

Housing Privatization as Intergenerational Redistribution

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Motivation

- Many emerging economies, such as Korea, Taiwan, and other East Asian Miracle economies, had experienced rapid growth of wage incomes, followed by an eventual slowdown;
- China's economic growth miracle seems to be slowing down significantly as well.

China's GDP Per Capita: 1993-2022



Question of the Paper

- The fast wage income growth in China over the recent decades has disproportionately benefited those working-age cohorts more than those old-age cohorts;
- **General Question:** How could the re-distributive policy be designed such that those old-age cohorts can also enjoy the benefits?
- **Main Idea:** subsidized housing purchase during housing market privatization plays the role of inter-generational redistribution.
- **Main Channel:** Initial generations may sell their houses later on to reap the *capital gains* to finance larger houses and consumption.

Asset Price as Intergenerational Transfer

- Housing prices, in general asset prices, incorporate future growth expectations;
- Initial generations, i.e. generations at the 1994 public housing reform, received the IPO of “Chinese housing”, which was then “floated” in 1998 when residential mortgages were introduced into the commercial housing market in China
- Housing prices grew at about 10% per year from 1998 to 2018, followed by housing market corrections

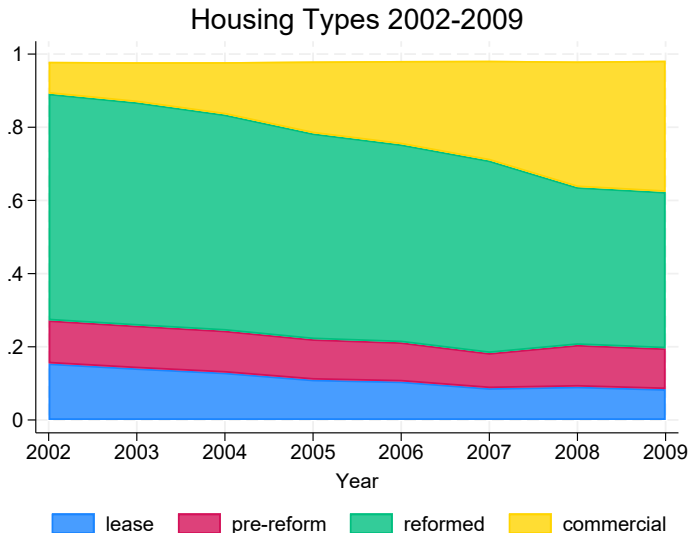
Asset Price vs. PAYGO as Intergenerational Transfer

- To transfer wealth from future generations to the initial generations in the 1990s via PAYGO, very generous replacement rates are needed, but it will be financially unsustainable when the growth rates slow down eventually.
- Problem exacerbated by the increasing dependency ratio due to population aging.
- In contrast, asset prices as intergenerational redistribution *automatically* stop when growth slows down.

What We Do

- Develop a quantitative general equilibrium framework to explore the distributional impacts and welfare implications from the housing privatization reform.
 - We incorporate the two major housing market privatization reforms during 1994-2000;
 - Model is calibrated into the Chinese economy.
- Counterfactual analysis:
 - Evaluate the role of housing purchase subsidy as IPO for initial cohorts;
 - Evaluate the desirability of housing reform to an alternative pension reform as intergenerational transfer;

Housing Types in the Chinese Housing Market



Sketch of the Model

- Multi-period OLG framework with housing in utility and two phase in life: work and retirement
- Warm-glow transfer & accidental bequests
- Long-term mortgages (to be repaid before death)
- Intertemporal constraints on government's both non-pension and pension budgets.
- **Key model ingredient:**
 - Two types of housings: reformed (smaller but cheaper) and commercial houses
 - a transition phase to capture the process of housing privatization, where different cohorts have different housing tenure options.

Demographics

- Each period a continuum of household is born; population grows at a exogenous rate n_t .
- Households live a maximum J periods. They are active as workers until age J_w .
- All households face a probability $S_j = \prod_{k=1}^j \psi_k$ of surviving up to age j , and ψ_k is the conditional survival probability from age $k - 1$ to age k .
- The fraction of households of age $j \in \{2, \dots, J\}$ at the calendar time t evolves according to

$$\mu_{t,j} = \frac{\psi_j}{1 + n_t} \mu_{t-1,j-1},$$

Preferences

For a household born at period \tilde{t} , his/her lifetime utility is

$$U_{\tilde{t}} = \sum_{j=1}^J \beta^{j-1} S_j \left\{ \log(c_{\tilde{t}+j-1,j}) + \phi \log(s_{\tilde{t}+j-1,j} - \underline{s}) + \iota \log(\mathbf{b}_{\tilde{t}+j-1,j} + \underline{\mathbf{b}}) \right\}$$

where

- $\underline{s} > 0$ is subsistence housing services.
- $\iota \log(\mathbf{b}_{\tilde{t}+j-1,j} + \underline{\mathbf{b}})$ denotes the *warm-glow* utility from leaving the *inter vivos* bequest to children.
- $\underline{\mathbf{b}} > 0$.
- For simplicity, parents do not obtain utility from accidental bequests.
- Total collectible bequests in period t include both inter vivos transfers and accidental bequests. They are transferred to the newborn equally by the government.

Income Process

- Household i 's after-tax income in period t at age j is

$$y_{t,j}^i = \begin{cases} (1 - \tau^{ss} - \tau)w_t e_{t,j}^i & \text{for } j \leq J_w \\ b_{t,j}^i & \text{for } j > J_w \end{cases}$$

where w_t is the wage rate per unit of efficiency labor supply; and the efficiency units are given by:

$$e_{t,j}^i = \lambda^i \varepsilon_j z_t^i \epsilon_{t,j}^i$$

- λ^i is skill-specific component
- ε_j is life-cycle income profile
- $z_{t,j}^i$ is a persistent shock that follows transitions $\pi(z_{t+1}|z_t)$
- $\epsilon_{t,j}^i$ is a transitory shock drawn from log-normal distribution

Social Security

- Social security benefit each period is determined as

$$b_{t,j}^i = \theta[\omega y_{t-j+J_w, J_w}^i + (1 - \omega)\bar{y}_{t-1}],$$

where

- θ is the replacement rate at the time of retirement.
 - y_{t-j+J_w, J_w} is the pre-retirement wage;
 - \bar{y}_{t-1} is the “social average wage” in the previous year
 - ω and $1 - \omega$ are respective weights on pre-retirement wage and current social average wage
- Assume that a constant social security tax τ^{SS} is determined to balance the government's intertemporal budget on social security.

Prior to Housing Reform

- Before housing reforms, the government owned all apartments.
- The government assigns to each household an apartment whose size is determined by household's skill type: more skilled households are assigned larger houses.
- No households born before period 1 own any house. The households pay a rental rate determined by the government.
- The economy is in the pre-reform steady state. The households' idiosyncratic shocks are assumed to be perfectly insured in the pre-reform era.

