

# Global Production Networks with Global Uncertainty

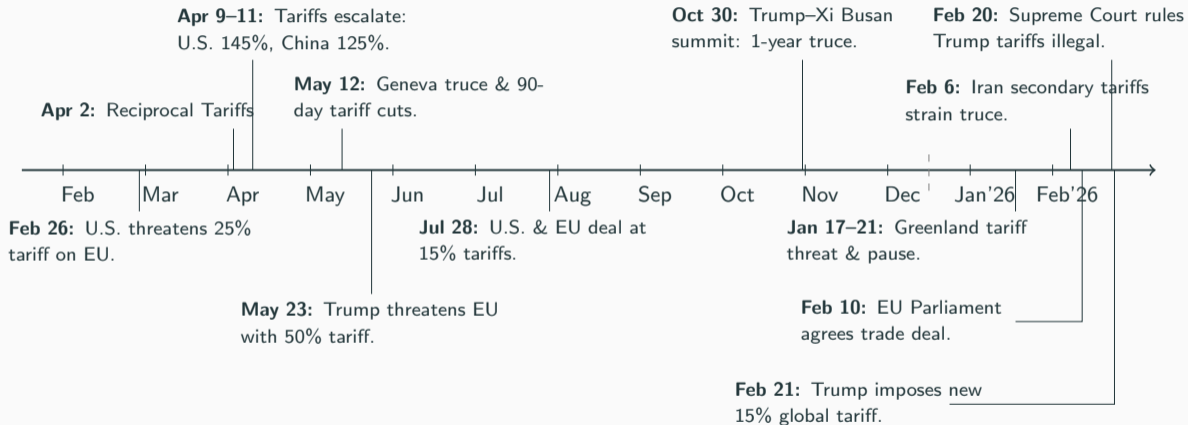
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# Introduction: Global Uncertainty in Tariffs



“The damage may have been done.” — Washington Post, 4/27/2025

- **Q:** How does **aggregate uncertainty** in productivity and tariffs impact global trade and welfare?
- **Theory:** incorporate aggregate uncertainty and **anticipatory sourcing** into trade models
  - ★ Impact of global uncertainty captured by bilateral sufficient statistics and enter as cost shifters
  - ★ Perturbation solution characterizing impact of uncertainty with formula of observable statistics

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  - ★ Perturbation solution characterizing impact of uncertainty with formula of observable statistics
- **Quantification:** apply to global economy with prod. uncertainty and uncertain tariff wars
  - ★ Productivity uncertainty increases trade for diversification incentive
  - ★ Uncertain tariffs reduce trade; anticipatory sourcing important; losses even tariffs not realized
  - ★ Trade wars increase country's exposure to prod. uncertainty due to reduced diversification

- **Trade and aggregate uncertainty:** Ruffin (1974), Helpman and Razin (1978), Caselli et al. (2020), Allen and Atkin (2022), Adamopoulos and Leibovici (2024), Castro-Vincenzi et al. (2024), Fan et al. (2023), Huo, Levchenko and Pandalai-Nayar (2025), Fitzgerald (2024), Caliendo, Kortum and Parro (2025)

**New: Impact of uncertainty by affecting ex-ante sourcing (incomplete markets)**

- **Perturbation solution in trade models:** Allen et al. (2020), Huo et al. (2025), Baqaee and Farhi (2024), Kleinman, Liu and Redding (2024a,b), Kleinman, Liu and Redding (2025)

**New: First-order ex-ante + Second-order ex-post; Local + Global for different shocks**

- **Trade policy uncertainty:** Handley and Limão (2015, 2017), Pierce and Schott (2016), and Alessandria et al. (2025)

**New: Multi-country multi-sector GE model. Interaction of prod. and tariff uncertainty**

- **Supply chain resilience:** Carvalho et al. (2021), Boehm et al. (2019), Grossman et al. (2024), Kopytov et al. (2024), Grossman et al. (2023)

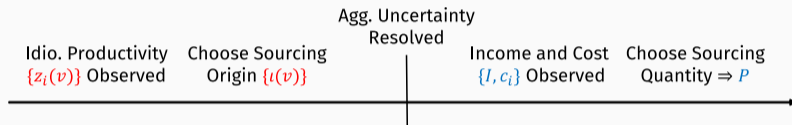
**New: Multi-country multi-sector GE model**

