The Intangible Gap

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Motivation

- Corporate investment in the U.S. is shifting toward intangible capital (Corrado and Hulten, 2010; Falato et al., 2022; Crouzet and Eberly, 2019)
- Intangible investment is known to be **risky**
  - Patent-value dispersion has tripled since the 1980s
  - Innovation pipelines (e.g., LLMs, RL training) generate highly skewed outcomes
- **This paper**: who invests in intangibles, and why?
  - Cross-sectionally: small firms invest *disproportionately* more in intangibles—this gap has tripled since 2000
  - Mechanism: intangible *quality* shock + financing channel produces a size-dependent intangible gap

Intangible Gap

Intangible gap $\equiv$ difference in intangible-to-physical capital ratio (H/K) between small and large firms.

1. **The intangible gap is large and rising.**
   - Median H/K of smallest firms grew **7.1**$\times$ since 1980, vs. only **2.2**$\times$ for largest firms
   - Pre-2000 H/K spread $=$ 0.467; post-2000 H/K spread $=$ 1.374 (nearly tripled)

2. **Intangible investment has become increasingly risky.**
   - Firm-level *hkratio* volatility rose from 0.15 (1980) to 2.55 (2023) for small firms; stable for large firms
   - Cross-sectional dispersion in patent values rose by $\sim$37.5% post-2000

This Paper

Theory: a firm-dynamics model with two capital types and a novel intangible-quality shock.

Two channels jointly produce the intangible gap:

(i) **Real option**: intangible-quality shock ω + exit option $\Rightarrow$ convex payoff to intangible investment. Especially valuable for small firms near the exit margin.

(ii) **Financing**: Intangible capital can be financed through the internal capital market, while physical capital still relies on external financing collateralized by tangible assets.

Quantitatively: rising riskiness of intangible capital alone explains $\sim$ 60% of the post-2000 H/K spread.

Related Literature

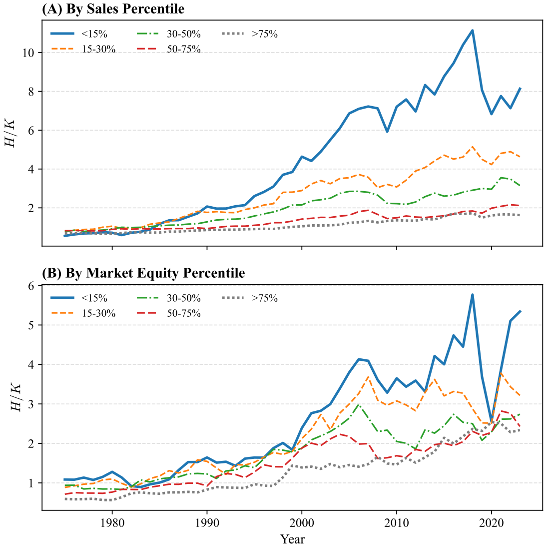
- **Firm dynamics with financial frictions**: Hopenhayn (1992), Cooley and Quadrini (2001), Gomes (2001), Albuquerque and Hopenhayn (2004), Rampini and Viswanathan (2013)
  - size-dependent heterogeneity in H/K
- **Intangible capital**: Eisfeldt and Papanikolaou (2013), Ai, Croce, and Li (2013), Gourio and Rudanko (2014), Sun and Xiaolan (2019), Falato et al. (2022), Crouzet and Eberly (2023), Eisfeldt, Falato, and Xiaolan (2023)
  - new evidence: rising cross-sectional dispersion in intangible intensity
- **Uncertainty-investment / sign-flipping channels**: Bernanke (1983), Pindyck (1991), Caballero and Pindyck (1996), Leahy and Whited (1996), Bloom (2009); Eisdorfer (2008), Ferreira, Manso, and Silva (2014)
  - *positive* uncertainty-investment relationship for intangibles and a *negative* one for physical capital *within the same firm* simutaneously
- **Corporate borrowing constraints**: Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Sun and Xiaolan (2019), Lian and Ma (2021),
  - internal financing channel for intangibles