Major Housing Reforms

- **Subsidized Purchase of Reformed Houses** Assume all households born before year 1994 are eligible to purchase the house they currently rent at a discounted price (reformed house). The discounted price depends on their age in 1994: **Illustration**

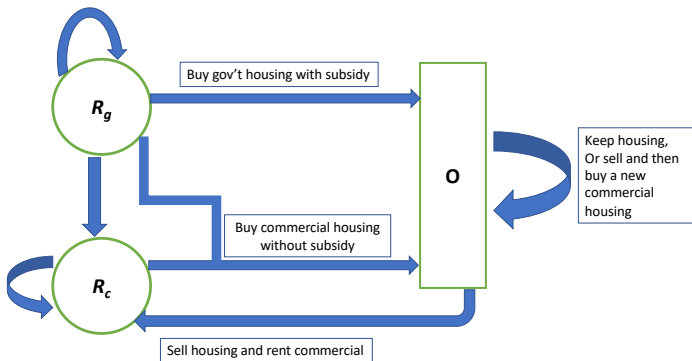
$$p_{gt,j} = p_t \left[1 - \frac{\min[\text{Age in 1994}, J_w]}{J_w} \times 65 \times 0.9\% \right]$$

- **Elimination of Government Rental Houses** The government declared to eliminate the rental-discounted houses for household born after period 1998.
- **Access to mortgage market** Residential mortgages with 30% down payment rate were officially introduced.

Differences b/w Renting and Owning

- **House size options** for commercial rental units (\mathcal{H}_R) and owner-occupied units (\mathcal{H}_O) may be different, e.g., commercial rental units tend to be smaller on average than owner-occupied units.
- **Ownership utility premium:** $s = \zeta h$, with $\zeta \geq 1$.
- **Mortgage:** From 1998 forward, home buyers can borrow via mortgages subject to down payment requirements (30%), while renters do not have this option.
- **Maintenance cost** $\delta_h p_t h_t$, expressed in units of the numeraire good, which fully compensates for any physical depreciation of the dwelling.
- **Selling transaction cost:** $\tau_s p_t h_t$ that is proportional to the value of the house transactions; renters can adjust their sizes freely.

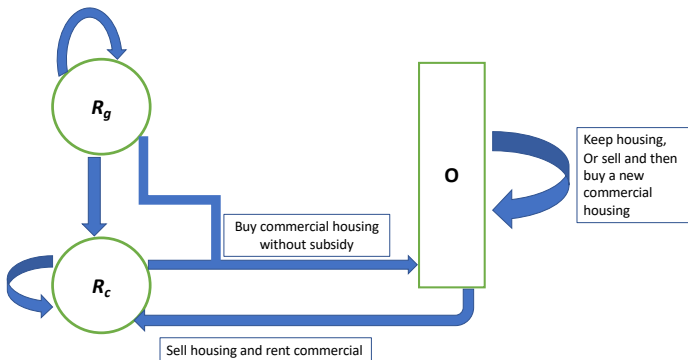
Housing Options for Initial Cohorts Before 1994



value function for renters of reformed houses

value function for renters of commercial houses

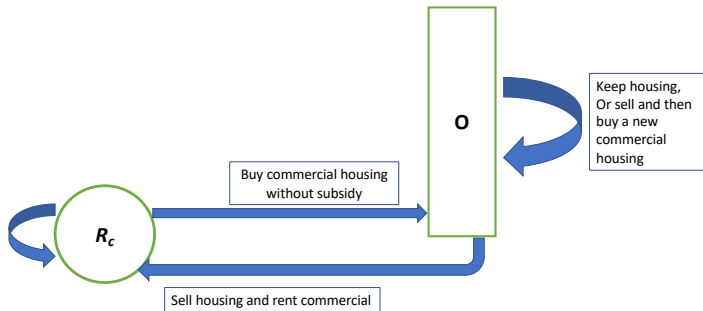
Housing Options for Cohorts 1994-1997



value function for buyers of reformed houses

value function for buyers of commercial houses

Housing Options for Cohorts On or After 1998



value function for owners of reformed houses

value function for owners of commercial houses

Production Sectors

- **Final goods sector:**

$$Y_t = A_t N_{ct}$$

- **Residential construction sector** sells apartments at price p_t produced from new land L_{ht} issued by the government and labor N_{ht} using a constant return to scale technology:

$$Y_{ht} = A_{ht} L_{ht}^{1-\alpha} N_{ht}^{\alpha}.$$

- Absentee rental companies buy apartments from construction sector and lease them to renters at price ρ_t . No-arbitrage condition implies:

$$p_t = \rho_t + \frac{1 - \delta_h}{1 + r} p_{t+1}.$$

Government

- Government imposes social security tax τ_{ss} to balance intertemporal budget constraint of social security system:

$$\tau^{ss} \sum_{t=1}^{\infty} \frac{\sum_{j=1}^{J_w} w_t \mu_{t,j} \int_i e_{t,j}^i di}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{\sum_{j=J_w+1}^J \mu_{t,j} b_{t,j}}{(1+r)^t},$$

- Government chooses income tax τ to balance between the present value of housing subsidy and the present value of wage income tax:

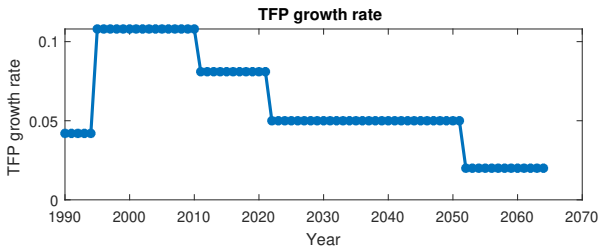
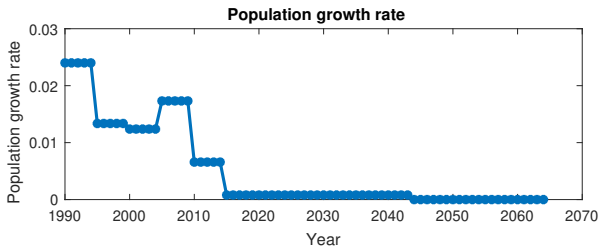
$$\begin{aligned} & \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \sum_{j=1}^J \sum_i \left[\int (p_t - p_{g,t}(j)) h_g^i \mathbf{I}^{B_g}(\mathbf{x}_t^R) d\mu_{t,j}^{R_g}(\mathbf{x}_t^R) \right. \\ & \left. + \int (1-\nu) \rho_t h_g^i \mathbf{I}^{R_g}(\mathbf{x}_t^R) d\mu_{t,j}^{R_g}(\mathbf{x}_t^R) \right] \\ & = B_0 + \tau \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \sum_{j=1}^{J_w} w_t \mu_{t,j} \sum_i \mathbb{E} e_{t,j}^i, \end{aligned}$$

where B_0 denotes the government's initial asset position.

