 1955 using

$$Z_{i,1955} = \frac{I_{i,1955}^{Z}}{\log\left(I_{i,1960}^{Z}/I_{i,1955}^{Z}\right)/5 + \delta}, \tag{39}$$

where $Z \in \{G, K\}$. We then construct capital series using the perpetual inventory method:

$$Z_{it+1} = I_{it}^{Z} + (1-\delta)Z_{it}. \tag{40}$$

A.3 Government Debt and Government Bonds

A.3.1 Government Debt and Effective Interest Rate

Government debt comprises two components: central government debt (CGD) and local government debt (LGD). Our CGD calculation includes general bonds, borrowings, financial bills, and government-guaranteed debt, collected from the Ministry of Finance for 1973-2023. We exclude Fiscal Investment and Loan Program (FILP) bonds from the CGD calculation for two reasons: (1) the 2001 FILP reform makes the data before and after that year incomparable, (2) to avoid double-counting with LGD.

Our LGD calculation encompasses both prefecture-level and municipal-level debt, obtained from the "Local Government Finance Settlement" for 1974-2022.

We calculate the effective interest rate by dividing interest payments by the outstanding debt amount. Since interest payment data are available only for general bonds, which constitute more than half of total government debt (56% during 1983-2012), we use the effective interest rate on general bonds as a proxy for the rate on total government debt.

A.3.2 Issuance Rate and Maturity Composition of Government Debt

We collect Japanese government bond auction results from the Ministry of Finance (link). Each auction record contains information on maturity, yield to maturity (YTM), and issuance amount. We classify bonds into three maturity categories: long-term (40, 30, 20, 15, and 10 years), medium-term (6, 5, 3, and 2 years), and short-term (1 year and 6 months). For each category, we calculate the annual issuance-weighted average YTM. The earliest observations begin in 1987, 1979, and 1985 for long-term, medium-term, and short-term bonds, respectively.

The Ministry of Finance publishes outstanding amounts of long-term, medium-term, and short-term government debt for 2005-2023 (link). For years prior to 2005, we assume that each category's share of total government debt remains constant at its 2005 level.

When we apply equation (35) to estimate the future effective interest rate, we set $m = 10, 5, 1$ for long-term, medium-term, and short-term bonds, respectively.

A.4 Others

A.4.1 Interest Rate

The uncollateralized overnight call rate is available from 1985 onward. For earlier years, we follow Hayashi and Koeda (2013) and use the collateralized call rate as a proxy.

A.4.2 Government Revenue

Total government revenue comprises tax and non-tax revenues from both central and local governments. We collect these data from the Ministry of Finance for 1975–2022.

A.5 Constructing Ten-Year Variables

This appendix details the conversion of annual variables to ten-year variables. For time-invariant annual parameters $X \in \{\beta, \beta_C, 1-\delta\}$, the conversion is straightforward: the ten-year counterpart is X^{10}. Time-variant variables require a more involved process. For expositional ease, we denote X_{yt} as the annual variable X in year yt, and X_t as the corresponding ten-year variable at period t in the model.

For capital stocks and nominal government debt, we set them to their corresponding annual variable at the beginning of the ten year at period t. For interest rates $X_{yt} \in \{1 + r^D_{yt}, 1 + r_{yt}\}$, $X_t = \prod_{yt} X_{yt}$. For nominal variables NX_t where $X_t \in \{I^G_{it}, I^K_{it}, Y_{it}, C^G_{it}, C^H_{it}\}$, ten-year variables are obtained by simple summation: $NX_t = \sum_{yt} NX_{yt}$.

The ten-year price index for output follows $P^Y_t = \frac{NY_t}{\sum_{yt} Y_{yt}}$, and we apply this to obtain ten-year real output and consumption: $X_t = \frac{NX_t}{P^Y_t}$, $X_t \in \{Y_{it}, C^G_{it}, C^H_{it}\}$. Ten-year price index for investment, P^{FCF}_t, is inferred from $P^{\text{FCF}}_t = \frac{NI^G_t + NI^K_t}{(G_{t+1}+K_{t+1})-(1-\delta)(G_t+K_t)}$, which ensures the consistency with ten-year total capital stocks and depreciation rate. We apply this to obtain ten-year real investment: $X_t = \frac{NX_t}{P^{\text{FCF}}_t}$, $X_t \in \{I^G_{it}, I^K_{it}\}$.