Stylized Facts

Data

- **Sample**: Compustat Fundamentals Annual, 1975–2023
  - Exclude financials (SIC 6000–6999), utilities (4900–4999), public service (9000+)
- **Intangible capital** Peters and Taylor (2017): $H_{it} = (1 - \delta_h)H_{i,t-1} + I_{H,it}$ using R&D (XRD) and 30% of net SG&A
  - Alternatives: patent value (Kogan et al., 2017) (KPSS, CPC-subclass $\times$ year normalized); equity-based compensation (STKCO) (Eisfeldt, Falato, and Xiaolan, 2023); balance sheet intangibles (INTAN)
- **Physical capital**: PPEGT

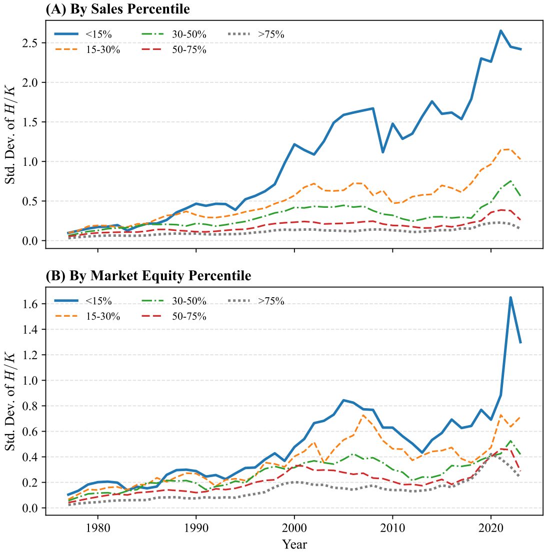
Fact 1: Intangible Gap by Firm Size



Median H/K ratio by sales-quintile (top) and market-equity-quintile (bottom), 1975–2023.

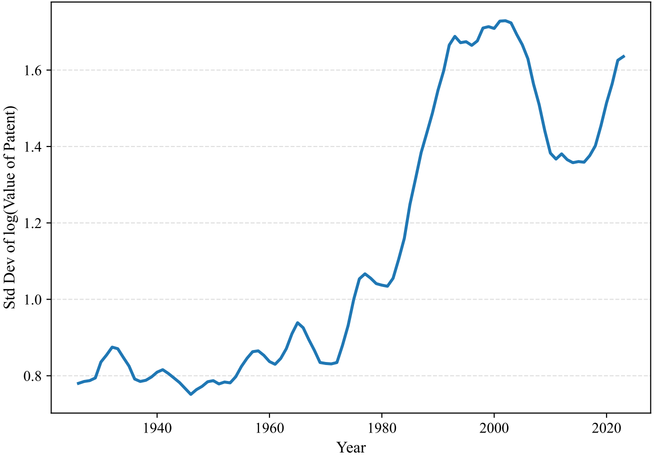
Investment rates

Fact 2: Rising Volatility of H/K Ratio



Firm-level standard deviation of *hkratio*, 5-year rolling window. Small firms: from 0.15 (1980) to 2.55

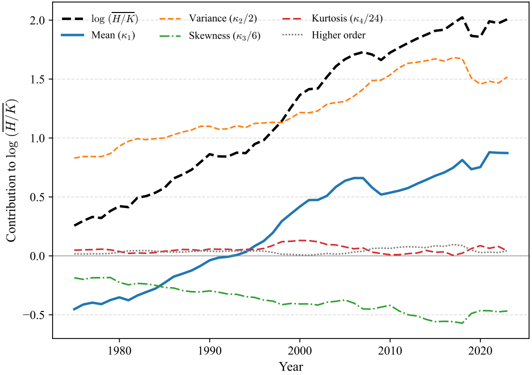
Fact 2 (cont.): Rising Patent-Value Dispersion



Standard deviation of log normalized patent value (CPC subclass × year median), 3-year rolling window. Kogan et al. (2017) data; 1975 normalized to 1.

Cumulant Decomposition: Variance, Not Mean

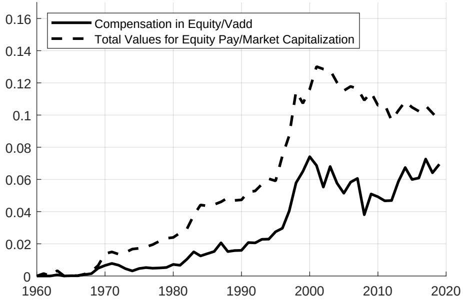
$$\log \overline{H/K}_{it} = \underbrace{\kappa_1(t)}_{\text{Mean}} + \underbrace{\frac{\kappa_2(t)}{2}}_{\text{Variance}} + \underbrace{\frac{\kappa_3(t)}{6}}_{\text{Skewness}} + \cdots$$



Variance term $(\kappa_2/2)$ now rivals the mean in driving aggregate H/K.