Calibration Strategy

- **Initial steady state:** the pre-1994 period where the government allocates houses across households for subsidized rent.
- **Final steady state:** all cohorts who are eligible for either purchase subsidies or rental discount have exited the economy, and all exogenously fed-in series have stabilized.
- Each period is five years. $J = 14$, $J_w = 8$ corresponding to entering labor force at age 21, retiring at age 60 and exiting at age 90.
- Model calibrated to match ownership rate (overall, reformed and commercial houses) in 2000.

Exogenous Series



Parameterization: Pre-determined Parameters

Description	Parameter	Value	Sources
Life time	J	14	Age 21-90
Working time	J_w	7	Age 21-60
Depreciation rate	δ_h	0.02	OECD estimates
Replacement rate	θ	0.6	Song et al. (2015)
Discount Factor	β	0.90	standard
auto-corr.of persistent shock	ρ	0.84	He et al. (2017)
std. dev.of persistent shock	σ_v	0.055	He et al. (2017)
std. dev.of transitory shock	σ_ϵ	0.055	Fan et al. (2010)
Minimum Down Payment Ratio	γ	0.3	government policy
Interest Rate for Savings	r	0.02	Government policy
Mortgage Interest Rate	r_m	0.06	Government policy
Land share	α	0.7	Wang et al. (2012)
Seller Transaction Cost	τ_s	0.12	Guren et al.(2020)
Conditional survival prob	$\{S\}_j$		Imrohorglu and Zhao (2018)
Life-cycle income profile	$\{\epsilon\}_j$		He et al. (2017)
Initial TFP level	A_1	1.3	avg. wage in 2005 equals 1
Land supply	ℓ_0	1	normalization

Parameterization: Jointly Determined Parameter

Para	Description	Value	Target	Data	Model
ζ	util. premium owning	1.65	own'p rate in 2000	0.697	0.713
ι	coef. on IV transfer	0.85	own'p rate of old in 2000	0.634	0.60
\underline{b}	“luxury” par. on IV transfer	0.12	med. wealth-to-income	0.634	0.60
ϕ	coef. on housing service	5.6	expend. share on rents	0.20	0.20
\underline{h}	subsistence housing service	0.17	price elas. of rental demand	-0.8	-0.8
h_1^g	size small reform house	9.1	price to income ratio	9.0	9.0
h_2^g	size large reform house	12.7	share reform h'owners 2000	0.513	0.486
λ^H	Skill premium	1.49	Own'p rate of skilled 2000	0.749	0.738
B_0	Initial gov assets	5.3	income tax	0.1	0.1
L_{ht}	Land supply	Figure	Housing price dynamics		

Model Fit by Year

	2000		2005	
	Data	Model	Data	Model
Ownership	0.697	0.713	0.753	0.741
Pop share own ref. house	0.513	0.486	0.431	0.344
Pop share own com. house	0.184	0.227	0.322	0.397

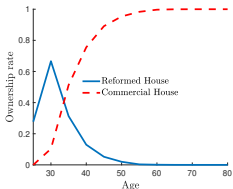
Model Fit by Skill

	2000		2005	
	Data	Model	Data	Model
Overall	0.697	0.713	0.753	0.741
Unskilled	0.672	0.692	0.738	0.691
Skilled	0.749	0.738	0.793	0.800

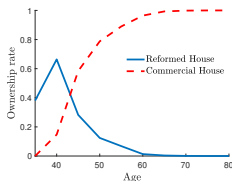
Model Fit by Age

	2000		2005	
	Data	Model	Data	Model
21-30	0.570	0.603	0.545	0.630
31-60	0.705	0.728	0.777	0.738
61-90	0.634	0.600	0.736	0.658

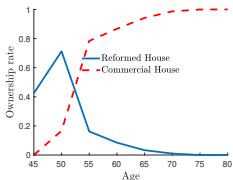
Ownership over Life Cycle among Initial Cohorts



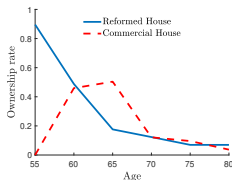
(a) Cohort born
1965-1969



(b) Cohort born
1955-1959

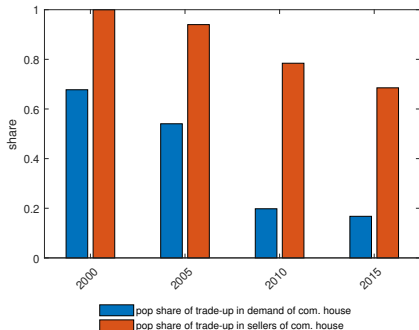


(c) Cohort born
1945-1949

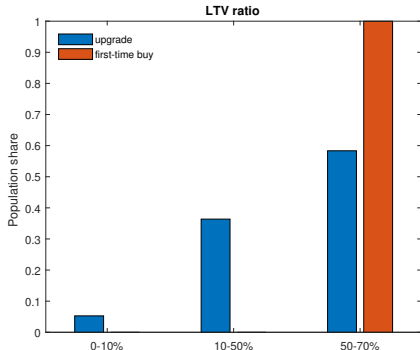


(d) Cohort born
1935-1939

Trade-up Mechanism At Work



(a) Trade-up in demand and supply



(b) LTV distribution

Quantifying Housing Purchase Subsidy as IPO for Initial Cohorts

Mechanism

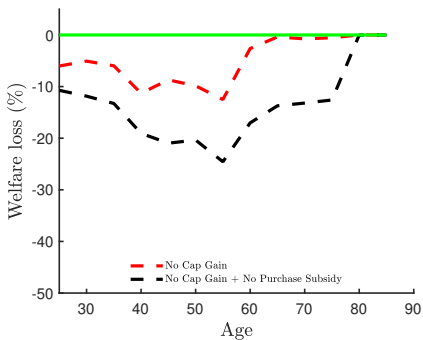
housing purchase subsidy \implies early ownership & trade up later \implies house price growth $\uparrow \implies$ capital gains for initial cohorts

Two steps for quantifying the role of housing reforms as IPO

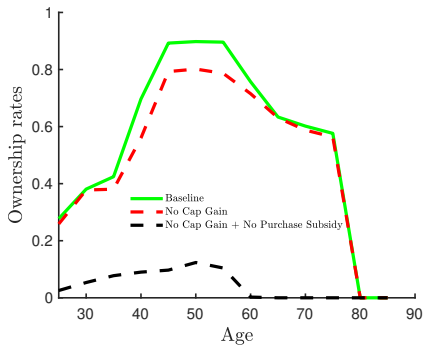
- Step 1: Taking house price growth as given (in the benchmark), explore the role of housing subsidy via capital gain and early ownership
- Step 2: Explore the role of purchase subsidy for house price growth

Step 1: Partial Equilibrium Analysis

- Counterfactual A: initial cohorts sell house at their purchase price (no capital gain).
- Counterfactual B: no capital gain & purchase subsidy.
- On average, capital gain contributes approximately 54% to the welfare gain in the initial cohort.

