Global Production Networks with Global Uncertainty*

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Abstract

We investigate how global uncertainty affects sourcing decisions and aggregate trade. We develop a quantitative trade model with anticipatory sourcing building on the Eaton-Kortum/Caliendo-Parro framework. The model incorporates uncertainty in productivity and tariffs, enabling anticipatory effects and interactions between shocks. We show that bilateral sourcing is summarized by pairwise sufficient statistics that capture the influence of fundamental factors and global uncertainty. These sufficient statistics form a fixed-point problem whose solution yields closed-form sourcing expressions. We characterize the solution analytically and implement the model quantitatively. We find that the mere possibility of future tariff hikes reduces trade through anticipatory sourcing shifts, and that these anticipatory shifts account for most of the trade effect of an uncertain Trump trade war. Shocks from different sources interact: an uncertain trade war amplifies losses from productivity uncertainty by hindering hedging through trade.

Keywords: input-output linkages, international trade, uncertainty, international risk diversification, uncertain trade war

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1 Introduction

The networks of global production, forged over four decades of integration, now face an era of unprecedented uncertainty. With pandemic and war-related production disruptions and shipping delays, sanctions, geopolitical uncertainty, and a "Trump trade war 2.0," firms around the world can no longer take the production and arrival of their inputs—and a stable tariff schedule upon arrival—for granted. How does this global rise in uncertainty affect firms' sourcing behavior and aggregate trade? How does uncertainty from different sources, such as Trump's trade policies and productivity shocks, compare in their impacts on trade and welfare? How do they interact with each other?

Answering these questions requires taking seriously two important aspects of how uncertainty shapes the global production networks. First, firms and consumers may change their sourcing behavior in anticipation of shocks. Thus, even if the most devastating policy does not materialize, its mere possibility can distort firms' supplier choice, and hence global trade. Second, global uncertainty comes from multiple sources, from production at the country-sector level to pairwise tariff changes, with their impacts intertwined due to production and trade linkages.

We incorporate these two features tractably into a class of quantitative trade models that build on the Eaton and Kortum (2002) structure. We show that in the presence of global uncertainty, the appeal of products from any country i to any other country n can be characterized by one sufficient statistic. In deterministic trade models, this sufficient statistic depends on comparative advantage, factor prices, and geography. In our model, it is also shaped by global uncertainty. Since the fundamental factors and uncertainty in different parts of the world are connected through trade and production, these pairwise sufficient statistics are linked to each other through a fixed point. The solution to this fixed-point problem informs when, relative to a deterministic model, uncertainty increases or decreases the appeal of a supplier to users. Solving the fixed point is challenging in the presence of high-dimensional sources of uncertainty. We develop a novel solution method, which can handle many shocks with continuous support alongside large, discrete shocks (such as different trade policy regimes).

Quantification of the model reveals three key findings. First, while both the risk of a trade war and productivity shocks represent fundamental economic uncertainty, they exert distinct influences on trade. In particular, anticipating a possible tariff war, firms switch from foreign suppliers to domestic ones. Thus, even if in the end the trade war does not materialize, trade would be dampened. On the other hand, facing uncertain future productivity, firms have an incentive to diversify risk by sourcing more from foreign suppliers, which tends to increase trade intensity. Second, unilateral Trump tariffs benefit the U.S. through the standard terms-of-trade effect. An unrealized threat, on the other hand, causes damage:

if all threatened tariffs end up being waived, the U.S. would lose 0.3% in real income due to anticipatory sourcing behavior. Even if these tariffs remain, a quarter of the terms-of-trade benefits would be offset by the change in ex-ante sourcing decisions. Third, uncertainty from different sources can either insure against or amplify the impacts of each other. In our sample, China is among the countries whose productivity shocks are less correlated with those of other countries, so its presence in the world economy provides insurance value. Tariff uncertainty also interacts with productivity uncertainty. In particular, by making it harder for countries to hedge against their own productivity risks through trade, an uncertain trade war amplifies the impacts of productivity uncertainty on welfare and income volatility.

In Section 2, we characterize the sourcing decision under uncertainty in a bare-bones, one-sector Eaton and Kortum (2002) model, which we later extend. The representative consumer in each country chooses the supplier country for each of the varieties $v \in [0, 1]$ to maximize expected utility. This choice is made after learning about the idiosyncratic sourcing efficiency of each variety from each country—a variable governed by a Fréchet distribution—but before the realization of tariffs and the productivity of origin countries. The choice of suppliers is therefore shaped by the structure of global uncertainty. After this choice, uncertainty resolves, and firms produce and sell to their customers. Given the realization, market clearing conditions determine the equilibrium prices—wages, production costs, and the price of the consumption bundle, which in turn shape the quantity purchased by consumers of each variety.

Solving this sourcing problem presents a key challenge: in choosing the supplier for each variety, the buyer must consider the relative costs of origin countries under all tariff and productivity scenarios—and in each scenario the costs are determined in general equilibrium. Moreover, the choice for each variety is shaped by the covariance between the cost of that variety and consumer real income. Since the real income depends on the price of other varieties, this introduces interdependence in supplier choice among varieties. We construct a probabilistic solution to the sourcing problem. We show that, under the standard CES aggregator, the sourcing decisions are characterized by a set of pairwise sufficient statistics, which measure the average appeal of products from an origin country for a destination country. These sufficient statistics capture the combined general equilibrium effects of tariff and productivity uncertainty from that origin and around the world. *Conditioning on these statistics*, sourcing decisions are independent across varieties and take the same analytical form as the sourcing probability in the Eaton and Kortum (2002) model. Our model therefore generalizes the Eaton and Kortum model to accommodate sourcing under arbitrary uncertainty regarding model fundamentals.

Our setup leads to a natural distinction between short-term and long-term trade elasticities. The short-term elasticity governs, after the resolution of uncertainty, how the quantity purchased of each variety responds to the variety's relative price. Since at that stage, consumers have decided where to source each variety, this elasticity arises from the substitution by consumers among varieties. It is therefore governed by σ, the elasticity of substitution between varieties. In the long term, consumers can adjust both the quantity of purchase and their supplier choice. The long-term elasticity of trade flows with respect to sourcing cost is thus governed by the producer heterogeneity parameter θ, the dispersion of the Fréchet distribution. With this distinction, we will be able to map the model to recent estimates of both short-term and long-term elasticities.

In Section 3, we close the one-sector model in general equilibrium and solve the equilibrium through perturbation to derive insights on how global uncertainty shapes sourcing decisions and equilibrium prices. Our closed-form characterization of sourcing decisions illuminates when a supplier is more appealing because of uncertainty. All else equal, larger (mean-preserving) uncertainty in the cost of a supplier can reduce the appeal of the supplier due to risk aversion or increase it due to the option value of uncertainty under CES aggregation. The former force is more likely to dominate for suppliers with a large expenditure share. Higher correlation between a supplier's cost and that of other suppliers tends to decrease its appeal due to a lack of diversification value. Trade cost and productivity uncertainty interact: An increase in trade costs can lower the correlation in production costs between countries arising from production linkages. This, in turn, increases the diversification benefit of trade, partially offsetting the reduction in trade due to higher trade costs.

We quantify the model in Section 4. Building on the Eaton and Kortum micro-foundation, our setup for sourcing under uncertainty can be readily integrated into workhorse quantitative trade models relying on similar underlying structures, such as the model of input-output linkages by Caliendo and Parro (2015), the model of multinational production by Ramondo and Rodríguez-Clare (2013), the model of global value chains by Antràs and De Gortari (2020), and the routing model by Allen and Arkolakis (2022). To incorporate the role of global production networks, we extend the baseline model to include input-output linkages à la Caliendo and Parro (2015). Our quantification incorporates two types of uncertainty: country-industry-specific productivity shocks, and the "Reciprocal Tariffs" policy initiated by the Trump administration as a case study of heightened tariff uncertainty.

We construct productivity uncertainty based on the estimated productivity covariance between pairs of country-sectors from the World Input-Output Database. We incorporate tariff risks by assuming that each country independently draws a tariff schedule vis-à-vis the U.S. Each schedule features a binary outcome: the tariff is lowered to 10% with a probability inferred from prediction markets regarding the likelihood of a country achieving successful negotiations with the U.S.; with the complementary probability, the tariff will be set at the "Reciprocal Tariffs." These scenarios represent large discrete shocks, so for accuracy, we solve the sourcing problem for tariff risk through enumeration of possible scenarios. In contrast, the shocks to productivity have continuous support and are generally much smaller, so they can be approximated with a local method. Our solution method handles productivity shocks through a perturbation solution and tariff risks through enumeration in a unified setting, enabling us to capture both types of shocks accurately and tractably.

Both sources of uncertainty have tangible impacts. Import shares are about 0.2% higher on average due to productivity uncertainty alone, with large heterogeneity that depends on the covariance of global shocks—about 0.8% for Turkey and around 0% for Norway. Even if no additional tariffs are imposed, the possibility of facing the "Reciprocal Tariffs" lowers import shares of Canada and Mexico by 5% and that of China by 2%. Factoring in the average realization of tariffs between the ceiling (i.e., April 2nd tariff levels) and the floor (10% extra compared to before April 2nd), the impacts increase to around 8% for Mexico and Canada and 3% for China. For other countries, the effects are smaller but remain nontrivial.

In total, the "Reciprocal Tariffs" lead to around 0.7% gains for the U.S., and around 1.5% losses for Canada, Ireland, and Mexico. However, if the tariffs are not realized, the U.S. will experience a 0.3% loss due to ex-ante sourcing adjustments. This occurs because the heightened tariffs, if realized, will increase U.S. production costs through higher prices for imported goods and higher wages through terms-of-trade adjustments. In anticipation of the elevated prices of U.S. goods, foreign buyers switch away from U.S. suppliers, which leads to U.S. losses if the U.S. does not follow through with the tariff threats. The U.S. achieves net gains only if tariffs are realized and generate tariff revenue; even in this case, the adjustments in ex-ante sourcing offset the ex-post gains by more than a quarter.

This paper contributes to the literature on the relationship between trade and uncertainty, dating at least as far back as Ruffin (1974) and Helpman and Razin (1978). Recent works have examined how, in the presence of shocks, trade affects the level and volatility of income (Caselli, Koren, Lisicky and Tenreyro, 2020; Allen and Atkin, 2022), how uncertainty affects sourcing (Adamopoulos and Leibovici, 2024; Castro-Vincenzi, Khanna, Morales and Pandalai-Nayar, 2024) and spatial reallocation (Fan, Hong and Parro, 2023), how global production networks shape business-cycle co-movement (Huo, Levchenko and Pandalai-Nayar, 2025), and the relationship between intra- and inter-temporal trade (Fitzgerald, 2024; Alessandria, Bai and Woo, 2024; Caliendo, Kortum and Parro, 2025).

Within this literature, our modeling approach is closely related to Caselli, Koren, Lisicky and Tenreyro (2020) and Allen and Atkin (2022). In both works, producers commit to factor allocation (labor in Caselli et al., 2020 and land in Allen and Atkin, 2022) before uncertainty resolves. Since producers do not make ex-ante sourcing decisions, uncertainty affects bilateral trade mainly through origin and destination effects. Also related are Adamopoulos and Leibovici (2024) and Castro-Vincenzi et al. (2024), in which firms choose both the suppliers and the order quantity before the shock materializes. Differing from these works, in our model, agents choose their supplier ex ante but are free to adjust their order size ex post. This gives rise to two features. Relative to Caselli et al. (2020) and Allen and Atkin (2022), our novel feature is to allow *bilateral* trade intensity to respond to bilateral uncertainty beyond the origin and destination effects. Relative to Adamopoulos and Leibovici (2024) and Castro-Vincenzi et al. (2024), our model differs in that it allows ex-post quantity responses and accommodates vertical linkages.[1] Both features are important for understanding the effect of uncertainty on the structure of global production networks.

Second, our perturbation solution of the economy builds on works such as Allen, Arkolakis and Takahashi (2020), Adao, Arkolakis and Esposito (2019), Huo, Levchenko and Pandalai-Nayar (2025), Baqaee and Farhi (2024), and Kleinman, Liu and Redding (2024a,b). We extend this framework to incorporate sourcing in anticipation of uncertainty. Our closed-form perturbation solution admits a formula that takes as inputs the covariance of fundamental shocks and the risk aversion parameter, as well as observable shares and trade elasticities. This allows researchers to gauge the influence of global uncertainty on equilibrium outcomes without solving the nonlinear model. In quantifying the full model, we show how to obtain the perturbation solution by combining implicit differentiation and automatic differentiation, circumventing the derivation of second-order exposure matrices in large trade models as well as the practical challenge of constructing and storing these matrices.

In contemporary work, Kleinman, Liu and Redding (2025) also develop a second-order perturbation solution to study trade under uncertainty. Their paper offers one perspective on anticipatory trade decisions, developing a constant elasticity trade model where trading partners invest in bilateral capital ex ante to facilitate trade ex post. Building on the Eaton and Kortum (2002) framework, our paper provides a complementary perspective. Here, the ex-ante decision is not a bilateral investment, but rather an extensive-margin sourcing choice. This fundamental difference in modeling the ex-ante decision naturally leads to distinct calibration strategies and possible avenues for future generalizations.[2] The two papers conduct different and complementary quantitative exercises as well.

Third, we add to recent studies on the welfare implications of the "Reciprocal Tariffs" (Ignatenko, Lashkaripour, Macedoni and Simonovska, 2025; Rodríguez-Clare, Ulate and Vasquez, 2025) by showing that a sizable part of the impact materializes through anticipatory sourcing. The key mechanism—that firms make sourcing decisions in anticipation of future trade environments—is supported by recent empirical and quantitative evidence on how policy uncertainty or shipping delays affect bilateral trade, see, e.g., Handley and Limão (2015, 2017), Pierce and Schott (2016), Alessandria, Khan, Khederlarian, Ruhl and Steinberg

[1] In models where firms make ex-ante quantity choices, when different production stages face different shock realizations, one needs to impose a rationing rule for market clearing. In our model, ex-post prices allocate the quantity choice at each production sector, as in deterministic trade models.

[2] For example, in the model of bilateral capital, trade costs have two components: one exogenous and one depending on investment; in our model of ex-ante sourcing, trade costs are exogenous, and ex-post trade depends on the ex-ante choice because the extensive-margin sourcing decision is made before the resolution of uncertainty.

(2025), and Esposito, Heise and Blaum (2024). We contribute to this body of evidence by developing a tractable multi-country framework. Instead of focusing on uncertainty specific to one origin or one destination country, our framework incorporates global uncertainty, which stems from multiple exogenous sources and is shaped endogenously by trade and production linkages.

Last but not least, recent works on supply chain resilience have studied the impacts of adverse shocks in production networks (Carvalho, Nirei, Saito and Tahbaz-Salehi, 2021; Boehm, Flaaen and Pandalai-Nayar, 2019; Grossman, Helpman and Redding, 2024; Kopytov, Mishra, Nimark and Taschereau-Dumouchel, 2024) and discussed the implications for optimal policy (Grossman, Helpman and Lhuillier, 2023). Relative to these works, our main difference is the focus on a multi-country general equilibrium setup.[3]

2 Sourcing Decision under Global Uncertainty

For expositional clarity, we begin by presenting a one-sector, partial equilibrium sourcing model with uncertainty in consumer income and origin-specific sourcing costs. This model and its solution serve as the building block for subsequent general equilibrium models. Without aggregate uncertainty, the model reduces to the standard sourcing problem in Eaton and Kortum (2002) (EK).

There are N regions, indexed by n or i. Representative consumers in each region n have stochastic income I. Their preferences follow Constant Relative Risk Aversion (CRRA), represented by the following utility function (suppressing the destination index n for clarity):

$$U(C) = \mathbb{E}\Big[\frac{1}{1-\gamma}C^{1-\gamma}\Big],$$

where $\gamma > 1$ is the relative risk aversion parameter,[4] and the expectation $\mathbb{E}$ is taken with respect to the aggregate uncertainty introduced below. C is the consumption bundle aggregated from a unit mass of varieties $v \in [0,1]$ through the following CES function:

$$C = \Big[\int_0^1 [q(v)]^{\frac{\sigma-1}{\sigma}} dv\Big]^{\frac{\sigma}{\sigma-1}},$$

where $q(v)$ is the quantity consumed and $\sigma > 1$ is the consumer elasticity of substitution.

[3]Among these works, our quantitative focus is closer to Kopytov et al. (2024), who study in a closed economy how uncertainty affects network linkages and welfare. In addition to the multi-country setting, our paper differs in two aspects. First, our two-stage sourcing setup naturally generates short- and long-term elasticities, whereas Kopytov et al. (2024) assumes unit elasticity. Our theory shows that the differences in the two elasticities, which we discipline with empirical estimates, are crucial in determining the impact of uncertainty on sourcing. Second, Kopytov et al. (2024) focuses on the efficient benchmark, whereas in our international setting with incomplete markets, outcomes for individual countries (and the aggregate economy) need not be efficient. A country's response to global uncertainty thus generates welfare effects on other countries.

[4]$\gamma > 1$ ensures that agents' ex-ante sourcing is a convex programming problem under arbitrary global uncertainty.

Each variety can be sourced from one of the N regions. The cost of sourcing variety v from region i is $c_i / z_i(v)$, where c_i is a stochastic origin-specific *aggregate* cost component, and $z_i(v)$ is the origin-variety-specific *idiosyncratic* efficiency that is independent of $\{c_i\}$ and independently drawn across (i, v).[5]

We make the following timing assumption: For each variety v, the sourcing location is chosen after observing variety-specific productivity $\{z_i(v)\}_{i=1,v\in[0,1]}^{N}$ but before the realization of aggregate shocks $\{c_i\}_{i=1}^{N}$. This captures the idea that buyer–supplier relationships are typically long-term: they are formed with some knowledge of the counterparty, yet under considerable uncertainty about macroeconomic conditions. The sourcing decision unfolds in two stages: (1) ex ante, before the realization of aggregate shocks, the sourcing location for each variety is determined; (2) ex post, after shocks are realized and given the chosen sourcing locations, the quantity of each variety is determined.

Formally, the ex-ante sourcing problem is to select sourcing locations for each variety, denoted by a measurable function $\{\iota(v)\}_{v\in[0,1]}$ that maps variety v to its source origin $\iota(v) \in \{1, 2, ..., N\}$, to maximize the expected utility:

$$\max_{\{\iota(v)\}_{v\in[0,1]}} U \equiv \mathbb{E}\Big[\frac{1}{1-\gamma} C^{1-\gamma}\Big]$$
$$s.t. \quad C = \frac{I}{P}, \quad P = \Big[\int_0^1 \Big(\frac{c_{\iota(v)}}{z_{\iota(v)}(v)}\Big)^{1-\sigma} \mathrm{d}v\Big]^{\frac{1}{1-\sigma}}, \tag{1}$$

where I is total income, P is the ideal price index under optimal ex-post sourcing quantities. The expectations are taken over income I and sourcing costs $\{c_i\}_{i=1}^{N}$. Following EK, to simplify aggregation, we assume the productivities $z_i(v)$ are independently and identically distributed according to a Fréchet distribution:

$$F_{z,i}(z) = e^{-T_i z^{-\theta}},$$

with $\theta > \sigma - 1$ and $T_i > 0$ a scale parameter.

This problem is challenging because the aggregate cost component $c_{\iota(v)}$ leads to correlation in the price of varieties sourced from the same location. Under this correlation, the choice of where to source a variety v depends on where all other varieties are sourced. This interdependence makes the solution strategy in the standard Eaton-Kortum framework, which solves the sourcing decision independently across varieties, inapplicable.

One of our contributions is to demonstrate that, given the multiplicative structure between aggregate and idiosyncratic cost components and the CES aggregation, this interdependence can be captured by a set of finite-dimensional real numbers. These numbers reflect the additional hedging motive— beyond minimizing expected costs—for sourcing a marginal variety from each origin. Once determined, these real numbers enter the sourc-

[5] To be precise, c_i, short for c_{ni}, captures not only production costs in i but also the cost of delivering to n.

ing problem as non-stochastic cost shifters, and the remaining sourcing problem can then be solved as if the source location is chosen independently across varieties. These real numbers, in turn, are determined by a fixed-point problem in conjunction with the conditional sourcing decision.

Specifically, for a sourcing strategy $\{\iota(v)\}_{v\in[0,1]}$, determined after observing $\{z_i(v)\}$, let $\mathcal{V}_i = \{v \in [0,1] : \iota(v) = i\}$ denote the set of varieties sourced from i and define

$$Z_i \equiv \int_{\mathcal{V}_i} [z_i(v)]^{\sigma-1} \mathrm{d}v,$$

which measures the effective productivity among varieties sourced from i. Then, the price index can be written as

$$P = \left[\sum_i [c_i]^{1-\sigma} Z_i \right]^{\frac{1}{1-\sigma}}. \tag{2}$$

Note that conditional on the sourcing strategy, $\{Z_i\}$ are non-stochastic, allowing us to derive the marginal effect of Z_i on *expected* utility:

$$\lambda_i \equiv \frac{\mathrm{d}U}{\mathrm{d}Z_i} = \mathbb{E}\left[I^{1-\gamma} P^{\gamma+\sigma-2} \frac{[c_i]^{1-\sigma}}{\sigma-1} \right].$$

The optimal sourcing rule must assign each variety v to the origin i that provides the largest marginal contribution to expected utility. A marginal shift of a set of varieties $\mathcal{B}$ from origin i to origin m changes expected utility by $-\lambda_i \int_{\mathcal{B}} [z_i(v)]^{\sigma-1} dv + \lambda_m \int_{\mathcal{B}} [z_m(v)]^{\sigma-1} dv$. For optimality, no such beneficial reshuffling can exist. Therefore, for any measurable $\mathcal{B} \subset \mathcal{V}_i$, it must be that:

$$\lambda_i \int_{\mathcal{B}} [z_i(v)]^{\sigma-1} \mathrm{d}v \geq \lambda_m \int_{\mathcal{B}} [z_m(v)]^{\sigma-1} \mathrm{d}v, \forall m.$$

Choosing $\mathcal{B}$ to be the neighborhood around a particular v and letting the measure of $\mathcal{B}$ converge to zero, it follows that i is chosen for variety v if and only if:[6]

$$\lambda_i [z_i(v)]^{\sigma-1} \geq \lambda_m [z_m(v)]^{\sigma-1}, \forall m. \tag{3}$$

Since $\{\lambda_i\}$ are non-stochastic, conditional on $\{\lambda_i\}$, the remainder of the sourcing problem follows the strategy of EK. In particular, applying properties of the Fréchet distribution to inequality (3), we obtain the shares of varieties sourced from i and the average value of $\lambda_i [z_i(v)]^{\sigma-1}$ among these varieties:

$$\mathbb{E}_v \left[\mathcal{I}(i = \mathrm{argmax}_m \lambda_m [z_m(v)]^{\sigma-1}) \right] = \frac{T_i [\lambda_i]^{\frac{\theta}{\sigma-1}}}{\sum_m T_m [\lambda_m]^{\frac{\theta}{\sigma-1}}} \equiv s_i$$

$$\mathbb{E}_v \left[\max_m \lambda_m [z_m(v)]^{\sigma-1} \right] = [\gamma_\theta]^{1-\sigma} \Big(\sum_m T_m \lambda_m^{\frac{\theta}{\sigma-1}} \Big)^{\frac{\sigma-1}{\theta}} \equiv \Lambda,$$

[6] Lemma 2 in Appendix A.1 formally establishes the necessity and sufficiency of this condition for optimality using the Lebesgue differentiation theorem.

where $\{T_m\}$ are the location parameters of the Fréchet distribution and $\gamma_\theta = [\Gamma(1 - \frac{\sigma-1}{\theta})]^{\frac{1}{1-\sigma}}$.

These sourcing outcomes imply that $\{Z_i\}$ as previously defined must be consistent with

$$Z_i \equiv \int_{\mathcal{V}_i} [z_i(v)]^{\sigma-1} \mathrm{d}v = \mathbb{E}_v \left[\mathcal{I}(\iota(v) = i) \cdot [z_i(v)]^{\sigma-1} \right]$$
$$= \frac{s_i}{\lambda_i} \cdot \mathbb{E}_v \left[\lambda_i [z_i(v)]^{\sigma-1} \middle| \mathcal{I}(\iota(v) = i) \right] = \frac{s_i}{\lambda_i} \Lambda.$$

where the last equality applies a property of the Fréchet distribution that the expectation conditional on choices equals the unconditional expectation of the maximum. Plugging this into the price index (2), we establish a fixed-point problem for the marginal utilities $\{\lambda_i\}$, summarized in the following proposition.

Proposition 1. *The solution to problem (1) is characterized by the mass of varieties sourced from i*

$$s_i = \frac{T_i [\lambda_i]^{\frac{\theta}{\sigma-1}}}{\sum_m T_m [\lambda_m]^{\frac{\theta}{\sigma-1}}}, \tag{4}$$

Here, $\{\lambda_i\}$ are positive real numbers that solve the following system of equations

$$\lambda_i = \mathbb{E} \left[I^{1-\gamma} P^{\gamma+\sigma-2} \frac{[c_i]^{1-\sigma}}{\sigma - 1} \right], \qquad \textit{where} \tag{5}$$

$$P = \left[\sum_i [c_i]^{1-\sigma} \frac{s_i}{\lambda_i} \Lambda \right]^{\frac{1}{1-\sigma}}, \tag{6}$$

treating $\{s_i\}$ and $\Lambda \equiv [\gamma_\theta]^{1-\sigma} \left(\sum_m T_m \lambda_m^{\frac{\theta}{\sigma-1}} \right)^{\frac{\sigma-1}{\theta}}$ as functions of $\{\lambda_i\}$. At the solution, the effective productivity of varieties from i satisfies:

$$Z_i \equiv \int_{\mathcal{V}_i} [z_i(v)]^{\sigma-1} \mathrm{d}v = \frac{s_i}{\lambda_i} \Lambda. \tag{7}$$

Proof. Combine the derivation above and Lemma 2 in Appendix A.1, which shows that conditional on $\{\lambda_i\}$, choosing the origin variety by variety is necessary and sufficient for maximizing the expected utility. □

The fixed point highlights the trade-offs inherent in sourcing under uncertainty. The shadow values, $\{\lambda_i\}$, reflect the appeal of each region's goods to consumers in n, balancing cost considerations with the desire to hedge against aggregate shocks. These sourcing decisions, in aggregate, influence the price level, which in turn affects the appeal of each region.

To illustrate the mechanism, rewrite equation (5) using the definition of covariance:

$$\lambda_i = \mathbb{E} \left[[P/I]^{\gamma+\sigma-2} \frac{[c_i/I]^{1-\sigma}}{\sigma - 1} \right] = \frac{1}{\sigma - 1} \mathbb{E} \left[[\tilde{P}]^{\gamma+\sigma-2} [\tilde{c}_i]^{1-\sigma} \right]$$
$$= \frac{1}{\sigma - 1} \left[1 + CM\left([\tilde{P}]^{\gamma+\sigma-2}, [\tilde{c}_i]^{1-\sigma} \right) \right] \mathbb{E}[\tilde{P}]^{\gamma+\sigma-2} \cdot \mathbb{E}[\tilde{c}_i]^{1-\sigma}$$

where $\tilde{c}_i \equiv c_i/I$ and $\tilde{P} \equiv P/I$, and $CM(X,Y) \equiv \frac{Cov(X,Y)}{\mathbb{E}[X]\mathbb{E}[Y]}$ is the covariance-to-mean ratio. We can then compare the sourcing shares from two locations i and m as:

$$\frac{s_i}{s_m} = \frac{T_i}{T_m}\left(\frac{\lambda_i}{\lambda_m}\right)^{\frac{\theta}{\sigma-1}}, \quad \frac{\lambda_i}{\lambda_m} = \frac{\mathbb{E}[\tilde{c}_i]^{1-\sigma}}{\mathbb{E}[\tilde{c}_m]^{1-\sigma}} \frac{1 + CM([\tilde{P}]^{\gamma+\sigma-2}, [\tilde{c}_i]^{1-\sigma})}{1 + CM([\tilde{P}]^{\gamma+\sigma-2}, [\tilde{c}_m]^{1-\sigma})}. \tag{8}$$

Several observations emerge: (1) Since $\sigma > 1$, we have $\mathrm{d}^2\tilde{c}_i^{1-\sigma}/\mathrm{d}\tilde{c}_i^2 = -\sigma(1-\sigma)\tilde{c}_i^{-\sigma-1} > 0$ which is a convex function. This creates a love-of-risk effect arising from the term $\frac{\mathbb{E}[\tilde{c}_i]^{1-\sigma}}{\mathbb{E}[\tilde{c}_m]^{1-\sigma}}$ in (8) such that origins with higher dispersion in cost distribution tend to attract higher ex-ante sourcing shares. This love-of-risk motive stems from the convexity of the aggregate price index with respect to each variety's prices. (2) Risk aversion operates through the aggregate price index: through the CM term, higher covariance between $\tilde{P}$ and $\tilde{c}_i$ decreases the sourcing share (since $\gamma + \sigma - 2 > 0$ and $1 - \sigma < 0$). This can be due to higher variance of $\tilde{c}_i$—a riskier origin, or higher covariance between $\tilde{c}_i$ and $\{\tilde{c}_m\}_{m \neq i}$—a lack of risk diversification.