Finally, real government debt is given by $D_t = \frac{ND_t}{P^Y_{t-1}}$, where ND_t denotes nominal government debt at the beginning of period t. This allows us to express the budget constraint in real terms as in equation (29).

B Proofs

B.1 Proof for Minimal Government Wealth

As government wealth W^G approaches its lower bound $\underline{W}^G$ (for optimized T) or $\underline{W}^G(T)$ (for exogenous T), we have $C^G \to 0$, so the shadow price $\frac{(1-\rho)C}{C^G}u'(C)$ diverges to infinity.

For optimized T, equation (15) requires R_i^G to converge to 1.

For exogenous T, equation (15) requires τR_i^G to converge to 1.

Both cases require that $G_i \propto A_i^{\frac{1}{1-\alpha}}$.

B.2 Proof for Abundant Government Wealth

When $W^G \to \infty$, G_i will converge to ∞. We prove this by contradiction. Suppose not, then the right-hand side of equation (15) will converge to 0 because the shadow price will converge to 0, while the left-hand side remains positive. This yields a contradiction.

Since G_i converges to ∞, R_i^G converges to zero. For optimized T, we have $\lim_{W^G\to\infty} \kappa_i v'(G_i) = \frac{(1-\rho)C}{C^G}u'(C)$.

For exogenous T, equation (15) can be rewritten as

$$\kappa_i v'(G_i) = u'(C)C\left\{\frac{(1-\rho)}{C^G}\left(1-\tau R_i^G\right) - R_i^G(1-\tau)\frac{\rho}{C^H}\right\}.$$

Since $\lim_{W^G\to\infty} R_i^G = 0$, when $W^G \to \infty$, $\{G_i\}$ converges to a hybrid formula that balances efficiency and political factors: $\kappa_i v'(G_i) = \left(\frac{1-\rho}{C^G} - \frac{\rho}{C^H}(1-\tau)R_i^G\right)u'(C)\,C$.

B.3 Proof of Proposition 1

Notice that when the transfer is chosen optimally, or when τ is large enough (for simplicity, we consider $\tau \to 1$) under exogenous transfer, the first-order conditions (11) and (14) are consistent, and we recall the condition here:

$$\left[1+\frac{\kappa_i C^G}{(1-\rho)\,\alpha Y_i}\right]R_i^G = 1 \tag{41}$$

B.3.1 Level of Government Capital

Since $\kappa_i > 0$, $(1-\rho) > 0$, $C^G > 0$, and $G_i > 0$, we have $R_i^G < 1$.

Taking the total differential of equations (9), (10), and (41) yields:

$$\frac{dG_i}{dC^G} = \frac{\frac{\kappa_i}{1-\rho}}{1-\alpha^2 A_i G_i^{\alpha-1}} > 0$$

$$\frac{dC^G}{dW^G} = \frac{1}{1 + \frac{\rho}{1-\rho} + \sum_i \frac{\kappa_i (1-R_i^G)}{(1-\rho)(1-\alpha R_i^G)}} > 0$$

Therefore:

$$\frac{dG_i}{dW^G} = \frac{dG_i}{dC^G}\frac{dC^G}{dW^G} > 0$$

Hence, the discrepancy between G_i and its first-best level increases monotonically with government wealth W^G.

B.3.2 Spatial Allocation of Government Capital

Notice that:

$$\begin{aligned}\frac{d\log\left(\overline{A}\right)}{dW^G} &= \alpha \sum_i \frac{G_i}{G}\left(\frac{\sum_i G_i}{\sum_i Y_i} A_i G_i^{\alpha-1} - 1\right)\frac{d\log G_i}{dW^G} \\ &= \mathrm{Cov}_{\frac{G_i}{G}}\left(\frac{\sum_i G_i}{\sum_i Y_i} A_i G_i^{\alpha-1}, \frac{d\log G_i}{dW^G}\right)\end{aligned}$$