Fact 3: Financing Intangibles—Internal Credit Market

- Aggregate share of corporate equity allocated to employee stock-based compensation has **roughly doubled** since 1980 (Eisfeldt, Falato, and Xiaolan, 2023)
- Strongly size-dependent: stkco/AT is **6.7**× larger for smallest firms (5.69%) than for largest (0.85%) Summary statistics
- Size gradient closely mirrors the intangible gap



⇒ Motivates the assumption "intangibles can be financed internally" in the model.

The Model

- Two-Period Model
- Dynamic Model

Two-Period Model without Financial Constraint

Setup. Firm enters with h_t, chooses h_{t+1}
Quality shock. ω_{t+1} ($\log \omega \sim N(\mu, \sigma^2)$). Firm operates if value ≥ 0 at time t:

$$V_t = \max_{h_{t+1}} E_t \left[\max\{(\omega_{t+1} h_{t+1})^\alpha - K(h_{t+1}),\ 0\} \right]$$

BSM form: $V_t = \max_{h_{t+1}} h_{t+1}^\alpha N(d_1) - K(h_{t+1}) N(d_2)$.

- **Abandon option:** A call-option-like payoff structure: the firm captures the upside when ω_{t+1} is high, while the exit option truncates the downside when ω_{t+1} is low.

Proposition (Volatility raises investment)

h_{t+1}^* is strictly increasing in σ.

Proposition (Size shrinks the option)

The exit option value $V_t - V_{0,t}$ is strictly decreasing in h_t.

Why an Asymmetric Mechanism?

- **Standard Abel–Dixit–Pindyck real-options:** uncertainty $\uparrow \Rightarrow$ investment $\downarrow$ (irreversibility, waiting option).
- **Our mechanism:** multiplicative *intangible-quality* shock + exit option $\Rightarrow$ convex payoff *for intangibles only*.
  - Physical investment $\Rightarrow$ standard real-options logic ($\downarrow$ with σ)
  - Intangible investment $\Rightarrow$ option payoff ($\uparrow$ with σ, especially for small firms)

Net effect: small firms tilt toward intangibles when σ_ω rises; large firms unaffected.

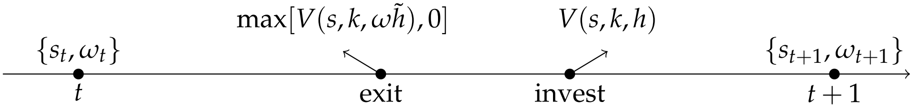
Dynamic Model: Setup

- **Production:** $F(s_t, k_t, h_t) = s_t k_t^{\alpha_k} h_t^{\alpha_h}$, s_t total productivity shock $\sim$ AR(1).
- **Capital laws of motion:** standard with quadratic adjustment costs ρ_k, ρ_h.
- **Multiplicative intangible quality shock:** $\log \omega_t \sim N(-\sigma_\omega^2/2, \sigma_\omega^2)$

$$h_{t+1} = \omega_{t+1} \tilde{\boldsymbol{h}}_{\boldsymbol{t+1}}, \qquad \tilde{\boldsymbol{h}}_{\boldsymbol{t+1}} = (1 - \delta_h)\boldsymbol{h}_{\boldsymbol{t}} + i_{h,t}$$

- **Budget:** $d_t = F(s_t, k_t, h_t) - i_{kt} - i_{ht} - g_h(\cdot) - g_k(\cdot)$.
- **Financing constraint:** $d_t \geq -\theta h_t$
  - $\theta > 0$ measures the effective borrowing capacity generated by intangible capital (e.g. equity-based comp or collateralized patents).