Relation to EK. The current model generalizes the sourcing model in EK to incorporate aggregate risk in sourcing costs and income. Indeed, Corollary 1 in Appendix A.1 shows that the solution to the sourcing problem reduces to the classical EK solution when aggregate uncertainty is removed. Relative to extensions of EK with uncertainty such as Caselli et al. (2020) and Allen and Atkin (2022), we incorporate anticipatory sourcing behavior by allowing agents to choose suppliers based on their idiosyncratic productivity, but before all aggregate uncertainty resolves.

Short-term and long-term trade elasticities. The impact of the ex-ante sourcing decision on the ex-post equilibrium is summarized by $\{Z_i\}$, which is a function of $\{\lambda_i\}$ through (7). In particular, the ideal price index for the buyer is:

$$P = \left[\sum_i [c_i]^{1-\sigma} Z_i\right]^{\frac{1}{1-\sigma}}.$$

This in turn implies an ex-post (i.e., after the realization of $\{c_i\}$) *expenditure* share of

$$\pi_i = \frac{[c_i]^{1-\sigma} Z_i}{\sum_m [c_m]^{1-\sigma} Z_m}.$$

Therefore, after the realization of aggregate shocks, substitution across regions is driven by the short-term trade elasticity $\sigma - 1$.

Before the realization of aggregate shocks, the heterogeneity in idiosyncratic productivity, governed by parameter θ, determines long-run substitution. This result echoes the classic EK where θ becomes the elasticity of substitution, with a subtle difference: in the presence of uncertainty, this elasticity applies to *risk-adjusted* sourcing costs—a result illustrated more clearly by the perturbation solution in Section 3.2.[7]

Having different long-term and short-term elasticities is necessary for the ex-ante sourc-

[7] There, we show that the elasticity of Z_i with respect to the risk-adjusted sourcing cost is exactly $\theta - (\sigma - 1)$, so the long-term elasticity is the sum of the sourcing choice elasticity $\theta - (\sigma - 1)$ and the quantity elasticity $\sigma - 1$.

ing decision to matter for ex-post equilibrium determination. Indeed, by combining equations (4) and (7), we can see that in the limit of $\theta \downarrow \sigma - 1$, Z_i/Z_m depends on exogenous technology shifters $\{T_m\}$ only. Featuring both elasticities also gives the model empirical flexibility in matching the estimated short- and long-term elasticities. Importantly, since consumers and firms can adjust quantity ex-post, our model can incorporate vertical linkages under standard competitive equilibrium, without having to impose a rationing rule.

Assumptions and generalizations. It is instructive to discuss which model assumptions are essential and which are not. Our strategy of characterizing sourcing decisions based on sufficient statistics relies on the CES structure. In standard deterministic trade models, such as the Armington model, the competitiveness of an origin country can be summarized by its price relative to that of the aggregate price index. Here, a similar intuition applies—except now the competitiveness measures $\{\lambda_i\}$ are risk-adjusted. Although this broad strategy does not rely on particular distributional assumptions regarding idiosyncratic efficiencies, the analytical sourcing probability does rely on the Fréchet assumption. The CRRA utility function is not essential.

Our timing assumption on sourcing is designed to capture the long-term and sticky nature of buyer-supplier relations amidst macroeconomic uncertainty. We motivate this assumption based on the fact that, while firms can and do conduct due diligence to find out the quality of their suppliers before forming a relationship, they often lack means to hedge against macroeconomic risks.

The sourcing block of our model can be readily integrated into important workhorse trade models with EK-style sourcing, such as those featuring input-output linkages (Caliendo and Parro, 2015), multinational production (Ramondo and Rodríguez-Clare, 2013), transport routing (Allen and Arkolakis, 2022), or global value chains (Antràs and De Gortari, 2020). In Section 4, we demonstrate this by incorporating our sourcing block into a multi-country, multi-sector trade model with input-output linkages (Caliendo and Parro, 2015) to study the impact of global productivity and tariff uncertainty on trade and welfare.

3 General Equilibrium and Analytical Characterization

To illuminate the mechanisms, in this section we incorporate the sourcing block into a one-sector general equilibrium model and provide a perturbation solution to the equilibrium. This solution expresses the equilibrium outcomes in an economy with global uncertainty as deviations from those in a deterministic equilibrium, using a formula involving the covariance matrix of fundamental productivities and the outcomes in the deterministic equilibrium, including bilateral expenditure and income shares, and value added shares. The solution highlights how uncertainty shapes the ex-ante sourcing decision and affects ex-post trade patterns. We start by describing the general equilibrium model.

3.1 Competitive Equilibrium with Ex-ante and Ex-post Sourcing Decisions

Endowments, production and trade. Each region i has a fixed mass of labor, L_i. The production function is given by

$$Y_i = A_i \ell_i,$$

where A_i is the total factor productivity and ℓ_i is labor demand. Trade from i to n is subject to iceberg trade costs d_{ni}. This implies that the sourcing cost from i to n, before the idiosyncratic productivity component, is given by $c_{ni} = \frac{w_i d_{ni}}{A_i}$, where w_i is the wage in the origin region i.

Ex-post sourcing problem. We now add back the destination region index n to the sourcing problem. Given the ex-ante choices of location for each variety, $\{\iota_n(v)\}_{v\in[0,1]}$, after realizations of per capita income I_n and region specific sourcing costs $\{c_{ni}\}$, consumers in region n solve the ex-post sourcing decision by choosing the *quantities* of each variety sourced:

$$\begin{aligned} \max_{\{q_{n,\iota(v)}(v)\}} \quad & \left(\int_v [q_{n,\iota_n(v)}(v)]^{\frac{\sigma-1}{\sigma}} dv \right)^{\frac{\sigma}{\sigma-1}} \\ s.t. \quad & \int_v q_{n,\iota_n(v)} \frac{c_{n,\iota_n(v)}}{z_{n,\iota_n(v)}(v)} = I_n, \end{aligned}$$

where $\{z_{n,\iota_n(v)}\}$ are the idiosyncratic sourcing efficiency draws distributed according to Fréchet distributions with location parameter $\{T\}$.

The solution delivers the ideal price index

$$\begin{aligned} P_n &= \left(\int_v \left(\frac{c_{n,\iota_n(v)}}{z_{n,\iota_n(v)}(v)} \right)^{1-\sigma} dv \right)^{\frac{1}{1-\sigma}} \\ &= \left(\sum_i [w_i d_{ni} / A_i]^{1-\sigma} Z_{ni} \right)^{\frac{1}{1-\sigma}}, \end{aligned}$$

where the last line uses the definition of $Z_{ni} = \int_{\mathcal{V}_{ni}} [z_{ni}(v)]^{\sigma-1} \mathrm{d}v$. Similarly, the ex-post import share is

$$\pi_{ni} = \frac{[w_i d_{ni} / A_i]^{1-\sigma} Z_{ni}}{\sum_m [w_m d_{nm} / A_m]^{1-\sigma} Z_{nm}}. \tag{9}$$

Market clearing. We assume balanced trade, so total income equals labor income: $I_n = w_n L_n$. Combining the labor and goods market clearing conditions (in expenditure) gives

$$w_i L_i = \sum_n \pi_{ni} w_n L_n. \tag{10}$$

Source of aggregate uncertainty. To simplify notation, we assume that the productivities $\{A_i\}$ are stochastic but the trade costs are not. All results can be derived for a model with additional trade cost uncertainty analogously, and we explore the interaction between productivity uncertainty and tariff uncertainty in Section 4.

Combining the ex-ante sourcing decision and the ex-post equilibrium conditions, we formally define a competitive equilibrium with global uncertainty below.

Definition 1. *Given the distributions of $\{A_i\}$, a competitive equilibrium consists of ex-ante sourcing decisions $\{\iota_n(v), Z_{ni}\}$ and ex-post random equilibrium variables $\{w_n\}$ such that*

1. *Given the distributions of $\{w_n, A_i\}$, $\{\iota_n(v)\}$ solves the ex-ante sourcing problem*

$$\max_{\{\iota_n(v)\}_{v\in[0,1]}} U_n = \mathbb{E}\Big[\frac{1}{1-\gamma}\Big(\frac{w_n}{P_n}\Big)^{1-\gamma}\Big]$$
$$s.t. \qquad P_n = \Big[\int_0^1 \Big(\frac{c_{n\iota(v)}}{z_{n\iota(v)}(v)}\Big)^{1-\sigma} \mathrm{d}v\Big]^{\frac{1}{1-\sigma}},$$

   where $c_{ni} = \frac{w_i d_{ni}}{A_i}$, with optimal Z_{ni} characterized in Proposition 1.

2. *Given the ex-ante sourcing decision summarized by $\{Z_{ni}\}$, for any realization of $\{A_i\}$, the random variables $\{w_n\}$ solve the ex-post equilibrium system*

$$w_i L_i = \sum_n \frac{[w_i d_{ni}/A_i]^{1-\sigma} Z_{ni}}{\sum_m [w_m d_{nm}/A_m]^{1-\sigma} Z_{nm}} w_n L_n. \tag{11}$$

The equilibrium definition highlights the two-way feedback between the ex-ante sourcing decision and the ex-post equilibrium determination. On the one hand, the ex-ante sourcing decision depends on the joint income and sourcing cost processes, $\{w_n, c_{ni}\}$, which are shaped by wages in the ex-post equilibrium. On the other hand, the ex-ante sourcing decisions, summarized by $\{Z_{ni}\}$, determine the ex-post equilibrium by acting as a demand shifter for goods from different locations, as equation (11) illustrates.

In the next subsection, we characterize these feedback effects in the perturbation solution to the general equilibrium. In Section 4, we show in the quantitative model that these feedback linkages play an important role in shaping the interaction between productivity uncertainty and tariff uncertainty in determining trade patterns and welfare.

3.2 Perturbation Solution

To characterize the effects of uncertainty on aggregate outcomes, we derive approximate solutions up to the order of the variance of shocks around a deterministic equilibrium.[8]

Assume $\{\ln A_i\}_{i=1}^N$ are jointly Gaussian and generated by $\ln A_i = \ln \bar{A}_i + \sqrt{\varepsilon}\zeta_i$, where

[8]Our perturbation solution is closely related to the first-order portfolio choice solution in the international finance literature (Devereux and Sutherland, 2011), viewing $\{Z_i\}$ as a portfolio of varieties sourced from different origins. It differs in two important aspects: (1) Due to the idiosyncratic efficiency components as in standard EK, this "portfolio choice" is determined even in the non-stochastic economy. (2) Conditional on marginal utilities $\{\lambda_i\}$, the "portfolio choice" can be solved in closed form *without approximations*. Crucially, our terminology of "first-order" is with respect to the *variance of shocks*, and should not be confused with the order of approximations to equilibrium conditions. Indeed, to obtain the solution accurate to the first order of the variance of shocks, we need to construct second-order approximations to the ex-post equilibrium conditions. See Lemma 1.

$\mathbb{E}[\zeta_i] = 0$ and $\Sigma_{im} \equiv Cov(\zeta_i, \zeta_m)$. The parameter $\varepsilon > 0$ controls the size of the variance.[9] Denote $\bar{x}$ as the equilibrium value of a variable x in a deterministic economy, where $\ln A_i = \ln \bar{A}_i$ (i.e., log productivities are deterministic at their means). Denote $\widehat{\ln x} \equiv \ln x - \ln \bar{x}$.

We start by characterizing the perturbation solution for the ex-ante sourcing decision, given the distributions of income and sourcing costs.

Proposition 2. *Consider the ex-ante sourcing problem stated in (1) with destination index n suppressed. Assume sourcing costs* $\ln c_i = \ln \bar{c}_i + \sqrt{\varepsilon} c_i^{(1)} + \varepsilon c_i^{(2)}$ *and income* $\ln I = \ln \bar{I} + \sqrt{\varepsilon} I^{(1)} + \varepsilon I^{(2)}$*, where* $\mathbb{E} c_i^{(1)} = 0$*,* $\mathbb{E} I^{(1)} = 0$*, and* $\{c_i^{(1)}\}, I^{(1)}$ *are jointly Gaussian. Assume* $\{c_i^{(2)}\}$ *and* $I^{(2)}$ *have bounded first and second moments. Then, up to terms of order ε, the log share of varieties sourced from i satisfies*

$$\ln s_i = \ln \bar{s}_i + \theta \varepsilon \Delta_i + o(\varepsilon),$$

and the total effective productivities $Z_i \equiv \int_{\mathcal{V}_i} [z_i(v)]^{\sigma-1} \mathrm{d}v$ *satisfy*

$$\ln Z_i = \ln \bar{Z}_i + (\theta + 1 - \sigma) \varepsilon \Delta_i + o(\varepsilon).$$

Here $\bar{s}_i$ *and* $\bar{Z}_i$ *are the corresponding variables for when* $\varepsilon = 0$*,*[10] *and*

$$\Delta_i = \underbrace{\Big(\sum_m \bar{s}_m \mathbb{E} c_m^{(2)} - \mathbb{E} c_i^{(2)}\Big)}_{\text{Mean cost difference}} + \underbrace{\Big[\frac{(\sigma-1)}{2}\big(\Omega_{ii} - \sum_m \bar{s}_m \Omega_{mm}\big) - (\sigma + \gamma - 2)\Big(\sum_m \bar{s}_m \Omega_{im} - \sum_m \sum_l \bar{s}_m \bar{s}_l \Omega_{ml}\Big)\Big]}_{\text{Risk aversion effect}}, \tag{12}$$

where $\Omega_{im} = Cov(c_i^{(1)} - I^{(1)}, c_m^{(1)} - I^{(1)})$ *is the covariance of the first-order shocks to the log of income-adjusted sourcing costs.*

Proof. Appendix A. □

The assumed processes for sourcing costs and income distributions will be verified to hold in equilibrium. As the proposition shows, to the order of the variance of the shock, stochastic productivity adds a correction term, $\varepsilon\theta\Delta_i$, to the log ex-ante sourcing share relative to its value in the non-stochastic equilibrium. Here, θ is the long-term trade elasticity. Δ_i is composed of two terms, with the first term capturing the effect of mean cost differences, and the second term arising from the CES aggregation and the risk aversion. The second term is fully characterized by the covariance structure of income and sourcing costs, the expenditure shares in the non-stochastic equilibrium, and the trade elasticity and risk aversion parameters.

Collecting the coefficient of the term Ω_{ii} in (12) gives: $\frac{\sigma-1}{2} - \frac{\sigma-1}{2}\bar{s}_i - (\sigma+\gamma-2)\bar{s}_i + (\sigma+\gamma-2)\bar{s}_i^2 = (1-\bar{s}_i)\Big[\frac{\sigma-1}{2} - (\sigma+\gamma-2)\bar{s}_i\Big]$. Echoing the discussion after Proposition 1, the

[9]This is equivalent to assuming $\{\ln A_i\}_{i=1}^N$ are jointly Gaussian with mean $\{\ln \bar{A}_i\}$ and covariance matrix $\varepsilon\Sigma$ with $\varepsilon\Sigma_{im} \equiv Cov(\ln A_i, \ln A_m)$.

[10]When $\varepsilon = 0$, these variables correspond to the solution of a standard EK sourcing model without aggregate uncertainty, characterized in Corollary 1 in Appendix A.1.

sign of this coefficient, which governs how the sourcing share responds to the variance of the sourcing cost, depends on the relative size of γ and σ. This reflects the tradeoff between two forces: the love-of-risk effect arising from the CES aggregation and the risk aversion to the induced fluctuation in real income. When the sourcing share is moderately high, $\bar{s}_i > \frac{1}{2}\frac{\sigma-1}{\sigma+\gamma-2}$, the latter effect dominates and the own variance of the sourcing cost of an origin has a negative effect on the sourcing share from that origin. The coefficient of the covariance term $\sum_{m\neq i}\bar{s}_m\Omega_{im}$ is negative when $\sigma+\gamma>2$ as assumed, suggesting that when risk aversion is high, a positive correlation between the sourcing cost of one origin with other locations diminishes the diversification benefit and reduces the sourcing share.

Bringing this perturbation solution of the ex-ante sourcing decision to the general equilibrium, we note that both the mean cost difference, which involves $\{c_m\}$, and the risk aversion effects, which involve the covariance structure of ex-post equilibrium variables, are endogenous. Furthermore, the associated correction terms in $\{\ln Z_i\}$ due to uncertainty will have first-order effects on the ex-post equilibrium determination. Hence, these objects must be solved in conjunction with ex-post equilibrium variables.

We solve this joint system in two steps. First, taking as given the sourcing decisions, we solve the ex-post general equilibrium through perturbation, expressing outcomes in the ex-post equilibrium as functions of shock realizations and sourcing decisions. In the second step, we combine this solution with the ex-ante sourcing decision, which depends on the distribution of ex-post outcomes as characterized in Proposition 2, to obtain the full solution.

To clarify our solution strategy, consider a generic equilibrium system

$$F(x, y(x), \sqrt{\varepsilon}\zeta) = 0. \tag{13}$$

Here, ζ is an exogenous Gaussian vector with mean zero and covariance Σ, and $\varepsilon > 0$ controls the size of the variance. $y(x)$ is a predetermined functional that maps a stochastic vector x to a real vector.

Mapping this system to the general equilibrium model, y corresponds to the *ex-ante* decision variables, $\{\ln Z_{ni}\}$; x corresponds to the *ex-post* equilibrium variables, $\{\ln w_n\}$; $\sqrt{\varepsilon}\zeta$ is the deviation of exogenous Gaussian shocks, $\{\ln A_n\}$, from their means. For any realization of $\{\zeta\}$ and any prices $\{\ln w_n\}$, $\{\ln Z_{ni}\}$ shapes the distribution of world demand. The equilibrium prices must clear all markets, represented by $F = 0$, where

$$F(\ln w, \ln Z, \sqrt{\varepsilon}\zeta) \equiv \ln\Big(w_i L_i\Big) - \Big[\ln\Big(\sum_n \frac{[d_{ni}w_i/(A_i e^{\sqrt{\varepsilon}\zeta_i})]^{1-\sigma}e^{\ln Z_{ni}}}{\sum_m [d_{nm}w_m/(A_m e^{\sqrt{\varepsilon}\zeta_m})]^{1-\sigma}e^{\ln Z_{nm}}} w_n L_n\Big) + \ln\Big(\sum_n w_n L_n\Big)\Big]. \tag{14}$$

The F function is transformed from equilibrium system (11), adding $\ln(\sum_n w_n L_n) = 0$ as a normalization condition. The first step of the solution perturbs (14) to obtain wages as functions of $\{\ln Z_{ni}\}$ and the realization of $\{\zeta\}$. The second step plugs in the dependence of $\{\ln Z_{ni}\}$ on the distribution of wages and other outcomes and solves for the fixed point.

The following lemma characterizes the full model solution under the generic F system.

Lemma 1. *Consider the solution to (13) up to terms of order ε, taking the form of $x = \bar{x} + x^{(1)}\sqrt{\varepsilon} + x^{(2)}\varepsilon + o(\varepsilon)$, where $x^{(1)}$ and $x^{(2)}$, respectively, represent the first- and second-order effects of the fundamental shocks on equilibrium outcomes. Suppose $y(x)$ depends on the distribution of $\{x\}$ as*

$$y(x) = \bar{y} + \phi(\varepsilon \mathbb{E} x^{(2)}) + \psi(\varepsilon\Omega) + o(\varepsilon), \tag{15}$$

where $\varepsilon\Omega$ is the covariance matrix of $\sqrt{\varepsilon}(\Gamma_1 x^{(1)} + \Gamma_2\zeta)$ with matrices Γ_1 and Γ_2, ϕ is a matrix, and ψ is a linear operator.

Then, the solution to (13) is characterized by

$$\begin{aligned}
x^{(1)} &= \Xi \cdot \zeta \\
\mathbb{E}x^{(2)} &= -[F_1 + F_2\phi]^{-1}\Big[F_2\psi((\Gamma_1\Xi + \Gamma_2)\Sigma(\Gamma_1\Xi + \Gamma_2)^T) + \frac{1}{2}\mathcal{H}_{11} vec(\Xi\Sigma\Xi^T) \\
&+ \frac{1}{2}\mathcal{H}_{33} vec(\Sigma) + \mathcal{H}_{13} vec(\Xi\Sigma)\Big].
\end{aligned}$$

Here, $\Xi \equiv -F_1^{-1}F_3$. $\mathcal{H}_{11}$ is the $n_F \times (n_x)^2$ matrix with the k-th row being $vec(F_{11}^{(k)})^T$, with $F_{11}^{(k)} \equiv \frac{\partial^2 F^{(k)}}{\partial x \partial x^T}$ and $vec(\cdot)$ the vectorization operator, and $\mathcal{H}_{33}$ and $\mathcal{H}_{13}$ are similarly defined. All derivatives are evaluated at $(x, y, \varepsilon) = (\bar{x}, \bar{y}, 0)$, where $(\bar{x}, \bar{y})$ are the solutions to the system with $\varepsilon = 0$.

Proof. Appendix A. □

The form of $y(x)$, given by (15), is consistent with the sourcing decision characterized in Proposition 2, where $\phi(\varepsilon\mathbb{E}x^{(2)})$ corresponds to the "mean cost difference" term in $\{\Delta_{ni}\}$ and $\psi(\varepsilon\Omega)$ corresponds to the "risk aversion" term. The lemma characterizes equilibrium outcomes as a function of the realized ζ and—through ϕ, ψ, and the second-order derivatives in the expression for $\mathbb{E}x^{(2)}$—the pre-determined functionals, accounting for the feedback between ex-ante choices and equilibrium outcomes.

For the equilibrium system of the trade model specified in (14), the first and second derivatives can be expressed as functions of observable statistics that include import expenditure shares, income shares, value added shares, and trade elasticities. Applying the lemma to the system (14) and plugging in the ex-ante sourcing decision from Proposition 2 yields the solution to the equilibrium in Definition 1, described in the following proposition.

Proposition 3. *Assume $\{\ln A_i\}_{i=1}^N$ are jointly Gaussian and generated by $\ln A_i = \ln \bar{A}_i + \sqrt{\varepsilon}\zeta_i$, where $\mathbb{E}[\zeta_i] = 0$ and $\Sigma_{im} \equiv Cov(\zeta_i, \zeta_m)$. The solution to the equilibrium in Definition 1 with equilibrium system F given by (14), approximated up to terms of order ε, is characterized by ex-ante sourcing effective productivities $Z_{ni} \equiv \int_{\mathcal{V}_{ni}} [z_{ni}(v)]^{\sigma-1} \mathrm{d}v$ that satisfy*

$$\widehat{Z}_{ni} \equiv \ln Z_{ni} - \ln \bar{Z}_{ni} = (\theta + 1 - \sigma)\varepsilon\Delta_{ni} + o(\varepsilon), \tag{16}$$

ex-ante sourcing shares that satisfy

$$\widehat{s}_{ni} \equiv \ln s_{ni} - \ln \bar{s}_{ni} = \theta\varepsilon\Delta_{ni} + o(\varepsilon), \tag{17}$$

and ex-post equilibrium normalized wages that satisfy

$$\widehat{w}_n \equiv \ln w_n - \ln \bar{w}_n = \underbrace{\Psi_{n,im}\widehat{Z}_{im}}_{\text{Ex-ante sourcing feedback}} + \underbrace{\sqrt{\varepsilon}\sum_i \Xi_{ni}\zeta_i}_{\text{FO direct exposure}} + \underbrace{\varepsilon\sum_{i,m}\Phi_{n,im}\zeta_i\zeta_m}_{\text{SO direct exposure}} + o(\varepsilon), \tag{18}$$

where

$$\varepsilon\Delta_{ni} = (\sum_m \bar{s}_{nm}\mathbb{E}\widehat{w}_m - \mathbb{E}\widehat{w}_i) \tag{19}$$
$$+ \varepsilon\Big[\frac{(\sigma-1)}{2}(\Omega_{ni,ni} - \sum_m \bar{s}_{nm}\Omega_{nm,nm}) - (\sigma+\gamma-2)\Big(\sum_m \bar{s}_{nm}\Omega_{ni,nm} - \sum_k\sum_l \bar{s}_{nk}\bar{s}_{nl}\Omega_{nk,nl}\Big)\Big]$$

is the risk-adjusted relative sourcing cost from i to n. $\Psi \equiv -F_1^{-1}F_2$ *and* $\Xi \equiv -F_1^{-1}F_3$ *are the general equilibrium exposure matrices of* $\ln \boldsymbol{w}$ *w.r.t.* $\ln \boldsymbol{Z}$ *and* $\ln \boldsymbol{A}$ *at the ex-post equilibrium.* Φ_n *are coefficient matrices that involve* F_1^{-1}, Ξ *and the Hessian of F.* $\varepsilon\Omega_{ni,nm} \equiv Cov(\ln \frac{w_i/A_i}{w_n}, \ln \frac{w_m/A_m}{w_n})$, *which can be constructed from* Ξ *and* Σ. *All coefficients are evaluated at the non-stochastic equilibrium.* $\{\bar{Z}_{ni}, \bar{s}_{ni}, \bar{w}_n\}$ *are their values at the non-stochastic equilibrium.*

All coefficients are functions of import expenditure shares, income shares, value-added shares, and parameters $\{\sigma, \theta, \gamma\}$. *Given observable statistics, all coefficients are linear transformations of* Σ.

Proof. Appendix A. The proof also presents the formulae for all coefficients explicitly. □

This proposition characterizes the full solution of the model: the ex-ante choice as a function of the covariance structure of the fundamental shocks; and the ex-post equilibrium as a function of this covariance structure and the realization of the fundamental shocks.

Equation (16) is the ex-ante sourcing solution characterized in Proposition 2. Equation (18) shows how aggregate uncertainty and sourcing under risk aversion affect ex-post equilibrium determination.[11] The second term, "FO direct exposure," is the only term that appears in the first-order perturbation solution without the ex-ante sourcing stage, as in Kleinman et al. (2024a) and Huo et al. (2025). The third term, "SO direct exposure," is mechanical, arising from the fact that we are now keeping terms up to the first order of the size of the variance, which would appear in the second-order solution of a model without uncertainty.

The key term is the one labeled "Ex-ante sourcing feedback." The equation makes clear that ex-ante sourcing under uncertainty generates a first-order level effect on equilibrium variables, through the effect on Z_{ni}. This level effect captures the two-way feedback between ex-ante sourcing and ex-post equilibrium determination, as evident from the fact that $\widehat{Z}$ enters the determination of $\widehat{w}$ in (18) while $\mathbb{E}\widehat{w}$ enters the determination of $\widehat{Z}$ through Δ in

[11] To simplify notation, we only report the formula for normalized wages. The analysis holds for other ex-post equilibrium variables.

(19). This level effect is thus determined by a fixed point system for $\mathbb{E}\widehat{w}$, which we explicitly characterize in the proof.

Note that the anticipatory sourcing decision, influenced by uncertainty, further changes the *covariance* structure of the ex-post equilibrium, in turn feeding back into the sourcing problem. While Proposition 3 shows that this effect is of order $o(\varepsilon)$, for exercises focusing on how a *large* policy change affects the economy's endogenous uncertainty, it must be accounted for. In Section 4, when studying the impact of a large tariff shock on the degree of risk-sharing against global productivity uncertainty, we design numerical methods that capture this feedback effect.

Finally, since given other observable statistics, all terms in the formulae are linear functions of the covariance matrix of the fundamentals, we can decompose the uncertainty effect into variance and insurance components by splitting the fundamental covariance matrix. We implement this decomposition in the quantitative analysis.

3.3 Comparative Statics in a Symmetric Economy

Before proceeding to the quantitative evaluation, we conduct comparative statics exercises to shed light on the model mechanisms. Our focus is on how productivity uncertainty affects trade, and how trade cost changes can lead to changes in endogenous uncertainty that feed back into ex-ante sourcing decisions.

Throughout this section, we assume $N \geq 2$ symmetric countries, with $T_{ni} = 1$, $\bar{A}_n = 1$ for all (i, n), and $d_{ni} = d > 1$ for $i \neq n$. We also assume a symmetric covariance matrix between country-pairs: $Var(\ln A_n) = \varepsilon\sigma_A^2 > 0$ and $Cov(\ln A_n, \ln A_i) = \varepsilon\rho\sigma_A^2$ with $\rho \in [-1, 1]$ being the correlation coefficient. Lemma 3 in Appendix A.2 presents the closed-form perturbation solution to this symmetric economy equilibrium, with which we assess the comparative statics with respect to parameters σ_A^2, ρ, and d.