# One-sector Sourcing Model with Aggregate Uncertainty

- Stochastic income  $I$ . CES agg. across varieties  $v \in [0, 1]$ , elasticity  $\sigma$
- $N$  origins. Stochastic aggregate cost  $\{c_i\}_{i=1}^N$ . Idiosyncratic prod.  $\{z_i(v)\}$  i.i.d. Frechet

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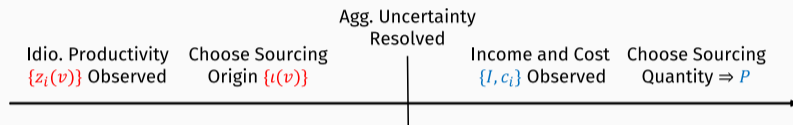
- **Ex-ante**: choose sourcing origin  $\{l(v)\}$  after observing  $\{z_i(v)\}$
- **Ex-post**: choose sourcing quantity, delivering ideal price index  $P$

$$\max_{\{l(v)\}_{v \in [0,1]}} U = \mathbb{E} \tilde{U}\left(\frac{I}{P}\right), \quad \tilde{U}(C) = \frac{C^{1-\gamma}}{1-\gamma}$$

$$\text{s.t. } P = \left[ \int_0^1 \left( \frac{c_{l(v)}}{z_{l(v)}(v)} \right)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}.$$

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- No uncertainty  $\Rightarrow$  EK (2002). Stochastic  $\{I, c_i\} \Rightarrow$  independent sourcing by  $v$  do not hold

## Characterizing Optimal Sourcing

- $Z_i \equiv \int_{t(v)=i} [z_i(v)]^{\sigma-1} dv$  effective varieties from  $i$
- $\lambda_i \equiv \frac{dU}{dZ_i}$ : marginal *expected* utility of sourcing from  $i$

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- Conditional on  $\{\lambda_i\}$  (EK), and definition of  $\lambda_i$  give a fixed point in  $\{\lambda_i\}$ :

$$\text{(EK): } Z_i \propto \left(\frac{\lambda_i}{\Lambda}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}}, \quad \Lambda \propto \left(\sum_m \lambda_m^{\frac{\theta}{\sigma-1}}\right)^{\frac{\sigma-1}{\theta}}$$

$$\text{(Def of } \lambda) : \lambda_i \propto \mathbb{E}[I^{1-\gamma} P^{\gamma+\sigma-2} [c_i]^{1-\sigma}], \quad P = \left[\sum_i [c_i]^{1-\sigma} Z_i\right]^{\frac{1}{1-\sigma}}$$

## Global Uncertainty as Cost Shifters

- Given ex-ante sourcing summarized by  $\{Z_i\}$ , sourcing after uncertainty resolution follows

$$\text{Trade share: } \pi_i = \frac{[c_i]^{1-\sigma} Z_i}{\sum_m [c_m]^{1-\sigma} Z_m}$$

$$\text{Price index: } P = \left[ \sum_i [c_i]^{1-\sigma} Z_i \right]^{\frac{1}{1-\sigma}}$$

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- Global uncertainty encoded in  $\{Z_i\}$  and enters as cost shifters
- Short-term trade elasticity:  $\sigma - 1$ . Long-term trade elasticity:  $\theta$ .

# Perturbation Solution to the Ex-ante Sourcing Solution

## Proposition (First Order Ex-ante Sourcing Solution)

$I, \{c_i\}$  jointly log normal, variance scaled by  $\varepsilon > 0$ . Up to  $O(\varepsilon)$ :

$$\ln Z_i = \ln \bar{Z}_i + (\theta + 1 - \sigma)\varepsilon \Delta_i + o(\varepsilon),$$

$$\Delta_i = \underbrace{\dots}_{\text{Mean cost difference}} + \underbrace{\left[ \frac{(\sigma - 1)}{2} (\Omega_{ii} - \sum_m \bar{s}_m \Omega_{mm}) - (\sigma + \gamma - 2) \left( \sum_m \bar{s}_m \Omega_{im} - \sum_m \sum_l \bar{s}_m \bar{s}_l \Omega_{ml} \right) \right]}_{\text{Risk aversion effect}}$$

