Impact Trickles Down: A General Equilibrium Theory of Stakeholder Exit and Engagement

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Motivation

- Two important ways that stakeholders affect a firm's decision
  - Exit or threat to exit
  - Engagement and voice
- The literature has examined stakeholders' choices and their impacts, but focus on a single firm
  - They take external environment and outside options as fixed.

Motivation

- Two important ways that stakeholders affect a firm's decision
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- The literature has examined stakeholders' choices and their impacts, but focus on a single firm
  - They take external environment and outside options as fixed.
- However, when a stakeholder exits, it gets involved in another firm rather than disappear.
  - A stakeholder's exit and engagement in one firm depends on and also affects other firms.
- The GE effect of exit and engagement is understudied in the literature
  - It may account for a large part of the aggregate impact of exit.

This Paper

- The paper intends to capture the GE effect
  - naturally motivated from a theory perspective and has significant practical relevance.
- The paper introduces a multi-sided matching model
  - Matching models are a bit unusual in finance but very appropriate here
- The GE effect operates through matching.
  - Need a framework with multiple firms, multiple stakeholders
  - Consider matching between firms and stakeholders, rather than simply exit from a firm or not
- Matching here is multi-sided and thus non-trivial
  - Players differ in two dimensions: productivity & purpose

The Model

- N types of stakeholders and one type of firm, each with productivity x_l.
- A firm hires one of each type of stakeholders, and output depends multiplicatively on their productivity

$$y(\mathbf{x}) = \prod_{l=1}^{N+1} x_l$$

  - Complementarity across types: positive assortative matching maximizes output
- Production harm

$$\phi(y) = \phi_0 + \sigma y$$

- Mitigation incurs pecuniary cost and reduces harm
  - A firm's pecuniary value: $y - cm$
  - A firm's harm: $\phi(y) - m$
- A stakeholder's utility

$$u = p - \theta_l \psi(\phi(y) - m)$$

  - positively linear in the transfer received from his firm
  - decreasing in his firm's harm if purpose-driven—$\theta_l = 1$, not if profit-driven—$\theta_l = 0$
  - A stakeholder of type l is purpose-driven with probability λ_l.

The Model

- Due to transferable utility,
  - The mitigation maximizes the joint surplus of all matched players

$$\Omega(y,n) = \max_{m \geq 0} y - cm - n\psi(\phi(y) - m),$$

    $n \equiv \sum_{l=1}^{N+1} \theta_l$ denotes the stakeholder-purpose index.
  - Any stable outcome is optimal—maximizes the total surplus of all players

$$\int \Omega(y,n)$$

- Three steps
  1. solve for optimal mitigation, and see how the joint surplus depends on (y,n)
  2. solve for optimal matching, based on how matching affects (y,n)
  3. examine exit and engagement

Joint Surplus under Optimal Mitigation

- For $n = 0$,
  - Optimal mitigation
$$m^* = 0$$
  - Joint surplus
$$\Omega(y, n) = y$$

- For $n > 0$,
  - Optimal mitigation
$$m^* = \phi_0 + \sigma y - \psi'^{(-1)}\left(\frac{c}{n}\right)$$
  - Joint surplus
$$\Omega(y, n) = (1 - c\sigma)y - c\left[\phi_0 - \psi'^{(-1)}\left(\frac{c}{n}\right)\right] - n\psi\left(\psi'^{(-1)}\left(\frac{c}{n}\right)\right)$$

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- $\Omega(y, n)$ is not continuous in n. Why?
  - For $n > 0$, optimal mitigation is an interior solution, with $m \geq 0$ not binding.
  - For $n = 0$, optimal mitigation is a corner solution constrained by $m \geq 0$.

Joint Surplus under Optimal Mitigation

- A unified representation of joint surplus

$$\Omega(y,n) = z(y,n) - w(n)$$

where

$$z(y,n) \equiv \begin{cases} y, & n=0 \\ (1-c\sigma)y, & n=1 \end{cases}$$

and

$$w(n) \equiv \begin{cases} 0, & n=0 \\ c\left[\phi_0 - \psi'^{(-1)}\left(\frac{c}{n}\right)\right] + n\psi\left(\psi'^{(-1)}\left(\frac{c}{n}\right)\right), & n=1 \end{cases}$$

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- Given the allocation of productivity, how to improve the total surplus through that of purpose
  - Lemma 1: $\Omega(y,n)$ is convex in n, so let purpose-driven stakeholders cluster
- Given the allocation of purpose, how to improve the total surplus through that of productivity
  - Lemma 2: $z(y,n)$ depends multiplicatively on productivity, so let high-productivity stakeholders cluster
- However, the two lemmas do not speak to the joint allocation of purpose and productivity.