Proposition 4. *When $\rho < 1$ and for ε small*

(1) An increase in σ_A^2 increases the log foreign sourcing shares $\{\ln s_{ni}\}_{i\neq n}$. The marginal effect increases with $1 - \rho$ and the risk aversion parameter γ, and is non-monotone with respect to σ.

(2) A decrease in ρ increases the log foreign sourcing shares $\{\ln s_{ni}\}_{i\neq n}$. The marginal effect increases with σ_A^2 and the risk aversion parameter γ.

Proof. Appendix A.2. □

Part (1) of Proposition 4 states that a uniform increase in the variance of productivity increases trade openness by raising the ex-ante foreign sourcing shares.[12] This occurs because

[12]Note that an increase in $\ln s_{ni}$ for $i \neq n$ also translates into a larger *ex-post* expenditure share according to (9), given the same ex-post productivity realizations.

higher productivity volatility generates a stronger hedging-through-trade motive. This motive is stronger when the cross-country productivity correlation ρ is low—when hedging is more effective—or when the risk aversion parameter γ is high. The comparative statics with respect to the short-term trade elasticity σ reveal two competing effects: First, a higher σ leads to a smaller pass-through of productivity volatility to relative sourcing cost volatility.[13] Second, a higher σ adds curvature to the price index when it enters the utility, effectively raising the degree of risk aversion.[14] The net effect of σ on how productivity volatility affects the hedging motive is therefore non-monotone.

Part (2) states that an increase in the correlation of productivity lowers the foreign sourcing shares. This occurs because stronger co-movement of productivity diminishes the benefit of hedging-through-trade. The effect is more pronounced when the productivity variance itself is larger or when consumers are more risk averse.

We now characterize the interaction between trade costs and productivity uncertainty.

Proposition 5. *With $\rho < 1$ and ε small, when the trade cost d increases:*

(1) The foreign sourcing shares $\{s_{ni}\}_{i \neq n}$ decrease in the absence of productivity uncertainty.

(2) When foreign sourcing shares are not too low, the risk correction terms in foreign sourcing shares relative to own sourcing shares, $\{\theta\varepsilon(\Delta_{ni} - \Delta_{nn})\}_{i \neq n}$, increase.

(3) The volatility of real income increases.

Proof. Appendix A.2. □

Proposition 5 shows that an increase in the trade cost level has two opposing effects on foreign sourcing shares: Part (1) describes the standard level effect from a model without aggregate uncertainty—high trade costs reduce foreign sourcing shares. Part (2) states a new mechanism: Trade between countries generates an additional *positive* co-movement in wages and hence a positive co-movement in sourcing costs between countries.[15] This endogenous co-movement becomes weaker when trade is hampered by higher trade costs, which increases the risk diversification value of foreign suppliers. As a result, importers source more from foreign suppliers ex ante, partially offsetting the decline in trade due to the level effect of higher trade costs.

[13] This is because a larger trade elasticity induces a larger response of relative wage to relative productivity, which partly offsets the movement in relative productivity. See the proof for details.

[14] The discussions after Proposition 1 and 2 hint at a "love-of-risk" effect that increases with σ, which arises because the price index is a convex function with respect to the sourcing cost of each origin. This "love-of-risk" effect is always dominated by the risk aversion effect in this symmetric setting.

[15] This occurs because an increase in productivity in region n raises the wage and income in n. Consumers in n thus increase their expenditures on other regions' goods, which generates a positive spillover on other regions' wages. This spillover effect is larger when import shares are higher.

Part (3) shows that, despite this additional motive for hedging through trade and ex-ante sourcing adjustment, real-income volatility still increases because of the overall decline in trade. This increase in real income volatility translates into additional welfare losses for risk-averse consumers. In other words, gains from trade now also encompass a gain from risk diversification. This analysis highlights the interaction between changes in trade costs and the incidence of productivity uncertainty, which we quantify in the next section.

4 Quantitative Evaluation

To assess the impacts of productivity and tariff uncertainty on trade patterns and welfare, we extend the model to a multi-country, multi-sector model with input-output linkages. We use the World Input-Output Database (WIOD) to estimate the covariance structure of the country-sector productivities, and use the 2025 "Reciprocal Tariffs" policy initiated by the Trump administration as a case study of heightened tariff uncertainty.

Although the theoretical extension is straightforward, implementing these counterfactuals faces two challenges. First, the different regimes of trade wars represent major shocks, for which the local solution method described in Section 3.2 is not suitable. On the other hand, these shocks arise from a relatively limited number of regimes. We incorporate various scenarios of tariff outcomes through enumeration simulation and show how to combine the global method for dealing with tariff shocks with the local method for productivity shocks.

Second, with $N = 44$ countries and $J = 32$ sectors, implementing the perturbation solution with respect to high-dimensional productivity shocks requires computing second-order exposure terms that involve large Hessian matrices.[16] We show that these second-order exposure terms required can be constructed as nested Jacobian-vector-products, which we then compute using automatic differentiation, leveraging recent developments in computer science for this type of problem (Moses and Churavy, 2020; Moses et al., 2022).[17] Our Hessian calculation is *exact*—identical to manual derivations of Hessians in Proposition 3. However, it avoids both the derivation and the practical challenges of calculating and storing these matrices, and is readily applicable to other quantitative trade models.

4.1 The Extended Multi-Sector Model

We extend the one-sector model to a multi-sector model with input-output linkages à la Caliendo and Parro (2015). We describe here the additional model ingredients and the mod-

[16] For example, the Hessian of the bilateral sourcing decision λ_{ni}^{j}, defined below, with respect to productivity shocks is a dense matrix with a number of elements on the order of $N^2 \times J \times (N \times J)^2$, exceeding 1 TB in size.

[17] These developments employ a source-to-source automatic differentiation, so once the nested Jacobian-vector-product procedure is compiled, it can be evaluated as efficiently and as accurately as analytically derived expressions.

ified sourcing problem; the definition of the full equilibrium is delegated to Appendix A.

Production and trade. Each country has J sectors indexed by superscripts j or k. Intermediate goods production uses labor and sectoral composite goods as inputs according to the Cobb-Douglas production function. For sector j of region i, the quantity of intermediate goods produced is given by

$$Y_i^j = \eta_i^j A_i^j [\ell_i^j]^{\gamma_i^{jL}} \prod_{k=1}^{J} [m_i^{jk}]^{\gamma_i^{jk}},$$

where ℓ_i^j is labor input, m_i^{jk} is the input of sector-k composite goods, A_i^j is total factor productivity, $\gamma_i^{jL}, \{\gamma_i^{jk}\}$ are the input shares with $\gamma_i^{jL} + \sum_k \gamma_i^{jk} = 1$, and $\eta_i^j \equiv [\gamma_i^{jL}]^{\gamma_i^{jL}} \prod_{k=1}^{J} [\gamma_i^{jk}]^{\gamma_i^{jk}}$ is the normalization constant. This delivers the unit production cost

$$\kappa_i^j = \frac{1}{A_i^j} [w_i]^{\gamma_i^{jL}} \prod_{k=1}^{J} [P_i^k]^{\gamma_i^{jk}},$$

where P_i^k is the price of the composite goods in sector k.

The composite goods producer in each sector j of region n sources globally for each variety v and aggregates them with a CES function:

$$Q_n^j = \Big[\int_0^1 [q^j_{n,\iota_n^j(v)}(v)]^{\frac{\sigma-1}{\sigma}} dv \Big]^{\frac{\sigma}{\sigma-1}},$$

where Q_n^j denotes the output quantity of the composite goods and $q^j_{n,\iota_n^j(v)}(v)$ denotes the quantity of intermediate goods sourced from the chosen origin $\iota_n^j(v)$ for variety v. Each variety is converted by the buyer from the origin's intermediate goods at an idiosyncratic productivity, $z_{ni}^j(v)$, that is independently and identically distributed across (n, i, j, v) following a Fréchet distribution with dispersion parameter θ.[18] The effective unit cost of sourcing from i for variety v is thus $\frac{\kappa_i^j d_{ni}^j}{z_{ni}^j(v)}$.

Sourcing takes two stages as in the previous section. In the first stage, before fundamental shocks and equilibrium variables are realized, $z_{ni}^j(v)$ are observed and the composite goods producers choose the sourcing origin of each variety, $\iota_n^j(v)$. In the second stage, fundamental shocks are realized and given the ex-ante sourcing decision, the composite goods producers can optimally choose the sourcing quantities.

The multi-sector sourcing problem. Consumers aggregate the sectoral composite goods

[18]In the original formulation of Eaton-Kortum/Caliendo-Parro, producers are heterogeneous with productivity characterized by Fréchet distributions. We reformulate the model to feature homogeneous intermediate goods producers and interpret the idiosyncratic Fréchet draws as representing sourcing efficiency borne by buyers. This choice simplifies the measurement of sectoral TFP, see Section 4.2. Both setups lead to identical equilibrium conditions and perturbation solutions.

using a Cobb-Douglas function:

$$C_n = \prod_{j=1}^{J} [C_n^j]^{\beta_n^j},$$

where β_n^j is the expenditure share with $\sum_j \beta_n^j = 1$. We assume the sourcing origin of each variety for country n is determined by its consumers, who maximize their expected utility under risk aversion:[19]

$$\max_{\{\iota_n^j(v)\}_{v\in[0,1]}} \quad U_n = \mathbb{E}\Big[\frac{1}{1-\gamma} C_n^{1-\gamma}\Big] \tag{20}$$

$$s.t. \quad C_n = \frac{I_n}{P_n}, \quad P_n = \prod_j [P_n^j]^{\beta_n^j}$$

$$P_n^j = \Big[\int_0^1 \Big(\frac{c^j_{n,\iota_n^j(v)}}{z_{n,\iota_n^j(v)}(v)}\Big)^{1-\sigma} \mathrm{d}v\Big]^{\frac{1}{1-\sigma}}, \text{ where } c_{ni}^j = \kappa_i^j d_{ni}^j.$$

Following a similar strategy as in Section 2, we denote $Z_{ni}^j \equiv \int_{\mathcal{V}_{ni}^j} [z_{ni}^j(v)]^{\sigma-1} \mathrm{d}v$ as the total effective productivities, where $\mathcal{V}_{ni}^j$ is the set of varieties sourced from i by (n, j). We can then write $P_n^j = \Big[\sum_i [c_{ni}^j]^{1-\sigma} Z_{ni}^j\Big]^{\frac{1}{1-\sigma}}$ and derive the marginal value of an additional variety in sector j from i to consumers in n as:

$$\lambda_{ni}^j \equiv \frac{\partial U_n}{\partial Z_{ni}^j} = -\mathbb{E}\left[I_n^{1-\gamma} P_n^{\gamma-2} \frac{\partial P_n}{\partial P_n^j} \frac{\partial P_n^j}{\partial Z_{ni}^j}\right], \tag{21}$$

where the expectation operator is with respect to I_n, $\{c_{ni}^j\}_{i=1,j=1}^{N,J}$, and P_n^j. Conditional on λ_{ni}^j, the remaining ex-ante sourcing decision compares the risk-adjusted sourcing cost variety by variety, and a similar fixed-point system for λ_n^j can be derived analogously.

Unlike the one-sector model characterized in Sections 2 and 3, consumers' sourcing decisions now also take into account the co-movement of sectoral prices and the sourcing decisions across sectors are jointly determined. Nevertheless, we can derive a similar formula for the perturbation solution of the ex-ante sourcing problem, which now is augmented with a term capturing the hedging incentive arising from cross-sector sourcing cost correlations. Proposition 6 in Appendix A.3 presents this perturbation solution and further discussion.

Other equilibrium conditions. Labor market clearing requires that the sum of labor used across sectors equals the exogenous labor supply in each country. Intermediate goods market clearing requires that the quantity of intermediate goods produced in each country-

[19] In principle, the sourcing decisions of composite goods producers that sell to producers may differ from those selling to consumers: large producers may have better risk management capacity and act as if they were risk neutral; small producers may be even more risk-averse than consumers in the face of supply chain disruptions. In the absence of further evidence on the risk characteristics of producers compared to consumers, we make the stated assumption. Note that this distinction is not essential in a model without aggregate uncertainty, as all sourcing decisions are based on the non-stochastic sourcing costs.

Table 1: Summary of Model Parameters

Parameters	Descriptions	Value	Target/Source
A. Externally Calibrated			
$\gamma_n^{jk}, \gamma_n^{jL}, \beta_n^j$	IO Structure and Consumption Share	-	WIOT; $N = 44, J = 32$
L_n	Labor Endowment	-	Penn World Table
$(\sigma - 1, \theta)$	Short- and Long-term Trade Elasticity	$(0.76, 2.12)$	Boehm et al. (2023)
γ	Relative Risk Aversion	5	Literature estimate
B. Calibrated in Non-stochastic Equilibrium			
$\bar{A}_n^j$	Mean Productivity	-	Countries' output shares in each sector
$\bar{d}_{ni}^j$	Pre-shock Iceberg Trade Costs	-	Bilateral trade shares (WIOT)
C. Productivity and Tariff Uncertainty			
Σ_{ni}^{jk}	Covariance of Productivity	-	Constructed from WIOD-SEA (Annual)
τ_{ni}^j	Stochastic Tariff Shock	See text	Negotiation outcome on "Reciprocal Tariffs"

sector equals the total quantity sourced. Composite goods market clearing requires that the composite goods assembled in each country-sector equal the total quantity used by intermediate goods producers and consumers. We assume balanced trade. We consider uncertainty in both productivity A_i^j and tariffs τ_{ni}^j. Tariffs raise trade costs and generate revenue that is rebated to consumers. Specifically, the trade costs are affected by tariffs according to

$$\ln d_{ni}^j = \ln \bar{d}_{ni}^j + \ln(1 + \tau_{ni}^j)$$

and the total income of a country equals wages plus tariff revenue. Definition 2 in Appendix A.3 presents the equilibrium definition in detail.

To facilitate the presentation of the quantitative results, we denote s_{ni}^j as the ex-ante sourcing share—the fraction of varieties sourced from i by region n in sector j; π_{ni}^j as the ex-post import share—the share of import expenditures from i by region n in sector j after the realization of shocks, given by

$$\pi_{ni}^j = \frac{[c_{ni}^j]^{1-\sigma} Z_{ni}^j}{\sum_m [c_{nm}^j]^{1-\sigma} Z_{nm}^j}, \tag{22}$$

and X_{ni}^j as the total trade value from i to n in sector j.

4.2 Calibration

Table 1 summarizes the parameter values. The input-output structure and consumption share parameters $\{\gamma_n^{jk}, \gamma_n^{jL}, \beta_n^j\}$ are constructed from the World Input-Output Table (WIOT; Timmer et al., 2015) data for 2016. We retain all 44 regions, which include 43 countries/regions and a synthetic Rest-of-World (ROW). For sectors, we retain all manufacturing sectors and group all other sectors at the one-digit level; we also group the government sector, household self-employed sector and activities of extraterritorial organizations (code N to U) into one sector. This gives us 32 sectors in total. We construct labor endowments $\{L_n\}$ using the Penn World Table (Feenstra et al., 2015) for 2016.

The short-term and long-term trade elasticities, and the consumer risk aversion parameter, are crucial for sourcing under uncertainty. Following the estimates of Boehm, Levchenko and Pandalai-Nayar (2023), we set the short-term elasticity $\sigma - 1$ to 0.76 and the long-run elasticity θ to 2.12. The difference between these elasticities is at the lower end of the literature. Since the strength of anticipatory sourcing in response to uncertainty is governed by $\theta - (\sigma - 1)$, as shown in (16), this calibration leads to conservative estimates for the importance of anticipatory sourcing. We set the relative risk aversion level to be 5, which lies in the range of literature estimates (see e.g., the meta-analysis by Elminejad, Havranek and Irsova, 2025).

We normalize the shifters of idiosyncratic productivities, T^j_{ni}, to one, and calibrate the mean productivities $\bar{A}^j_n$ and deterministic pre-trade war trade costs $\bar{d}^j_{ni}$ in the non-stochastic economy to match the bilateral import shares and countries' sectoral output value shares.[20]

To construct the covariance of productivity shocks between country-sectors, we measure productivity and define its time changes as productivity shocks. Under the setup presented in Section 4.1, where the Fréchet idiosyncratic draws represent sourcing efficiency and are borne by the buyer, the Solow residuals of a country-sector's output are a theory-consistent measure for A^j_i. To construct this measure, we use data on output, capital and labor inputs, and intermediate goods from the 2016 World Input Output Database-Social Economic Accounts (WIOD-SEA), all of which are deflated using corresponding price indices. Assuming that within each country-sector the cost shares of inputs are constant, which we measure using the average cost shares of each country-sector, we calculate productivity as the difference between log output and the sum of cost-share-weighted log inputs. The necessary data are available for all countries and sectors for 2000-2014. For the synthetic ROW, for which input expenditures are not reported, we construct the Solow residuals for each sector by averaging across all other regions. Appendix 4.4 describes the details.

Our choice of uncertain tariff shocks is motivated by the "Reciprocal Tariffs" order initiated by the Trump administration in April, 2025. The order imposes a $\underline{\tau} = 10\%$ minimum tariff on all imports and country-specific tariff ceilings, $\bar{\tau}_i$. We assume that the tariff that actually takes effect depends on whether each country reaches an agreement with Washington during the 90-day window. In our sample, there are 27 EU countries that face a tariff ceiling of 20% and 10 non-EU countries that face a ceiling higher than 10%. We assume that these 27 EU countries bargain as one bloc and that each of the remaining 10 countries bargains independently. A successful negotiation brings the tariff to 10%, whereas a failure leaves the

[20] Note that trade and output shares do not separately identify all $\{\bar{A}^j_n\}$ and all $\{\bar{d}^j_{ni}\}$ jointly. Focusing on proportional changes of these variables and the volatility of log values, our counterfactual exercises are invariant to the normalization choice in the calibration of these parameters.

tariff at the ceiling.[21] To calibrate the probabilities of negotiation success, we rely on the implied probability from Kalshi's market "Which countries will Trump make new trade deals with this year?" as of July 8, 2025, when most outcomes remained unresolved.[22] The seven remaining countries in our sample have a tariff ceiling equal to 10%. We assume these countries face 10% tariffs with certainty. This binary structure gives $2^{11} = 2048$ tariff vectors, with each vector occurring with probability equal to the product of the success/failure probabilities for individual countries. Appendix B.3 reports the tariff distribution by country in more detail.

4.3 Numerical Implementation with Small Shocks and Large Shocks

We solve the equilibrium by formulating it as a fixed-point problem for $\{\lambda_{ni}^j\}$ defined in (21). Given $\{\lambda_{ni}^j\}$ and the implied $\ln Z_{ni}^j$, we then solve the stochastic ex-post equilibrium detailed in Definition 2 in Appendix A.3. We compute the implied $\{\lambda_{ni}^j\}$ following its definition and iterate to find the fixed point.

A key step in this algorithm is calculating the expectation term in equation (21), which involves integration over the realizations of the ex-post equilibrium. Our algorithm combines local solutions to the ex-post equilibrium with respect to productivity shocks, conditional on tariff realizations, with global nonlinear simulations for tariff shocks. This strategy recognizes the nature of different shocks—small and dense productivity shocks, and large but sparse tariff shocks. It is both necessary for computational feasibility and essential for capturing the core model mechanism.

Regarding computational feasibility, the productivity shock is at the country-sector level and has one fewer dimension than the bilateral tariffs, which makes it feasible for a local solution under the new technique we describe below. However, the tariff shocks are essentially sparse, which allows us to enumerate and solve nonlinearly the ex-post equilibrium for all 2048 cases.

In implementing the local solution for productivity shocks, a remaining obstacle is obtaining the exposure matrices of general equilibrium outcomes with respect to productivity shocks (Jacobian and Hessian matrices), which are required for solving λ_{ni}^j up to the order of

[21] The 2025 April–May window was marked by dramatic shifts that justify modeling tariffs as genuinely uncertain. A trade meeting in Geneva on May 11-12 produced a joint U.S.–China statement promising a 90-day freeze and "substantial progress" toward rolling back the 84% ceiling. Barely three weeks later, President Trump declared that Beijing had "totally violated" the Geneva agreement and mused that an 80% tariff "seems right," signaling possible re-escalation. Parallel reversals showed up in his stance toward the EU: on May 25th he postponed a threatened 50% steel-and-aluminum duty until July 9th to allow talks, only to revive the threat five days later before hinting at another delay. These rapid pivots motivate our assumption that each negotiation succeeds or fails independently.

[22] We use the July 8, 2025, quotes because negotiations were ongoing. The implied probability of success is inferred directly from the market price: if a "Yes" contract trades at price p (in dollars), the corresponding probability is $p/100$. See B.3 for details about the Kalshi prediction market and the inference procedure.

the variance of shocks. In this multi-sector model with input-output linkages, these exposure matrices cannot be expressed as *simple* functions of observable statistics as in Proposition 3 and can be too large for standard desktop computers to handle. We address this challenge by formulating the problem in a way that enables the use of implicit differentiation and automatic differentiation. Rather than constructing the Hessian matrices manually and multiplying them with the covariance of shocks to obtain the second-order exposure terms, we transform the product of these two components into the form of nested Jacobian-vector-products, which modern automatic differentiation routines can evaluate efficiently. We further design an efficient algorithm for this evaluation. Appendix B.1 contains the details.

Regarding capturing the model mechanism, a novel finding in this section is the interaction between trade cost shocks and productivity uncertainty: increases in the level and uncertainty of trade costs weaken countries' risk sharing against productivity shocks through trade, resulting in additional welfare losses. As shown in Proposition 3 and the discussion thereafter, this uncertainty feedback effect does not appear in the perturbation solutions, but emerges in the solution when the non-linearity w.r.t. trade cost shocks is accounted for.

4.4 The Impact of Productivity Uncertainty

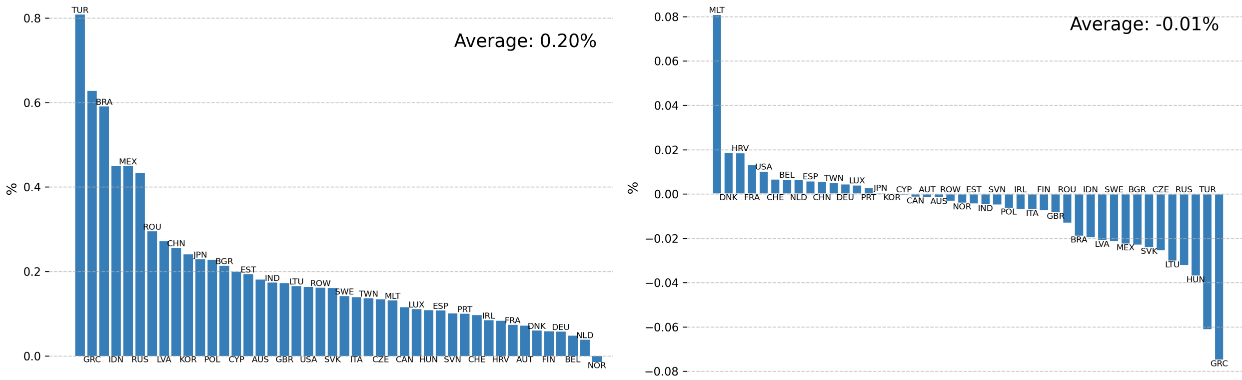
We first assess the impact of productivity uncertainty, by comparing the calibrated economy with a counterfactual economy without productivity uncertainty (setting $\Sigma_{ni}^{jk} = 0$). To isolate the mechanisms, we assume there is no tariff uncertainty for now.

4.4.1 Global Redistribution Shaped by Countries' Productivity Risk Profiles

We start by examining the volatility of real income and import share (imports / total expenditures ratio) in the calibrated economy. Since in this subsection, the only aggregate uncertainty comes from productivity, this also represents the contribution of productivity uncertainty to the volatility of these variables. On average, the model generates a standard deviation of log(Real Income) of 5.3% and a standard deviation of log(Import Share) of 0.7%, with substantial cross-country variations as detailed in Figure 11 of Appendix B.4. These numbers roughly reflect the volatility of the constructed TFP residuals.

The volatility due to productivity uncertainty affects the *level* of openness and income through ex-ante sourcing. Panel (a) of Figure 1 plots the change in import share in the ex-post equilibrium due to productivity uncertainty. We set the realization of ex-post log productivity at its mean value in this comparison, so the change results entirely from ex-ante sourcing decisions. In all countries except Norway, global productivity uncertainty increases the import share, with an average magnitude of 0.2%.

Why does global productivity uncertainty increase trade openness? As long as countries' productivities are not perfectly correlated, by trading with other countries, consumers can



(a) Import Share

(b) Real Income

Note: The figure shows the percentage difference in the labeled variable between the calibrated economy with productivity uncertainty and the one without productivity uncertainty. Variables are evaluated in the ex-post equilibrium with log productivities realized at their means.

Figure 1: Impact of Global Productivity Uncertainty Through Ex-ante Sourcing

diversify their own productivity risks and benefit from reduced real income volatility. This echoes the classical "risk diversification" motive of trade that dates back to Ruffin (1974) and Helpman and Razin (1978).

Countries facing higher domestic productivity uncertainty, such as Turkey and Greece, shift more towards imports than countries facing less uncertainty. To balance trade, therefore, the terms of trade depreciate for Turkey and Greece and appreciate for countries with smaller uncertainty. This terms-of-trade adjustment, in turn, leads to distributional effects on real income across countries, as Panel (b) of Figure 1 shows.

The impact of uncertainty on ex-post trade patterns can be represented by the changes in Z^j_{ni}, which summarize the risk-adjusted effective sourcing productivities. In particular, as equation (22) shows, Z^j_{ni} enters the determinants of the trade shares as a cost shifter, capturing the comparative advantage of a country due to its covariance with other countries. In our experiment, the changes in $\ln Z^j_{ni}$ due to productivity uncertainty range from -0.025 to 0.066, suggesting significant and heterogeneous uncertainty-induced comparative advantage. However, when we average the changes across sectors by exporting country, they drop to negligible levels, suggesting that uncertainty-induced comparative advantages differ across sectors, largely averaging out. The *standard deviation* of the changes in $\ln Z^j_{ni}$ differs significantly across sectors, ranging from 0.003 to 0.010, indicating that the uncertainty-induced comparative advantage is more important in some sectors than others.

4.4.2 The Welfare Effect of Productivity Uncertainty

As the above discussion alludes to, productivity uncertainty can affect the welfare of a country through both a risk aversion effect and a trade diversion effect. How important are these effects? We calculate the consumption equivalent variation (CEV) resulting from productivity uncertainty. Moving from the non-stochastic economy to the stochastic economy (with variables denoted by prime), the CEV for country n, denoted by μ_n, solves

$$\frac{1}{1-\gamma}\mathbb{E}[((1+\mu_n)C_n)^{1-\gamma}] = \frac{1}{1-\gamma}\mathbb{E}[{C'_n}^{1-\gamma}]$$

and $\mu_n \approx \frac{1}{1-\gamma}\Big(\ln \mathbb{E}[{C'_n}^{1-\gamma}] - \ln \mathbb{E}[{C_n}^{1-\gamma}]\Big)$ when the welfare changes are small. We can further decompose μ as

$$\begin{aligned}
&\mu_n = \mu_{n,\text{Ex-ante}} + \mu_{n,\text{Ex-post, Certainty Equivalent}} + \mu_{\text{n,Risk_Aversion}}, \text{ where} \\
&\mu_{n,\text{Ex-ante}} \equiv \ln \bar{C}'_n - \ln \bar{C}_n, \text{with } \bar{C}_n \text{ and } \bar{C}'_n \text{ evaluated at } \ln A_i^j = \ln \bar{A}_i^j, \forall i \\
&\mu_{n,\text{Ex-post, CE}} \equiv \ln \mathbb{E}[C'_n] - \ln \mathbb{E}[C_n] - \mu_{n,\text{Ex-ante}} \\
&\mu_{n,\text{RA}} \equiv \Big(\frac{1}{1-\gamma}\ln \mathbb{E}[{C'_n}^{1-\gamma}] - \ln \mathbb{E}[C'_n]\Big) - \Big(\frac{1}{1-\gamma}\ln \mathbb{E}[{C_n}^{1-\gamma}] - \ln \mathbb{E}[C_n]\Big).
\end{aligned} \tag{23}$$

$\mu_{n,\text{Ex-ante}}$ arises purely from the changes in the ex-ante sourcing decision, since the ex-post equilibria are evaluated at the same productivity realizations. $\mu_{n,\text{RA}}$ is the change in welfare resulting from ex-post uncertainty due to risk aversion—with either $\gamma = 0$ or no uncertainty, this term would be zero. $\mu_{n,\text{Ex-post, CE}}$ is a residual component that captures the change in the ex-post expected consumption, beyond the change due to the ex-ante sourcing decision.