$\bar{Z}_i, \bar{s}_i$ : values at  $\varepsilon = 0$ ;  $\varepsilon \Omega_{im} = \text{Cov}(\ln(c_i/I), \ln(c_m/I))$

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- When  $\bar{s}_i$  is moderately large, a higher  $\Omega_{ii}$  (higher volatility) lowers  $\Delta_i$
- A higher  $\sum_{m \neq i} \bar{s}_m \Omega_{im}$  (lower diversification benefit) always lowers  $\Delta_i$

# General Equilibrium

- $N$  countries (subscript  $n, i$ ),  $J$  sectors (superscript  $j, k$ ), with input-output linkages
- Stochastic productivity  $\{A_n^j\}$  and tariff  $\{\tau_{ni}^j\}$
- $\{c_{ni}^j\}$  depend on origin's productivity, wages, input costs, and *bilateral* trade costs
- $\{I_n\}$  depend on wages and tariff revenue

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- **Ex-ante** and **Ex-post** feedback in equilibrium determination:
  - ★ Optimal ex-ante sourcing  $\{Z_i\}$  in anticipation of ex-post distribution
  - ★ Given ex-ante sourcing, trade equilibrium determined after shock realizations, shaping  $\{c_{ni}^j\}, I_n$

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- Ex-ante sourcing + Second-order approx. of ex-post equilibrium  $\Rightarrow$  Full eqm solution GE perturb
  - ★ Effect of uncertainty characterized by measurable stats (elasticity, trade shares, covariance)

## Impact of Productivity Uncertainty on Trade - Analytical Examples

- Symmetric countries w/o IO linkages.  $Var(A_i) = \sigma_A^2$ ;  $Corr(A_i, A_n) = \rho$ . Trade cost:  $d$

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- (Diversification incentive increases trade) A decrease in  $\rho$  increases foreign sourcing shares
- When trade cost  $d$  increases
  - ★ (Level effect) Foreign sourcing shares decrease in the absence of uncertainty
  - ★ (Endogenous uncertainty) The risk adjustment term ( $\Delta_i$ ) increases, offsetting level effect
  - ★ (Gains from diversification) Volatility of real income increases

## Extensions: Loading on Effective Elasticity and Covariances

- **Inventory** (real hedging)
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- **Ex-post adjustment**

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- **Inventory** (real hedging)
  - ★ Setup: inventory  $\Rightarrow$  constant marginal inventory cost + iceberg storage cost & time value
  - ★ Result: shrink **effective covariance** to:  $(1 - \bar{s}_m^I)(1 - \bar{s}_i^I)\Omega_{mi}$ ;  $\bar{s}_i^I$ : inventory expenditure share
- **Financial assets** (financial hedging)
  - ★ Setup: arbitrary set of assets with stochastic return  $R = \{R_\ell\}_{\ell=1}^L$ , traded ex-ante
  - ★ Result: change **effective covariance** to:  $\text{Cov}(c_i^{(1)} - W^{(1)}, c_m^{(1)} - W^{(1)})$   
 $\bar{W}^{(1)}$ : residual income process risk orthogonal to the asset-return span of  $R$
- **Ex-post adjustment**
  - ★ Setup: option to re-optimize suppliers ex-post with probability  $\pi$
  - ★ Result: does not change ex-ante sourcing; Change ex-post elas. to  $(\sigma - 1) + \pi(\theta - (\sigma - 1))$

# Quantification: Impact of Productivity Uncertainty on Global Trade and Welfare

# Summary of Calibration

Parameters	Descriptions	Value	Target/Source
A. Externally Calibrated			
$\gamma_n^{jk}, \gamma_n^{jL}, \beta_n^j$	IO Structure and Consumption Share	-	WIOT; $N = 44, J = 32$
$L_n$	Labor Endowment	-	PWT
$(\sigma - 1, \theta)$	Short- and Long-term Trade Elasticity	(0.76, 2.12)	Boehm et al. (2023)
$\gamma$	Relative Risk Aversion	5	Literature estimate
B. Calibrated in Non-stochastic Equilibrium			
$\bar{A}_n^j$	Mean Productivity	-	Countries' output shares in each sector
$\bar{d}_{ni}^j$	Pre-shock Iceberg Trade Costs	-	Bilateral trade shares (WIOT)
C. Productivity and Tariff Uncertainty			
$\tilde{\Sigma}_{ni}^{jk}$	Covariance of Productivity	-	Solow residuals from WIOD-SEA (Annual)
$\tau_{ni}^j$	Uncertain Tariffs	See text	Negotiation outcome on "Reciprocal Tariffs"