This can be interpreted as a covariance between $\frac{\sum_i G_i}{\sum_i Y_i} A_i G_i^{\alpha-1}$ and $\frac{d\log G_i}{dW^G}$ with weights $\frac{G_i}{G}$. Notice that:

$$\begin{aligned}\frac{d\log G_i}{dW^G} &= \frac{1}{G_i}\frac{\frac{\kappa_i}{1-\rho}}{1-\alpha^2 A_i G_i^{\alpha-1}} \\ &= \frac{1}{C^G}\frac{1-\alpha A_i G_i^{\alpha-1}}{1-\alpha^2 A_i G_i^{\alpha-1}}\end{aligned}$$

meaning that $A_i G_i^{\alpha-1}$ and $\frac{d\log G_i}{dW^G}$ are monotonically negatively correlated. So, we have:

$$\frac{d\log\left(\overline{A}\right)}{dW^G} = \mathrm{Cov}_{\frac{G_i}{G}}\left(\frac{\sum_i G_i}{\sum_i Y_i} A_i G_i^{\alpha-1}, \frac{d\log G_i}{dW^G}\right) < 0$$

B.3.3 Government to Household Consumption Ratio (for exogenous transfers T only)

Since $\lim_{W^G \downarrow \underline{W^G}(T)} Y > 0$ and $\lim_{W^G \downarrow \underline{W^G}(T)} C^G = 0$, we have:

$$\lim_{W^G \downarrow \underline{W^G}(T)} \frac{C^G}{C^H} = \lim_{W^G \downarrow \underline{W^G}(T)} \frac{C^G}{(1-\tau)\, Y + W^H} = 0$$

When W^G becomes sufficiently large, $\frac{G}{W^G}$ must be smaller than a finite number, and due to the decreasing returns in the production function, $\lim_{W^G \to \infty} \frac{Y}{W^G} = 0$. Therefore, $\lim_{W^G \to \infty} \frac{C^H}{W^G} = 0$.

Based on equations (9), (41), and $\lim_{W^G\to\infty} \frac{Y}{W^G} = 0$:

$$\lim_{W^G\to\infty} \frac{C^G}{W^G} = \frac{(1-\rho)}{(1-\rho+\kappa)}$$

Therefore, when $W^G \to \infty$: $\frac{C^G}{C^H} \to \infty$.

By continuity, this ensures the existence of the threshold.

Since we are considering the case where τ is sufficiently large, dC^H will be dominated by dC^G, and it is straightforward to see that: $\frac{d\left(\frac{C^G}{C^H}\right)}{dW^G} > 0$.

B.4 Model Details

B.4.1 Local Production and Aggregate Output

Firm's profit-maximization problem is

$$\max_{K_{it},L_{it}} \Pi_{it} = (1-\tau_t)\,Y_{it} - \left(1+\tau_{it}^K\right)\left(1+r_t^K\right)K_{it} - \left(1+\tau_{it}^L\right)w_t L_{it}, \tag{42}$$

where $Y_{it} = A_{it}G_{it}^{\alpha_G}K_{it}^{\alpha_K}L_{it}^{\alpha_L}$. The first-order conditions for firms are

$$(1-\tau_t)\,\alpha_K\frac{Y_{it}}{K_{it}} = \left(1+\tau_{it}^K\right)\left(1+r_t^K\right), \tag{43}$$

$$(1-\tau_t)\,\alpha_L\frac{Y_{it}}{L_{it}} = \left(1+\tau_{it}^L\right)w_t. \tag{44}$$

Thus firm profit equals to

$$\Pi_{it} = (1-\tau_t)\,(1-\alpha_K-\alpha_L)\,Y_{it}. \tag{45}$$

Combine Equation (43) and (44), we obtain the capital labor ratio as

$$\frac{K_{it}}{L_{it}} = \frac{\alpha_K}{\alpha_L}\cdot\frac{w_t}{\left(1+r_t^K\right)}\cdot\frac{1+\tau_{it}^L}{1+\tau_{it}^K}, \tag{46}$$

