

# Optimal Trade Policies and Market Power in Quantitative Trade Models

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# Motivation

- A large number of general-equilibrium quantitative models analyze the impact of trade
- What are the policy implications of these models?
- Challenging to characterize optimal policies within GE quantitative trade models
  - Complex interdependency of demand and supply across goods and countries
  - Tariff imposed in one sector affects demand and supply in all sectors and countries

# Classic Literature

- Literature of optimal tariff:  $\tau_j^m = \frac{1}{\epsilon_j^*}$ 
  - Using *simple general equilibrium* or *partial equilibrium* to reduce to a problem of a single-foreign-country single-good monopsonist/monopolist
  - Export supply elasticity  $\epsilon_j^*$
  - Get a simple formula: optimal tariff should equal the inverse of the (own-price) elasticity of the foreign export supply curve
  - Terms of trade manipulation: Bagwell and Staiger (1999), Broda, Limao, and Weinstein (2008), and Feenstra (2015)

# Optimal Policies within Modern GE Trade Models

- GE models emphasize interdependencies  $\Rightarrow$  difficult to characterize optimal policies
- Most quantitative work studies alternative policies but not optimal policies, or computes optimal policies
- Recent development
  - Costinot-Donaldson-Vogel-Werning (2015): canonical Ricardian model (DFS)
    - Optimal export taxes increase with comparative advantage
    - Optimal import tariffs should be *uniform* across sectors
  - Lashkaripour-Lugovskyy (2023): multi-sector Krugman with economies of scale
    - Approximate optimal policies
    - Wrong formulas

# Simple Intuition

Standard models assume **perfectly mobile labor** across sectors

DFS, multi-sector EK, multi-sector Armington

- Wages equalized across sectors in Foreign country,  $w_{2j} = w_{2k} = w_2$
- Relative import price of two foreign goods  $\frac{w_2/z_{2j}}{w_2/z_{2k}}$  determined by technology ratios
- Home government cannot manipulate relative foreign prices using tariffs
- Optimally, **zero/uniform tariffs across sectors**

# Simple Intuition

**Example: Multi-sector models with fixed labor in each sector (specific factor)**

- A tariff on sector  $j$  decreases sector demand faced by Foreign
- Sectoral labor fixed; the sectoral wage in Foreign will decrease
- Foreign price  $w_{2j}/z_{2j}$  decreases, improving terms of trade of Home imports good  $j$
- We prove optimal tariffs increase with Home net import share in Foreign income

**Thus: differential tariffs if they can affect relative prices  $\frac{w_{2j}/z_{2j}}{w_{2k}/z_{2k}}$**

- Wages are different across sectors (imperfectly mobile/substitutable labor)
- Endogenous technology

# This Paper

- Build a multi-country, multi-sector model with a **generic supply system** nesting
  - Extended multi-sector Armington, Eaton-Kortum, Krugman
  - Different economies of scale across sectors
  - Labor market specifications: perfectly mobile labor, fixed labor, and imperfectly substitutable labor across sectors as in the Ricardo-Roy model

▶ Model

## Proposition: Two-Country Optimal Policies under *GIIA Supply System*

Models with *CES demand system* with trade elasticities  $\epsilon_j$  and our defined *GIIA supply system* with supply elasticities  $\eta_j$ , optimal policies:

$$\text{Domestic tax: } 1 + \tau_j^d = (1 + \bar{\tau}^d) \frac{\eta_j}{\eta_j - 1}$$

$$\text{Export tax: } 1 + \tau_j^x = \frac{1 + \tau_j^d}{1 + \tau_j^m} \left( 1 + \frac{1}{\epsilon_j \pi_{22,j}} \right)$$

$$\text{Import tariff: } \tau_j^m - \tau_i^m = \frac{1}{\eta_j} \frac{\beta_j x_1 - (1 + \tau_j^d) Y_{1j}}{Y_{2j}} - \frac{1}{\eta_i} \frac{\beta_i x_1 - (1 + \tau_i^d) Y_{1i}}{Y_{2i}}$$