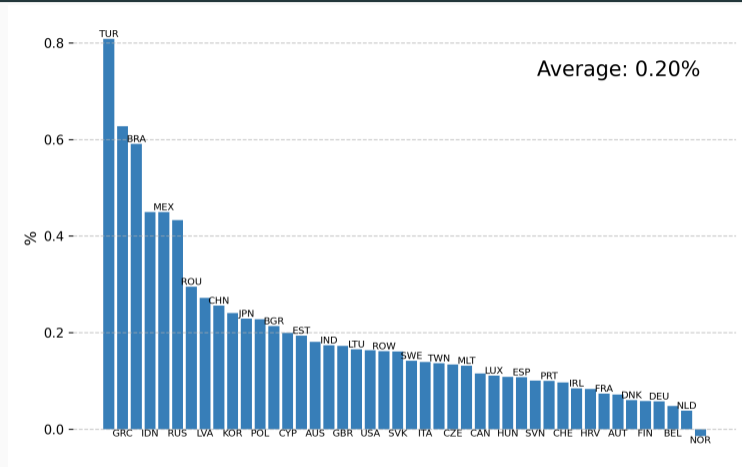
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- Numerical implementation

- ★ Local solution w.r.t. small dense prod. shocks + enumeration across large sparse tariff shocks
- ★ Automatic Differentiation for Hessian-vector Products to calculate second-order approximation

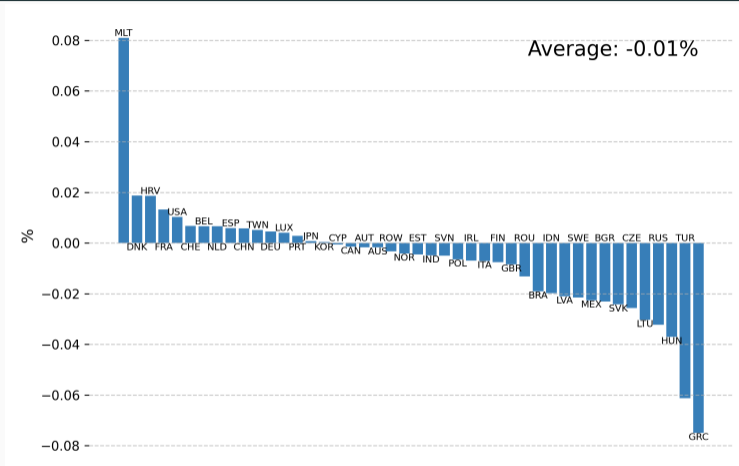
# Productivity Uncertainty Increases Trade



Plotted are the impacts of productivity uncertainty on *import shares* at mean shock realizations

- Productivity uncertainty increases trade due to the risk diversification motivation

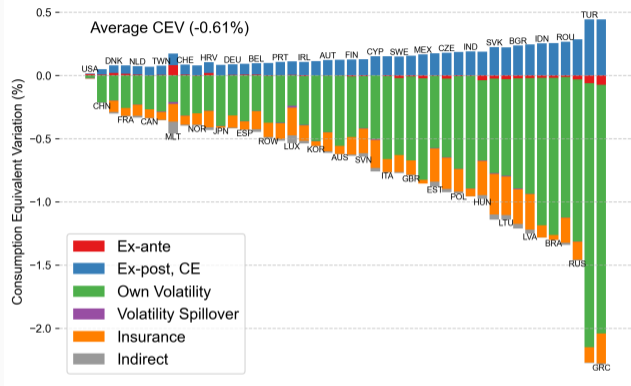
# Productivity Uncertainty Redistributes Real Income