Combine Equation (43), (44) and (46), firm's inputs in private capital and labor are functions of G_{it}, r_t^K and w_t:

$$K_{it} = \left((1-\tau_t)\left(\frac{\alpha_K}{1+r_t^K}\right)^{1-\alpha_L}\left(\frac{\alpha_L}{w_t}\right)^{\alpha_L}\right)^{\frac{1}{1-\alpha_K-\alpha_L}}\frac{1+\tau_{it}^L}{1+\tau_{it}^K}Z_{it}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}, \tag{47}$$

$$L_{it} = \left((1-\tau_t)\left(\frac{\alpha_K}{1+r_t^K}\right)^{\alpha_K}\left(\frac{\alpha_L}{w_t}\right)^{1-\alpha_K}\right)^{\frac{1}{1-\alpha_K-\alpha_L}}Z_{it}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}. \tag{48}$$

where we denote $Z_{it} \equiv \left(\frac{A_{it}}{\left(1+\tau_{it}^{K}\right)^{\alpha_K} \left(1+\tau_{it}^{L}\right)^{1-\alpha_K}} \right)^{\frac{1}{1-\alpha_K-\alpha_L}}$

Then, the output follows:

$$Y_{it} = \left((1-\tau_t)^{\alpha_K+\alpha_L} \left(\frac{\alpha_K}{1+r_t^K} \right)^{\alpha_K} \left(\frac{\alpha_L}{w_t} \right)^{\alpha_L} \right)^{\frac{1}{1-\alpha_K-\alpha_L}} \left(1+\tau_{it}^{L}\right) Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}. \tag{49}$$

We assume the total labor supply L_t is exogenous. Then, the equation (43) can be rewritten as

$$\begin{aligned} Y_{it} &= \left(\frac{\alpha_K (1-\tau_t)}{\left(1+\tau_{it}^{K}\right)\left(1+r_t^K\right)} \right)^{\frac{\alpha_K}{1-\alpha_K}} A_{it}^{\frac{1}{1-\alpha_K}} G_{it}^{\frac{\alpha_G}{1-\alpha_K}} L_{it}^{\frac{\alpha_L}{1-\alpha_K}} \\ &= \left(\frac{\alpha_K (1-\tau_t)}{1+r_t^K} \right)^{\frac{\alpha_K}{1-\alpha_K}} \left(\frac{L_t}{\sum_{i=1}^{N} \left(Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}} \right)} \right)^{\frac{\alpha_L}{1-\alpha_K}} Z_{it} \left(1+\tau_{it}^{L}\right) G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}} \end{aligned}$$

Local output in Equation (49) is a function of prices and local government capital. We denote it by $Y_{it} = \mathcal{Y}_{it}\left(G_{it}, (1+r_t^K), w_t\right)$. An increase in G_{it} has two effects on output: a direct effect that boosts local output in region i, and a general equilibrium effect that reduces output in all regions. Specifically,

$$\frac{\partial \mathcal{Y}_{it}}{\partial G_{it}} = \frac{\alpha_G}{1-\alpha_K-\alpha_L} \frac{Y_{it}}{G_{it}} > 0, \tag{50}$$

$$\frac{\partial \mathcal{Y}_{jt}}{\partial (1+r_t^K)} \frac{\partial (1+r_t^K)}{\partial G_{it}} < 0, \quad \frac{\partial \mathcal{Y}_{jt}}{\partial w_t} \frac{\partial w_t}{\partial G_{it}} < 0. \tag{51}$$

We assume that the total labor supply, i.e. L_t, is exogenous. Then, $L_{it} \propto Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}$ implies

$$L_{it} = \frac{Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}}{\sum_{i=1}^{N} \left(Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}} \right)} L_t, \tag{52}$$

Combine with Equation (51), we express the wage rate as a function of $\{G_{it}\}_{i=1}^{N}$:

$$w_t = \frac{\left(\left(\frac{\alpha_K}{(1+\tau_{it}^K)(1+r_t^K)}\right)^{\alpha_K}\left(\frac{\alpha_L}{(1+\tau_{it}^L)}\right)^{1-\alpha_K}(1-\tau_{it})\,A_{it}\right)^{\frac{1}{1-\alpha_K}} G_{it}^{\frac{\alpha_G}{1-\alpha_K}}}{\left(\frac{Z_{it}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}}{\sum_{i=1}^{N}\left(Z_{it}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}\right)}L_t\right)^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K}}}$$
$$= \frac{\left(\left(\frac{\alpha_K}{(1+\tau_{it}^K)(1+r_t^K)}\right)^{\alpha_K}\left(\frac{\alpha_L}{(1+\tau_{it}^L)}\right)^{1-\alpha_K}(1-\tau_{it})\,A_{it}\right)^{\frac{1}{1-\alpha_K}}}{\left(\frac{Z_{it}L_t}{\sum_{i=1}^{N}\left(Z_{it}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}\right)}\right)^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K}}}. \tag{53}$$