Independence Benchmark

- Suppose $\sigma = 0$.

$$\int \Omega(y,n) = \int y - \int w(n)$$

  - Maximizing the total output $\int y$ entails maximizing the clustering of productivity
  - Minimizing the total purpose loss $\int w(n)$ entails maximizing the clustering of purpose

- Proposition 1: We can achieve them simultaneously by first sorting on productivity and then sorting on purpose

- The impact of some profit-driven stakeholders becoming purpose-driven
  - The composition of all teams in terms of productivity stay unchanged, so the distribution of output stays unchanged
  - Some teams have higher purpose indices and choose higher mitigation, while other teams choose the same mitigation as before.
  - Engagement only and no spillovers

General Case

- When $\sigma = 0$, PAM in productivity: high-productivity (low-productivity) stakeholders cluster
- When $\sigma > 0$ and λ_l are not all equal, not the case.
- Consider the two-type case with workers and banks and workers are more likely to be purpose-driven
- If PAM in productivity is followed, for $x_1 > x_2$, there must be
  - a productivity-x_1 team with a profit-driven bank and a purpose-driven worker, and
  - a productivity-x_2 team with a profit-driven bank and a profit-driven worker
  - The total surplus of the two teams is

$$\Omega\left(x_1^2, 1\right) + \Omega\left(x_2^2, 0\right) = (1 - c\sigma)\, x_1^2 + x_2^2 - w(1)$$

General Case

- Now match
  - the productivity-x_1 profit-driven bank with the productivity-x_2 profit-driven worker, and
  - the productivity-x_2 profit-driven bank with the productivity-x_1 purpose-driven worker
  - The total surplus of the two teams is

$$\Omega(x_1 x_2, 0) + \Omega(x_1 x_2, 1) = x_1 x_2 + (1 - c\sigma) x_1 x_2 - w(1)$$

- The second match is socially preferred if

$$x_2 > (1 - c\sigma) x_1$$

  - The second match results in lower total output but also lower mitigation cost
- Tradeoff between productivity loss and mitigation cost
  - $\Omega(y, n)$ depends on y and n in a nonseparable way.

Exit

- Proposition 2: full separation at the top
  - high-productivity profit-driven (purpose-driven) banks match exclusively with high-productivity profit-driven (purpose-driven) workers
- Consider a high-productivity profit-driven bank that is in a purely profit-driven team.
- Exit: if he becomes purpose-driven, he will exit the purely profit-driven team and join a more purpose-driven team.
- Trickle-down:
  - He may not make the team more purpose-driven if he replaces another purpose-driven bank
  - However, the displaced purpose-driven bank will join a team with lower productivity and may make the team more purpose-driven.
- Key message:
  - Exit may have indirect effects through the displacement of stakeholders.
  - The aggregate impact of exit is larger than what any firm-level comparison implies

Comment: Use Simple Concrete Examples to Illustrate

- The model setup is clean and parsimonious, but what's going on is really complicated.
- Why complicated?
  - Matching involves multiple types, and each type has multiple stakeholders
  - Stakeholders differ in two dimensions: productivity and purpose
  - A team's joint surplus depends on productivity and purpose in a nonseparable way

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- Why complicated?
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  - A team's joint surplus depends on productivity and purpose in a nonseparable way
- For illustration purpose, the paper should show simple and concrete examples
  - Two types
  - Each type has four stakeholders: two possible preferences, at least two possible productivity
  - Explicitly pin down the equilibrium matching
  - Great if one example can be used consistently to illustrate all results

Comment: Constrained Mitigation at $n = 0$

- $\Omega(y, n)$ is not continuous in n. Why?
  - For $n > 0$, optimal mitigation is an interior solution, with $m \geq 0$ not binding.
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- Some of the paper's proof does not take the discontinuity into consideration
  - The results are correct, just some modification of the proof
- The paper's key message depends on that mitigation is constrained by $m \geq 0$ at $n = 0$.
  - Consider the alternative that all players care about the harm to some extent, or environmental regulation makes firms care
  - n is always positive, and optimal mitigation is always an interior solution.
  - Then the total surplus depends on y and n separately

$$\int \Omega(y, n) = \int (1 - c\sigma) y - \int w(n)$$

Similar to the independence benchmark, we observe engagement only and no exit.

Comment: Constrained Mitigation at $n = 0$

- Given the importance of this feature, the paper should take it more seriously
- Great to clearly state it as an assumption.
- Discuss why this feature is somehow prevalent in practice.
  - I do not think the feature is weird but not clear about its prominence.
- Here, the constraint is binding when n is low because m is bounded from below. In principle, m could also be bounded from above so that the constraint is binding when n is high.
  - The paper can check whether the results are similar in this case.
  - Or the paper can discuss why this case is less relevant than the current setting.

Comment: Equilibrium Transfers

- The paper does not fully characterize the equilibrium transfers.
  - Propositions 1 and 3 have statements about the difference in transfers between profit-driven and purpose-driven stakeholders with identical productivity.
  - I think it is not enough to determine the level.
- I am curious about how transfers are determined in such a matching setting.
  - In a typical setting with decreasing marginal output, the transfer to a stakeholder equals his marginal output, and the sum of all transfers is smaller than the output.
  - Here, due to the multiplicative structure of the output, a stakeholder's marginal output can be high.
  - Does it mean that the transfer to him can be high?
  - Is it possible that the sum of all transfers exceeds the output?

- A very innovative and intense paper
- A grand banquet for determined readers
  - I got stuck many times but felt excited every time I managed to move on
- Need creative writing to make it more accessible