Plotted are the impacts of productivity uncertainty on *Real GDP* evaluated at mean shocks

- Uncertainty leads to redistributive effects across countries (via cost shifters  $\{Z_{ni}^j\}$ )

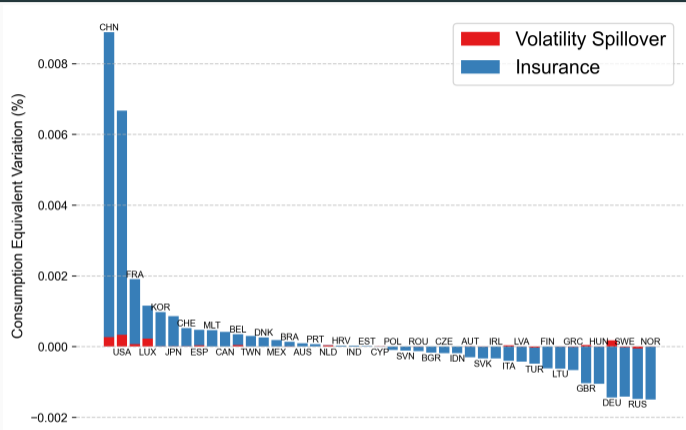
# The Welfare Impact of Productivity Uncertainty



$$\mu = \mu_{\text{Ex-ante}} + \mu_{\text{Ex-post, Certainty Equivalent}} + \mu_{\text{Risk\_Aversion}}$$

$$\mu_{\text{Risk\_Aversion}} = -\frac{\gamma}{2} \left\{ \underbrace{\sum_{j,k} D_{nn}^j \Sigma_{nn}^{jk} D_{nn}^k}_{\text{own volatility}} + \underbrace{\sum_{i \neq n} \sum_{j,k} D_{ni}^j \Sigma_{ii}^{jk} D_{ni}^k}_{\text{volatility spillover}} + 2 \underbrace{\sum_{i \neq n} \sum_{j,k} D_{nn}^j \Sigma_{ni}^{jk} D_{ni}^k}_{\text{insurance}} + \text{Indirect Effects} \right\},$$

# Impact of Each Country's Risk Profile on Global Welfare



Plotted are global CEV Risk Aversion terms by origin country, relative to avg

Raw Corr.

US detail

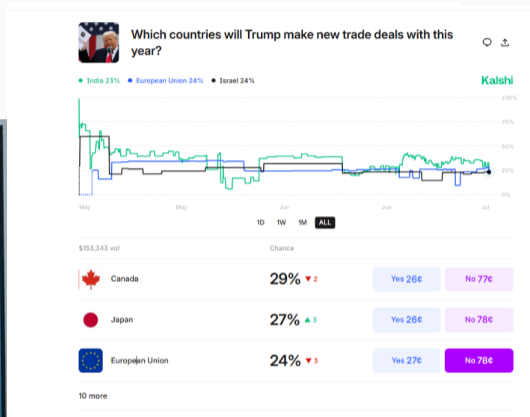
- China provides greater insurance benefits than others due to lower correlated prod. shocks  
★ average magnitude small ( $\sim 5\%$  compared to level effect); can be large for some countries

# Impact of Uncertain Tariffs on Global Trade and Welfare

# Uncertain Trump Tariffs



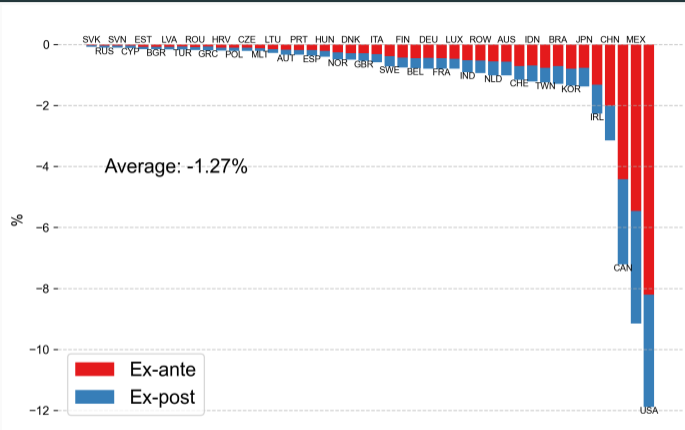
(a) Tariff Rate



(b) Chance of Successful Negotiation

- Successful Negotiation brings tariff to 10%; otherwise, reciprocal tariffs
- Independent negotiation by country (EU as one group)  $\Rightarrow 2^{11} = 2048$  scenarios