The derivative of w_t w.r.t. G_{it} is:

$$\frac{\partial w_t}{\partial G_{it}} = \frac{1-\alpha_K-\alpha_L}{1-\alpha_K}\frac{w_t}{\sum_{i=1}^{N}\left(Z_{it}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}\right)}\cdot Z_{it}\frac{\alpha_G}{1-\alpha_K-\alpha_L}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}-1}$$
$$= \frac{\alpha_G}{1-\alpha_K}\frac{Z_{it}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}}{\sum_{i=1}^{N}\left(Z_{it}G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}\right)}\frac{w_t}{G_{it}} \tag{54}$$
$$= \frac{\alpha_G}{1-\alpha_K}\frac{L_{it}}{L_t}\frac{w_t}{G_{it}}$$

thus the elasticity of wage rate to government investment, i.e. $\frac{\partial \log w_t}{\partial \log G_{it}}$, is increasing in region's size.

An increase in government capital in region i would affect the output in all regions through the general equilibrium effect. Specifically, we have:

$$\frac{\partial \mathcal{Y}_{nt}}{\partial w_t}\frac{\partial w_t}{\partial G_{it}} = \left(-\frac{\alpha_L}{1-\alpha_K-\alpha_L}\frac{Y_{nt}}{w_t}\right)\cdot\left(\frac{\alpha_G}{1-\alpha_K}\frac{L_{it}}{L_t}\frac{w_t}{G_{it}}\right)$$
$$= -\frac{\alpha_L\alpha_G}{(1-\alpha_K-\alpha_L)(1-\alpha_K)}\frac{L_{it}}{L_t}\frac{Y_{nt}}{G_{it}}. \tag{55}$$

Combine with the direct effect $\frac{\partial \mathcal{Y}_{it}}{\partial G_{it}} = \frac{\alpha_G}{1-\alpha_K-\alpha_L}\frac{Y_{it}}{G_{it}}$, we can express the impact of G_{it} on Y_{nt} as:

$$\frac{\partial Y_{nt}}{\partial G_{it}} = \begin{cases} -\frac{\alpha_G\alpha_L}{(1-\alpha_K-\alpha_L)(1-\alpha_K)}\frac{L_{it}}{L_t}\frac{Y_{nt}}{G_{it}} & n \neq i \\ \frac{\alpha_G}{1-\alpha_K-\alpha_L}\left(1-\frac{\alpha_L}{1-\alpha_K}\frac{L_{it}}{L_t}\right)\frac{Y_{it}}{G_{it}} & n = i \end{cases} \tag{56}$$

Then,

$$
\begin{aligned}
Y_t &= \sum_{i=1}^{N} Y_{it} \\
&= \left(\frac{\alpha_K (1-\tau_t)}{1+r_t^K}\right)^{\frac{\alpha_K}{1-\alpha_K}} \left(\frac{L_t}{\sum_{i=1}^{N}\left(Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}\right)}\right)^{\frac{\alpha_L}{1-\alpha_K}} \sum_{i=1}^{N} Z_{it}\left(1+\tau_{it}^L\right) G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial Y_t}{\partial G_{it}} &= \sum_{n=1}^{N} \frac{\partial Y_{nt}}{\partial G_{it}} \\
&= \frac{\alpha_G}{1-\alpha_K-\alpha_L} \frac{Y_{it}}{G_{it}} - \frac{\alpha_G \alpha_L}{(1-\alpha_K-\alpha_L)(1-\alpha_K)} \frac{L_{it}}{L_t} \frac{Y_t}{G_{it}}
\end{aligned}
$$