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- *Domestic tax*: Pigouvian tax to fix domestic distortions
- *Sector-specific export taxes* are higher (given other taxes)
  - in sectors where Home is a large seller to Foreign ( $\pi_{22,j}$  lower)
  - if the trade elasticity  $\epsilon_j$  is smaller

## Two-Country Optimal Tariffs under *GIIA Supply System*

$$\tau_j^m - \tau_i^m = \frac{1}{\eta_j} \frac{\overbrace{\beta_j x_1 - (1 + \tau_j^d) Y_{1j}}^{\text{Home's net import in sector } j}}{\underbrace{Y_{2j}}_{\text{foreign income in } j}} - \frac{1}{\eta_i} \frac{\beta_i x_1 - (1 + \tau_i^d) Y_{1i}}{Y_{2i}}$$

*supply elasticity para.* }

- Relative tariffs depend on the difference in Home's net import share across sectors, each scaled inversely by its supply elasticity parameter  $\eta_j$

## Models Satisfying *GIIA Supply System*

$$\tau_j^m - \tau_i^m = \frac{1}{\kappa} \left( \frac{\beta_j x_1 - Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - Y_{1i}}{Y_{2i}} \right)$$

*Home's net import in sector j*

- **Ricardo-Roy or multi-sector Armington model with Imperfect Substitutable Labor**
  - Perfectly mobile labor,  $\kappa \rightarrow \infty \Rightarrow$  uniform tariffs in standard models
  - Fixed labor,  $\kappa = 1$
  - Imperfectly substitute labor with CES elasticity  $\kappa$
  - Ricardo-Roy with one factor ▶ RR ▶ Multi-RR
  - $\eta = \kappa \geq 0$ , labor market elasticity of substitution

Tariffs increase with Home net import share, but decrease with labor substitutable

## Models Satisfying *GIIA Supply System*

$$\tau_j^m - \tau_i^m = -\psi \left( \frac{\beta_j x_1 - Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - Y_{1i}}{Y_{2i}} \right)$$

*Home's net import in sector j*

- **Production specifications:**

- Endogenous technology: steady-state of Bai, Jin, and Lu (2025),  $\psi = \frac{1}{\theta}$
- Increasing return to scale (IRS  $\psi$ )
- Multi-sector Krugman (same ES parameter for all sectors),  $\psi = \frac{1}{\sigma-1}$
- $\eta = -\frac{1}{\psi} < 0$

Tariffs negatively related to Home net import share

## Models Satisfying *GIIA Supply System*

$$\tau_j^m - \tau_i^m = \underbrace{\left( \frac{1}{\kappa} - \psi_j \frac{\kappa - 1}{\kappa} \right)}_{\frac{1}{\eta_j}} \frac{\beta_j x_1 - (1 + \tau_j^d) Y_{1j}}{Y_{2j}} - \underbrace{\left( \frac{1}{\kappa} - \psi_i \frac{\kappa - 1}{\kappa} \right)}_{\frac{1}{\eta_i}} \frac{\beta_i x_1 - (1 + \tau_i^d) Y_{1i}}{Y_{2i}}$$

- **Generalized multisector Krugman, multisector Armington and Eaton-Kortum with EES:**  $p_{n,j} \propto w_{n,j} (L_{n,j})^{\psi_j}$ , and imperfectly substitute labor ( $\kappa$ )
- For example, generalized multisector Krugman:  $\frac{1}{\eta_j} = \frac{1}{\kappa} - \frac{1}{\sigma_j - 1} \frac{\kappa - 1}{\kappa}$

## *GIIA Supply System*

### **Definition (*GIIA Supply System*)**

We define a *GIIA Supply System* which satisfies

$$\frac{\partial \ln Y_s}{\partial \ln p_j} \frac{Y_s}{Y_j} \frac{1}{\eta_j} - \frac{\partial \ln Y_s}{\partial \ln p_i} \frac{Y_s}{Y_i} \frac{1}{\eta_i} = \begin{cases} 0 & \text{for } s \neq i, j \\ 1 & \text{for } s = j, \end{cases}$$

for any sector  $s, i, j$ , where  $Y_j$  is the income of a sector  $j$ , and  $\eta_j$  is sector specific constant related to supply elasticity.