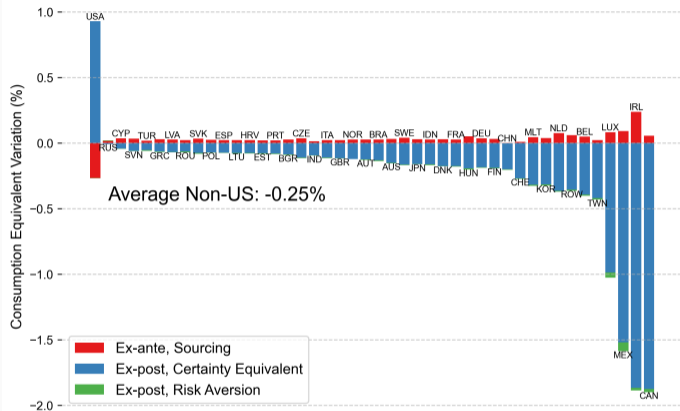
# Impact of Uncertain Tariffs on Trade - Anticipatory Sourcing Effect



Plotted are changes in *log import shares* led by uncertain tariffs

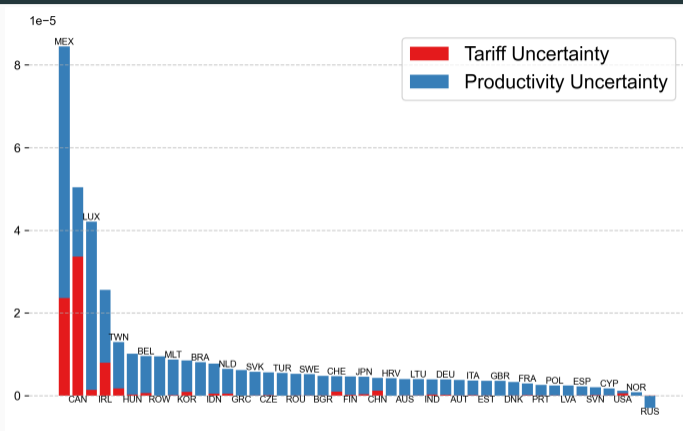
- **Ex-ante:** evaluated at 0 tariff realizations  $\Rightarrow$  effect purely arising from ex-ante sourcing
- **Ex-post:** additional changes to reach means across tariff realizations

# Impact of Uncertain Tariffs on Welfare



- US gains due to tariff revenue
- US **loses** due to ex-ante sourcing in anticipation of tariffs and US ToT appreciation
- US consumers lose if tariffs are not realized

# Interactions Between Productivity and Tariff Uncertainty



Plotted are changes in **Variance of Real GDP** led by uncertain tariffs

- Tariff uncertainty itself only contributes moderately to heightened GDP volatility
- Exposure to prod. uncertainty increases due to reduced diversification through trade

# Conclusion

- A tractable approach to introduce anticipatory sourcing under global aggregate uncertainty
  - ★ Sufficient statistics characterized by a FP problem and uncertainty enters as cost shifters
  - ★ Applicable to any models with an EK structure (global value chain; multinational; transport)
- Productivity uncertainty increases trade and uncertain tariffs reduce trade
- Anticipatory sourcing is important in shaping trade and welfare impact of uncertain tariffs

# Perturbation Recap

## A generic fixed point system

$$F(x, y(x), \sqrt{\varepsilon}\zeta) = 0, \quad (1)$$

where  $\zeta$  is an exogenous Gaussian vector with mean zero and covariance  $\Sigma$ . Here,  $y(x)$  is a functional that maps a stochastic vector  $x$  to a real vector, and has the property that if  $x$  is given by  $x = \bar{x} + \sqrt{\varepsilon}x^{(1)} + \varepsilon x^{(2)}$ , then

$$y(x) = \bar{y} + \phi(\varepsilon \mathbb{E}x^{(2)}) + \psi(\varepsilon \Omega) + o(\varepsilon), \quad (2)$$

where  $(\bar{x}, \bar{y})$  are solutions to the system with  $\varepsilon = 0$ . Here,  $\varepsilon \Omega$  is the covariance matrix of  $\sqrt{\varepsilon}(\Gamma_1 x^{(1)} + \Gamma_2 \zeta)$  with matrices  $\Gamma_1$  and  $\Gamma_2$ , and linear operators  $\phi$  and  $\psi$ .