If we further assume $\tau_{it}^L = \tau^L$, we redefine $Z_{it} = \left(\frac{A_{it}}{(1+\tau_{it}^K)^{\alpha_K}}\right)^{\frac{1}{1-\alpha_K-\alpha_L}}$ and then

$$
Y_t = \left(\frac{\alpha_K (1-\tau_t)}{1+r_t^K}\right)^{\frac{\alpha_K}{1-\alpha_K}} L_t^{\frac{\alpha_L}{1-\alpha_K}} \left(\sum_i \left(Z_{it} G_{it}^{\frac{\alpha_G}{1-\alpha_K-\alpha_L}}\right)\right)^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K}}. \tag{57}
$$

When G_{it} is efficiently allocated – i.e., $G_{it} \propto Z_{it}^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K-\alpha_L-\alpha_G}}$, we obtain the maximized A_t as

$$
\bar{A}_t = \frac{\left(\sum_{i=1}^{N} Z_{it}^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K-\alpha_G-\alpha_L}}\right)^{1-\alpha_G-\alpha_L}}{\left(\sum_{i=1}^{N} \frac{1}{1+\tau_{it}^K} Z_{it}^{\frac{1-\alpha_K-\alpha_L}{1-\alpha_K-\alpha_G-\alpha_L}}\right)^{\alpha_K}}. \tag{58}
$$

B.4.2 Central Planner Problem

Central planner chooses $\left\{C_{t+j}^G, G_{it+j+1}, D_{t+j+1}\right\}_{i=1,j=0}^{N,\infty}$ to maximize

$$
U_t^C = \omega \log\left(\left(C_{t-1,t}^H\right)^{\rho}\left(C_t^G\right)^{1-\rho}\right) + \sum_{j=0}^{\infty} \beta_C^j \left[U_{t+j}^H + \sum_{i=1}^{N} \kappa_{it+j} \log G_{it+j+1}\right],
$$

where ω is assumed to be $\frac{\beta}{\beta_C}$

$$
U_{t+j}^H = \log\left(\left(C_{t+j,t+j}^H\right)^{\rho}\left(C_{t+j}^G\right)^{1-\rho}\right) + \beta \log\left(\left(C_{t+j,t+j+1}^H\right)^{\rho}\left(C_{t+j+1}^G\right)^{1-\rho}\right)
$$

and the budget constraint is:

$$C^G_{t+j}+\sum_{i=1}^{N} q^G_{t+j} I^G_{it+j}+T_{t+j}+\Phi_{t+j}=\tau_{t+j}Y_{t+j}+D_{t+j+1}-\left(1+r^D_{t+j}\right)D_{t+j}-\frac{\psi^D_{t+j}}{2}\left(D_{t+j+1}-\bar{D}_{t+j+1}\right)^2$$

The objective function can be rewritten as

$$\sum_{j=0}^{\infty}\beta_C^j\left[(1-\rho)\left(1+\frac{\beta}{\beta_C}\right)\log C^G_{t+j}+\rho\left(\frac{\beta}{\beta_C}\log C^H_{t+j-1,t+j}+\log C^H_{t+j,t+j}\right)+\sum_{i=1}^{N}\kappa_{it+j}\log G_{it+j+1}\right]$$

where $C^H_{t+j-1,t+j}=\frac{\beta(1+r_{t+j})}{1+\beta}\left(Y^H_{t+j-1}+T_{t+j-1}\right)$, $C^H_{t+j,t+j}=\frac{1}{1+\beta}\left(Y^H_{t+j}+T_{t+j}\right)$. Notice that households total income include both firm profits and wages, so it follows

$$\begin{aligned}Y^H_{t+j}&=\sum_i\left(\Pi_{it+j}+w_{t+j}L_{it+j}\right)\\&=(1-\tau_{t+j})\sum_i\left(\left(1-\alpha_K-\frac{\tau^L_{it+j}}{1+\tau^L_{it+j}}\alpha_L\right)Y_{it+j}\right)\end{aligned}\tag{59}$$

We prepare the marginal effect of G_{it+j} on Y^H_{t+j} as follows for further analysis

$$\frac{\partial Y^H_{t+j}}{\partial G_{it+j}}=(1-\tau_{t+j})\sum_{n=1}^{N}\left(\left(1-\alpha_K-\frac{\tau^L_{nt+j}}{1+\tau^L_{nt+j}}\alpha_L\right)\frac{\partial Y_{n,t+j}}{\partial G_{it+j}}\right).\tag{60}$$

The initial state is W^H_t, G_{it}, K_{it}, D_t. This problem is time-consistent with log-preference for the households.