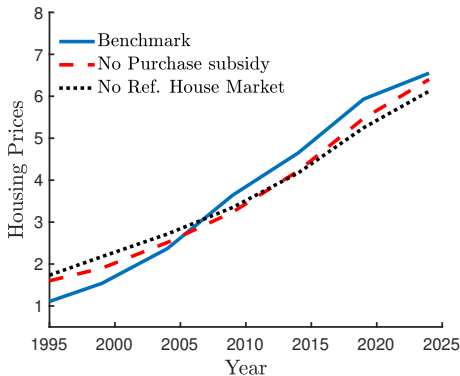


(a) Welfare loss (CEV)



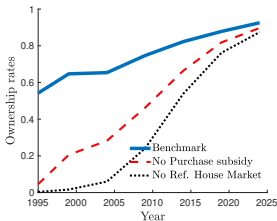
(b) Ownership rate in 1995

Step 2: Role of Housing Reforms for Housing Prices

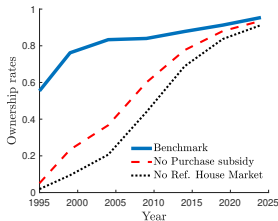


- By removing purchase subsidy (reformed housing option) to initial cohorts, housing price growth between 2000-2015 drops from 8.21% to 5.85% (4.76%).
- Early ownership of reformed houses (and later trade-up) raises growth rate of housing price (capital gain).

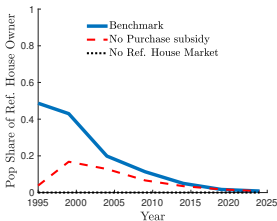
Ownership Rates



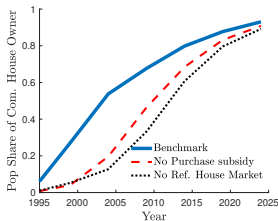
(a) Unskilled



(b) Skilled



(c) Reformed House



(d) Commercial House

Housing vs. Social Security as Intergenerational Transfer

Question

For newborns experiencing fast wage income growth, is housing or social security reform more desirable as inter-generational transfers?

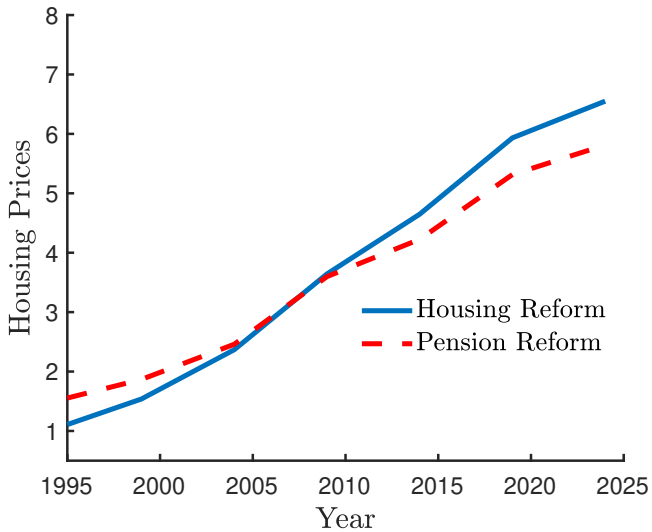
- Compare welfare effects of housing reforms with SS reforms for newborns
- Evaluate the budgetary consequences of an unexpected slowdown under the housing reform vs. under an alternative pension reform.

Counterfactual I: Alternative Pension Reform

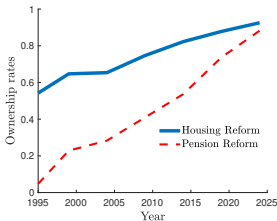
- Remove housing purchase subsidy to initial cohorts
- Replace by a higher replacement rate θ so that the average welfare among initial cohorts are the same as in the housing reform. θ is found to be $0.82 > 0.6$ in the pension reform.
- Due to downward-rigidity, the same replacement rate is carried over to all future cohorts.

	Income tax	Social Security tax
Baseline	0.100	0.231
No purchase subsidy	0.076	0.231
No 94 reform	0.076	0.231
Pension reform	0.076	0.340