## In the one-sector GE model:

$$F(\ln w, \ln Z, \sqrt{\varepsilon}\zeta) \equiv \ln(w_i L_i) - \left[ \ln \left( \sum_n \frac{[d_{ni} w_i / (A_i e^{\sqrt{\varepsilon}\zeta_i})]^{1-\sigma} Z_{ni}}{\sum_m [d_{nm} w_m / (A_m e^{\sqrt{\varepsilon}\zeta_m})]^{1-\sigma} Z_{nm}} w_n L_n \right) + \ln \left( \sum_n w_n L_n \right) \right], \quad (3)$$

which adds a normalization equation  $\ln(\sum_n w_n L_n) = 0$  to the log-transformed equations.

## Lemma

The solution to the system (1) approximated up to  $O(\varepsilon)$  is characterized by

$$x = \bar{x} + x^{(1)}\sqrt{\varepsilon} + x^{(2)}\varepsilon + o(\varepsilon) \text{ with}$$

$$x^{(1)} = \Xi \cdot \zeta$$

$$\begin{aligned} \mathbb{E}x^{(2)} = & -[F_1 + F_2\phi]^{-1} \left( F_2\psi((\Gamma_1\Xi + \Gamma_2)\Sigma(\Gamma_1\Xi + \Gamma_2)^T) + \frac{1}{2}\mathcal{H}_{11}\text{vec}(\Xi\Sigma\Xi^T) \right. \\ & \left. + \frac{1}{2}\mathcal{H}_{33}\text{vec}(\Sigma) + \mathcal{H}_{13}\text{vec}(\Xi\Sigma) \right) \end{aligned}$$

where  $\Xi \equiv -F_1^{-1}F_3$ . Here,  $\mathcal{H}_{11}$  is the  $n_F \times (n_x)^2$  matrix with the  $k$ -th row being  $\text{vec}(F_{11}^{(k)})^T$ , with  $F_{11}^{(k)} \equiv \frac{\partial^2 F^{(k)}}{\partial x \partial x^T}$  and  $\text{vec}(\cdot)$  the vectorization operator, and  $\mathcal{H}_{33}$  and  $\mathcal{H}_{13}$  are similarly defined. All derivatives are evaluated at  $(x, y, \varepsilon) = (\bar{x}, \bar{y}, 0)$ , where  $(\bar{x}, \bar{y})$  are the solutions to the system with  $\varepsilon = 0$ .

## Proposition (Perturbation Solution to the General Equilibrium)

Assume  $\{\ln A_i\}_{i=1}^N$  are jointly Gaussian and generated by  $\ln A_i = \ln \bar{A}_i + \sqrt{\varepsilon}\zeta_i$ , where  $\mathbb{E}[\zeta_i] = 0$  and  $\Sigma_{im} \equiv \text{Cov}(\zeta_i, \zeta_m)$ . The solution to the general equilibrium, approximated up to terms of order  $\varepsilon$ , is characterized by ex-ante sourcing effective productivities  $Z_{ni} \equiv \int_{\mathcal{V}_{ni}} [z_{ni}(v)]^{\sigma-1} dv$  that satisfy

$$\hat{Z}_{ni} \equiv \ln Z_{ni} - \ln \bar{Z}_{ni} = (\theta + 1 - \sigma)\varepsilon\Delta_{ni} + o(\varepsilon), \quad (4)$$

ex-ante sourcing shares that satisfy

$$\hat{s}_{ni} \equiv \ln s_{ni} - \ln \bar{s}_{ni} = \theta\varepsilon\Delta_{ni} + o(\varepsilon), \quad (5)$$