Using the local approximation and the assumption of Gaussian distributed log productivities, for a country n, $\mu_{n,\text{RA}}$ can be further written as (see Lemma 6 in Appendix B.5)

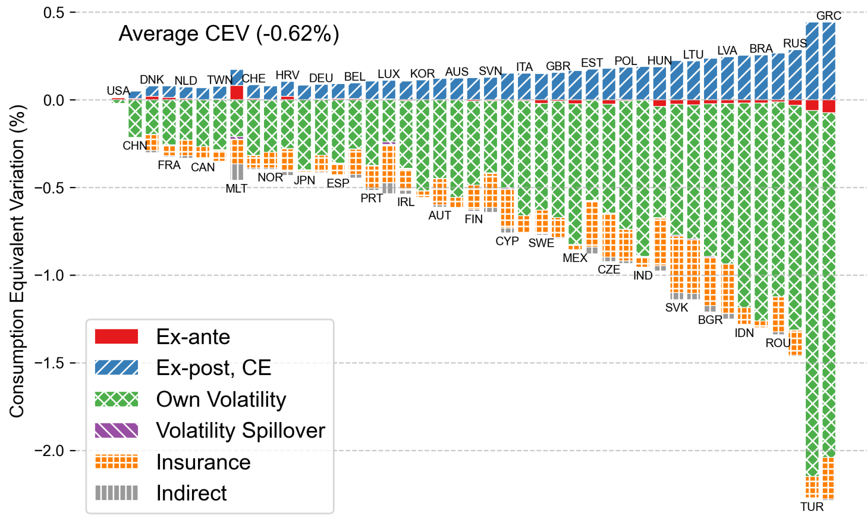
$$\mu_{n,\text{RA}} \approx -\frac{\gamma}{2}\sum_i \sum_m \sum_{j,k} \mathcal{D}_{ni}^j \Sigma_{im}^{jk} \mathcal{D}_{nm}^k,$$

where $\mathcal{D}_{ni}^j$ is the general equilibrium exposure matrix of $\ln C_n$ with respect to $\ln A_i^j$ and $\Sigma_{im}^{jk} = Cov(\ln A_i^j, \ln A_m^k)$ as previously defined. We can thus apply a variance decomposition by writing

$$\mu_{n,\text{RA}} = -\frac{\gamma}{2}\Big\{\underbrace{\sum_{j,k}\mathcal{D}_{nn}^j\Sigma_{nn}^{jk}\mathcal{D}_{nn}^k}_{\text{own volatility}} + \underbrace{\sum_{i\neq n}\sum_{j,k}\mathcal{D}_{ni}^j\Sigma_{ii}^{jk}\mathcal{D}_{ni}^k}_{\text{volatility spillover}} + \underbrace{2\sum_{i\neq n}\sum_{j,k}\mathcal{D}_{nn}^j\Sigma_{ni}^{jk}\mathcal{D}_{ni}^k}_{\text{insurance}} + \text{Others}\Big\}, \tag{24}$$

where "own volatility" collects the variance and covariance terms of productivities of n only, "volatility spillover" collects other countries' variance terms, and "insurance" collects the covariance terms between country n's productivities and other countries' productivities. The remaining terms capture the indirect spillover of insurance effects.

Figure 2 plots the CEV induced by global productivity uncertainty, and its decomposi-



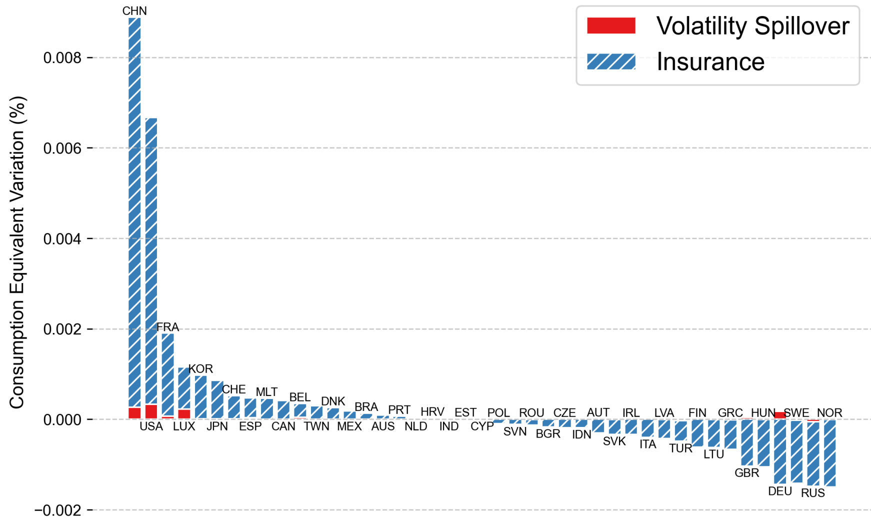
Note: Plotted are the consumption equivalent variations driven by global productivity uncertainty. "Ex-ante" refers to effects led by changes in ex-ante sourcing decisions. "Ex-post, CE" refers to effects through changing the expected consumption levels. All other terms correspond to the ex-post effects arising from risk aversion, with decompositions described in Section 4.4.2.

Figure 2: Impact of Global Productivity Uncertainty on Welfare

tion, for each country. On average, the CEV amounts to -0.61% with substantial heterogeneity: countries that went through major crises like Greece and Turkey lose approximately 2%, whereas countries with more stable and balanced economies like the U.S., U.K., and Canada lose less than 0.3%.

The red solid bars, $\mu_{\text{Ex-ante}}$, represent the effect resulting from the changes in the ex-ante sourcing decision, which by construction coincides with the numbers in Panel (b) of Figure 1. As discussed before, this effect causes redistribution across countries through trade diversion. Although this effect is on average small, it can be important. For Malta, for example, this effect is positive and offsets a third of the welfare loss due to own volatility (green bar).

The blue dashed bars, $\mu_{\text{Ex-post, CE}}$, reflect the additional effect through the change in expected consumption levels. This effect is positive for most countries, reflecting the fact that a mean-preserving spread of log productivities generally raises the consumption level, due to the convexity of the CES aggregation. This love-of-risk effect, however, is dominated by the losses from the overall risk aversion preference, as reflected by the large negative value of the remaining terms that arise from risk aversion. Among these terms, the risk induced by a country's own productivity variance is dominant, whereas the risk spillover from other countries, either direct or indirect, is negligible. The insurance effect—shaped by other countries' productivity covarying with domestic productivity—is on average negative, reflecting the fact that yearly productivity growth is in general positively correlated between countries.



Note: The figure shows the average welfare impact of each country's productivity fluctuations on other countries' welfare by affecting real income volatility. "Volatility Spillover" captures the effect through propagating via the trade and input-output linkages. "Insurance" captures the effect through covarying with other countries' productivity.

Figure 3: Productivity Insurance and Volatility Spillover of Each Country

Some countries, with weaker productivity correlation with other countries, provide greater insurance benefits to the world economy. We assess the insurance value of each country. For a destination country n and sector pair (j,k), we calculate the average covariance of country n's productivity with other countries, denoted by $\overline{\Sigma}_n^{jk} \equiv \frac{1}{N-1}\sum_{i\neq n}\Sigma_{ni}^{jk}$. We then construct each origin country i's covariance with n relative to this average as $\widetilde{\Sigma}_{ni}^{jk} \equiv \Sigma_{ni}^{jk} - \overline{\Sigma}_n^{jk}$, and calculate the welfare impact relative to the average, analogous to (24), as:

$$\mu_{ni,\text{Rel Insurance}} = -\frac{\gamma}{2}\Big\{2\sum_{j,k}\mathcal{D}_{nn}^{j}\widetilde{\Sigma}_{ni}^{jk}\mathcal{D}_{ni}^{k}\Big\}. \tag{25}$$

The thought experiment considers what additional welfare impact country i brings to country n through its covariance structure relative to the average covariance that country n faces. We then use the average of $\mu_{ni,\text{Rel Insurance}}$ across $n \neq i$ to represent the additional insurance benefit that country i provides to the globe through its covariance structure. The blue dashed bars in Figure 3 plot these values.

Countries differ in their insurance values. China notably has the largest positive influence on global welfare through this insurance effect, increasing global welfare by an average of approximately 0.01% relative to other countries. By construction, this occurs because China's productivities, after transformation by the general equilibrium exposure matrix, covary less with other countries' productivities on average. The magnitude of this insurance effect is small but not negligible compared to the level effect,[23] and can be one order of magnitude larger for countries whose domestic productivities are uncertain. Interestingly, as a major trade partner of the U.S. with weakly correlated productivity, China provides the largest insurance benefits to the U.S. among its trading partners. See Appendix B.5 for the detailed analysis. Our analysis thus uncovers a novel dimension of welfare gain from adding China to world trade, beyond the level effect extensively studied in the literature (di Giovanni et al., 2014; Hsieh and Ossa, 2016; Caliendo et al., 2019).

Finally, following a similar approach, we calculate how much additional welfare impact a country brings to the globe through its productivity variance relative to other countries. The red solid bars in Figure 3 represent these values. As shown, their magnitude is one order of magnitude smaller than the heterogeneous insurance effects.

4.5 Tariff Uncertainty

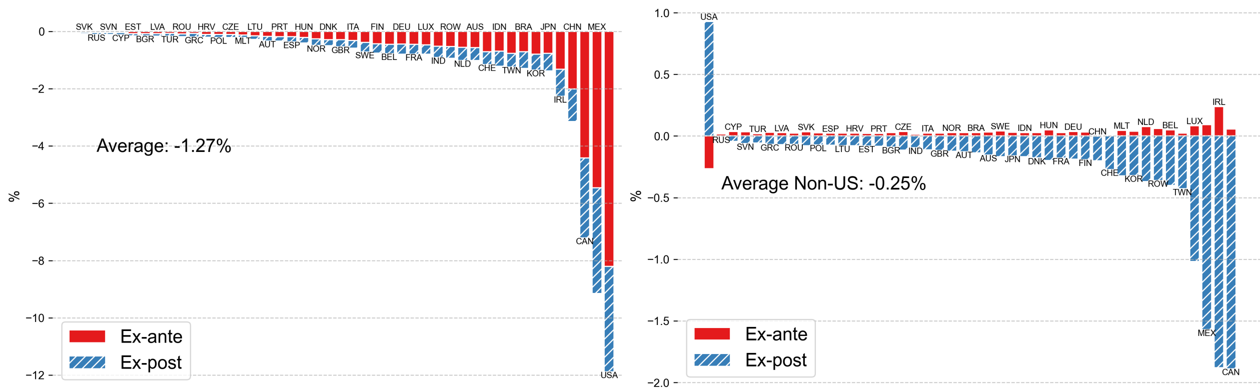
This section incorporates the tariff uncertainty resulting from the 2025 "Reciprocal Tariffs" order, constructed following the procedure in Section 4.2, on top of the productivity uncertainty. This exercise reveals the impacts of the tariff uncertainty and its interaction with productivity uncertainty.

4.5.1 The Anticipatory Effect and Adjustment in Ex-ante Sourcing

We begin by examining how the uncertain tariff shock impacts trade openness and real income across countries. Figure 4 plots changes in the import share (i.e., Imports/Total Expenditures) and real income when we introduce the uncertain tariff shock to a baseline model containing only productivity uncertainty. In the model, since the origin of each variety is chosen before tariffs are realized, the uncertain tariff affects trade and income through two channels. First, anticipating potential tariff escalation, importers adjust the origin country of each variety even before actual tariffs materialize. We label this the "Ex-ante" effect. Second, the realized tariffs, which depend on negotiation outcomes between each country and the U.S., further influence trade patterns and income. We label this the "Ex-post" effect. Since the "Ex-post" effect depends on actual realizations, we report its mean across different realizations.

Panel (a) of Figure 4 shows that trade openness, measured by the import share, declines for all countries, averaging -1.27%. The U.S. experiences a drop as large as 12%, while Mexico, Canada, and China suffer the most among other countries. The ex-ante sourcing decision

[23] To gauge the importance of this insurance effect from China, we benchmark it against the level effect of China's trade integration estimated in the literature using structural models. di Giovanni et al. (2014) finds that the mean welfare gain from adding China to world trade is 0.13%, using a static model without uncertainty calibrated to around 2007. Hsieh and Ossa (2016) finds that China's productivity growth during 1995-2007 has a modest average effect of 0.1% on global welfare through trade spillover.



(a) Import Share (b) Real Income

Note: The figure shows the percentage changes in variables when adding the tariff uncertainty induced by the "Reciprocal Tariffs" to a baseline model with productivity uncertainty only. "Ex-ante" refers to the change when actual realized tariffs are at the pre-shock level, which results from changes in the ex-ante sourcing decision. "Ex-post" refers to the average additional change to reach the ex-post mean, when realized tariffs follow the actual post-shock distribution.

Figure 4: Impact of Uncertain "Reciprocal Tariffs" on Trade and Real Income

drives the majority of this effect, as indicated by the "Ex-ante" component. This component captures what would happen if the announced tariff escalation never materializes: having already contracted to source fewer varieties from likely affected countries in anticipation of tariff increases, U.S. buyers would buy less from these countries ex post. Consequently, ex-post trade is affected by the anticipated but unrealized tariff escalation. When tariffs do materialize according to the constructed distribution, they further reduce trade, though with limited additional impact due to the relatively small short-term trade elasticity.

The heightened trade cost—in both level and uncertainty—translates into real income losses for all non-U.S. economies except Russia, as Panel (b) demonstrates. Expected real income falls by -0.25% on average, with Canada, Ireland, and Mexico experiencing the largest losses. The U.S. achieves expected real income gains of 0.66%, primarily due to the increase in tariff revenue.

Interestingly, if the U.S. does not follow through on its tariff threats, it would incur losses from the ex-ante sourcing decisions while other countries would benefit. This ex-ante redistribution effect is significant, offsetting approximately one-quarter of U.S. ex-post gains. To understand this result, recall that ex-ante sourcing decisions affect the ex-post equilibrium through total effective sourcing productivities $\{Z_{ni}^{j}\}$, which function as cost shifters (see equation (22)). All else equal, larger Z_{ni}^{j} increases the "appeal" of products from producer i, thereby raising i's real income.

Our discussion thus centers around how uncertainty alters the relative values of Z_{ni}^{j}. Three competing channels are at play. First, anticipating potentially higher tariffs, U.S.

importers increase ex-ante sourcing from domestic suppliers, raising Z^j_{nn} while lowering $\{Z^j_{ni}\}_{i \neq n}$ for $n = USA$. Second, with input-output linkages, heightened tariffs would increase U.S. production costs, discouraging other countries from sourcing from the U.S. Third, a more subtle channel operates through tariff revenue. Once realized, tariffs lead to an appreciation in the U.S. terms of trade. This translates into higher U.S. production costs, driving foreign buyers away from U.S. suppliers ex ante. Quantitatively, the third channel dominates: if we eliminate tariff revenue redistribution while maintaining tariffs' effect on trade costs alone, ex-ante sourcing decisions would instead lead to income gains for the U.S. at the expense of other countries.

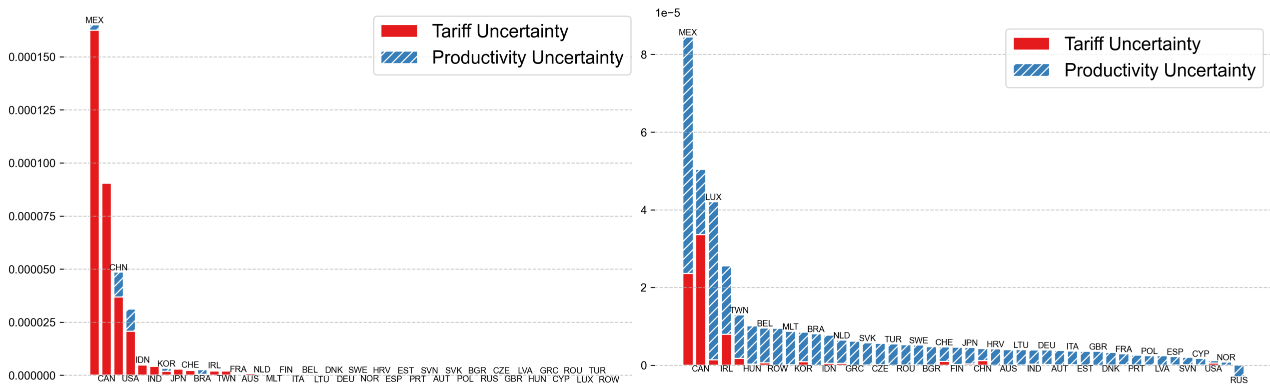
This analysis shows that anticipated but unrealized tariffs can shift income distribution due to their ex-post impact on terms of trade. In static non-stochastic models, this mechanism is entangled with other general equilibrium forces. Here, it operates independently of the realized tariffs, allowing us to gauge its importance directly.

4.5.2 The Impact on Trade and Income Volatility: Direct Effect and Interaction with Productivity Uncertainty

We next assess how the tariff shocks affect trade and output volatility. Figure 5 plots the changes in the variance of log(Import Share) and log(Real Income) when adding the tariff uncertainty into the baseline model with productivity uncertainty only. As shown in Panel (a), the trade volatility of the U.S. and several heavily exposed trading partners such as Mexico, Canada, and China increases substantially. The increase in the variance of log(Import Share) translates to a 31% increase relative to the pre-shock variance for the U.S.

Through a variance decomposition formula, we can decompose the increase in variance into contributions from tariff uncertainty and productivity uncertainty.[24] The figure shows that the tariff uncertainty itself contributes to the increase in the variance of trade share for most countries. However, for the U.S., the variance resulting from productivity uncertainty also increases significantly, accounting for half of the total increase in the variance of the log trade share. In other words, the tariff shocks increase the exposure of the U.S. to global productivity uncertainty. Intuitively, the tariff shocks—both the heightened trade cost and uncertainty—reduce trade between the U.S. and other countries, thereby limiting the diversification of the U.S. against productivity uncertainty. Panel (b) shows that the volatility of real

[24] Specifically, for a variable X we decompose $\text{Var}(X) = \text{Var}(\mathbb{E}[X|\{\tau^j_{ni}\}]) + \mathbb{E}[\text{Var}(X|\{\tau^j_{ni}\})]$, which we label as variance contributed by "Tariff Uncertainty" and "Productivity Uncertainty", respectively. Recognizing that the tariff shocks are independent of the productivity shocks, and using the perturbation solution of the ex-post equilibrium around the mean log productivities, the decomposition simplifies to $\text{Var}(X) = \text{Var}(\bar{X}(\{\tau^j_{ni}\})) + [\mathcal{D}_{X,\ln A}]\Sigma[\mathcal{D}_{X,\ln A}]^T$. Here, $\bar{X}(\{\tau^j_{ni}\})$ is the ex-post equilibrium at realized tariffs $\{\tau^j_{ni}\}$ and mean log productivities, which we enumerate for all possible tariff realizations and evaluate nonlinearly, and $\mathcal{D}_{X,\ln A}$ is the general equilibrium exposure matrix of X w.r.t. $\ln A$, which we construct at the ex-post equilibrium with mean tariff realization.



(a) Var(log(Import Share)) (b) Var(log(Real GDP))

Note: The figure shows the changes in the variance of variables when adding the tariff uncertainty induced by the "Reciprocal Tariffs" to a baseline model with productivity uncertainty only. The total variance is decomposed into the variance conditional on tariffs and the remainder, which we label as the "Tariff Uncertainty" term and "Productivity Uncertainty" term, respectively.

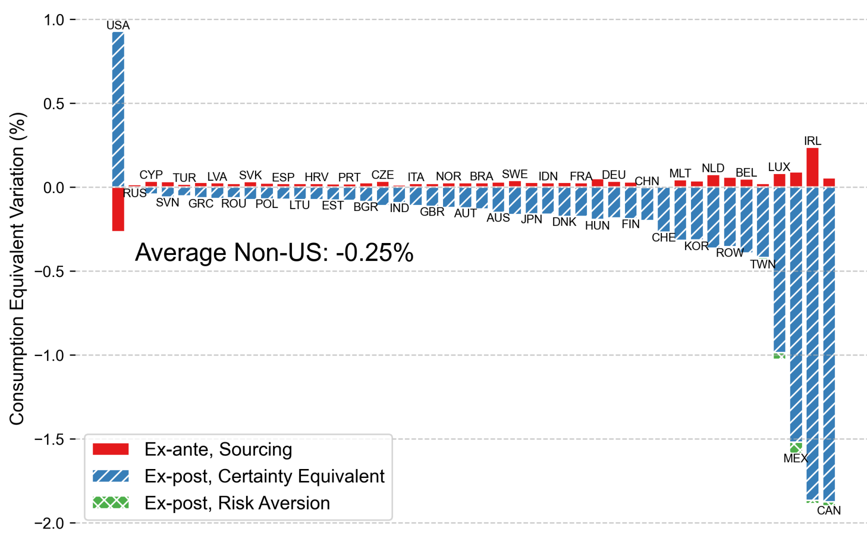
Figure 5: Impact of Uncertain "Reciprocal Tariffs" on Trade and Output Volatility

income increases in most countries, although the increase is an order of magnitude smaller.[25] Here, the change driven by productivity uncertainty actually plays a more important role. This analysis demonstrates the interaction between trade cost shocks and productivity uncertainty in shaping the policy impact.

4.5.3 The Impact on Welfare

Combining the level and uncertainty effects, we arrive at the welfare impact of the uncertain tariff shock, summarized in Figure 6. The global average welfare loss is approximately 0.23%, with the U.S. gaining 0.66% and non-U.S. countries losing on average 0.25%. The CEV is decomposed into three components following the formula (23), with each component shown in the figure. The "Ex-ante" effect corresponds to the real income change arising from adjustments in ex-ante sourcing decisions, which coincides with the "Ex-ante" component in Panel (b) of Figure 4. This component offsets one quarter of the welfare gains for the U.S. and mitigates the welfare loss for non-U.S. countries, for reasons discussed above. The "Ex-post, Certainty Equivalent" effect corresponds to the additional change in the log of expected real income, arising from realized trade cost increases and tariff revenue. This component accounts for the majority of the welfare impact. The "Ex-post, Risk Aversion" component corresponds to the effect through changes in real income volatility, and accounts for 1%-5% of total welfare losses. The majority of this increase in real income volatility arises because the heightened tariffs hamper global risk sharing against productivity uncertainty.

[25]For example, for the U.S., this increase in the variance of real income translates to a 1% increase over the pre-shock variance.



Note: The figure shows the changes in welfare when adding the tariff uncertainty induced by the "Reciprocal Tariffs" to a baseline model with productivity uncertainty only. "Ex-ante" refers to effects resulting from changes in ex-ante sourcing decisions. "Ex-post, CE" refers to effects through changing the expected consumption levels. "Ex-post, RA" refers to the effects arising from risk aversion. See equation (23) in the text for detailed definitions.

Figure 6: Impact of Uncertain "Reciprocal Tariffs" on Welfare

5 Conclusion

In this paper, we developed a quantitative trade model to demonstrate how global uncertainty, through anticipatory sourcing decisions, shapes trade patterns and welfare.

Our methodological contributions are threefold. First, we show that within a large class of trade models, the impact of arbitrary global uncertainty on anticipatory sourcing can be summarized by a deterministic country-pair-specific cost shifter, which can be solved via a fixed-point system given sourcing cost and income distributions. Second, we provide a perturbation solution to the general equilibrium model and derive analytical insights on how uncertainty affects sourcing and equilibrium outcomes. Third, we show how to implement quantitative trade models with global uncertainty through automatic differentiation, combining a local method for small shocks with a global method for large, discrete shocks.

Our main substantive findings highlight that the mere possibility of future adverse shocks, such as heightened tariffs from a "Trump trade war," can significantly depress trade and redistribute income as importers preemptively adjust their sourcing. Furthermore, we demonstrate that a trade war can amplify the negative welfare consequences of productivity shocks by limiting hedging opportunities through international trade.

Two directions for future research emerge from our work. First, global uncertainty poses challenges to the functioning of multinational firms and global value chains (GVCs). Our framework can be extended to incorporate these features of global commerce towards a better understanding of how they are shaped by uncertainty affecting different countries and production stages. Second, our framework offers a tractable foundation for analyzing the role of policies, such as trade agreements and industrial policies, in mitigating uncertainty. Incorporating and analyzing policy responses is a key direction for future work.

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Appendix for Online Publication Only

A Theory

This appendix presents details of model ingredients and proofs.

A.1 The One-sector Model

Corollary 1. *The solution to the sourcing model shutting down the aggregate uncertainty collapses to the solution to the classical EK model, with ex-ante sourcing share equals expenditure share*

$$\bar{s}_i = \frac{T_i \bar{c}_i^{-\theta}}{\sum_m T_m \bar{c}_m^{-\theta}},$$

and price index

$$\bar{P} = \gamma_\theta \left(\sum_m T_m \bar{c}_m^{-\theta} \right)^{-1/\theta},$$

where $\bar{c}_m$ are the non-stochastic sourcing costs.

Proof. Plugging in the non-stochastic sourcing costs $c_m = \bar{c}_m$ and income $I = \bar{I}$ and dropping the expectation term in λ_i

$$\begin{aligned} \Lambda &\equiv [\gamma_\theta]^{1-\sigma} \Big(\sum_m T_m \lambda_m^{\frac{\theta}{\sigma-1}} \Big)^{\frac{\sigma-1}{\theta}} \\ \lambda_i &= \bar{I}^{1-\gamma} P^{-2+\gamma+\sigma} \frac{[\bar{c}_i]^{1-\sigma}}{\sigma - 1} \\ s_i &= \frac{T_i [\lambda_i]^{\frac{\theta}{\sigma-1}}}{\sum_m T_m [\lambda_m]^{\frac{\theta}{\sigma-1}}} \end{aligned} \tag{26}$$

Therefore,

$$Z_i = \frac{s_i}{\lambda_i} \Lambda = [\gamma_\theta]^{1-\sigma} T_i [\lambda_i]^{\frac{\theta}{\sigma-1}-1} \Big[\sum_m T_m [\lambda_m]^{\frac{\theta}{\sigma-1}} \Big]^{\frac{\sigma-1}{\theta}-1}.$$

The price level

$$\begin{aligned} P &= \Big[\sum_i [\bar{c}_i]^{1-\sigma} Z_i \Big]^{\frac{1}{1-\sigma}} \\ &= \Big[\sum_i [\bar{c}_i]^{1-\sigma} [\gamma_\theta]^{1-\sigma} T_i [\lambda_i]^{\frac{\theta}{\sigma-1}-1} \Big[\sum_m T_m [\lambda_m]^{\frac{\theta}{\sigma-1}} \Big]^{\frac{\sigma-1}{\theta}-1} \Big]^{\frac{1}{1-\sigma}} \\ &= \gamma_\theta \Big[\sum_i T_i [\bar{c}_i]^{1-\sigma} \Big(\frac{\bar{c}_i^{1-\sigma}}{[\sum_m T_m \bar{c}_m^{-\theta}]^{\frac{\sigma-1}{\theta}}} \Big)^{\frac{\theta+1-\sigma}{\sigma-1}} \Big]^{\frac{1}{1-\sigma}} \\ &= \gamma_\theta [\sum_m T_m \bar{c}_m^{-\theta}]^{-\frac{1}{\theta}} \end{aligned}$$

where the third line cancels out the common factor in $\{\lambda_m\}$.

And the sourcing share

$$s_i = \frac{T_i[\bar{c}_i^{1-\sigma}]^{\frac{\theta}{\sigma-1}}}{\sum_m T_m[\bar{c}_m^{1-\sigma}]^{\frac{\theta}{\sigma-1}}} = \frac{T_i\bar{c}_i^{-\theta}}{\sum_m T_m\bar{c}_m^{-\theta}}$$

where the first equality cancels out the common factor in $\{\lambda_m\}$. □

Lemma 2. *The necessary and sufficient condition for $\{\mathcal{V}_i\}_{i=1}^N$ to be the optimal sourcing solution is that for all i,*

$$\lambda_i[z_i(v)]^{\sigma-1} \geq \lambda_m[z_m(v)]^{\sigma-1}, \forall m$$

a.e. for $v \in \mathcal{V}_i$.