Within-Period Timing



Firm enters with $(s_t, k_t, \tilde{h}_t)$, observes shocks, decides exit, then invests $(k_{t+1}, \tilde{h}_{t+1})$. Exit & Entry

Investment Wedge: Real Option + Financing

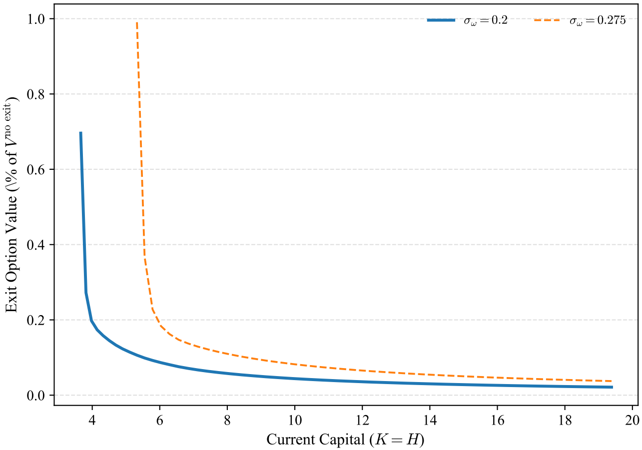
Euler equations (conditional on survival $\mathbf{1}\{\omega_{t+1} > \omega^*\}$):

$$(1+\lambda_t)q_{k,t} = \beta E_t\big[\text{MPK}_{t+1} \cdot \mathbf{1}\{\omega > \omega^*\}\big]$$
$$(1+\lambda_t)q_{h,t} = \beta E_t\Big[\boldsymbol{\omega_{t+1}}\big(\text{MPH}_{t+1} + \lambda_{t+1}\theta\big) \cdot \mathbf{1}\{\omega > \omega^*\}\Big]$$

Two asymmetries between h and k:

- Multiplicative ω_{t+1} on the intangible side $\Rightarrow$ convex payoff in ω

Investment Wedge: Real Option + Financing



Exit option value as % of firm value, along the diagonal $K = H$.

Policy function without financial friction

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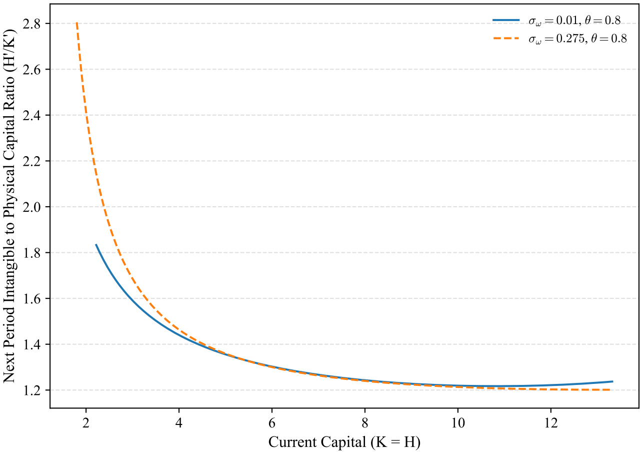
Two asymmetries between h and k:

- Multiplicative ω_{t+1} on the intangible side $\Rightarrow$ convex payoff in ω
- Additional $\lambda_{t+1}\theta$ term $\Rightarrow$ intangible capital relaxes the constraint

$\Rightarrow$ Small, financially-constrained firms tilt strongly toward intangibles.

Investment Wedge: Real Option + Financing



H'/K' along $K = H$; benchmark financing capacity ($\theta = 0.8$).

Calibration and Quantitative Analysis

Calibration

- **Sample period:** 1975–2000 (pre-shift baseline).
- **Targeted moments:**
  - Median $H/K = 1.36 \Rightarrow \alpha_h = 0.4, \alpha_k = 0.3$
  - Exit rate $= 0.046 \Rightarrow \xi = 2.45$
  - H/K spread (small vs. large) $= 0.467 \Rightarrow \sigma_\omega = 0.2$
  - Employee-equity / value share $= 0.05 \Rightarrow \theta = 0.8$
- **Externally set:** $\beta = 0.96$ (4% rate); $\rho_s = 0.8, \sigma_s = 0.1$ (Hennessy and Whited, 2005); $\delta_h = \delta_k = 0.1$, $\rho_h = \rho_k = 0.1$ (imposed symmetric to isolate the quality-shock mechanism).

Model fit

H/K Ratio by Size

	<15%	[15%,30%]	[30%,50%]	[50%,75%]	>75%
H/K Data	1.676	1.437	1.374	1.336	1.209
$\theta = 0.8,\ \sigma_\omega = 0.2$	**1.818**	1.375	1.348	1.333	1.315
$\theta = 0.8,\ \sigma_\omega = 0.01$	1.541	1.340	1.340	1.340	1.340
$\theta = 0.01,\ \sigma_\omega = 0.2$	1.123	1.158	1.355	1.327	1.370
$\theta = 0.01,\ \sigma_\omega = 0.01$	1.060	1.285	1.201	1.354	1.347

Benchmark ($\theta = 0.8, \sigma_\omega = 0.2$) matches the declining H/K gradient across size groups. Without the quality shock ($\sigma_\omega = 0.01$), H/K is flat across size.