and ex-post equilibrium normalized wages that satisfy

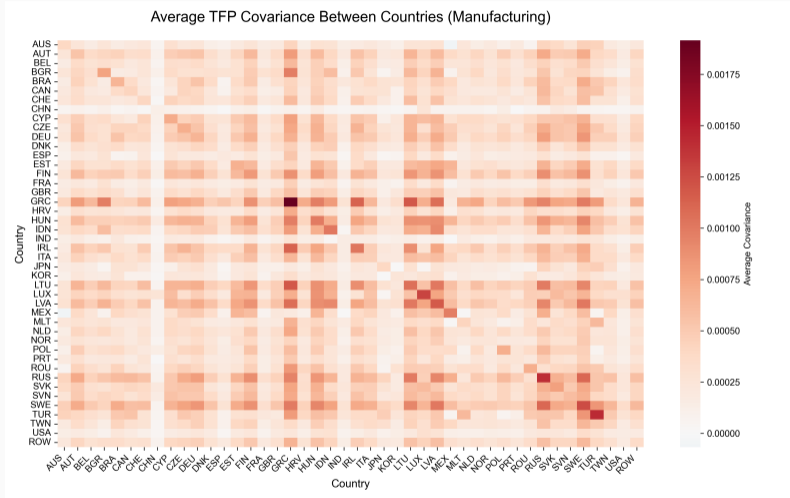
$$\hat{w}_n \equiv \ln w_n - \ln \bar{w}_n = \underbrace{\Psi_{n,im}\hat{Z}_{im}}_{\text{Ex-ante sourcing feedback}} + \underbrace{\sqrt{\varepsilon} \sum_i \Xi_{ni}\zeta_i}_{\text{FO direct exposure}} + \underbrace{\varepsilon \sum_{i,m} \Phi_{n,im}\zeta_i\zeta_m}_{\text{SO direct exposure}} + o(\varepsilon), \quad (6)$$

where  $\varepsilon\Delta_{ni}$  is the risk adjusted relative sourcing cost of region  $i$ .

$\Psi \equiv -F_1^{-1}F_2$  and  $\Xi \equiv -F_1^{-1}F_3$  are the general equilibrium exposure matrices of  $\ln \mathbf{w}$  w.r.t.  $\ln \mathbf{Z}$  and  $\ln \mathbf{A}$  at the ex-post equilibrium.  $\Phi_n$  are coefficient matrices that involve  $F_1^{-1}$ ,  $\Xi$  and the Hessian of  $F$ .

All coefficients are functions of import expenditure shares, income shares, value-added shares, and parameters  $\{\sigma, \theta, \gamma\}$ . Given observable statistics, all coefficients are linear transformations of  $\Sigma$ . [Back](#)

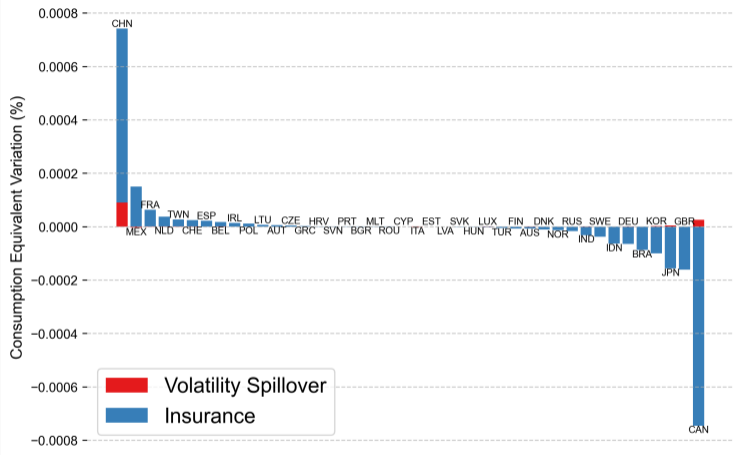
# Productivity Correlation Matrix



- China is among countries that have low productivity correlation with others

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# Contribution of Other Countries' Productivity on US Income Volatility



- China is the most important country in providing insurance benefits to the US [Back](#)
- US friends (high FO link) like CAN provide less insurance as costs are *endogenously* correlated

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