Proof. **Necessity.** Fix an i. Consider any measurable set $\mathcal{B} \subset \mathcal{V}_i$ with measure $\mu(\mathcal{B}) > 0$. Consider a perturbed partition where varieties in $\mathcal{B}$ are switched from i to m, then for $Z_i \equiv \int_{\mathcal{V}_i}[z_i(v)]^{\sigma-1}\mathrm{d}v$

$$\Delta Z_i(\mathcal{B}) = -\int_{\mathcal{B}}[z_i(v)]^{\sigma-1}\mathrm{d}v$$
$$\Delta Z_m(\mathcal{B}) = \int_{\mathcal{B}}[z_m(v)]^{\sigma-1}\mathrm{d}v$$
$$\Delta Z_k = 0, \text{ for } k \neq i, m.$$

By Taylor expansion with Lagrange remainder:

$$\Delta U = \lambda_i[\Delta Z_i(\mathcal{B})] + \lambda_m[\Delta Z_m(\mathcal{B})] + \frac{1}{2}\sum_{k,j}\frac{\partial^2 U}{\partial Z_k \partial Z_j}\Big|_{\mathbf{Z}+\xi\Delta\mathbf{Z}}\Delta Z_k(\mathcal{B})\Delta Z_j(\mathcal{B}),$$

where $\boldsymbol{\lambda}$ are evaluated at the optimal $\{\mathcal{V}_i\}$, and $\xi \in (0,1)$.

Since N is finite, and $\{z_m(v)\}_{m=1}^N$ are i.i.d. across v and have finite means, we have $\max_i \mathbb{E}[z_i(v)^{\sigma-1}] < \infty$. Denote $M \equiv \max_i \mathbb{E}[z_i(v)^{\sigma-1}]$, there is

$$|\Delta Z_i(\mathcal{B})| \leq \mu(\mathcal{B})M,$$

and thus

$$\Delta U(\mathcal{B}) = \lambda_i[\Delta Z_i(\mathcal{B})] + \lambda_m[\Delta Z_m(\mathcal{B})] + o(\mu(\mathcal{B})).$$

The optimality of $\mathcal{V}_i$ requires that for all such perturbation, there is $\Delta U(\mathcal{B}) \leq 0$. Since $z_i(v)^{\sigma-1}$ is integrable on any measurable set, taking $\mu(\mathcal{B}) \to 0$ and invoking the Lebesgue differentiation theorem we have

$$\lambda_i[z_i(v)]^{\sigma-1} \geq \lambda_m[z_m(v)]^{\sigma-1} \quad \text{for a.e. } v \in \mathcal{V}_i.$$

Sufficiency. Denote measurable function $\iota^*(v)$ the sourcing plan that generates the partition

$\{\mathcal{V}_i^*\}_{i=1}^N$ that satisfies the premise. From the premise we have that for a.e. v:

$$\lambda_{\iota^*(v)}[z_{\iota^*(v)}(v)]^{\sigma-1} = \max_{m\in 1,...,N}\left(\lambda_m[z_m(v)]^{\sigma-1}\right).$$

Therefore, for any other partitions $\{\mathcal{V}_i\}$ and the associated sourcing plan $\iota(v)$, for a.e. v:

$$\lambda_{\iota^*(v)}[z_{\iota^*(v)}(v)]^{\sigma-1} \geq \lambda_{\iota(v)}[z_{\iota(v)}(v)]^{\sigma-1}.$$

Integrating over all varieties we have

$$\begin{aligned}\int_0^1 \lambda_{\iota^*(v)}[z_{\iota^*(v)}(v)]^{\sigma-1}\mathrm{d}v &\geq \int_0^1 \lambda_{\iota(v)}[z_{\iota(v)}(v)]^{\sigma-1}\mathrm{d}v \\ &\Rightarrow \sum_i \lambda_i Z_i^* \geq \sum_i \lambda_i Z_i \\ &\Rightarrow \nabla\tilde{U}(\mathbf{Z}^*)\cdot(\mathbf{Z}-\mathbf{Z}^*) \leq 0,\end{aligned}$$

$\forall\mathbf{Z}$, where $\tilde{U}(\mathbf{Z})$ is the expected utility treated as a function of $\mathbf{Z}$, and $\nabla\tilde{U}(\mathbf{Z}^*) = (\lambda_1, \lambda_2, ..., \lambda_N)^T$ is the gradient of $\tilde{U}$ with respect to $\mathbf{Z}$ evaluated at $\mathbf{Z}^*$. To establish that $\mathbf{Z}^*$ is optimal, it suffices to prove that $\tilde{U}$ is jointly concave in $\mathbf{Z}$.

Denote $X(\mathbf{Z}) \equiv \sum_i [c_i]^{1-\sigma} Z_i$ and consider $U = \mathbb{E}\frac{1}{1-\gamma}(I/P)^{1-\gamma} = \mathbb{E}\frac{1}{1-\gamma}I^{1-\gamma}X^{\frac{1-\gamma}{\sigma-1}} \equiv \mathbb{E}g(X)$.

$$\begin{aligned}g'(X) &= \frac{1}{1-\gamma}I^{1-\gamma}\frac{1-\gamma}{\sigma-1}X^{\frac{1-\gamma}{\sigma-1}-1} \\ g''(X) &= \frac{1}{1-\gamma}I^{1-\gamma}\frac{1-\gamma}{\sigma-1}\frac{2-\gamma-\sigma}{\sigma-1}X^{-\frac{1-\gamma}{1-\sigma}-2} < 0,\end{aligned}$$

under the assumption that $\gamma > 1$ and $\sigma > 1$. Since X is a linear mapping of $\mathbf{Z}$ with linear coefficient $[c_i]^{1-\sigma} > 0$, hence given $\{c_i\}$ and I, $\frac{1}{1-\gamma}I^{1-\gamma}X^{\frac{1-\gamma}{\sigma-1}}$ is jointly concave in $\mathbf{Z}$. This holds for any realizations of $\{c_i\}$ and I, thus U is jointly concave in $\mathbf{Z}$.

□

Proof of Proposition 2.

Proof. Since λ_i are non-stochastic, let $\ln\lambda_i = \ln\bar{\lambda}_i + \varepsilon\lambda_i^{(2)} + o(\varepsilon)$. We first have

$$\widehat{\ln P} = \sum_m \bar{\omega}_m\left[\sqrt{\varepsilon}c_m^{(1)} + \varepsilon c_m^{(2)} + \frac{1}{1-\sigma}\widehat{\ln\frac{s_m}{\lambda_m}}\right] + \frac{1}{1-\sigma}\widehat{\ln\Lambda} + o(\varepsilon) \tag{27}$$

where $\bar{\omega}_m = \frac{\bar{c}_m^{1-\sigma}\bar{s}_m/\bar{\lambda}_m}{\sum_i \bar{c}_i^{1-\sigma}\bar{s}_i/\bar{\lambda}_i} = \bar{s}_m$, which applies that $[\bar{c}_m^{1-\sigma}]/\bar{\lambda}_m$ is a constant across m according to (26).

Consider

$$\widehat{\ln s_i} = \varepsilon\varphi[\lambda_i^{(2)} - \bar{\lambda}^{(2)}],$$

where $\varphi \equiv \frac{\theta}{\sigma-1}$ and $\bar{\lambda}^{(2)} = \sum_m \bar{s}_m \lambda_m^{(2)}$. We first have

$$\widehat{\ln \frac{s_m}{\lambda_m}} = \varepsilon\{(\varphi - 1)\lambda_m^{(2)} - \varphi\bar{\lambda}^{(2)}\} + o(\varepsilon).$$

Next consider

$$\widehat{\ln \Lambda} = \varepsilon \sum_m \bar{s}_m \lambda_m^{(2)} = \varepsilon\bar{\lambda}^{(2)} + o(\varepsilon)$$

So we have

$$\sum_m \bar{s}_m \widehat{\ln \frac{s_m}{\lambda_m}} + \widehat{\ln \Lambda} = o(\varepsilon)$$

Plugging this into (27) we have

$$\widehat{\ln P} = \sqrt{\varepsilon} \sum_m \bar{s}_m c_m^{(1)} + \varepsilon \sum_m \bar{s}_m c_m^{(2)} + o(\varepsilon).$$

Next consider $\lambda_i = \frac{1}{\sigma-1}\mathbb{E}[I^{1-\gamma}P^{-2+\gamma+\sigma}[c_i]^{1-\sigma}]$. Denote $g^{(1)} \equiv \sum_m \bar{s}_m c_m^{(1)}$ and $g^{(2)} \equiv \sum_m \bar{s}_m c_m^{(2)}$ then

$$\begin{aligned}
&\mathbb{E}[I^{1-\gamma}P^{-2+\gamma+\sigma}[c_i]^{1-\sigma}] \\
&= (\bar{I}^{1-\gamma}\bar{P}^{-2+\gamma+\sigma}[\bar{c}_i]^{1-\sigma})\mathbb{E}[e^{\sqrt{\varepsilon}[(1-\sigma)c_i^{(1)}+(-2+\gamma+\sigma)g^{(1)}+(1-\gamma)I^{(1)}]}e^{\varepsilon[(1-\sigma)c_i^{(2)}+(-2+\gamma+\sigma)g^{(2)}+(1-\gamma)I^{(2)}]}] + o(\varepsilon) \\
&= (\bar{I}^{1-\gamma}\bar{P}^{-2+\gamma+\sigma}[\bar{c}_i]^{1-\sigma})e^{\frac{1}{2}Var(\sqrt{\varepsilon}[(1-\sigma)c_i^{(1)}+(-2+\gamma+\sigma)g^{(1)}+(1-\gamma)I^{(1)}])}e^{\varepsilon[(1-\sigma)\mathbb{E}c_i^{(2)}+(-2+\gamma+\sigma)\mathbb{E}g^{(2)}+(1-\gamma)\mathbb{E}I^{(2)}]} + o(\varepsilon),
\end{aligned}$$

where the last line applies the expectation of the log-normal distributed variable, and uses the fact that the variance and covariance terms that involve $c_i^{(2)}$ or $I^{(2)}$ are of order $o(\varepsilon)$. Therefore

$$\begin{aligned}
\ln \lambda_i = \ln \frac{1}{\sigma - 1} &+ \ln[(\bar{I}^{1-\gamma}\bar{P}^{-2+\gamma+\sigma}[\bar{c}_i]^{1-\sigma})e^{\varepsilon[(1-\sigma)\mathbb{E}c_i^{(2)}+(-2+\gamma+\sigma)\mathbb{E}g^{(2)}+(1-\gamma)\mathbb{E}I^{(2)}]}] \\
&+ \frac{1}{2}Var(\sqrt{\varepsilon}[(1-\sigma)c_i^{(1)} + (-2+\gamma+\sigma)g^{(1)} + (1-\gamma)I^{(1)}]) + o(\varepsilon)
\end{aligned}$$

and hence

$$\begin{aligned}
\widehat{\ln \lambda_i} = \varepsilon[(1-\sigma)\mathbb{E}c_i^{(2)} + (-2+\gamma+\sigma)\mathbb{E}g^{(2)} + (1-\gamma)\mathbb{E}I^{(2)}] + \\
\frac{1}{2}Var(\sqrt{\varepsilon}[(1-\sigma)c_i^{(1)} + (-2+\gamma+\sigma)g^{(1)} + (1-\gamma)I^{(1)}])
\end{aligned}$$

Define

$$\ln \kappa_i \equiv \ln \lambda_i - \sum_m \bar{s}_m \ln \lambda_m$$

$$\varepsilon\Delta_i \equiv \frac{1}{\sigma - 1}\Big(\ln \kappa_i - \ln \bar{\kappa}_i\Big) + o(\varepsilon)$$

We have

$$\widehat{\ln \kappa_i} = \varepsilon(1-\sigma)(\mathbb{E}c_i^{(2)} - \mathbb{E}g^{(2)}) + \frac{1}{2}Var(\sqrt{\varepsilon}[(1-\sigma)c_i^{(1)} + (-2+\gamma+\sigma)g^{(1)} + (1-\gamma)I^{(1)}]) - \frac{1}{2}\sum_m \bar{s}_m Var(\sqrt{\varepsilon}[(1-\sigma)c_m^{(1)} + (-2+\gamma+\sigma)g^{(1)} + (1-\gamma)I^{(1)}]) + o(\varepsilon)$$

Denote $\tilde{c}_i^{(1)} \equiv c_i^{(1)} - I^{(1)}$ and $\Omega_{im} = cov(\tilde{c}_i^{(1)}, \tilde{c}_m^{(1)})$, rewrite

$$(1-\sigma)c_i^{(1)} + (-2+\gamma+\sigma)g^{(1)} + (1-\gamma)I^{(1)} = (1-\sigma)\tilde{c}_i^{(1)} + (-2+\gamma+\sigma)\tilde{g}^{(1)},$$

where $\tilde{g}^{(1)} = \sum_m \bar{s}_m \tilde{c}_m^{(1)}$. Then we have

$$Var(\sqrt{\varepsilon}[(1-\sigma)\tilde{c}_i^{(1)} + (-2+\gamma+\sigma)\tilde{g}^{(1)}]) = \varepsilon\Big[(1-\sigma)^2\Omega_{ii} + (\sigma+\gamma-2)^2(\sum_m\sum_l \bar{s}_m\bar{s}_l\Omega_{ml}) + 2(1-\sigma)(\sigma+\gamma-2)(\sum_m \bar{s}_m\Omega_{im})\Big].$$

Plugging this into $\ln\kappa_i - \ln\bar{\kappa}_i$ and assigning the first order term to $\varepsilon\Delta_i$ we have

$$\Delta_i = (\mathbb{E}g^{(2)} - \mathbb{E}c_i^{(2)}) + \frac{(\sigma-1)}{2}(\Omega_{ii} - \sum_m \bar{s}_m\Omega_{mm}) - (\sigma+\gamma-2)\Big(\sum_m \bar{s}_m\Omega_{im} - \sum_m\sum_l \bar{s}_m\bar{s}_l\Omega_{ml}\Big)$$

Next, from (4), $s_i = \frac{T_i[\kappa_i]^\varphi}{\sum_m T_m[\kappa_m]^\varphi}$. Denote $K \equiv \sum_m T_m[\kappa_m]^\varphi$ we have

$$\widehat{\ln K} = \sum_m \bar{s}_m \varphi \widehat{\ln \kappa_m} + o(\varepsilon) = \varepsilon\varphi(\sigma-1)\sum_m \bar{s}_m\Delta_m + o(\varepsilon) = o(\varepsilon)$$

Therefore,

$$\widehat{\ln s_i} = \varphi\widehat{\ln \kappa_i} - \widehat{\ln K} = \varepsilon\theta\Delta_i + o(\varepsilon).$$

From $Z_i = \frac{s_i}{\lambda_i}\Lambda$ we have that

$$\widehat{\ln Z_i} = \widehat{\ln s_i} + \sum_m \bar{s}_m\widehat{\ln\lambda_m} - \widehat{\ln\lambda_i} + o(\varepsilon) = \varepsilon\theta\Delta_i - \widehat{\ln\kappa_i} + o(\varepsilon) = \varepsilon(\theta+1-\sigma)\Delta_i + o(\varepsilon)$$

□

Proof of Lemma 1.

Proof. Denote $\bar{x}$ and $\bar{y}$ the solution to the system with $\varepsilon = 0$, and $\hat{x} \equiv x - \bar{x}$ and $\hat{y} \equiv y - \bar{y}$.

Expand $F(x, y(g(x)), \sqrt{\varepsilon}\zeta) = 0$ around $(\bar{x}, \bar{y}, 0)$ up to terms of order ε:

$$
\begin{aligned}
F(\bar{x}, \bar{y}, 0) + F_1\hat{x} + F_2\hat{y} + F_3\sqrt{\varepsilon}\zeta \\
+\frac{1}{2}D^2_{11}F[\hat{x}, \hat{x}] + \frac{1}{2}D^2_{22}F[\hat{y}, \hat{y}] + \frac{1}{2}D^2_{33}F[\sqrt{\varepsilon}\zeta, \sqrt{\varepsilon}\zeta] \\
+D^2_{12}F[\hat{x}, \hat{y}] + D^2_{13}F[\hat{x}, \sqrt{\varepsilon}\zeta] + D^2_{23}F[\hat{y}, \sqrt{\varepsilon}\zeta] + o(\varepsilon) = 0,
\end{aligned}
$$

where F_u is the Jacobian matrix, and $D^2_{uv}F[A, B]$ denotes the second partial derivative term which is a bilinear map of (A, B). In particular, the k-th element of $D^2_{11}F[\hat{x}, \hat{x}] = \hat{x}^T \cdot [F^{(k)}_{11}] \cdot \hat{x}$, where $F^{(k)}_{11}$ is the Hessian of the k-th element of F with respect to (x, x^T) evaluated at $(\bar{x}, \bar{y}, 0)$.

Denote $y^{(2)} \equiv \phi(\mathbb{E}x^{(2)}) + \psi(\Omega)$. Substituting in $\hat{x} = x^{(1)}\sqrt{\varepsilon} + x^{(2)}\varepsilon + o(\varepsilon)$ and $\hat{y} = \phi(\mathbb{E}x^{(2)}\varepsilon) + \psi(\Omega\varepsilon) + o(\varepsilon)$, term by term, collecting terms up to order ε, we have

$$
\begin{aligned}
F_1\hat{x} &= F_1[x^{(1)}\sqrt{\varepsilon} + x^{(2)}\varepsilon] + o(\varepsilon) \\
F_2\hat{y} &= F_2[y^{(2)}\varepsilon] + o(\varepsilon) \\
\frac{1}{2}D^2_{11}F[\hat{x}, \hat{x}] = \frac{1}{2}D^2_{11}F[x^{(1)}\sqrt{\varepsilon} + x^{(2)}\varepsilon, x^{(1)}\sqrt{\varepsilon} + x^{(2)}\varepsilon] &= \frac{1}{2}D^2_{11}F[x^{(1)}, x^{(1)}]\varepsilon + o(\varepsilon) \\
\frac{1}{2}D^2_{22}F[\hat{y}, \hat{y}] &= o(\varepsilon) \\
\frac{1}{2}D^2_{33}F[\sqrt{\varepsilon}\zeta, \sqrt{\varepsilon}\zeta] &= \frac{1}{2}D^2_{33}F[\zeta, \zeta]\varepsilon \\
D^2_{12}F[\hat{x}, \hat{y}] &= o(\varepsilon) \\
D^2_{13}F[\hat{x}, \sqrt{\varepsilon}\zeta] &= D^2_{13}F[x^{(1)}, \zeta]\varepsilon + o(\varepsilon) \\
D^2_{23}F[\hat{y}, \sqrt{\varepsilon}\zeta] &= o(\varepsilon)
\end{aligned}
$$

Collecting order $\sqrt{\varepsilon}$ terms:

$$
\begin{aligned}
\sqrt{\varepsilon}F_1x^{(1)} + \sqrt{\varepsilon}F_3\zeta = 0 \\
\Rightarrow x^{(1)} = -F_1^{-1}F_3\zeta \equiv \Xi\zeta.
\end{aligned}
$$

Collecting order ε terms:

$$
\begin{aligned}
F_1x^{(2)}\varepsilon + F_2[y^{(2)}]\varepsilon + \frac{1}{2}D^2_{11}F[x^{(1)}, x^{(1)}]\varepsilon + \frac{1}{2}D^2_{33}F[\zeta, \zeta]\varepsilon + D^2_{13}F[x^{(1)}, \zeta]\varepsilon = 0 \\
\Rightarrow F_1x^{(2)}\varepsilon + F_2[\phi(\mathbb{E}x^{(2)}) + \psi(\Omega)]\varepsilon + \frac{1}{2}D^2_{11}F[x^{(1)}, x^{(1)}]\varepsilon + \frac{1}{2}D^2_{33}F[\zeta, \zeta]\varepsilon + D^2_{13}F[x^{(1)}, \zeta]\varepsilon = 0
\end{aligned}
$$

Plugging in $\Omega = \mathbb{E}[[\Gamma_1x^{(1)} + \Gamma_2\zeta][\Gamma_1x^{(1)} + \Gamma_2\zeta]^T] = (\Gamma_1\Xi + \Gamma_2)\Sigma(\Gamma_1\Xi + \Gamma_2)^T$, we have

$$
F_1x^{(2)} + F_2\phi(\mathbb{E}x^{(2)}) + F_2\psi((\Gamma_1\Xi + \Gamma_2)\Sigma(\Gamma_1\Xi + \Gamma_2)^T) + \frac{1}{2}D^2_{11}F[\Xi\zeta, \Xi\zeta] + \frac{1}{2}D^2_{33}F[\zeta, \zeta] + D^2_{13}F[\Xi\zeta, \zeta] = 0 \tag{28}
$$

Now we are to take expectations both sides. Let $\mathcal{H}_{11}$ be the $n_F \times (n_x)^2$ matrix with the k-th row being $vec(F^{(k)}_{11})^T$ (i.e., flatten the Hessian of the k-th element of F w.r.t. (x, x) into the

k-th row of $\mathcal{H}_{11}$), then

$$\mathbb{E}D_{11}^2 F[\Xi\zeta, \Xi\zeta] = \mathcal{H}_{11} \cdot vec(\Xi\Sigma\Xi^T).$$

Similarly, we have

$$\mathbb{E}D_{33}^2 F[\zeta, \zeta] = \mathcal{H}_{33} vec(\Sigma),$$
$$\mathbb{E}D_{13}^2 F[\Xi\zeta, \zeta] = \mathcal{H}_{13} vec(\Xi\Sigma).$$

Taking expectations both sides, and solving out $\mathbb{E}x^{(2)}$ we have

$$\mathbb{E}x^{(2)} = -[F_1 + F_2\phi]^{-1}\Big(F_2\psi((\Gamma_1\Xi + \Gamma_2)\Sigma(\Gamma_1\Xi + \Gamma_2)^T) + \frac{1}{2}\mathcal{H}_{11} vec(\Xi\Sigma\Xi^T) + \frac{1}{2}\mathcal{H}_{33} vec(\Sigma) + \mathcal{H}_{13} vec(\Xi\Sigma)\Big).$$

Finally, we can solve out $x^{(2)}$ by substituting in $\mathbb{E}x^{(2)}$ into (28).

□

Proof of Proposition 3.

Proof. Starting from the equilibrium system (14) restated below:

$$F(\ln w, \ln Z, \sqrt{\varepsilon}\zeta) \equiv \ln\Big(w_i L_i\Big) - \Big[\ln\Big(\sum_n \frac{[d_{ni} w_i/(A_i e^{\sqrt{\varepsilon}\zeta_i})]^{1-\sigma} e^{\ln Z_{ni}}}{\sum_m [d_{nm} w_m/(A_m e^{\sqrt{\varepsilon}\zeta_m})]^{1-\sigma} e^{\ln Z_{nm}}} w_n L_n\Big) + \ln\Big(\sum_n w_n L_n\Big)\Big]$$

Linearize the system w.r.t. $(\ln \boldsymbol{w}, \ln \boldsymbol{Z}, \ln \boldsymbol{A})$ around the non-stochastic solution (i.e., $\varepsilon = 0$), we have

$$\begin{aligned}
\mathrm{d}\ln w_i = \sum_n \mathcal{E}_{in}\{(1-\sigma)\mathrm{d}\ln w_i - (1-\sigma)\sum_m \pi_{nm}\mathrm{d}\ln w_m + \mathrm{d}\ln w_n\} + \sum_n \chi_n \mathrm{d}\ln w_n \\
- \sum_n \mathcal{E}_{in}\{(1-\sigma)\mathrm{d}\ln A_i - (1-\sigma)\sum_m \pi_{nm}\mathrm{d}\ln A_m\} \\
+ \sum_n \mathcal{E}_{in}\{\mathrm{d}\ln Z_{ni} - \sum_m \pi_{nm}\mathrm{d}\ln Z_{nm}\},
\end{aligned}$$

where $\pi_{ni} \equiv \frac{[d_{ni} w_i/A_i]^{1-\sigma} Z_{ni}}{\sum_m [d_{nm} w_m/A_m]^{1-\sigma} Z_{nm}}$ is region n's ex-post expenditure share and $\mathcal{E}_{in} \equiv \frac{\pi_{ni} w_n L_n}{\sum_m \pi_{mi} w_m L_m}$ is region i's income share. $\chi_n \equiv \frac{w_n L_n}{\sum_m w_m L_m}$ is region n's value added share. Coefficient matrices $\Pi = (\pi_{ni})_{n,i}, \mathcal{E} = (\mathcal{E}_{ni})_{n,i}, \chi = (\chi_n)_n$ are all evaluated at the ex-post equilibrium with $\varepsilon = 0$.

Collecting terms, we have

$$[F_1]_{im} \equiv [F_{\ln w}]_{im} = \delta_{im} - [(1-\sigma)\delta_{im} - (1-\sigma)\sum_n \mathcal{E}_{in}\pi_{nm} + \mathcal{E}_{im} + \chi_m], \tag{29}$$

where $\delta_{im} = 1$ if and only if $i = m$ and 0 otherwise. Similarly,

$$[F_2]_{i,nm} \equiv [F_{\ln Z}]_{i,nm} = -(\mathcal{E}_{in}\delta_{im} - \mathcal{E}_{in}\pi_{nm}). \tag{30}$$

And

$$[F_3]_{im} \equiv [F_{\ln A}]_{im} = (1-\sigma)(\delta_{im} - \sum_n \mathcal{E}_{in}\pi_{nm}). \tag{31}$$

Next, we derive the Hessian matrices. First, consider

$$\mathrm{d}\ln\pi_{ni} = (1-\sigma)\mathrm{d}\ln w_i - (1-\sigma)\mathrm{d}\ln A_i - [(1-\sigma)\sum_m \pi_{nm}\mathrm{d}\ln w_m - (1-\sigma)\sum_m \pi_{nm}\mathrm{d}\ln A_m] + \mathrm{d}\ln Z_{ni} - \sum_m \pi_{nm}\mathrm{d}\ln Z_{nm}.$$

Therefore,

$$\frac{\partial \ln \pi_{ni}}{\partial \ln w_k} = (1-\sigma)(\delta_{ik} - \pi_{nk}), \quad \frac{\partial \ln \pi_{ni}}{\partial \ln Z_{kj}} = \delta_{nk}(\delta_{ij} - \pi_{kj}), \quad \frac{\partial \ln \pi_{ni}}{\partial \ln A_k} = -(1-\sigma)(\delta_{ik} - \pi_{nk}).$$

Then consider differentiate $\ln \mathcal{E}_{in}$ with respect to $\ln \boldsymbol{w}$, keeping only terms that involve $\mathrm{d}\ln \boldsymbol{w}$:

$$\begin{aligned}
\mathrm{d}\ln\mathcal{E}_{in} &= \mathrm{d}\ln\pi_{ni} + \mathrm{d}\ln w_n - \sum_m \mathcal{E}_{im}[\mathrm{d}\ln\pi_{mi} + \mathrm{d}\ln w_m] \\
&= (1-\sigma)[\mathrm{d}\ln w_i - \sum_j \pi_{nj}\mathrm{d}\ln w_j] + \mathrm{d}\ln w_n \\
&\quad - \sum_m \mathcal{E}_{im}\{(1-\sigma)[\mathrm{d}\ln w_i - \sum_j \pi_{mj}\mathrm{d}\ln w_j] + \mathrm{d}\ln w_m\} \\
&= \mathrm{d}\ln w_n - \sum_m \mathcal{E}_{im}\mathrm{d}\ln w_m - (1-\sigma)(\sum_j \pi_{nj}\mathrm{d}\ln w_j - \sum_m\sum_j \mathcal{E}_{im}\pi_{mj}\mathrm{d}\ln w_j)
\end{aligned}$$

Therefore,

$$\frac{\partial \ln \mathcal{E}_{in}}{\partial \ln w_k} = \delta_{nk} - \mathcal{E}_{ik} - (1-\sigma)(\pi_{nk} - \sum_m \mathcal{E}_{im}\pi_{mk}).$$

Similarly, differentiate $\ln \mathcal{E}_{in}$ with respect to $\ln \boldsymbol{Z}$, keeping only terms that involve $\mathrm{d}\ln \boldsymbol{Z}$:

$$\begin{aligned}
\mathrm{d}\ln\mathcal{E}_{in} &= \mathrm{d}\ln\pi_{ni} - \sum_m \mathcal{E}_{im}\mathrm{d}\ln\pi_{mi} \\
&= \mathrm{d}\ln Z_{ni} - \sum_j \pi_{nj}\mathrm{d}\ln Z_{nj} - \sum_m \mathcal{E}_{im}[\mathrm{d}\ln Z_{mi} - \sum_j \pi_{mj}\mathrm{d}\ln Z_{mj}]
\end{aligned}$$

and therefore

$$\frac{\partial \ln \mathcal{E}_{in}}{\partial \ln Z_{kj}} = \delta_{nk}(\delta_{ij} - \pi_{kj}) - \mathcal{E}_{ik}(\delta_{ij} - \pi_{kj}).$$

Differentiate $\ln \mathcal{E}_{in}$ with respect to $\ln \boldsymbol{A}$, keeping only terms that involve $\mathrm{d}\ln \boldsymbol{A}$:

$$\begin{aligned}
\mathrm{d}\ln\mathcal{E}_{in} &= \mathrm{d}\ln\pi_{ni} - \sum_m \mathcal{E}_{im}\mathrm{d}\ln\pi_{mi} \\
&= -(1-\sigma)\Big[\mathrm{d}\ln A_i - \sum_j \pi_{nj}\mathrm{d}\ln A_j - \sum_m \mathcal{E}_{im}[\mathrm{d}\ln A_i - \sum_j \pi_{mj}\mathrm{d}\ln A_j]\Big] \\
&= (1-\sigma)\Big[\sum_j \pi_{nj}\mathrm{d}\ln A_j - \sum_m\sum_j \mathcal{E}_{im}\pi_{mj}\mathrm{d}\ln A_j\Big]
\end{aligned}$$

and therefore

$$\frac{\partial \ln \mathcal{E}_{in}}{\partial \ln A_k} = (1-\sigma)[\pi_{nk} - \sum_m \mathcal{E}_{im}\pi_{mk}].$$

Finally,

$$\frac{\partial \ln \chi_n}{\partial \ln w_k} = \delta_{nk} - \chi_k, \quad \frac{\partial \ln \chi_n}{\partial \ln Z_{kj}} = 0, \quad \frac{\partial \ln \chi_n}{\partial \ln A_k} = 0.$$

These partial derivatives of log shares are all functions of shares themselves and the elasticity parameter σ. Therefore

$$(F_{11}^{(i)})_{mn} = (1-\sigma)\sum_n \Big(\frac{\partial \mathcal{E}_{in}}{\partial \ln w_n}\pi_{nm} + \mathcal{E}_{in}\frac{\partial \pi_{nm}}{\partial \ln w_n}\Big) - \frac{\partial \mathcal{E}_{im}}{\partial \ln w_n} - \frac{\partial \chi_m}{\partial \ln w_n} \tag{32}$$

$$(F_{33}^{(i)})_{mn} = -(1-\sigma)\sum_n \Big(\frac{\partial \mathcal{E}_{in}}{\partial \ln A_n}\pi_{nm} + \mathcal{E}_{in}\frac{\partial \pi_{nm}}{\partial \ln A_n}\Big) \tag{33}$$

and

$$(F_{13}^{(i)})_{mn} = (1-\sigma)\sum_n \Big(\frac{\partial \mathcal{E}_{in}}{\partial \ln A_n}\pi_{nm} + \mathcal{E}_{in}\frac{\partial \pi_{nm}}{\partial \ln A_n}\Big) - \frac{\partial \mathcal{E}_{im}}{\partial \ln A_n}, \tag{34}$$

where the super script (i) denotes the i-th equation and subscripts denote the indices of variables to be differentiated with respect to. These second derivatives are also functions of shares and σ.