The Lagrangian equation is

$$\begin{aligned}\mathcal{L}=\sum_{j=0}^{\infty}\beta_C^j\Bigg[&(1-\rho)\left(1+\frac{\beta}{\beta_C}\right)\log C^G_{t+j}+\rho\left(\frac{\beta}{\beta_C}\log C^H_{t+j-1,t+j}+\log C^H_{t+j,t+j}\right)+\sum_{i=1}^{N}\kappa_{it+j}\log G_{it+j+1}\\&+\lambda_{t+j}\left(\tau_{t+j}\sum_{i=1}^{N}Y_{it+j}+D_{t+j+1}-\left(1+r^D_{t+j}\right)D_{t+j}-\frac{\psi^D_{t+j}}{2}\left(D_{t+j+1}-\bar{D}_{t+j+1}\right)^2-C^G_{t+j}-\sum_{i=1}^{N}q^G_{t+j}I^G_{it+j}-T_{t+j}-\Phi_{t+j}\right)\Bigg]\end{aligned}$$

The first-order condition w.r.t C^G_{t+j} is

$$0=(1-\rho)\left(1+\frac{\beta}{\beta_C}\right)\frac{1}{C^G_{t+j}}-\lambda_{t+j},\tag{61}$$

The first-order conditions w.r.t. D_{t+j+1} is

$$0=\lambda_{t+j}\left(1-\psi^D_{t+j}\left(D_{t+j+1}-\bar{D}_{t+j+1}\right)\right)-\beta_C\lambda_{t+1}\left(1+r^D_{t+j+1}\right),$$

which can be simplified to:

$$\frac{C_{t+j+1}^{G}}{C_{t+j}^{G}} = \beta_C \frac{1 + r_{t+j+1}^{D}}{1 - \psi_{t+j}^{D}\left(D_{t+j+1} - \bar{D}_{t+j+1}\right)}. \tag{62}$$

The first-order conditions w.r.t. G_{it+j+1} for $i = 1, \ldots, N$ are

$$\begin{aligned} 0 =& \beta_C \rho \left(1+\beta\right) \frac{1}{Y_{t+j+1}^{H}} \frac{\partial Y_{t+j+1}^{H}}{\partial G_{it+j+1}} + \kappa_{it+j} \frac{1}{G_{it+j+1}} \\ & - \lambda_{t+j} q_{t+j}^{G} + \beta_C \lambda_{t+j+1} \left(\tau_{t+j+1} \sum_{n=1}^{N} \frac{\partial Y_{nt+j+1}}{\partial G_{it+j+1}} + q_{t+j+1}^{G}\left(1-\delta_G\right) \right) \end{aligned}$$

where $\frac{\partial Y_{nt}}{\partial G_{it}}$ follows equation (56) and

$$\frac{\partial Y_t^H}{\partial G_{it}} = \sum_{n=1}^{N} \left(\left(1-\tau_t\right) \left(1 - \alpha_K - \frac{\tau_{n,t}^{L}}{1+\tau_{n,t}^{L}} \alpha_L \right) \frac{\partial Y_{nt}}{\partial G_{it}} \right),$$

The first-order condition w.r.t. G_{it+j+1} can be further simplified to

$$\begin{aligned} \frac{C_{t+j+1}^{G}}{C_{t+j}^{G}} =& \frac{1}{\left(1-\rho\right)\left(1+\frac{\beta}{\beta_C}\right)} \left(\beta_C \rho \frac{C_{t+j+1}^{G}}{C_{t+j+1,t+j+1}^{H}} \frac{\partial Y_{t+j+1}^{H}}{q_{t+j}^{G} \partial G_{it+j+1}} + \kappa_{it+j} \frac{C_{t+j+1}^{G}}{q_{t+j}^{G} G_{it+j+1}} \right) \\ & + \beta_C \left(\sum_{n=1}^{N} \tau_{t+j+1} \frac{\partial Y_{n,t+j+1}}{q_{t+j}^{G} \partial G_{it+j+1}} + \frac{q_{t+j+1}^{G}}{q_{t+j}^{G}} \left(1-\delta_G\right) \right) \end{aligned} \tag{63}$$