A sector's partial supply elasticities with respect to

- prices in any two other sectors (scaled by the size and supply parameters) are the same
- its own price relative to the elasticity to any other sector's price equals one.

## *GIIA Supply System*

- *GIIA Supply System* is **not equivalent to** constant supply elasticity
  - A sector's export supply elasticity does *NOT* equal to  $\eta$
  - For example, in a CES labor supply model

$$\frac{\partial \ln Y_s}{\partial \ln p_j} \frac{Y_s}{Y_j} = \begin{cases} -(\kappa - 1)\lambda_s & \text{for } s \neq j \\ \kappa - (\kappa - 1)\lambda_j & \text{for } s = j. \end{cases}$$

- $\lambda_j$  share of sector  $j$  income in total income
- Self-elasticity is not a constant
- Cross-elasticity is not zero
- **Different from PE setups** that assume exogenous constant foreign export supply elasticity and overlook cross-sector interdependencies
- CES supply is sufficient but not necessary for *GIIA Supply System*

## Multi-Country Case Differs from Two-Country Case

- **Supply across country-sector:** Labor may be more substitutable within domestic markets than across countries. This asymmetry in factor mobility breaks the symmetry of cross-elasticities across sectors and countries even under GIIA supply within a country
- **Trade among foreign countries:** Home government lacks instruments to directly influence trade flows between any two foreign countries

$$Y_{n,j} - \underbrace{\sum_{i \neq 1}^N \beta_{i,j} \pi_{in,j} x_i}_{E_{1n,j}} = \beta_{1,j} \frac{1}{1 + \tau_{n,j}^m} \pi_{1n,j} x_1,$$

# Multi-Country Optimal Import Tariffs

Additional interdependencies across countries within the trade network

$$\begin{pmatrix} \frac{\partial E_{12}}{\partial \ln p_2} \circ \frac{1}{Y_2} & \frac{\partial E_{13}}{\partial \ln p_2} \circ \frac{1}{Y_2} & \dots & \frac{\partial E_{1N}}{\partial \ln p_2} \circ \frac{1}{Y_2} \\ \frac{\partial E_{12}}{\partial \ln p_3} \circ \frac{1}{Y_3} & \frac{\partial E_{13}}{\partial \ln p_3} \circ \frac{1}{Y_3} & \dots & \frac{\partial E_{1N}}{\partial \ln p_3} \circ \frac{1}{Y_3} \\ \dots & \dots & \dots & \dots \\ \frac{\partial E_{12}}{\partial \ln p_N} \circ \frac{1}{Y_N} & \frac{\partial E_{13}}{\partial \ln p_N} \circ \frac{1}{Y_N} & \dots & \frac{\partial E_{1N}}{\partial \ln p_N} \circ \frac{1}{Y_N} \end{pmatrix} \begin{pmatrix} 1 + \tau_2^m \\ 1 + \tau_3^m \\ \dots \\ 1 + \tau_N^m \end{pmatrix} = \begin{pmatrix} \beta_1 x_1 \circ \pi_{12} \circ \frac{1}{Y_2} - \sum_{k=1}^J (1 + \tau_k^d) \frac{\partial E_{11,k}}{\partial \ln p_2} \circ \frac{1}{Y_2} \\ \beta_1 x_1 \circ \pi_{13} \circ \frac{1}{Y_3} - \sum_{k=1}^J (1 + \tau_k^d) \frac{\partial E_{11,k}}{\partial \ln p_3} \circ \frac{1}{Y_3} \\ \dots \\ \beta_1 x_1 \circ \pi_{1N} \circ \frac{1}{Y_N} - \sum_{k=1}^J (1 + \tau_k^d) \frac{\partial E_{11,k}}{\partial \ln p_N} \circ \frac{1}{Y_N} \end{pmatrix}$$