The linear operator Γ_1, Γ_2 maps $\{\ln w_m\}, \{\ln A_m\}$ to $\ln \tilde{c}_{ni} = \ln w_i - \ln A_i - \ln w_n$:

$$\Gamma_{1,ni,m} = \delta_{im} - \delta_{nm} \tag{35}$$

$$\Gamma_{2,ni,m} = -\delta_{im} \tag{36}$$

The linear operator ϕ maps $\{\ln w_m\}$ to $(\theta + 1 - \sigma)[\sum_m \pi_{nm} \ln w_m - \ln w_i]$:

$$\phi_{ni,m} = (\theta + 1 - \sigma)[\pi_{nm} - \delta_{mi}] \tag{37}$$

The linear operator ψ maps the covariance matrix of $\ln \tilde{c}_{ni}$, Ω, to

$$\begin{aligned}[\psi(\Omega)]_{ni} = (\theta + 1 - \sigma)\Big[&\frac{(\sigma-1)}{2}(\Omega_{ni,ni} - \sum_m \bar{s}_{nm}\Omega_{nm,nm}) \\ &-(\sigma+\gamma-2)\Big(\sum_m \bar{s}_{nm}\Omega_{ni,nm} - \sum_k\sum_l \bar{s}_{nk}\bar{s}_{nl}\Omega_{nk,nl}\Big)\Big]\end{aligned} \tag{38}$$

With these characterizations, combining the ex-ante sourcing solution characterized in Proposition 2 and invoking Lemma 1 we have

$$\ln Z_{ni} = \ln \bar{Z}_{ni} + \varepsilon(\theta + 1 - \sigma)\Delta_{ni} + o(\varepsilon)$$

with

$$\begin{aligned}\Delta_{ni} = &(\sum_m \bar{s}_{nm}\mathbb{E}w_m^{(2)} - \mathbb{E}w_i^{(2)}) \\ &+ \frac{(\sigma-1)}{2}(\Omega_{ni,ni} - \sum_m \bar{s}_{nm}\Omega_{nm,nm}) - (\sigma+\gamma-2)\Big(\sum_m \bar{s}_{nm}\Omega_{ni,nm} - \sum_k\sum_l \bar{s}_{nk}\bar{s}_{nl}\Omega_{nk,nl}\Big),\end{aligned}$$

$$\widehat{w} \equiv \ln w - \ln \bar{w} = -\varepsilon(\theta + 1 - \sigma) F_1^{-1} F_2 \cdot vec(\Delta) + \sqrt{\varepsilon}\Xi\zeta$$
$$- \varepsilon F_1^{-1} \Big(\frac{1}{2} D_{11}^2 F[\Xi\zeta, \Xi\zeta] + \frac{1}{2} D_{33}^2 F[\zeta, \zeta] + D_{13}^2 F[\Xi\zeta, \zeta] \Big),$$

where

$$\mathbb{E} w^{(2)} = -[F_1 + F_2\phi]^{-1} \Big(F_2 \psi(\Omega) + \frac{1}{2} \mathcal{H}_{11} vec(\Xi\Sigma\Xi^T) + \frac{1}{2} \mathcal{H}_{33} vec(\Sigma) + \mathcal{H}_{13} vec(\Xi\Sigma) \Big).$$

Here, $\Xi \equiv -F_1^{-1} F_3$, $\mathcal{H}_{11}$ is the $N \times N^2$ matrix with the k-th row being $vec(F_{11}^{(k)})^T$, and $\mathcal{H}_{33}$ and $\mathcal{H}_{13}$ are similarly defined. $F_1, F_2, F_3, F_{11}, F_{33}, F_{13}$ are defined in (29)-(34); $\Omega = (\Gamma_1 \Xi + \Gamma_2)\Sigma(\Gamma_1 \Xi + \Gamma_2)^T$ with Γ_1, Γ_2 defined in (35) and (36); ϕ is defined in (37) and ψ is defined in (38). Note that $\bar{s}_{ni}$ equals the expenditure share π_{ni} evaluated at the non-stochastic equilibrium (see Corollary 1).

Note that all terms that involve Σ are compound linear transformations and thus all resulted coefficients are linear transformations of Σ, given other observable statistics.

□

A.2 Comparative Statics in the One-Sector Model

We first state a series of lemmas that express the solution to symmetric economy in closed forms.

Lemma 3. *For the symmetric economy stated in Section 3.3, the equilibrium effective sourcing productivities $\{Z_{in}\}$ satisfy*

$$\ln Z_{ni} = \ln \bar{Z}_{ni} + (\theta + 1 - \sigma)\varepsilon\Delta_{ni} + o(\varepsilon),$$

where

$$\Delta_{ni} = \begin{cases} \sigma_A^2(1-\chi)(1-\rho)\tilde{\pi}\Big\{(\gamma-1)[1-N\pi(1-\chi)] + (\sigma-1)(1-\chi)(1-N\pi)\Big\}, & \text{for } i \neq n \\ -\sigma_A^2(1-\chi)(1-\rho)(N-1)\pi\Big\{(\gamma-1)[1-N\pi(1-\chi)] + (\sigma-1)(1-\chi)(1-N\pi)\Big\}, & \text{for } i = n \end{cases}$$

Here, $\pi < 1/N$ is the import share and $\tilde{\pi} = 1 - (N-1)\pi$ is the own expenditure share. $\chi = \frac{(\sigma-1)(1-N\pi)+\sigma-1}{(\sigma-1)(1-N\pi)+\sigma} \in (0,1)$. As a result,

$$\ln s_{ni} - \ln s_{nn} = (\ln \bar{s}_{ni} - \ln \bar{s}_{nn}) + \theta\varepsilon(\Delta_{ni} - \Delta_{nn}) + o(\varepsilon),$$

where for $i \neq n$,

$$\Delta_{ni} - \Delta_{nn} = \sigma_A^2(1-\chi)(1-\rho)\Big\{(\gamma-1)(1-N\pi(1-\chi)) + (\sigma-1)(1-\chi)(1-N\pi)\Big\} > 0.$$

Proof. Starting from the market clearing condition

$$w_i L_i = \sum_n \frac{[d_{ni} w_i / A_i]^{1-\sigma}}{\sum_m [d_{nm} w_m / A_m]^{1-\sigma}} w_n L_n,$$

where $\sigma > 1$ is the trade elasticity. With productivity shocks only, consider

$$\mathrm{d}\ln w_i = \sum_n \mathcal{E}_{in}\Big[(1-\sigma)\mathrm{d}\ln w_i - (1-\sigma)\sum_m \pi_{nm}\mathrm{d}\ln w_m + \mathrm{d}\ln w_n\Big]$$
$$- \sum_n \mathcal{E}_{in}\Big[(1-\sigma)\mathrm{d}\ln \hat{A}_i - (1-\sigma)\sum_m \pi_{nm}\mathrm{d}\ln \hat{A}_m\Big]$$

where $\mathrm{d}\ln \hat{A}_i = \mathrm{d}\ln A_i - \frac{1}{N}\sum_m \mathrm{d}\ln A_m$ and there is $\sum_i \mathrm{d}\ln \hat{A}_i = 0$ and

$$\mathcal{E}_{in} \equiv \frac{\pi_{ni} w_n L_n}{\sum_m \pi_{mi} w_m L_m}$$

is the income share. Denote $\pi_{ni} = \pi < 1/N$ for $i \neq n$ and thus $\mathcal{E}_{in} = \pi$ for $i \neq n$. Denote $\tilde{\pi} = 1 - (N-1)\pi$.

In matrix form this is

$$(I - \Pi)(\sigma I - (1-\sigma)\Pi)\mathrm{d}\ln w = (\sigma - 1)(I - \Pi^2)\mathrm{d}\ln \hat{A}.$$

Combining the normalization equation $\sum_m \mathrm{d}\ln w_m = 0 \Rightarrow \mathrm{d}\ln w_i = -\sum_{m\neq i}\mathrm{d}\ln w_m$, we have that

$$[\Pi \cdot \mathrm{d}\ln w]_i = \underbrace{(1 - N\pi)}_{\equiv \chi_2}\mathrm{d}\ln w_i$$
$$\Rightarrow \Pi \cdot \mathrm{d}\ln w = \chi_2 \cdot \mathrm{d}\ln w$$

Similarly, there is

$$\Pi^2 \cdot \mathrm{d}\ln \hat{A} = \chi_2^2 \mathrm{d}\ln \hat{A}.$$

Plugging in we have

$$(1-\chi_2)(\sigma - (1-\sigma)\chi_2)\mathrm{d}\ln w = (\sigma - 1)(1 - \chi_2^2)\mathrm{d}\ln \hat{A}$$
$$\Rightarrow \mathrm{d}\ln w = \underbrace{\frac{(\sigma-1)(1+\chi_2)}{\sigma - (1-\sigma)\chi_2}}_{\chi}\mathrm{d}\ln \hat{A}.$$

Apply the assumption $Var(\ln A_m) = \varepsilon\sigma_A^2$ and $Cov(\ln A_i, \ln A_m) = \varepsilon\rho\sigma_A^2$. We first calculate

$$\sum_m \pi_{nm} Cov\Big(\ln\frac{w_i/A_i}{w_n}, \ln\frac{w_m/A_m}{w_n}\Big) - \sum_k\sum_l \pi_{nk}\pi_{nl} Cov\Big(\ln\frac{w_k/A_k}{w_n}, \ln\frac{w_l/A_l}{w_n}\Big) \tag{39}$$

Fix an n, for $i \neq n$, denote $X_m \equiv \ln\frac{w_m/A_m}{w_n}$. Then for $m \neq n$:

$$X_m = (\chi - 1)\ln A_m - \chi \ln A_n$$

For $m = n$:

$$X_n = -\ln A_n.$$

Denote $M_{im} = Cov(X_i, X_m)$. Now for both $i, m \neq n$ and $i \neq m$:

$$\begin{aligned} M_{im} &= Cov((\chi-1)\ln A_i - \chi \ln A_n, (\chi-1)\ln A_m - \chi \ln A_n) \\ &= \varepsilon\sigma_A^2[(\chi-1)^2\rho - 2\chi(\chi-1)\rho + \chi^2] \equiv M_3 \end{aligned}$$

For one of i, m is n:

$$M_{in} = M_{ni} = Cov((\chi-1)\ln A_i - \chi \ln A_n, -\ln A_n) = \varepsilon\sigma_A^2\Big[-(\chi-1)\rho + \chi\Big] \equiv M_1 \tag{40}$$

And

$$\begin{aligned} Var(X_i) &= \varepsilon\sigma_A^2[(\chi-1)^2 + \chi^2 - 2\chi(\chi-1)\rho] \equiv M_2, \quad i \neq n \\ Var(X_n) &= \varepsilon\sigma_A^2. \end{aligned}$$

The first term in (39) thus equals

$$\sum_m \pi_{nm} M_{im} = \tilde{\pi} M_1 + \pi M_2 + (N-2)\pi M_3.$$

And the second term equals

$$\sum_k \sum_l \pi_{nk}\pi_{nl}M_{kl} = \tilde{\pi}^2 Var(X_n) + (N-1)\pi^2 M_2 + 2(N-1)\tilde{\pi}\pi M_1 + (N-1)(N-2)\pi^2 M_3,$$

where the four terms come from $(k,l) = (n,n), k = l \neq n, k = n \neq l, k \neq l \neq n$. Subtract the second term from the first term we have that (39) equals

$$-\varepsilon\sigma_A^2(1-\chi)(1-\rho)\tilde{\pi}[1 - N\pi(1-\chi)].$$

Since $\chi < 1$ and $\rho < 1$, this term is negative. And a decrease in π (an increase in $\tilde{\pi}$) increase the absolute value of this term.

Similarly, for $n = i$, (39) equals

$$\varepsilon\sigma_A^2(1-\chi)(1-\rho)(N-1)\pi[1 - N\pi(1-\chi)].$$

Next we calculate

$$Cov\Big(\ln\frac{w_i/A_i}{w_n}, \ln\frac{w_i/A_i}{w_n}\Big) - \sum_m \pi_{nm} Cov\Big(\ln\frac{w_m/A_m}{w_n}, \ln\frac{w_m/A_m}{w_n}\Big). \tag{41}$$

When $i \neq n$, this is

$$\begin{aligned} &M_2 - (N-1)\pi M_2 - \tilde{\pi} Var(X_n) \\ &= \tilde{\pi}[M_2 - Var(X_n)] = -2\varepsilon\sigma_A^2\tilde{\pi}\chi(1-\chi)(1-\rho). \end{aligned}$$

When $i = n$, this is

$$Var(X_n) - (N-1)\pi M_2 - \tilde{\pi} Var(X_n) = 2\varepsilon\sigma_A^2(N-1)\pi\chi(1-\chi)(1-\rho)$$

Restate the risk correction term in Proposition 3:

$$\varepsilon\Delta_{ni} = (\sum_m \pi_{nm}\mathbb{E}\widehat{w}_m - \mathbb{E}\widehat{w}_i) \tag{42}$$
$$+\varepsilon\Big[\frac{(\sigma-1)}{2}(\Omega_{ni,ni} - \sum_m \pi_{nm}\Omega_{nm,nm}) - (\sigma+\gamma-2)\Big(\sum_m \pi_{nm}\Omega_{ni,nm} - \sum_k\sum_l \pi_{nk}\pi_{nl}\Omega_{nk,nl}\Big)\Big]$$

Under the symmetric equilibrium, $(\sum_m \pi_{nm}\mathbb{E}\widehat{w}_m - \mathbb{E}\widehat{w}_i)$ drops. Substituting in the value of (39) and (41) characterized above, and plugging in this risk correction term into the formula for $\ln Z_{ni}$ and $\ln s_{ni}$ characterized in Proposition 3, we arrive at the desired results.

□

Lemma 4. *For the symmetric economy stated in Section 3.3, to the first order of* ε*, the variance of equilibrium log real income satisfies*

$$Var(\ln C_n) = \varepsilon\sigma_A^2\left[\left(1-2(N-1)\chi_3 + N(N-1)\chi_3^2\right)(1-\rho)+\rho\right] + o(\varepsilon)$$

Here, $\pi < 1/N$ *is the import share and* $\tilde{\pi} = 1-(N-1)\pi$ *is the own expenditure share in the non-stochastic equilibrium.* $\chi_3 \equiv \frac{\pi}{\sigma-(1-\sigma)(1-N\pi)}$.

Furthermore, to the first order of ε*,* $Var(\ln C_n)$ *decreases with* π.

Proof. Consider

$$d\ln C_n = d\ln w_n - d\ln P_n$$
$$= d\ln w_n - \sum_i \pi_{ni}[d\ln w_i - d\ln A_i] = -\sum_i \pi_{ni} d\ln\frac{w_i/A_i}{w_n}$$

Fixing an n and using the definition of M_1, M_2, M_3 in the proof of Lemma 3, to the first order of ε, we have

$$Var(\ln C_n) \approx \sum_k\sum_l \pi_{nk}\pi_{nl} Cov\Big(\ln\frac{w_k/A_k}{w_n}, \ln\frac{w_l/A_l}{w_n}\Big)$$
$$= [\tilde{\pi}^2 Var(X_n) + (N-1)\pi^2 M_2 + 2(N-1)\tilde{\pi}\pi M_1 + (N-1)(N-2)\pi^2 M_3]$$
$$= \varepsilon\sigma_A^2\left[\left(1-2(N-1)\chi_3 + N(N-1)\chi_3^2\right)(1-\rho)+\rho\right],$$

where $\chi_3 \equiv \frac{\pi}{\sigma-(1-\sigma)(1-N\pi)} < \pi < \frac{1}{N}$, which increases with π. Rewrite

$$N(N-1)\chi_3^2 - 2(N-1)\chi_3 + 1 = N(N-1)[\chi_3^2 - \frac{2}{N}\chi_3] + 1 = N(N-1)(\chi_3 - \frac{1}{N})^2 + \frac{1}{N},$$

which is strictly positive and decreases with χ_3 since $\chi_3 < \frac{1}{N}$. Therefore, to the first order, $Var(\ln C_n)$ decreases with χ_3 and decreases with π. □

Proof of Proposition 4.

Proof. From Lemma 3 we have

$$\ln s_{ni} - \ln s_{nn} = (\ln\bar{s}_{ni} - \ln\bar{s}_{nn}) + \theta\varepsilon(\Delta_{ni} - \Delta_{nn}) + o(\varepsilon),$$

where for $i \neq n$, $\Delta_{ni} - \Delta_{nn} = (\sigma+\gamma-2)\sigma_A^2(1-\chi)(1-\rho)[1-N\pi(1-\chi)]$, and $\chi = \frac{(\sigma-1)(1-N\pi)+\sigma-1}{(\sigma-1)(1-N\pi)+\sigma} \in (0,1)$ which increases with σ.

Since changing σ_A^2 or ρ does not affect $\bar{s}_{ni}$, we have that to the first order of ε, for $i \neq n$, $\ln s_{ni} - \ln s_{nn}$ increases with σ_A^2 and the marginal effect is larger with a higher $1-\rho$ or γ. Similarly, $\ln s_{ni} - \ln s_{nn}$ increases with $1-\rho$ and the marginal effect is larger with a higher σ_A^2 or γ. Since $\sum_i s_{ni} = 1$, $\{\ln s_{ni}\}_{i\neq n}$ changes proportionally to $\{\ln s_{ni} - \ln s_{nn}\}_{i\neq n}$. Since $1-\chi$ decreases with σ and $\sigma+\gamma-2$ increases with σ, the marginal effect is non-monotone with respect to σ. □

Proof of Proposition 5.

Proof. Apply the characterization of the non-stochastic equilibrium in Corollary 1 we have that for the non-stochastic equilibrium, the foreign souring share equals the import share, and is given by

$$\bar{s}_{ni} = \frac{T_{ni}\bar{c}_{ni}^{-\theta}}{\sum_m T_{nm}\bar{c}_{nm}^{-\theta}}, \quad \forall i \neq n.$$

Apply the symmetric equilibrium property $\bar{c}_{ni} = \bar{w}_i d/\bar{A}_i$ for $i \neq n$ and $T_{ni} = T_{nm}, \bar{A}_i = \bar{A}_m, \bar{w}_i = \bar{w}_m, \forall i, m$, this is

$$\bar{s}_{ni} = \frac{d^{-\theta}}{1+(N-1)d^{-\theta}}, \quad \forall i \neq n.$$

Therefore, $\{\bar{s}_{ni}\}_{i\neq n}$ decrease with d. This proves part (1).

Next from Lemma 3, consider for $i \neq n$,

$$\Delta_{ni} - \Delta_{nn} = \sigma_A^2(1-\chi)(1-\rho)\Big\{(\gamma-1)(1-N\pi(1-\chi)) + (\sigma-1)(1-\chi)(1-N\pi)\Big\}$$

with $\chi = \frac{(\sigma-1)(1-N\pi)+\sigma-1}{(\sigma-1)(1-N\pi)+\sigma}$. Here by definition, $\pi = \frac{d^{-\theta}}{1+(N-1)d^{-\theta}}$, and $\tilde{\pi} = 1-(N-1)\pi$.

Denote $\tilde{\chi} \equiv 1-\chi = \frac{1}{(\sigma-1)(1-N\pi)+\sigma} \in (\frac{1}{2\sigma-1}, \frac{1}{\sigma})$ which increases with π, and $D \equiv \Delta_{ni} - \Delta_{nn}$. Rewrite

$$\begin{aligned} D &= \sigma_A^2\tilde{\chi}(1-\rho)\Big\{(\gamma-1)(1-N\pi\tilde{\chi}) + (\sigma-1)\tilde{\chi}(1-N\pi)\Big\} \\ &= \sigma_A^2\tilde{\chi}(1-\rho)\Big\{(\gamma-1)\Big(\frac{\sigma}{\sigma-1} - \frac{2\sigma-1}{\sigma-1}\tilde{\chi}\Big) + (1-\sigma\tilde{\chi})\Big\} \end{aligned}$$

Therefore,

$$\frac{dD}{d\tilde{\chi}} = \sigma_A^2(1-\rho)\Big\{\Big(-2\frac{(\gamma-1)(2\sigma-1)}{\sigma-1} - 2\sigma\Big)\tilde{\chi} + \frac{\gamma-1}{\sigma-1}\sigma + 1\Big\}.$$

which is negative if and only if

$$\tilde{\chi} > \frac{1}{2}\frac{\frac{\gamma-1}{\sigma-1}+\gamma}{\frac{\gamma-1}{\sigma-1}(2\sigma-1)+\sigma} \equiv \tilde{\chi}^* \equiv \frac{1}{(\sigma-1)(1-N\pi^*)+\sigma},$$

which implicitly defines a threshold π^* as a function of (N, γ, σ). Therefore, when $\pi > \pi^*$, there is $\frac{dD}{d\pi} < 0$. An increase in d thus decreases π and increases $\Delta_{ni} - \Delta_{nn}$. This proves part (2).

Finally, from Lemma 4, $Var(\ln C_n)$ decreases with π and thus increases with d. This proves part (3).

□

A.3 The Multi-Sector Model

Proposition 6. *Consider the ex-ante sourcing problem stated in (20) with destination index n suppressed. Assume sourcing costs* $\ln c^j_{ni} = \ln \bar{c}^j_{ni} + \sqrt{\varepsilon} c^{j(1)}_{ni} + \varepsilon c^{j(2)}_{ni}$ *and income* $\ln I_n = \ln \bar{I}_n + \sqrt{\varepsilon} I^{(1)}_n + \varepsilon I^{(2)}_n$*, where* $\mathbb{E} c^{j(1)}_{ni} = 0$*,* $\mathbb{E} I^{(1)}_n = 0$*, and* $\{c^{j(1)}_{ni}\}$*,* $I^{(1)}_n$ *are jointly Gaussian. Assume* $\{c^{j(2)}_{ni}\}$ *and* $I^{j(2)}_n$ *have bounded first and second moments. Then up to terms of order ε, the log share of varieties sourced from i satisfies*

$$\ln s^j_{ni} = \ln \bar{s}^j_{ni} + \varepsilon \theta \Delta^j_{ni} + o(\varepsilon),$$

and the total effective productivities $Z^j_{ni} \equiv \int_{\mathcal{V}^j_{ni}} [z^j_{ni}(v)]^{\sigma-1} \mathrm{d}v$ *satisfies*

$$\ln Z^j_{ni} = \ln \bar{Z}^j_{ni} + \varepsilon(\theta + 1 - \sigma)\Delta^j_{ni} + o(\varepsilon).$$

Here $\bar{s}^j_{ni}$ *and* $\bar{Z}^j_{ni}$ *are the corresponding variables with* $\varepsilon = 0$ *and*

$$\begin{aligned}\Delta^j_{ni} = &\underbrace{\Big(\sum_m \bar{s}^j_{nm} \mathbb{E} c^{j(2)}_{nm} - \mathbb{E} c^{j(2)}_{ni}\Big)}_{\text{Mean cost difference}} + \underbrace{\frac{(\sigma-1)}{2}\Big(\Omega^{jj}_{ni,ni} - \sum_m \bar{s}^j_{nm} \Omega^{jj}_{nm,nm}\Big)}_{\text{Risk aversion}} - \underbrace{(\sigma-1)\Big(\sum_m \bar{s}^j_{nm} \Omega^{jj}_{ni,nm} - \sum_m \sum_l \bar{s}^j_{nm} \bar{s}^j_{nl} \Omega^{jj}_{nm,nl}\Big)}_{\text{Hedging, within-sector}} \\ &\underbrace{- (\gamma-1)\Big(\sum_k \sum_m \beta^k_n \bar{s}^k_{nm} \Omega^{jk}_{ni,nm} - \sum_k \sum_m \sum_l \beta^k_n \bar{s}^j_{nm} \bar{s}^k_{nl} \Omega^{jk}_{nm,nl}\Big)}_{\text{Hedging, across-sector}},\end{aligned} \tag{43}$$

where $\Omega^{jk}_{ni,nm} \equiv Cov(c^{j(1)}_{ni} - I^{(1)}_n, c^{k(1)}_{nm} - I^{(1)}_n)$ *is the covariance of the first-order shocks to the log income-adjusted sourcing costs.*

Proof. The consumers solve

$$\max \mathbb{E} \frac{1}{1-\gamma} \Big(\frac{I_n}{\prod_j [P^j_n]^{\beta^j_n}} \Big)^{1-\gamma},$$

where $P^j_n = \Big[\sum_i [c^j_{ni}]^{1-\sigma} Z^j_{ni} \Big]^{\frac{1}{1-\sigma}}$ and $\sum_j \beta^j_n = 1$. The shadow value of Z^j_{ni} reads

$$\lambda^j_{ni} \equiv \frac{\partial \mathbb{E} \frac{1}{1-\gamma} \Big(\frac{I_n}{\prod_j [P^j_n]^{\beta^j_n}} \Big)^{1-\gamma}}{\partial Z^j_{ni}} = -\mathbb{E} I_n^{1-\gamma} P_n^{\gamma-2} \frac{\partial P_n}{\partial P^j_n} \frac{\partial P^j_n}{\partial Z^j_{ni}},$$

where $P_n \equiv \prod_j [P_n^j]^{\beta_n^j}$, and thus $\frac{\partial P_n}{\partial P_n^j} = \beta_n^j P_n / P_n^j$. Substituting in we have

$$\begin{aligned}
\lambda_{ni}^j &= -\mathbb{E} I_n^{1-\gamma} P_n^{\gamma-2}\, \beta_n^j \frac{P_n}{P_n^j} \frac{1}{1-\sigma} [c_{ni}^j]^{1-\sigma} [P_n^j]^{\sigma} \\
&= \frac{\beta_n^j}{\sigma-1} \mathbb{E} I_n^{1-\gamma} P_n^{\gamma-1} [P_n^j]^{\sigma-1} [c_{ni}^j]^{1-\sigma} \\
&= \frac{\beta_n^j}{\sigma-1} \mathbb{E} \tilde{P}_n^{\gamma-1} [\tilde{P}_n^j]^{\sigma-1} [\tilde{c}_{ni}^j]^{1-\sigma}, \qquad (44)
\end{aligned}$$

where $\tilde{c}_{ni}^j \equiv c_{ni}^j / I_n$, $\tilde{P}_n \equiv P_n / I_n$ and $\tilde{P}_n^j \equiv P_n^j / I_n$. With

$$\begin{aligned}
Z_{ni}^j &= \frac{s_{ni}^j}{\lambda_{ni}^j} \Lambda_n^j \\
s_{ni}^j &= \frac{T_i [\lambda_{ni}^j]^{\frac{\theta}{\sigma-1}}}{\sum_m T_m [\lambda_{nm}^j]^{\frac{\theta}{\sigma-1}}} \qquad (45) \\
\Lambda_n^j &= [\gamma_\theta]^{1-\sigma} \Big(\sum_m T_m [\lambda_{nm}^j]^{\frac{\theta}{\sigma-1}} \Big)^{\frac{\sigma-1}{\theta}}.
\end{aligned}$$

Therefore, we have

$$\sum_m \bar{s}_{nm}^j \widehat{\ln \frac{s_{nm}^j}{\lambda_{nm}^j}} + \widehat{\ln \Lambda_n^j} = o(\varepsilon)$$

and

$$\begin{aligned}
\widehat{\ln P_n^j} &= \sum_m \bar{s}_{nm}^j \Big[\sqrt{\varepsilon} c_{nm}^{j(1)} + \varepsilon c_{nm}^{j(2)} + \frac{1}{1-\sigma} \widehat{\ln \frac{s_{nm}^j}{\lambda_{nm}^j}} \Big] + \frac{1}{1-\sigma} \widehat{\ln \Lambda_n^j} + o(\varepsilon) \\
&= \sqrt{\varepsilon} \sum_m \bar{s}_{nm}^j c_{nm}^{j(1)} + \varepsilon \sum_m \bar{s}_{nm}^j c_{nm}^{j(2)} + o(\varepsilon).
\end{aligned}$$

So we have

$$\widehat{\ln \lambda_{ni}^j} = \varepsilon[(1-\sigma)\mathbb{E} c_{ni}^{j(2)} - Cons_n^j] + \frac{1}{2} Var(\sqrt{\varepsilon}[(1-\sigma)\tilde{c}_{ni}^{j(1)} + (\sigma-1)\tilde{g}_d^{j(1)} + (\gamma-1)\tilde{P}_n^{(1)}]),$$

where $Cons_n^j$ does not depend vary by i, and $\tilde{g}_n^{j(1)} \equiv \sum_m \bar{s}_{nm}^j \tilde{c}_{nm}^{j(1)}$. Consider

$$\ln \kappa_{ni}^j \equiv \ln \lambda_{ni}^j - \sum_m \bar{s}_{nm}^j \ln \lambda_{nm}^j$$

we have

$$\begin{aligned}
\widehat{\ln \kappa_{ni}^j} = \varepsilon(1-\sigma)[\mathbb{E} c_{ni}^{j(2)} - \sum_m \bar{s}_{nm}^j \mathbb{E} c_{nm}^{j(2)}] + \frac{1}{2}[Var(\sqrt{\varepsilon}[(1-\sigma)\tilde{c}_{ni}^{j(1)}) - \sum_m \bar{s}_{nm}^j Var(\sqrt{\varepsilon}[(1-\sigma)\tilde{c}_{nm}^{j(1)})] \\
+[Cov(\sqrt{\varepsilon}[(1-\sigma)\tilde{c}_{ni}^{j(1)}, (\sigma-1)\tilde{g}_d^{j(1)}) - \sum_m \bar{s}_{nm}^j Cov(\sqrt{\varepsilon}[(1-\sigma)\tilde{c}_{nm}^{j(1)}, (\sigma-1)\tilde{g}_d^{j(1)})] \\
+[Cov(\sqrt{\varepsilon}[(1-\sigma)\tilde{c}_{ni}^{j(1)}, (\gamma-1)\tilde{P}_n^{(1)}) - \sum_m \bar{s}_{nm}^j Cov(\sqrt{\varepsilon}[(1-\sigma)\tilde{c}_{nm}^{j(1)}, (\gamma-1)\tilde{P}_n^{(1)})] + o(\varepsilon).
\end{aligned}$$

Note that all terms that do not differ by i have been differenced out.