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Explaining the Post-2000 Widening

Scenario	H/K spread	Empl. equity/V	% of data
Data 1975–2000	0.467	4.8%	100%
Model, $\sigma_\omega = 0.2$, $\theta = 0.8$	0.503	10.5%	107.7%
Model, $\sigma_\omega = 0.2$, $\theta = 1.2$	0.160	16.2%	34.3%
Data 2001–2023	1.374	11.1%	100%
Model, $\sigma_\omega = 0.275$, $\theta = 0.8$	**0.812**	18.7%	**60.1%**

Key result: raising σ_ω by 37.5% (matched to patent-value dispersion) explains $\sim$ 60% of the post-2000 H/K spread.

Not driven by θ: raising θ alone *shrinks* the spread (option channel needed).

Robustness: Generalized Financial Frictions

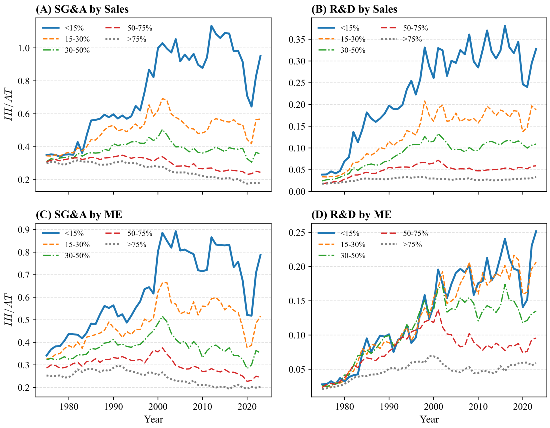
- **Extension:** $d_t \geq -\theta_h h_t - \theta_k k_t$ (allow physical capital as collateral).
- **Empirical pinning of** θ_k from Lian and Ma (2021):
  - Asset-based lending averages $\approx$ 2% of total assets (Table 8) $\Rightarrow \theta_k \approx 0.05$
  - Liquidation-value haircut: liquidation values for fixed assets are typically 30–60% of book value
- **Plausible range:** $\theta_k \approx 0.05$, an order of magnitude below $\theta_h = 0.8$.
- **Result:** throughout this range, the H/K amplification mechanism is robust. Mechanism only weakens modestly relative to the $\theta_k = 0$ baseline.

Conclusion

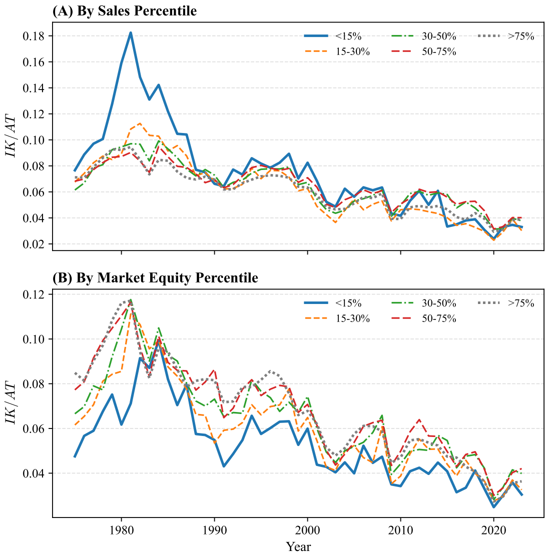
- Document a large and rising **intangible gap** between small and large U.S. firms: tripled since 1980, concentrated in H/K *volatility* more than in the mean
- Propose a mechanism that **differentiates** the effect of uncertainty between physical and intangible investment, via a multiplicative quality shock ω + endogenous exit option
- Combined with the intangible-financing channel ($d_t \geq -\theta h_t$), the model accounts for $\sim 60\%$ of the post-2000 widening, with the rest unexplained by movements in θ

Appendix

Investment Rates by Firm Size



Intangible investment / total assets by size, 1975–2023.