$\tau_{n,j}^m \uparrow \Rightarrow$

- Self-elasticity
  - $p_{n,j} \downarrow \Rightarrow$  affect  $nj$ 's supply
  - Other countries adjust in response to price changes  $\Rightarrow$  affect  $nj$ 's exports to Home
- Cross-elasticity
  - Affect prices globally  $\Rightarrow$  affect global supply and demand

► LHS

## Lemma (*GIIA Supply System & Multiple Countries*)

Home unilateral optimal tariffs satisfy

$$\sum_{n \neq 1}^N (\delta_n + \tau_{n,j}^m) Y_{n,j} = \frac{1}{\kappa} [\beta_{1,j} x_1 - Y_{1,j}], \quad \text{for any } j$$

where  $\delta_n$  is endogenous but independent of sectors;

- Average tariff on a sector is strongly related to Home's overall net import  $\beta_{1,j} x_1 - Y_{1,j}$
- Trade network shapes country interdependencies  $\Rightarrow$  tariffs convergence within a sector

## Lemma (*GIIA Supply System & Multiple Countries*)

- When foreign countries are symmetric or when they do not trade with each other, optimal policies only depend on bilateral trade shares:

$$\tau_{n,j}^m - \tau_{n,i}^m = \frac{1}{\kappa} \left[ \frac{\pi_{1n,j} \beta_{1,j} x_1 - \frac{1}{1+\tau_{n,j}^x} \pi_{n1,j} \beta_{n,j} x_n}{Y_{n,j}} - \frac{\pi_{1n,i} \beta_{1,i} x_1 - \frac{1}{1+\tau_{n,i}^x} \pi_{n1,i} \beta_{n,i} x_n}{Y_{n,i}} \right],$$

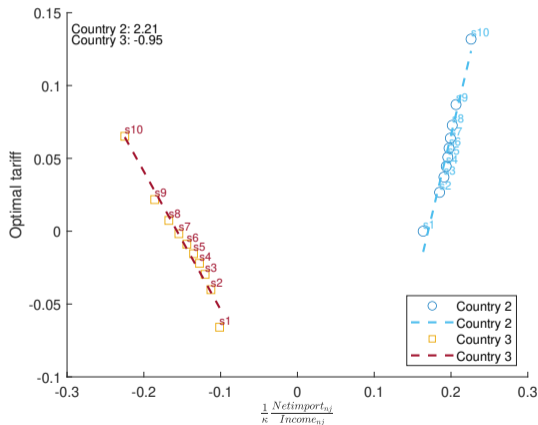
$$1 + \tau_{n,j}^x = \frac{1}{1 + \tau_{n,j}^m} \left( 1 + \frac{1}{\epsilon_j (1 - \pi_{n1,j})} \right).$$