Denoting

$$\varepsilon\Delta_{ni}^{j} \equiv \frac{1}{\sigma-1}\widehat{\ln\kappa_{ni}^{j}} + o(\varepsilon)$$

we have

$$\Delta_{ni}^{j} = (\sum_m \bar{s}_{nm}^{j}\mathbb{E}c_{nm}^{j(2)} - \mathbb{E}c_{ni}^{j(2)}) + \frac{(\sigma-1)}{2}(\Omega_{ni,ni}^{jj} - \sum_m \bar{s}_{nm}^{j}\Omega_{nm,nm}^{jj}) - (\sigma-1)\Big(\sum_m \bar{s}_{nm}^{j}\Omega_{ni,nm}^{jj} - \sum_m\sum_l \bar{s}_{nm}^{j}\bar{s}_{nl}^{j}\Omega_{nm,nl}^{jj}\Big)$$
$$-(\gamma-1)\Big(Cov(\tilde{c}_{ni}^{j(1)},\tilde{P}_n^{(1)}) - \sum_m \bar{s}_{nm}^{j}Cov(\tilde{c}_{nm}^{j(1)},\tilde{P}_n^{(1)})\Big),$$

from which by applying

$$P_n = \prod_j [P_n^j]^{\beta_n^j}, \quad P_n^j = \Big[\sum_i [c_{ni}^j]^{1-\sigma} Z_{ni}^j\Big]^{\frac{1}{1-\sigma}},$$

we can write

$$Cov(\tilde{c}_{ni}^{j(1)},\tilde{P}_n^{(1)}) = \sum_k\sum_m \beta_n^k \bar{s}_{nm}^k \Omega_{ni,nm}^{jk},$$

in which $\Omega_{ni,nm}^{jk} \equiv Cov(\tilde{c}_{ni}^{j(1)},\tilde{c}_{nm}^{k(1)})$.

□

Definition 2. *Given the distributions of $\{A_i^j\}$ and $\{\tau_{ni}^j\}$, a competitive equilibrium consists of ex-ante sourcing decisions $\{\iota_n(v), Z_{ni}^j\}$ and ex-post equilibrium random variables $\{\kappa_i^j, c_{ni}^j, \pi_{ni}^j, w_n, I_n, P_n^j, E_n^k, X_n^j\}$ such that*

1. *Given the distributions of $\{I_n, P_n^j, c_{ni}^j\}$, $\{\iota_n(v), Z_{ni}^j\}$ solve the ex-ante sourcing problem defined in* (20).

2. *Given the ex-ante sourcing decision summarized by $\{Z_{ni}^j\}$, random variables $\{\kappa_i^j, c_{ni}^j, \pi_{ni}^j, w_n, I_n, P_n^j, E_n^k, X_n^j\}$*

solve the ex-post equilibrium system

$$\kappa_i^j = \frac{1}{A_i^j} \prod_k [P_i^k]^{\gamma_i^{jk}} [w_i]^{\gamma_i^{jL}} \tag{46a}$$

$$c_{ni}^j = \kappa_i^j (1+\tau_{ni}^j) \bar{d}_{ni}^j \tag{46b}$$

$$P_n^j = \left(\sum_i [c_{ni}^j]^{1-\sigma} Z_{ni}^j \right)^{\frac{1}{1-\sigma}} \tag{46c}$$

$$\pi_{ni}^j = \frac{[c_{ni}^j]^{1-\sigma} Z_{ni}^j}{[P_n^j]^{1-\sigma}} \tag{46d}$$

$$I_n = w_n + \frac{1}{L_n} \sum_{i,k} \frac{\tau_{ni}^k \pi_{ni}^k E_n^k}{1+\tau_{ni}^k} \tag{46e}$$

$$E_n^k = \sum_j \gamma_n^{jk} X_n^j + \beta_n^k I_n L_n \tag{46f}$$

$$X_i^k = \sum_n \frac{\pi_{ni}^k E_n^k}{1+\tau_{ni}^k} \tag{46g}$$

$$w_n L_n = \sum_j \gamma_n^{jL} X_n^j / \left(\sum_m w_m L_m \right) \tag{46h}$$

for any realization of $\{A_i^j, \tau_{ni}^j\}$*, where* $\ln d_{ni}^j = \ln \bar{d}_{ni}^j + \ln(1+\tau_{ni}^j)$*. Here* κ_i^j *is the unit production cost of intermediate goods and* c_{ni}^j *is the aggregate component of the sourcing cost.* π_{ni}^j *is the import expenditure share,* E_n^k *is the total expenditures of composite goods of sector k by country n, and* X_i^k *is the total sales of intermediate goods produced by sector k of country i net of tariffs.*

(46a) *is given by the unit production cost.* (46b) *follows the definition of the sourcing cost.* (46c) *is given by the price index of composite goods.* (46d) *is given by the optimal ex-post sourcing decision.* (46e) *corresponds to total income equal to wage income plus tariff revenue.* (46f) *follows the definition of total expenditures of composite goods: sum of those spent as intermediate inputs and final goods.* (46g) *is the market clearing condition for intermediate goods stated in terms of value net of tariffs: total sales equals total expenditures.* (46h) *is the labor market clearing condition, each divided by the normalization equation* $\sum_m w_m L_m = 1$.

B Quantification

This appendix presents the numerical procedure and additional quantitative results.

B.1 Numerical Algorithm

The algorithm for solving the stochastic competitive equilibrium uses an iterative procedure to solve for $\{\lambda_{ni}^j\}$. For the ex-post equilibrium conditional on $\{\lambda_{ni}^j\}$, we combine the local solution with respect to log productivities, $\ln A_i^j$, conditional on tariffs τ_{ni}^j, and fully non-linear solutions when we alter τ_{ni}^j.

We start by describing the algorithm for solving the ex-post equilibrium non-linearly, *given* $\{\lambda_{ni}^j\}$ and $\{\tau_{ni}^j\}$, in Algorithm 1.

Algorithm 1 Solving the Nonlinear Ex-post Equilibrium Given $\{\lambda_{ni}^j\}$ and $\{\tau_{ni}^j\}$

1: Construct $\{Z_{ni}^j\}$ from $\{\lambda_{ni}^j\}$ according to equation (7).
2: Initiate $\{\ln w_n, \ln I_n, \ln P_n^j, \ln E_n^k\}$. Set converged to false.
3: **while** not converged **do**
4: Evaluate the ex-post equilibrium conditions stated in (46) given $\{\lambda_{ni}^j\}$ and $\{\tau_{ni}^j\}$, by substituting in $\{\ln w_n, \ln I_n, \ln P_n^j, \ln E_n^k\}$ to the right hand side, and collect these objects from the left hand side, denoted by $\{T \ln w_n, T \ln I_n, T \ln P_n^j, T \ln E_n^k\}$.
5: Calculate metric $=$ $\max\{\text{metric}_w, \text{metric}_I, \text{metric}_P, \text{metric}_E\}$, where metric_w $=$ $\max_n\{|T \ln w_n - \ln w_n|\}$, and similarly defined for metric_I, metric_P and metric_E.
6: Set converged to true if metric $< 1e-8$.
7: Update $\ln w_n \leftarrow \ln w_n + \text{step}_{w,n} \cdot (T \ln w_n - \ln w_n)$, with $\text{step}_{w,n}$ controlled by a fixed point solver, and similarly for $\ln I_n$, $\ln P_n^j$ and $\ln E_n^k$.
8: **end while**

We now describe the iterative algorithm for solving λ_{ni}^j. We first describe how to calculate the expectation term that enters the determination of λ_{ni}^j in (21). Define

$$K_{ni}^j \equiv \ln\Big(- I_n^{1-\gamma} P_n^{\gamma-2} \frac{\partial P_n}{\partial P_n^j} \frac{\partial P_n^j}{\partial Z_{ni}^j}\Big) \tag{47}$$

$$= (\gamma-1)\ln(P_n/I_n) + (\sigma-1)\ln(P_n^j/I_n) + (1-\sigma)\ln(c_{ni}^j/I_n) + \ln(\frac{\beta_n^j}{\sigma-1}),$$

Applying the law of iterative expectations to λ_{ni}^j

$$\lambda_{ni}^j = \mathbb{E}[\exp(K_{ni}^j)] = \mathbb{E}_{\boldsymbol{\tau}}\Big[\mathbb{E}_{\boldsymbol{A}}[\exp(K_{ni}^j(\boldsymbol{\tau}))]\Big]. \tag{48}$$

Our strategy of solving for λ_{ni}^j is to characterize $\mathbb{E}_{\boldsymbol{A}}[\exp(K_{ni}^j(\boldsymbol{\tau}))]$ through local solution, and then integrate over the solutions across $\boldsymbol{\tau}$ with nonlinear solutions.

1. **Characterizing** $\mathbb{E}_{\boldsymbol{A}}[\exp(K_{ni}^j(\boldsymbol{\tau}))]$
   Stack K_{ni}^j into a vector $\boldsymbol{K} \equiv vec(\{K_{ni}^j\})$ and similarly for $\ln \boldsymbol{A} \equiv vec(\{\ln A_i^j\})$. Conditional on tariffs, $\boldsymbol{\tau} = \{\tau_{ni}^j\}$, we apply a local approximation of the ex-post equilibrium around $\ln \boldsymbol{A} = \ln \bar{\boldsymbol{A}}$:

$$\boldsymbol{K}(\boldsymbol{\tau}) \approx \bar{\boldsymbol{K}}(\boldsymbol{\tau}) + [\mathcal{D}_{K,\ln A}(\boldsymbol{\tau}) \cdot \widehat{\ln \boldsymbol{A}}], \tag{49}$$

where $\bar{\boldsymbol{K}}(\boldsymbol{\tau})$ is the expected value of $\boldsymbol{K}$ at the ex-post equilibrium conditional on $\boldsymbol{\tau}$ (with expectation taken across $\widehat{\ln \boldsymbol{A}}$); $\widehat{\ln \boldsymbol{A}} = \ln \boldsymbol{A} - \ln \bar{\boldsymbol{A}}$; and $\mathcal{D}_{K,\ln A}(\boldsymbol{\tau})$ is the general equilibrium exposure matrix of $\boldsymbol{K}(\boldsymbol{\tau})$ w.r.t. $\ln \bar{\boldsymbol{A}}$, which we describe how to construct tractably below. In other words, we have written $\boldsymbol{K}(\boldsymbol{\tau})$ as a random variable around its mean. Then

$$
\begin{aligned}
\lambda_{ni}^{j} &= \mathbb{E}_{\boldsymbol{\tau}}\Big[\mathbb{E}_{\boldsymbol{A}}[\exp(K_{ni}^{j}(\boldsymbol{\tau}))]\Big] \\
&\approx \mathbb{E}_{\boldsymbol{\tau}}\Big[\exp(\bar{K}_{ni}^{j}(\boldsymbol{\tau}))\exp(\frac{1}{2}Var(K_{ni}^{j}|\boldsymbol{\tau}))\Big],
\end{aligned}
\tag{50}
$$

where one can view the approximation here as either from the log normal distribution of $\boldsymbol{K}(\boldsymbol{\tau})$ or a second-order approximation for the exponential function.

2. **Computing** $Var(K_{ni}^{j}|\boldsymbol{\tau})$
   For $Var(K_{ni}^{j}|\boldsymbol{\tau})$, we only need to calculate the diagonal elements of the following:

   $$Cov(\boldsymbol{K}|\boldsymbol{\tau}) \approx [\mathcal{D}_{K,\ln A}(\boldsymbol{\tau})]\Sigma[\mathcal{D}_{K,\ln A}(\boldsymbol{\tau})]^{T},$$

   where Σ is the covariance matrix of $\ln \boldsymbol{A}$.
   We calculate$\mathcal{D}_{K,\ln A}(\boldsymbol{\tau})$, the general equilibrium exposure matrix, by combining chain rules and implicit differentiation:

   $$
   \begin{aligned}
   \mathcal{D}_{K,\ln A}(\boldsymbol{\tau}) &= \mathcal{J}_{K,\ln A} + \mathcal{J}_{K,x}\cdot\mathcal{D}_{x,\ln A} \\
   \mathcal{D}_{x,\ln A} &= -\mathcal{J}_{F,x}^{-1}\mathcal{J}_{F,\ln A},
   \end{aligned}
   $$

   where $\mathcal{J}_{u,v}$ is the Jacobian matrix of u w.r.t. v, and all Jacobians are evaluated at the solution to the ex-post equilibrium with tariffs $\boldsymbol{\tau}$ and $\ln \boldsymbol{A} = \ln \bar{\boldsymbol{A}}$. Here, $x \equiv (\ln \boldsymbol{w}, \ln \boldsymbol{I}, \ln \boldsymbol{P}, \ln \boldsymbol{E})$ is the unknown that enters the system of equations that defines the ex-post equilibrium—the one that Algorithm 1 solves, and F is the residual vector of the system. We use an automatic differentiation (AD) algorithm implemented by Enzyme (Moses and Churavy, 2020; Moses et al., 2022) to construct the Jacobians.

3. **Expression for** $\bar{K}_{ni}^{j}(\boldsymbol{\tau})$
   The most involved step is to compute $\bar{K}_{ni}^{j}(\boldsymbol{\tau})$, the expected sourcing decision across $\widehat{\ln \boldsymbol{A}}$ conditioned on $\boldsymbol{\tau}$. Note that this expected value is different from the value of $K_{ni}^{j}(\boldsymbol{\tau})$ when $\ln \boldsymbol{A} = \ln \bar{\boldsymbol{A}}$, because the second-order effects of $\widehat{\ln \boldsymbol{A}}$ also affect $\bar{K}_{ni}^{j}(\boldsymbol{\tau})$ through the Hessian. Following similar conceptual steps in the proof of Proposition

3,[26] we can write:

$$\bar{\boldsymbol{K}}(\boldsymbol{\tau}) = \boldsymbol{K}(\boldsymbol{\tau})_{\widehat{\ln \boldsymbol{A}}=0} + \mathcal{J}_{K,x}\mathbb{E}\hat{x} + \frac{1}{2}\mathcal{H}_{K,(x,x^T)} \cdot vec(\Xi\Sigma\Xi^T) + \frac{1}{2}\mathcal{H}_{K,(\ln A,\ln A^T)} \cdot vec(\Sigma) + \mathcal{H}_{K,(x,\ln A^T)} \cdot vec(\Xi\Sigma) + o(\varepsilon). \tag{51}$$

Here, $\mathbb{E}\hat{x}$ is the expectation (over $\boldsymbol{A}$) of the unknowns of the ex-post equilibrium system deviated from their values at $\ln \boldsymbol{A} = \ln \bar{\boldsymbol{A}}$. This term is non-zero under the second-order approximation, and is characterized in Lemma 5.

The main computational complexity goes to the second-order exposure terms inside the calculation of $\mathbb{E}\hat{x}$ and other parts of equation (51), which take the form of the product between a Hessian matrix and a vector.[27]

For an example of such Hessian-vector products, consider the $\mathcal{H}_{K,(x,x^T)} \cdot vec(\Xi\Sigma\Xi^T)$ term in equation (51). Here, $\mathcal{H}_{K,(x,x^T)}$ is the $n_K \times (n_x)^2$ matrix with the k-th row being $vec(\frac{\partial^2 K^{(k)}}{\partial x \partial x^T})$, with $\frac{\partial^2 K^{(k)}}{\partial x \partial x^T}$ being the Hessian of the k-th equation of system (47) with respect to (x, x^T); $\Xi\Sigma\Xi^T$ is the covariance matrix of x, conceptually similar to the matrix in the one-sector model, but now expanded in dimension to feature multiple sectors and input-output linkages. Here, we have written this covariance matrix in the sandwich form of Σ—the covariance of fundamental productivities, and Ξ—the exposure matrix of x with respect to $\ln \boldsymbol{A}$ conditional on $\boldsymbol{\tau}$.

$\Xi\Sigma\Xi^T$ is straightforward to evaluate—it can be calculated using the first-order general equilibrium exposure matrix of x with respect to $\ln \boldsymbol{A}$ (i.e., the $\mathcal{D}_{x,\ln A}$ matrix constructed above). The challenge is to derive $\mathcal{H}_{K,(x,x^T)}$: since $\boldsymbol{K}$ has a dimension of $N^2 \times J$, the dimension of $\mathcal{H}_{K,(x,x^T)}$ is on the order of $N^2 \times J \times (n_x)^2$, where n_x is on the order of $N \times J$. The size of storage of this matrix exceeds 1 TB.

4. **Auto-Diff Computation of $\mathcal{H}_{K,(x,x^T)} \cdot vec(\Xi\Sigma\Xi^T)$**
   Explicitly constructing the Hessian matrix $\mathcal{H}_{K,(x,x^T)}$ is often tedious and, in this case, computationally infeasible. With the help of automatic differentiation, we can directly compute the Hessian-vector-products. Furthermore, recognizing that the number of rows of $\mathcal{H}_{K,(x,x^T)}$ is one order of magnitude larger than the number of columns, we group computations such that the Hessian-vector-products can be computed with nested Jacobian-vector-products, so the computational complexity is $O(NJ)$ instead of $O(N^2J)$.[28]

[26] As discussed below, in the computation we do not need to derive the matrix explicitly; so the similarity is only "conceptual."

[27] Under certain assumptions on the shock structure and specific choices of unknowns of the equilibrium system, a subset of these second-order exposure terms can be made zero by construction. However, in general, evaluating these terms are unavoidable in deriving the second-order perturbation solution.

[28] Directly computing the Hessian-vector-products for each entry of K would require N^2J calculations.

Let $V \equiv \Xi \Sigma \Xi^T$ be the $n_x \times n_x$ covariance matrix of x. Our goal is to compute the $n_K \times 1$ vector

$$\boldsymbol{h} \equiv \mathcal{H}_{K,(x,x^T)} \cdot \text{vec}(V).$$

By definition,

$$h_k = \sum_{i=1}^{n_x} \sum_{j=1}^{n_x} \frac{\partial^2 K^{(k)}}{\partial x_i \partial x_j} V_{ij} \tag{52}$$

We now construct $\boldsymbol{h}$ as nested Jacobian-vector-products of K w.r.t. x.

Define $\boldsymbol{v}_i$ as the i-th column of V, and define the auxiliary function $\mathcal{M}_i(x)$ as

$$\mathcal{M}_i(x) \equiv \mathcal{J}_{K,x}(x) \cdot \boldsymbol{v}_i.$$

Now consider the Jacobian of $\mathcal{M}_i$ with respect to x:

$$(\mathcal{J}_{M_i,x})_{km} = \frac{\partial (\mathcal{M}_i)_k}{\partial x_m} = \frac{\partial}{\partial x_m} \Big(\sum_{l=1}^{n_x} \frac{\partial K^{(k)}}{\partial x_l} (v_i)_l \Big) = \sum_{l=1}^{n_x} \frac{\partial^2 K^{(k)}}{\partial x_m \partial x_l} (v_i)_l$$

Setting m in the above equation to i and comparing with (52) , we see that

$$h_k = \sum_i (\mathcal{J}_{M_i,x})_{ki}.$$

Denote $\boldsymbol{u}_i$ as the i-th column of $\mathcal{J}_{M_i}$. We thus have

$$\boldsymbol{h} = \sum_{i=1}^{n_x} \boldsymbol{u}_i.$$

Note that $\boldsymbol{u}_i = \mathcal{J}_{M_i} \cdot \boldsymbol{e}_i$ where $\boldsymbol{e}_i$ is a size n_x vector with the i-th element being 1 and other elements being zero. Therefore, we can calculate $\boldsymbol{u}_i = JVP(JVP(K, \boldsymbol{v}_i), \boldsymbol{e}_i)$, where $JVP(f, \zeta)$ is to evaluate the product of the Jacobian of f (with respect to x) and vector ζ, evaluated at the ex-post equilibrium.

Efficiency and Parallelization. This algorithm circumvents the memory constraint by never forming the full Hessian $\mathcal{H}_{K,(x,x^T)}$. Furthermore, it is easily parallelizable. The main computational work occurs inside the loop over i, which runs n_x times. The loop's independence allows its n_x iterations to be distributed across multiple cores with minimal overhead. Each core computes its assigned $\boldsymbol{u}_i$ vectors simultaneously, and a final summation combines these results.

Lemma 5. *Denote $\hat{x}$ the ex-post equilibrium unknowns, deviated from their values in the non-stochastic equilibrium, given $\boldsymbol{\tau}$. To the order of the variance of productivity shocks,*

$$\mathbb{E}\hat{x} = -\varepsilon F_1^{-1} \Big(\frac{1}{2} \mathcal{H}_{11} vec(\Xi \Sigma \Xi^T) + \frac{1}{2} \mathcal{H}_{22} vec(\Sigma) + \mathcal{H}_{12} vec(\Xi \Sigma) \Big) + o(\varepsilon),$$

where $\mathcal{H}_{11}$ is the $n_F \times (n_x)^2$ matrix with the k-th row being $vec(F_{11}^{(k)})^T$ (i.e., flatten the hessian of the k-th element of F w.r.t. x into the k-th row of $\mathcal{H}_{11}$). $\mathcal{H}_{12}$ and $\mathcal{H}_{22}$ are similarly defined.

Proof. Starting from the ex-post equilibrium system

$$F(x, \ln A; \boldsymbol{\tau}) = 0,$$

where x is endogenous variable. We omit the dependence on $\boldsymbol{\tau}$; all expectations are taken conditional on $\boldsymbol{\tau}$. Denote the solution to the non-stochastic equilibrium as $\bar{x}$, so we have

$$F(\bar{x}, \ln \bar{A}) = 0.$$

Denote $\hat{x} \equiv x - \bar{x}$. Denote $\hat{a} = \ln A - \ln \bar{A} = \sqrt{\varepsilon}\zeta$. As a reminder, $\mathbb{E}\zeta = 0$ and $Cov(\zeta) = \Sigma$. Apply a second-order Taylor expansion of F with respect to $x, \ln A$, around $\bar{x}, \ln \bar{A}$ we have

$$F_1\hat{x} + F_2\hat{a} + \frac{1}{2}D_{11}^2F[\hat{x},\hat{x}] + \frac{1}{2}D_{22}^2F[\hat{a},\hat{a}] + D_{12}^2F(\hat{x},\hat{a}) = 0 \tag{53}$$

where the zero-th order term drops. D_{uv}^2F is a *bilinear* operator that represents the second derivatives of F. In particular, the k-th element of $D_{11}^2F[\hat{x},\hat{x}] = \hat{x}^T \cdot [F_{11}^{(k)}] \cdot \hat{x}$, where $F_{11}^{(k)}$ is the Hessian of the k-th element of F with respect to (x, x^T) evaluated at $(\bar{x}, \ln \bar{A})$. D_{22}^2F and D_{12}^2F are similarly defined.

We keep terms up to order ε. Denote $\hat{x} = \sqrt{\varepsilon}x^{(1)} + \varepsilon x^{(2)}$ where $x^{(1)}$ and $x^{(2)}$ are random variables to be determined. Expand each term of (53):

$$\begin{aligned}
F_1\hat{x} &= F_1\sqrt{\varepsilon}x^{(1)} + F_1\varepsilon x^{(2)} \\
F_2\hat{a} &= F_2\sqrt{\varepsilon}\zeta \\
\frac{1}{2}D_{11}^2F[\hat{x},\hat{x}] = \frac{1}{2}D_{11}^2F[\sqrt{\varepsilon}x^{(1)} + \varepsilon x^{(2)}, \sqrt{\varepsilon}x^{(1)} + \varepsilon x^{(2)}] &= \varepsilon\frac{1}{2}D_{11}^2F[x^{(1)}, x^{(1)}] + o(\varepsilon) \\
\frac{1}{2}D_{22}^2F[\hat{a},\hat{a}] &= \varepsilon\frac{1}{2}D_{22}^2F[\zeta,\zeta] \\
D_{12}^2F(\hat{x},\hat{a}) = D_{12}^2F(\sqrt{\varepsilon}x^{(1)} + \varepsilon x^{(2)}, \sqrt{\varepsilon}\zeta) &= \varepsilon D_{12}^2F(x^{(1)},\zeta) + o(\varepsilon)
\end{aligned}$$

Collecting terms of order $\sqrt{\varepsilon}$, we have

$$F_1\sqrt{\varepsilon}x^{(1)} + F_2\sqrt{\varepsilon}\zeta = 0,$$

so we have $x^{(1)} = -F_1^{-1}F_2\zeta \equiv \Xi\zeta$.