Physical investment (CAPX) / total assets by size,

Summary Statistics by Size Quintile

Market equity	<15%	15–30%	30–50%	50–75%	>75%
H(SGA)/K	1.93	1.48	1.21	0.93	0.77
H(RD)/K	1.35	0.95	0.76	0.49	0.37
Stkco / ME (%)	1.71	1.24	1.00	0.78	0.48
Stkco / AT (%)	5.69	3.70	2.22	1.60	0.85
R&D / AT (%)	13.96	7.54	4.82	3.30	3.43
Patent Value / AT (%)	66.93	24.21	15.22	13.31	7.18
Tobin's q	2.82	2.48	2.25	2.11	1.94
ROA (%)	−8.27	6.19	10.28	11.96	13.29
Book Leverage	0.41	0.39	0.42	0.48	0.56
Market Leverage	0.17	0.20	0.23	0.27	0.32

Back

Calibration of θ_k – Details

Direct anchor (Lian–Ma Table 8):

$$\text{ABL}/\text{Assets} \approx 0.02$$

With $\text{Assets}/k \approx 2.4$ (since $\text{Assets} \approx k + h$, $h/k \approx 1.36$):

$$\theta_k \approx 0.02 \times 2.4 \approx 0.05$$

Pledgeability haircut:

- Advance rate $\approx$ 60% of book inventory
- Liquidation value $\approx$ 30–60% of book for fixed assets
- Effective max: $0.60 \times 0.50 = 0.30$

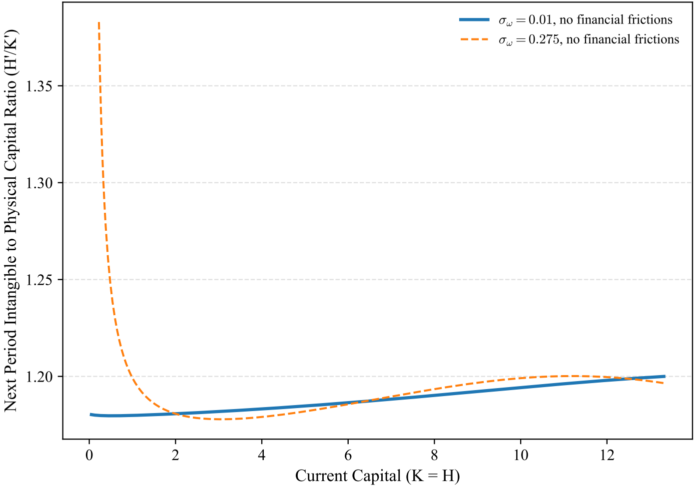
Plausible range: $\theta_k \in [0.05, 0.10]$. Both estimates are upper bounds (abstract from default/intermediation costs).

Model Fit at the Benchmark

	Data 1975–2000	$\theta = 0.8$ $\sigma_\omega = 0.2$	$\theta = 0.8$ $\sigma_\omega = 0.01$	$\theta = 0.01$ $\sigma_\omega = 0.2$	$\theta = 0.01$ $\sigma_\omega = 0.01$
Exit rate	0.046	0.035	0.022	0.215	0.261
H/K spread	0.47	**0.50**	0.20	−0.25	−0.29
Investment rate (K)	0.10	0.14	0.13	0.01	−0.04
Investment rate (H)	0.11	0.14	0.12	0.02	−0.05
Empl. equity / value	0.05	0.11	0.07	0.01	0.01

Both σ_ω and θ are needed: shutting down either channel destroys the H/K spread. Back

Policy Function: No Financial Frictions



H'/K' along the diagonal $K = H$; two levels of σ_ω (0.01 vs. 0.275), no financial frictions.

Exit and Entry: Details

Exit decision:

- Firm exits if $V(s_t, k_t, \omega_t \tilde{h}_t) \leq 0$
- Defines exit threshold $\omega^*(s_t, k_t, \tilde{h}_t)$:

$$\omega^* = \inf\{\omega : V(s, k, \omega\tilde{h}) > 0\}$$

- Firm bears fixed operating cost ξ each period it operates; ξ calibrated to match annual exit rate of 4.6%

Entry decision:

- Entrants draw (s_0, k_0) from entry distribution Γ_0 and pay sunk cost c_e
- Free-entry condition: $E_{\Gamma_0}[V(s_0, k_0, h_0)] = c_e$
- Mass of entrants M_e adjusts endogenously to clear the goods market

Back

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