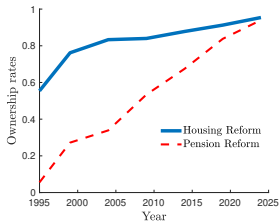
Housing Prices



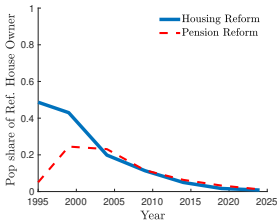
Ownership Rates



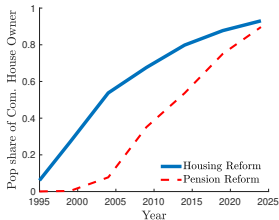
(a) Unskilled



(b) Skilled

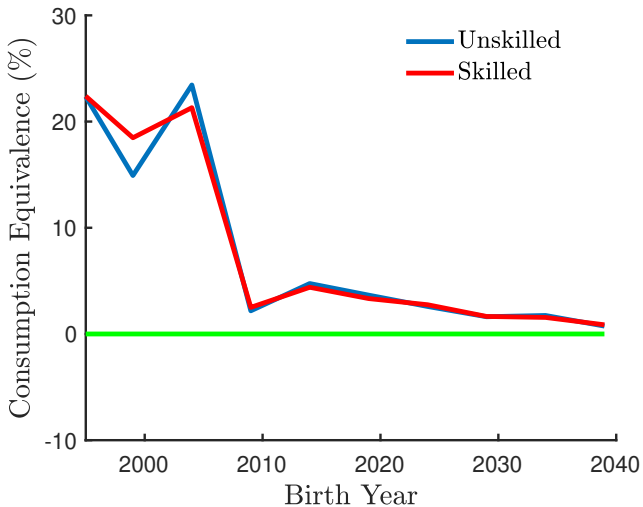


(c) Reformed Houses

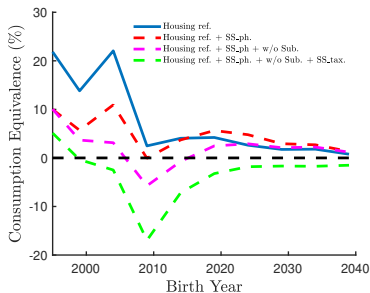


(d) Commercial Houses

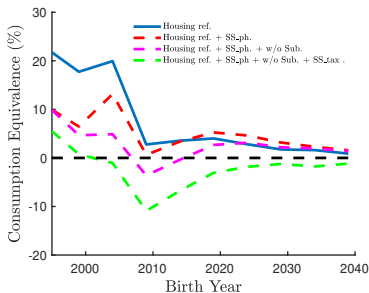
Welfare Comparison among Newborns



Decomposition of Welfare Gains among Newborns



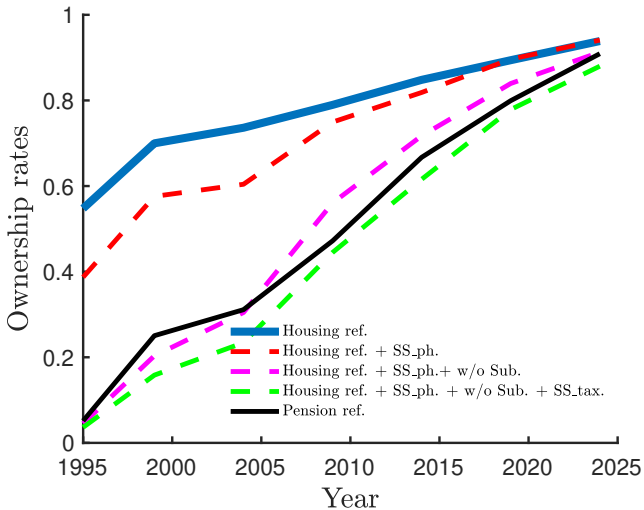
(a) Unskilled



(b) Skilled

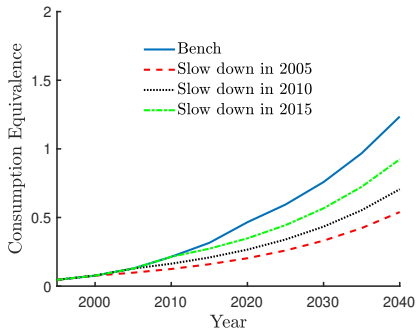
- Capital gains associated with housing reforms account for about half of the welfare gains of cohorts born before 2010 (especially for the skilled households).

Ownership Rates

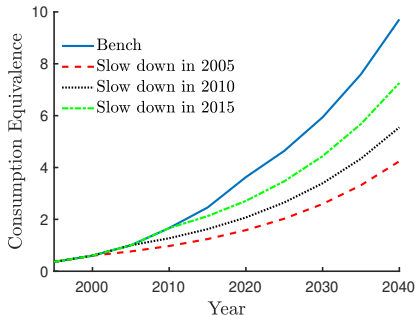


- Purchase subsidy per se facilitates early ownership.

Counterfactual II: Unexpected Slowdown



(a) TFP



(b) Average Wages

Government Budget

	No Slowdown		Slowdown at 2005		Slowdown at 2010		Slowdown at 2015	
	Housing	Pension	Housing	Pension	Housing	Pension	Housing	Pension
Tot subsidy	3.016	0.399	1.767	0.153	1.770	0.150	1.770	0.150
Tax income	6.753	4.138	3.687	2.809	4.616	3.516	5.662	4.313
Pension deficit	0.000	0.000	0.736	2.526	0.552	2.228	0.313	1.512
Total deficit	-0.000	-0.002	3.803	4.858	2.694	3.849	1.408	2.337
Tot deficit/Tax income	-0.000	-0.000	1.031	1.730	0.584	1.095	0.249	0.542

Conclusion

- Housing privatization can serve as an intergenerational transfer mechanism in fast-growing economies like China.
 - Earlier cohorts receive subsidized housing purchases and later realize capital gains to support retirement consumption.
- We develop a quantitative framework with heterogeneous households, financial constraints, mortgages, and housing market dynamics.
 - Housing purchase subsidies promote early homeownership, raise house prices through trade-up dynamics, and generate capital gains for initial cohorts.
 - Compared with PAYGO-style pension transfers, housing reform generates higher welfare for future cohorts and smaller long-run fiscal burdens.

Literature

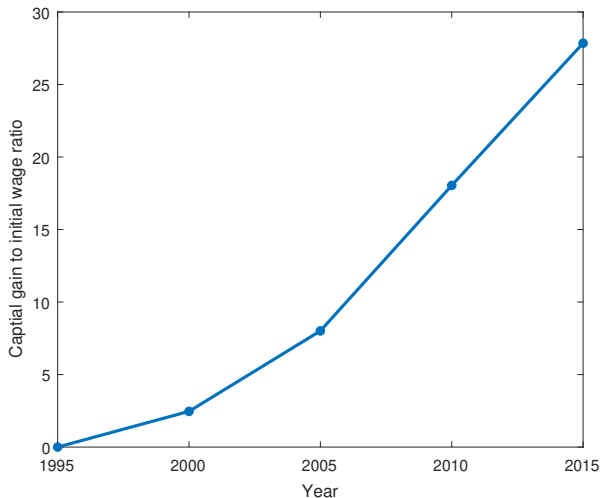
- Role of asset prices in intergenerational redistribution: Glover, Heathcote, Krueger and Rios-Rull (2020, JPE)
- China's housing reforms and housing booms: Wang (AER, 2011), Logan, Fang and Zhang (Housing Studies, 2010), Fang, Gu, Xiong and Zhou (NBER Marco Annual, 2016), Chen and Wen (AEJ: Macro, 2017)
- Social security reforms in China: Song, Storesletten, Wang and Zilibotti (AEJ: Macro, 2015)

Discussions

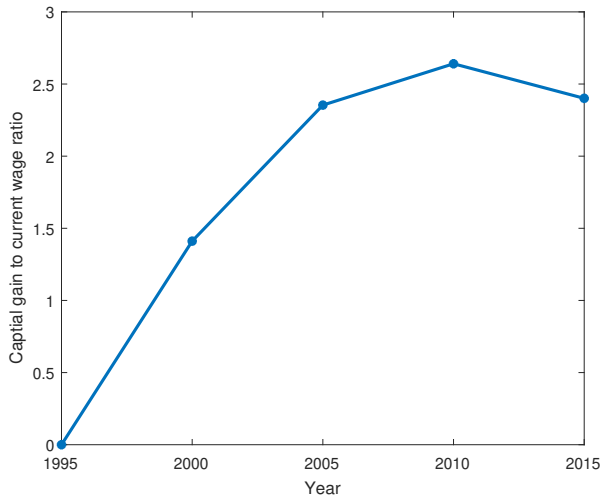
- Difference between housing assets vs. financial assets: USSR vs. China
- Partial average wage indexing: Germany and UK
- Relevant for other countries?