Collecting terms of order ε, we have

$$F_1\varepsilon x^{(2)} + \varepsilon\frac{1}{2}D_{11}^2F[x^{(1)}, x^{(1)}] + \varepsilon\frac{1}{2}D_{22}^2F[\zeta,\zeta] + \varepsilon D_{12}^2F(x^{(1)},\zeta) = 0$$

So we have

$$x^{(2)} = -F_1^{-1}\Big(\frac{1}{2}D_{11}^2F[x^{(1)}, x^{(1)}] + \frac{1}{2}D_{22}^2F[\zeta,\zeta] + D_{12}^2F(x^{(1)},\zeta)\Big)$$

Substituting in $x^{(1)} = \Xi\zeta$ this is

$$x^{(2)} = -F_1^{-1}\Big(\frac{1}{2}D_{11}^2F[\Xi\zeta,\Xi\zeta] + \frac{1}{2}D_{22}^2F[\zeta,\zeta] + D_{12}^2F(\Xi\zeta,\zeta)\Big).$$

Note that we can calculate

$$\mathbb{E}D_{11}^2F[\Xi\zeta,\Xi\zeta] = \mathcal{H}_{11}\cdot vec(\Xi\Sigma\Xi^T),$$

where $\mathcal{H}_{11}$ is the $n_F \times (n_x)^2$ matrix with the k-th row being $vec(F_{xx}^{(k)})^T$ (i.e., flatten the hessian of the k-th element of F w.r.t. x into the k-th row of $\mathcal{H}_{11}$). Similarly, we can calculate $\mathbb{E}D_{22}^2F[\zeta,\zeta]$ and $\mathbb{E}D_{12}^2F(\Xi\zeta,\zeta)$. Therefore, we have

$$\mathbb{E}x^{(2)} = -F_1^{-1}\Big(\frac{1}{2}\mathcal{H}_{11}vec(\Xi\Sigma\Xi^T) + \frac{1}{2}\mathcal{H}_{22}vec(\Sigma) + \mathcal{H}_{12}vec(\Xi\Sigma)\Big).$$

Therefore $\mathbb{E}\hat{x} = \mathbb{E}[\sqrt{\varepsilon}x^{(1)} + \varepsilon x^{(2)}] + o(\varepsilon) = \varepsilon\mathbb{E}x^{(2)} + o(\varepsilon)$. □

With these procedures ready, we continue with Algorithm 2 to solve the stochastic equilibrium.

Algorithm 2 Solving the Stochastic Equilibrium by Solving $\{\lambda_{ni}^j\}$

1: Initiate $\{\lambda_{ni}^j\}$. Set converged to false.
2: **while** not converged **do**
3: Enumerate for each possible realization of tariff vector $\boldsymbol{\tau}$, solve for the ex-post equilibrium nonlinearly using Algorithm 1. Construct $\exp(\bar{K}_{ni}^j(\boldsymbol{\tau}))$ and $\exp(\frac{1}{2}Var(K_{ni}^j|\boldsymbol{\tau}))$ following the procedure described above.
4: Calculate the implied λ_{ni}^j using (48), by taking the expectation across all possible realizations of $\boldsymbol{\tau}$. Denote the implied λ_{ni}^j by $T\lambda_{ni}^j$.
5: Set converged to true if $\max|\ln T\lambda_{ni}^j - \ln\lambda_{ni}^j| < 1e-5$.
6: Update $\ln\lambda_{ni}^j \leftarrow \ln\lambda_{ni}^j + \text{step}_{\lambda,ni}^j \cdot [\ln T\lambda_{ni}^j - \ln\lambda_{ni}^j]$ with $\text{step}_{\lambda,ni}^j$ controlled by a fixed-point solver.
7: **end while**

For calibration, we shut down the aggregate uncertainty and solve for trade costs absent tariffs $\{\bar{d}_{ni}^j\}$ in conjunction with the equilibrium, to match the bilateral import expenditure shares $\{\pi_{ni}^j\}$ constructed from the WIOT. It modifies Algorithm 1 by adding a set of calibration equations, described in Algorithm 3.[29]

[29]Note that the mean productivities $\{\bar{A}_n^j\}$ can not be separately identified from $\{\bar{d}_{ni}^j\}$ by the import shares. Our counterfactual exercises—focusing on the proportional changes of variables and the volatility of their log values—are not affected when using additional moments (e.g., relative real income) to isolate $\{\bar{d}_{ni}^j, \bar{A}_n^j\}$.

Algorithm 3 Calibration Based on the Non-stochastic Equilibrium

1: Initiate $\{\ln w_n, \ln I_n, \ln P_n^j, \ln E_n^j, \ln \bar{d}_{ni}^j\}$, with $\ln \bar{d}_{nn}^j = 0$. Set converged to false.

2: **while** not converged **do**

3: Evaluate the equilibrium conditions for the non-stochastic equilibrium, which can be modified from (46) by setting $Z_{ni}^j = 1$ and $\sigma - 1 = \theta$ (i.e., use θ for the elasticity and remove the ex-ante sourcing stage). Substitute in $\{\ln w_n, \ln I_n, \ln P_n^j, \ln E_n^j, \ln \bar{d}_{ni}^j\}$ to the right hand side, and collect these objects from the left hand side, denoted by $\{T \ln w_n, T \ln I_n, T \ln P_n^j, T \ln E_n^j\}$, combined with an updated $\ln \bar{d}_{ni}^j$ adjusted toward matching the import shares:

$$T \ln \bar{d}_{ni}^j = \ln \bar{d}_{ni}^j + 0.2 \Big[\ln \Big(\frac{\pi_{ni}^j}{\pi_{nn}^j} \Big) - \ln \Big(\frac{\pi_{ni}^{j,Data}}{\pi_{nn}^{j,Data}} \Big) \Big].$$

4: Calculate metric $=$ $\max\{\text{metric}_w, \text{metric}_I, \text{metric}_P, \text{metric}_E, \text{metric}_{\bar{d}}\}$, where $\text{metric}_w = \max_n\{|T \ln w_n - \ln w_n|\}$, and similarly defined for metric_I, metric_P, metric_E and $\text{metric}_{\bar{d}}$.

5: Set converged to true if metric $< 1e-8$.

6: Update $\ln w_n \leftarrow \ln w_n + \text{step}_{w,n} \cdot (T \ln w_n - \ln w_n)$, with $\text{step}_{w,n}$ controlled by a fixed point solver, and similarly for $\ln I_n$, $\ln P_n^j$, $\ln E_n^j$ and $\ln \bar{d}_{ni}^j$.

7: **end while**

B.2 Estimating Productivity Covariance Structure

We estimate the productivity covariance matrix using Solow residuals for each country-sector, constructed from the 2016 World Input Output Database-Social Economic Accounts (WIOD-SEA) data. We retain all 44 regions (including one ROW) and group industries into 32 sectors according to Table 2.

Table 2: List of Sectors

Sector ID	Description	WIOT Industry Code
1	Agriculture, Forestry, Fishing	A01,A02,A03
2	Mining & Quarrying	B
3	Food, Beverages, Tobacco	C10-C12
4	Textiles, Apparel, Leather	C13-C15
5	Wood, Cork, Straw Products (excl. Furniture)	C16
6	Paper & Paper Products	C17
7	Printing & Media Reproduction	C18
8	Coke & Refined Petroleum	C19
9	Chemicals & Chemical Products	C20
10	Pharmaceuticals	C21
11	Rubber & Plastic Products	C22
12	Non-Metallic Mineral Products	C23
13	Basic Metals	C24
14	Fabricated Metal Products (excl. Machinery)	C25
15	Computer, Electronic, Optical Products	C26
16	Electrical Equipment	C27
17	Machinery & Equipment	C28
18	Motor Vehicles, Trailers	C29
19	Other Transport Equipment	C30
20	Furniture; Other	C31_C32
21	Machinery Repair & Installation	C33
22	Electricity, Gas, Steam, AC Supply	D35
23	Water Supply, Sewerage, Waste	E36,E37-E39
24	Construction	F
25	Wholesale & Retail Trade	G45,G46,G47
26	Transportation & Storage	H49,H50,H51,H52,H53
27	Accommodation & Food Services	I
28	Information & Communication	J58,J59_J60,J61,J62_J63
29	Financial & Insurance Activities	K64,K65,K66
30	Real Estate Activities	L68
31	Professional, Scientific, Technical Activities	M69_M70,M71,M72,M73,M74_M75
32	Other Service Activities	N,O84,P85,Q,R_S

Notes: All two-digit manufacturing industries are retained. Other industries are aggregated into the one-digit level. The government sector and extraterritorial organizations (code *N* to *S*) are aggregated into one sector.

The data report nominal output (GO), labor input (in population; EMP), capital (K), and intermediate input (II) expenditures for 2000-2014. We deflate output and intermediate inputs using the corresponding deflators provided by the dataset (GO_PI and II_PI), and use the capital price deflator (pl_n) from the Penn World Table 10.0 (Feenstra, Inklaar and Timmer, 2015). The data also report wage compensation $(COMP)$ from which we can construct factor input shares.

We construct input shares for each country-sector, by first calculating the shares as input costs divided by nominal output (i.e., $COMP/GO$ for labor and II/GO for intermediate input) for each year, then taking a simple average of these shares across years for each country-sector. We assume a constant-return-to-scale production function and obtain the capital share by subtracting the labor and intermediate input shares from one. We treat these input shares as time invariant and use them to back out the Solow residuals according to

$$\ln TFP^j_{nt} = \ln(y^j_{nt}) - \text{lsh}^j_n \cdot \ln(EMP^j_{nt}) - \text{iish}^j_n \cdot \ln(ii^j_{nt}) - \text{ksh}^j_n \cdot \ln(k^j_{nt}),$$

where lsh^j_n, iish^j_n, ksh^j_n are the constructed input shares for country-sector (n, j), and y^j_{nt}, ii^j_{nt}, k^j_{nt} are the deflated output, intermediate inputs and capital.

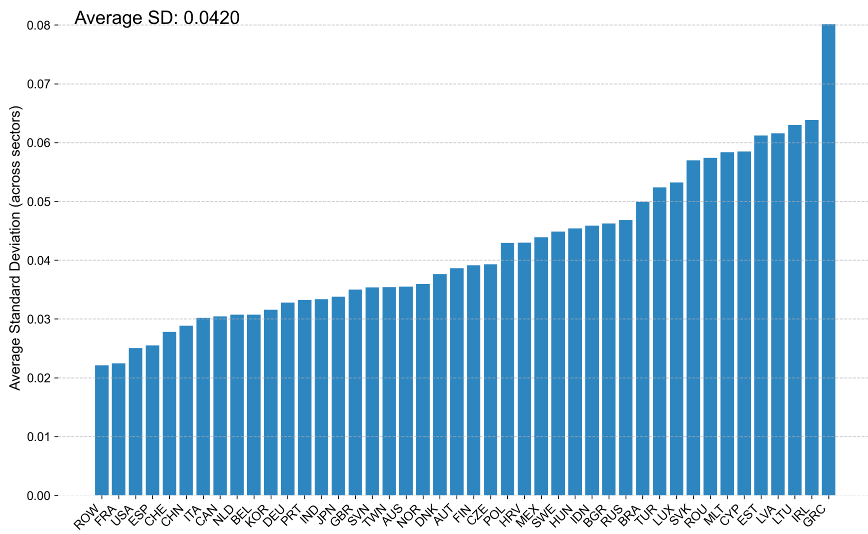
We then estimate the productivity covariance between any two country-sectors using the first difference of the Solow residual series. That is:

$$\Sigma^{jk}_{ni} = Cov(\Delta \ln TFP^j_{nt}, \Delta \ln TFP^k_{it}),$$

where Δ is the first order difference operator. We winsorize $\Delta \ln TFP^j_{nt}$ at 1% and 99% to remove outliers. For ROW, where inputs are not reported, we calculate its $\Delta \ln TFP^j_{ROW,t}$ as the average value of this variable across all countries within the same sector year (j, t).

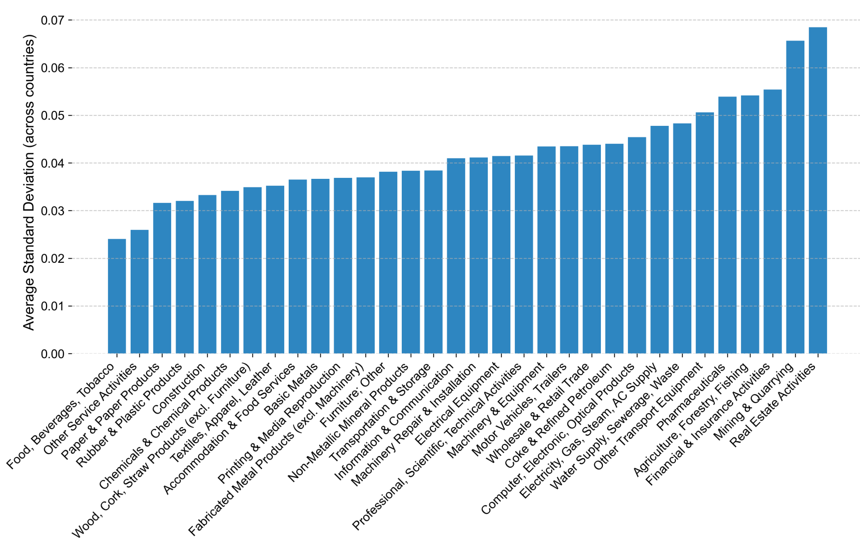
Figure 7 reports the estimated standard deviation of TFP growth rates, averaged across sectors by country. The TFP growth rate has an average volatility of 4.2%, with economically stable countries like France, the U.S., and Canada at the lower end (around 2%), while countries that experienced crises, structural changes, or that heavily rely on service sectors such as Lithuania, Ireland and Greece are at the higher end. Figure 8 reports the average standard deviation by sector. Manufacturing sectors generally exhibit lower productivity volatility, while mining and service sectors have higher volatility.

Besides differences in own productivity volatility, countries also differ in the extent to which their productivity co-varies with others. Figure 9 reports the bilateral TFP growth covariance between country-pairs. Notably, China and the U.S. are among countries whose productivities are less correlated with other countries (indicated by lighter colors), while countries like Greece and Luxembourg have productivities that are highly correlated with others, possibly due to their high dependence on international trade and international capital flows. These covariance structures are important for understanding the redistribution effects of ex-ante sourcing decisions across countries, as discussed in Section 4.4.



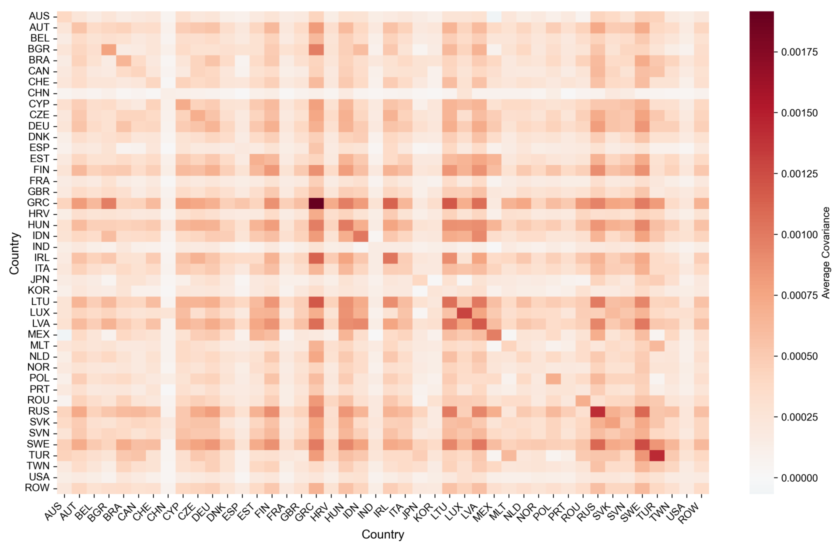
Note: The figure plots the average standard deviation of TFP growth rates across sectors for each country. The TFP series are constructed using WIOD-SEA data for 2010-2014.

Figure 7: Average TFP Volatility by Country



Note: The figure plots the average standard deviation of TFP growth rates across countries for each sector. The TFP series are constructed using WIOD-SEA data for 2010-2014.

Figure 8: Average TFP Volatility by Sector



Note: The figure plots the covariance of TFP growth rates between countries, first calculated at the country-sector level and then averaged across all sector-pairs for each country-pair. The TFP series are constructed using WIOD-SEA data for 2010-2014.

Figure 9: Productivity Covariance Between Countries

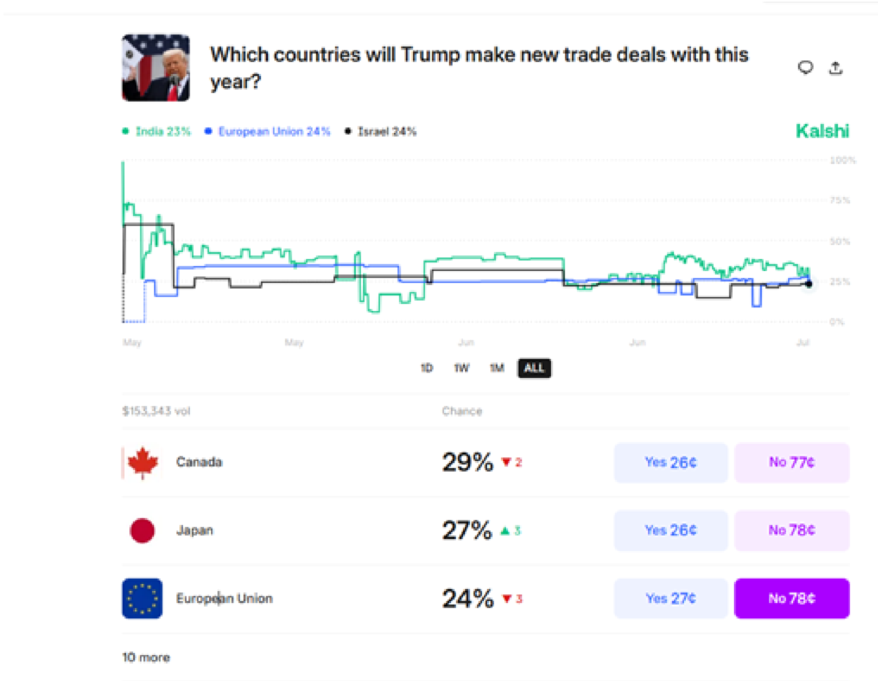


Figure 10: Example of Prediction Markets Run by Kalshi

B.3 Tariff Shocks

As discussed in Section 4.2, we assume that countries negotiate independently with the U.S., with the exception of the EU countries, which negotiate as a bloc. We model the tariff outcome as a binary process: if negotiations succeed, the tariff drops to 10%; otherwise, the tariff remains at the "Reciprocal Tariffs" announced around April 2nd, as listed in Column 2 of Table 3.

To estimate the probability of successful negotiations, we use betting prices from prediction markets run by Kalshi, specifically for the question "Which countries will Trump make new trade deals within this year?", as shown in the example in Figure 10. Kalshi is a federally regulated U.S. exchange for event contracts: its affiliated clearinghouse (Kalshi Klear) has been registered as a derivatives clearing organization since August 2024, so trading on the venue is lawful under the Commodity Exchange Act. U.S. individuals and institutions can open accounts; participants on the exchange include retail traders (who make up the majority at the present), professional/institutional traders, and market makers. Contracts are $1 binary "Yes/No" contracts, and quoted prices are conventionally interpreted as market-implied probabilities of the event. The platform is sizable and liquid—valued at about $2B in June 2025. We therefore treat Kalshi prices as reflecting the market's assessment of event likelihood.

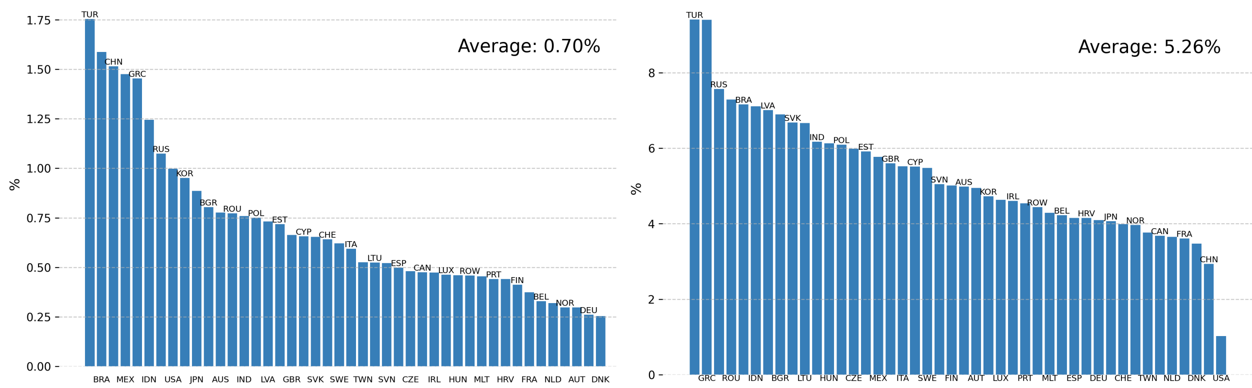
As of July 8, 2025, the probability of a new trade deal with the U.S. ranges from 34% for China, to 29% for Canada and the EU, and 8% for Russia. For regions without a corresponding prediction market, namely IDN, TWN, CHE, NOR and TUR, we use the average probability across other countries. Column 3 of Table 3 reports these probabilities.

Table 3: List of Regions and Tariff Shocks

ISO	Tariffs	Negotiation Probability
CHN	0.84	0.34
IDN	0.32	0.24
TWN	0.32	0.24
CHE	0.31	0.24
IND	0.27	0.33
CAN	0.25	0.29
KOR	0.25	0.26
MEX	0.25	0.23
JPN	0.24	0.25
EU	0.2	0.29
NOR	0.16	0.24
AUS	0.1	0.23
BRA	0.1	0.21
GBR	0.1	0.18
RUS	0.1	0.08
TUR	0.1	0.24
ROW	0.1	0.24

Notes: The probability of a successful negotiation is inferred from the betting price of prediction markets run by Kalshi, accessed on July 8, 2025. All 27 EU countries (as of 2025) are included in the model, subject to a common tariff shock of 20%. The 84% tariff for China was announced on April 9, 2025. The 25% tariffs for Mexico and Canada were announced before April 2025. Tariffs for other countries were announced on April 2, 2025.

B.4 Additional Quantitative Results



(a) S.d. log(Import Share) (b) S.d. log(Real Income)

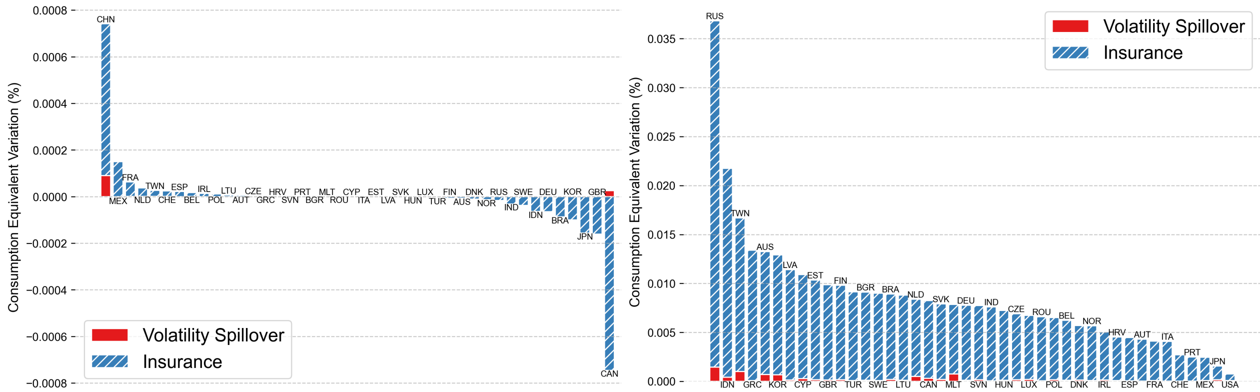
Note: The figure shows the change in variables between the calibrated economy with productivity uncertainty and the one without productivity uncertainty.

Figure 11: Impact of Productivity Uncertainty on the Volatility of Import Share and Income

Trade and real income volatility Figure 11 plots the volatility of log(Import Share) and log(Real Income) in the baseline model with aggregate productivity shocks. Since productivity shocks are the only source of uncertainty in this economy, these numbers are also the impact of global productivity uncertainty on the volatility of these variables. Their magnitude roughly reflects the magnitude of the calibrated productivity volatilities themselves and are a useful benchmark when we understand the importance of the additional volatility induced by tariff uncertainty.

The uncertainty spillover: The U.S. and China As discussed in Section 4.4.2, the productivity uncertainty of a country can spillover to other countries by transmitting volatility through trade and input-output linkages, or by co-varying with other countries' productivities to provide insurance. Panel (a) of Figure 12 plots the impact of each country's productivity uncertainty on the welfare of U.S. consumers, compared to an average country, constructed through the procedure described by (25). As shown, countries that are more geographically distant from the U.S., such as China and a few European countries provide the largest insurance benefits, whereas those that share close geographic, economic and cultural bonds with the U.S. like Canada and the UK generate negative insurance effects. Although the insurance effect is overall small, it is one order of magnitude larger than the direct volatility spillover effect.

We note that the insurance effect reflects not just raw productivity correlations but also the global trade and production structure, captured by the general equilibrium exposure matrix in decomposition equation (24). For instance, the close trade linkages between the U.S., Canada and the UK endogenously generate a positive co-movement in their produc-



(a) Other Countries' Impact on the U.S. (b) China's Impact on Other Countries

Note: The figure shows the impact of a country's productivity fluctuations on other countries' welfare by affecting real income volatility, relative to an average country. "Volatility Spillover" captures the effect through propagating the trade and input-output linkages. "Insurance" captures the effect through co-varying with other countries' productivity.

Figure 12: Impact of Productivity Uncertainty on Welfare, By-Country Examples

tion costs. Consequently, international "friends" in levels – countries with strong positive mutual exposure of economic activities (Kleinman et al., 2024a)—can paradoxically become "enemies" in uncertainty spillovers, as they provide each other with smaller insurance benefits.

Panel (b) plots the impact of China's productivity uncertainty on other countries' welfare. As shown, China provides a positive insurance benefit for all trading partners, compared to an average country; although the average effect is small, it can be one order of magnitude larger for countries that have highly uncertain domestic productivities.

B.5 Welfare Decomposition

Lemma 6. *The welfare loss for consumers in region n due to risk aversion, in an economy with log-normal distributed productivities, can be approximated by*

$$\mu_{RA,n} \approx -\frac{\gamma}{2}\sum_i \sum_m \sum_{j,k} \mathcal{D}^j_{ni}\Sigma^{jk}_{im}\mathcal{D}^k_{nm},$$

where $\mathcal{D}^j_{ni}$ *is the general equilibrium exposure matrix of* $\ln C_n$ *with respect to* $\ln A^j_i$ *and* $\Sigma^{jk}_{im} = Cov(\ln A^j_i, \ln A^k_m)$.

Proof. Recall

$$\mu_{RA,n} \equiv \frac{1}{1-\gamma}\ln \mathbb{E}[C_n^{1-\gamma}] - \ln \mathbb{E}[C_n].$$

Stacking $\boldsymbol{C} = vec(\{C_n\})$, $\boldsymbol{A} = vec(\{A^j_{ni}\})$, and treating $\mathcal{D}$ as a matrix of $N \times (NJ)$. Denote

$\widehat{\ln \boldsymbol{A}} = \ln \boldsymbol{A} - \ln \bar{\boldsymbol{A}}$. Under log-linearized approximation

$$\ln \boldsymbol{C} \approx \ln \bar{\boldsymbol{C}} + \mathcal{D} \cdot \widehat{\ln \boldsymbol{A}}.$$

Therefore

$$\begin{aligned} Cov(\ln \boldsymbol{C}) &\approx \mathbb{E}[(\mathcal{D} \cdot \ln \boldsymbol{A})(\mathcal{D} \cdot \ln \boldsymbol{A})^T] \\ &= \mathcal{D}\Sigma\mathcal{D}^T. \end{aligned}$$

Collecting the diagonal term of $Cov(\ln \boldsymbol{C})$ we have $Var(\ln C_n) \approx \sum_i \sum_m \sum_{j,k} \mathcal{D}^j_{ni} \Sigma^{jk}_{im} \mathcal{D}^k_{nm}$. Applying $\mathbb{E}X = \bar{X} \exp(\frac{1}{2} Var(X))$ for X a log-normal distributed variable we have

$$\begin{aligned} \mu_{RA,n} &\approx \left[\ln \bar{C} + \frac{1}{1-\gamma} \frac{1}{2} Var((1-\gamma) \ln C_n) \right] - \left[\ln \bar{C} + \frac{1}{2} Var(\ln C_n) \right] \\ &= -\frac{\gamma}{2} Var(\ln C_n). \end{aligned}$$

□