Capital Gains Relative to Initial Wages

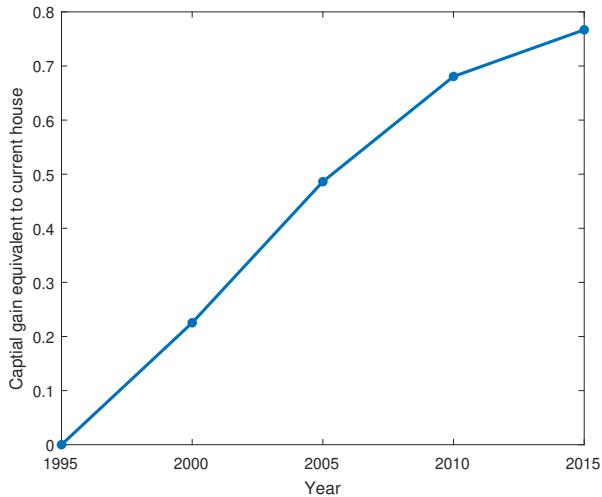
Suppose a household bought a house of 30 square meters in 1995.



Capital Gains Relative to Current Wages



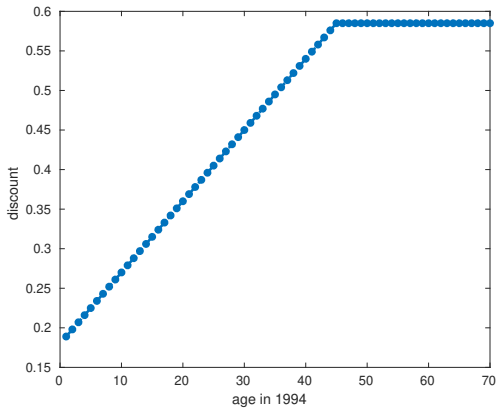
Capital Gains Relative to Current House Prices



Housing Privatization

- The government first sold all the rental apartments it owned to absentee rental companies when housing reform starts.
- When housing reform starts, all households can choose to rent a commercial apartment $h_R \in \mathcal{H}_R = \{h_R^1, h_R^2, \dots, h_R^{N_R}\}$ that provides housing service $s = h_R$ at the market rental rate ρ_t , where \mathcal{H}_R is the set of size options among the rental units;
- All households can also choose to purchase an owner-occupied apartment $h_O \in \mathcal{H}_O = \{h_O^1, h_O^2, \dots, h_O^{N_O}\}$ at market price p_t .
- An owner-occupied apartment provides housing service $s = \zeta h$, with $\zeta \geq 1$.

Housing Subsidy Illustration



Back

Value Function: Renter of Reformed Houses

$$V_t^{Rg}(\lambda^i, j, h_g^i, y, z) = \max_{\{c, a', \mathbf{b}\}} u(c, h_g^i, \mathbf{b}) + \beta \psi_{j+1} \mathbb{E} \left[\max \left\{ V_{t+1}^{Rg}(\lambda^i, j+1, h_g^i, y', z'), V_{t+1}^{Bg} \right. \right. \\ \left. \left. V_{t+1}^{Rc}(\lambda^i, j+1, y', z'), V_{t+1}^{Bc}(\lambda^i, j+1, y', z') \right\} \right]$$

$$\text{s.t. } c + \mathbf{b} + a' + \nu \rho_t h_g^i = y$$

$$y' = \begin{cases} (1 - \tau^{ss} - \tau) \lambda^i w_{t+1} \varepsilon_{j+1} z' + a'(1+r) + \mathcal{T}_{t+1} & \text{for } j \leq J_w \\ b_{t+1, j+1}^i + a'(1+r) + \mathcal{T}_{t+1} & \text{for } j > J_w \end{cases}$$

$$a' \geq 0$$

Back

Value Function: Renter of Commercial Houses

$$V_t^{Rc}(i, j, y, z) = \max_{\{c, a', h_R \in \mathcal{H}_R, \mathbf{b}\}} u(c, h_R, \mathbf{b}) + \beta \psi_{j+1} \mathbb{E} \left[\max \left\{ V_{t+1}^{Rc}(y', z', j+1), V_{t+1}^{Bc}(i, j) \right\} \right]$$

$$\text{s.t. } c + \mathbf{b} + a' + \rho_t h_R = y$$

$$y' = \begin{cases} (1 - \tau^{ss} - \tau) \lambda^i w_{t+1} \varepsilon_{j+1} z' + a(1+r) + \mathcal{T}_{t+1} & \text{for } j \leq J_w \\ b_{t+1, j+1}^i + a'(1+r) + \mathcal{T}_{t+1} & \text{for } j > J_w \end{cases}$$

$$a' \geq 0$$

Back

Value Function: Buyer of Reformed Houses

$$V_t^{Bg}(i, j, y, z) = \max_{c, a', d', \mathbf{b}} u(c, \zeta h_g^i, \mathbf{b}) + \beta \psi_{j+1} \mathbb{E} \left[\max \left\{ V_{t+1}^{Og}(i, j+1, y', z', \zeta h_g^i, d'), \right. \right. \\ \left. \left. V_{t+1}^{Rc}(i, j+1, y'_{\text{sell}}, z'), V_{t+1}^{Bc}(i, j+1, y'_{\text{sell}}, z') \right\} \right]$$

$$\text{s.t. } c + \mathbf{b} + a' + p_{gt}(j)h_g^i = y + d'$$

$$y' = \begin{cases} (1 - \tau^{ss} - \tau)\lambda^i w_{t+1} \varepsilon_{j+1} z' + a'(1+r) & \text{for } j \leq J_w \\ b_{t+1, j+1}^i + a'(1+r) & \text{for } j > J_w \end{cases}$$

$$y'_{\text{sell}} = y' + (1 - \tau_s)p_{t+1}h_g^i - d'(1+r_m) + \mathcal{T}_{t+1}$$

$$d' \leq (1 - \gamma)p_{gt}(j)h_g^i$$

$$a' \geq 0$$

Value Function: Buyer of Commercial Houses

$$V_t^{Bc}(i, j, y, z) = \max_{c, a', d', h_o \in \mathcal{H}_O, \mathbf{b}} u(c, \zeta h_o, \mathbf{b}) + \beta \psi_{j+1} \mathbb{E} \left[\max \left\{ V_{t+1}^{Oc}(i, j+1, y', z', h_o, d'), \right. \right. \\ \left. \left. V_{t+1}^{Rc}(i, j+1, y'_{\text{sell}}, z'), V_{t+1}^{Bc}(i, j+1, y'_{\text{sell}}, z') \right\} \right]$$

$$\text{s.t. } c + \mathbf{b} + a' + p_t h_o = y + d'$$

$$y' = \begin{cases} (1 - \tau^{ss} - \tau) \lambda^i w_{t+1} \varepsilon_{j+1} z' + a'(1+r) & \text{for } j \leq J_w \\ b_{t+1, j+1}^i + a'(1+r) & \text{for } j > J_w \end{cases}$$

$$y'_{\text{sell}} = y' + (1 - \tau_s) p_{t+1} h_o - d'(1 + r_m) + \mathcal{T}_{t+1}$$

$$d' \leq (1 - \gamma) p_t h_o$$

$$a' \geq 0$$

Value Function: Owner of Reformed Houses

$$V_t^{Og}(i, j, y, z, h_g^i, d) = \max_{c, a', m, \mathbf{b}} u(c, \zeta h_g^i, \mathbf{b})$$

$$+ \beta \psi_{j+1} \mathbb{E} \left[\max \{ V_{t+1}^{Og}(i, j+1, y', z', h_g^i, d'), V_{t+1}^{Rc}(i, j+1, y'_{\text{sell}}, z'), V_{t+1}^{Bc}(i, j+1, y'_{\text{sell}}, z') \} \right]$$

$$\text{s.t. } c + \mathbf{b} + a' + \delta_h p_t h_g^i + m = y$$

$$m \geq \frac{r_m(1+r_m)^{J+1-j}}{(1+r)^{J+1-j} - 1} d$$

$$d' = d(1+r_m) - m$$

$$y' = \begin{cases} (1 - \tau^{ss} - \tau) \lambda^i w_{t+1} \varepsilon_{j+1} z' + a'(1+r) & \text{for } j \leq J_w \\ b_{t+1, j+1}^i + a'(1+r) & \text{for } j > J_w \end{cases}$$

$$y'_{\text{sell}} = y' + (1 - \tau_s) p_{t+1} h_g^i - d'(1+r_m) + \mathcal{T}_{t+1}.$$

$$a' \geq 0$$

Back

Value Function: Owner of Commercial Houses

$$V_t^{O_c}(i, j, y, z, h_o, d) = \max_{c, a', m, \mathbf{b}} u(c, \zeta h_o, \mathbf{b})$$

$$+ \beta \psi_{j+1} \mathbb{E} \left[\max \{ V_{t+1}^{O_c}(i, j+1, y', z', h_o, d'), V_{t+1}^{R_c}(i, j+1, y'_{\text{sell}}, z'), V_{t+1}^{B_c}(i, j+1, y'_{\text{sell}}, z') \} \right]$$

$$\text{s.t. } c + \mathbf{b} + m + a' + \delta_h p_t h_o = y$$

$$m \geq \frac{r_m(1+r_m)^{J+1-j}}{(1+r)^{J+1-j} - 1} d$$

$$d' = d(1+r_m) - m$$

$$y' = \begin{cases} (1 - \tau^{ss} - \tau) \lambda^i w_{t+1} \varepsilon_{j+1} z' + a'(1+r) & \text{for } j \leq J_w \\ b_{t+1, j+1}^i + a'(1+r) & \text{for } j > J_w \end{cases}$$

$$y'_{\text{sell}} = y' + (1 - \tau_s) p_{t+1} h_o - d'(1+r_m) + \mathcal{T}_{t+1}$$

$$a' \geq 0$$

Back

Equilibrium

- Given government policies $\langle r_m, \theta, \nu, \gamma, \omega, p_{g,t}(j) \rangle$, and a sequence of aggregate variables $\langle A_t, L_t, n_t \rangle_{t=1}^{\infty}$, a dynamic equilibrium of our model consists of quantities $\langle N_{ct}^*, N_{ht}^*, L_{h,t}^* \rangle$, prices $\langle p_t^*, \rho_t^*, w_t^*, p_{Lt}^* \rangle$, taxation policies $\langle \tau, \tau^{ss} \rangle$, household value functions $\mathbb{V} \equiv \langle V_t^{Rg}, V_t^{Rc}, V_t^{Bg}, V_t^{Bc}, V_t^{Og}, V_t^{Oc} \rangle$ and associated policy functions $\mathbb{P}^* \equiv \langle \mathbf{I}^{Bg}(\cdot), \mathbf{I}_{Rg}^{Rg}(\cdot), \mathbf{I}_{Rg}^{Rc}(\cdot), \mathbf{I}^{Bc}(\cdot), \mathbf{I}^{Sg}(\cdot), \mathbf{I}^{Sc}(\cdot), \mathbf{I}_O^{Rc}(\cdot), \mathbf{I}^{Tc}(\cdot), c_t^*(\cdot), a_t^*(\cdot), \mathbf{b}_t^*(\cdot), h_t^*(\cdot), m_t^*(\cdot) \rangle$, and distributions of the state variables among renters and owners of government-owned and commercial houses $\boldsymbol{\mu}_t \equiv \langle \mu_t^{Rg}, \mu_t^{Rc}, \mu_t^{Og}, \mu_t^{Oc} \rangle$, measured at the beginning of period t , that satisfy the following conditions:

Equilibrium

1. Given the prices $\langle p_t^*, \rho_t^*, w_t^* \rangle$ and the household value functions \mathbb{V} , the household policy functions \mathbb{P}^* solve the households' recursive problems.
2. The household value functions \mathbb{V} are consistent with the households' policy functions \mathbb{P}^* and the evolution of the state variables.
3. Given $\langle A_t, L_t \rangle$ and prices $\langle p_t^*, \rho_t^*, w_t^*, p_{L_t}^* \rangle$, the demand for labor N_{ht}^* and the demand for land L_{ht}^* solves the problem of profit maximization of construction firms.
4. The labor market clears at the wage rate $w_t^* = A_t$, i.e., $N_{ct}^* + N_{ht}^* = N_t$.
5. The land market clears in each period, i.e., $L_{ht}^* = L_t$, for all t where L_t is the exogenous land supply.

Equilibrium

6. Given the households' policy functions and state distributions, and the new housing supply from construction firms, housing price p_t clears the housing market in each period.
7. The no-arbitrage condition between housing price p_t^* and rental rates ρ_t^* is satisfied all all period t .
8. The social security tax τ^{SS} balances the intertemporal budget of the social security system.
9. The evolution of state variables, encapsulated by μ_t , is consistent with exogenous stochastic processes including income shocks and aging/death processes and all endogenous household decision rules \mathbb{P}^* .

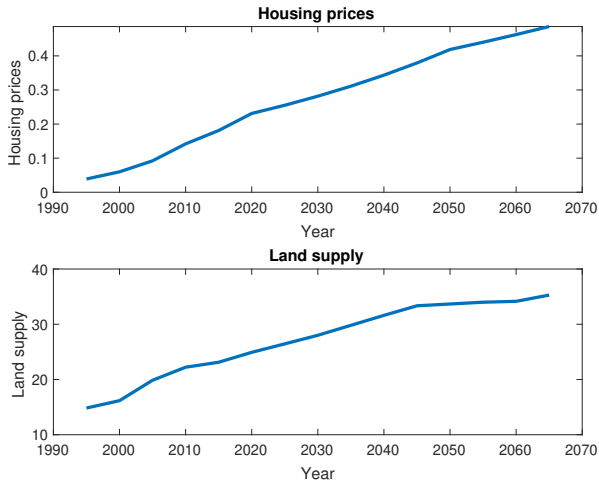
details

Equilibrium Details

- Denote the vector of individual states for homeowners and renters as $\mathbf{x}_t^R := (\lambda^i, h_t, j, y_t, z_t^i, t)$ and $\mathbf{x}_t^O := (\lambda^i, h_t, j, y_t, d_t, z_t^i, t)$.
- Let $\mathbb{I}^{B_k}, \mathbb{I}^{S_k}, k \in \{c, g\}$ be the decision to buy a house among renters, and the decision to sell a house among owners.
- Housing market clears at price p .

$$\begin{aligned}
 & \sum_{j=1}^J \left[\int h_a (1 - \mathbb{I}_j^{B_c}(x^R)) d\mu_{t,j}^{R_c} + \int h_g (1 - \mathbb{I}_j^{B_g}(x^R)) d\mu_{t,j}^{R_g} \right. \\
 & \left. + \int \delta_a h_g (1 - \mathbb{I}_j^{S_g}(x^O)) d\mu_{t,j}^{O_g} \right] + \sum_{j=1}^J \left[\int h \mathbb{I}_j^{B_c}(x^R) d\mu_{t,j}^{R_c} \right. \\
 & \left. + \int h \mathbb{I}_j^{B_c}(x^R) d\mu_{t,j}^{R_g} + \int h \mathbb{I}_j^{B_c}(x^O) d\mu_{t,j}^{O_g} + \int \delta_h h (1 - \mathbb{I}_j^{S_c}(x^O)) d\mu_{t,j}^{O_c} \right] \\
 & = H_{a,t-1} (1 - \delta_a) + \int h_g \mathbb{I}_j^{S_g}(x^O) d\mu_{t,j}^{O_g} + \int h \mathbb{I}_j^{S_c}(x^O) d\mu_{t,j}^{O_c} + Y_{ht},
 \end{aligned}$$

Construction TFP and Housing Prices



Detrend

- The growth rate of housing investment and housing price at BGP is:

$$1 + g_I = [(1 + g)(1 + n)]^\alpha$$
$$1 + g_p = [(1 + g)(1 + n)]^{1-\alpha}$$

- For individual variables, we detrend as follows

$$\begin{aligned}\widehat{c}_{t,j} &= c_{t,j}(1 + g)^{T+j-t-1} \\ \widehat{h}_{t+1,j+1} &= h_{t+1,j+1}(1 + g_h)^{T+j-t-1} \\ \widehat{b}_{t,j} &= b_{t,j}(1 + g)^{T+j-t-1} \\ \widehat{y}_{t,j} &= y_{t,j}(1 + g)^{T+j-t-1}\end{aligned}$$

- For aggregate variables, we detrend as follows

$$\begin{aligned}\widehat{w}_t &= w_t(1 + g)^{T-t} \\ \widehat{p}_t &= p_t(1 + g_p)^{T-t} \\ \widehat{I}_t &= I_t(1 + g_I)^{T-t} \\ \widehat{L}_t &= L_t \\ \widehat{H}_t &= H_t(1 + g_I)^{T-t} \\ \widehat{\rho}_h &= \rho_h(1 + g_p)^{T-t} \\ \widehat{\tilde{H}} &= \tilde{H}'(1 + g_I)^{T-t} \\ \widehat{Y}_t &= Y_t[(1 + g)(1 + n)]^{T-t}\end{aligned}$$

Computation Algorithm

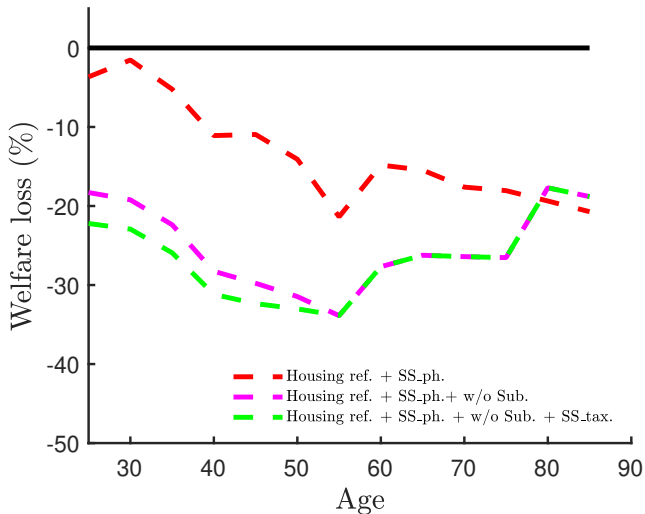
Given government policy, $\{\tau_t, b_t\}_{t=1}^{\infty}$, discount housing policy, land supply, $\{H_t\}_{t=1}^{\infty}$, and interest rate $\{r_t\}_{t=1}^{\infty}$ and wage rate $\{w_t\}_{t=1}^{\infty}$ on the path, and the initial distribution of household on the state space (initial state).

1. Choose the number of transition periods T .
2. De-trend the economy by the time T variables.
3. Provide an initial guess for income tax rate τ .
4. Given all policy variables, solve for the final steady state housing price that clear the housing market by bisection method.
5. Provide an initial guess for housing price on the path, $\{p_t\}_{t=0}^T$, and solve household's problem backwards:
At period t , compute the value functions and policy functions for the new born at t , which has a perfect foresight.

Computation Algorithm

6. Compute the transition path: Compute the optimal path for consumption, housing, and saving by forward induction given the initial state in period $t = 1$. In initial state, households receive assignment of public housing from the government, \bar{H} . The bequests for period t newborn are collected from the household passing away at period t .
7. Aggregate household's net housing demand each period. Check if housing market in each period is clear. If not, update the guess of $\{p_t\}_{t=1}^T$, and go to step 5.
8. Aggregate government tax revenue, housing sale revenue, pension expenditure on the path. Combined with government's deficit/surplus in the steady state, check whether government's intertemporal budget is balanced. If not, update the guess of τ , and go to step 4.
9. Check whether p_T is close enough with the final steady state housing market price. If not, increase T , and go to step 2.

Welfare Comparison among Initial Cohorts



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