# Effects of Interdependencies Between Foreign Countries

Country 2 has a comparative advantage in s10 relative to Home

Country 3 has a comparative advantage in s1 relative to Home

Asymmetric trade costs ▶ Parameter ▶ Alternative



- Optimal tariffs on a sector **converge** across countries

# Quantitative Optimal Policies

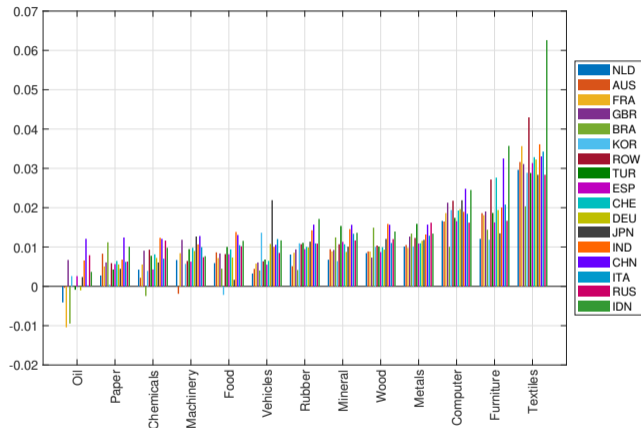
- Select the 19 largest countries based on 2014 GDP
- **Optimal policy formula:** use trade shares and sectoral production data & Exact hat method with **FOCs of optimal policies and eqm conditions**
  - US and China's unilateral optimal policies and implied welfare change
  - Nash optimal policies and welfare change

▶ Hat Method

▶ Nash

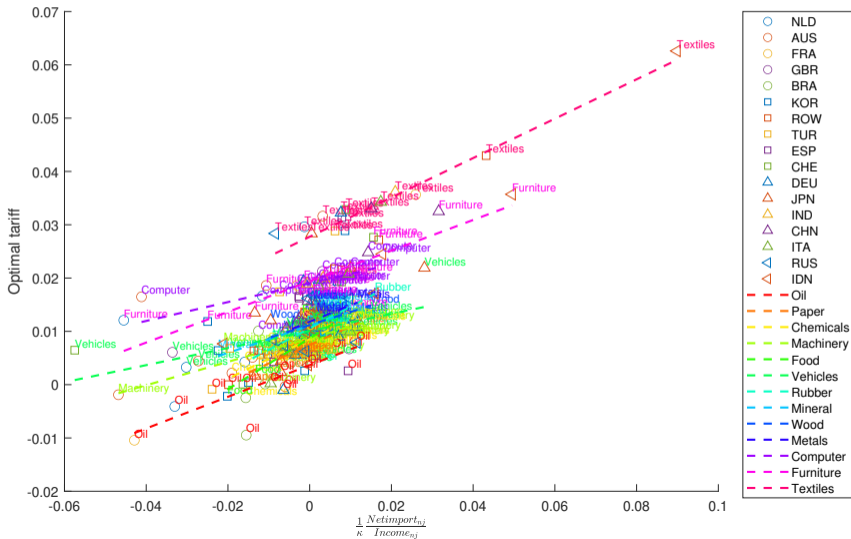
▶ Model and Data

# U.S. Optimal Policies: Import Tariff



- Sectors are ordered by U.S. net imports relative to world output before policy
- On average, import tariffs increase with a sector's net imports
- Interdependency: textiles from Russia also face higher tariffs

# U.S. Unilateral Optimal Tariffs and Net Import Share over Foreign Income



# Welfare Changes

|      | U.S. Unilateral Policy (%) |                               |                             | China's Unilateral Policy (%) |                               |                             | Nash Policy (%) |
|------|----------------------------|-------------------------------|-----------------------------|-------------------------------|-------------------------------|-----------------------------|-----------------|
|      | Optimal                    | Endogenous<br>Bilateral Share | Observed<br>Bilateral Share | Optimal                       | Endogenous<br>Bilateral Share | Observed<br>Bilateral Share | Optimal         |
| U.S. | 1.307                      | 1.286                         | 1.052                       | -0.386                        | -0.418                        | -0.521                      | -0.888          |
| NLD  | -0.373                     | -0.411                        | -0.443                      | -0.724                        | -0.877                        | -1.186                      | -7.985          |
| AUS  | -0.284                     | -0.321                        | -0.376                      | -1.399                        | -1.712                        | -2.089                      | -4.150          |
| FRA  | -0.263                     | -0.276                        | -0.285                      | -0.331                        | -0.350                        | -0.409                      | -4.610          |
| GBR  | -0.517                     | -0.536                        | -0.574                      | -0.498                        | -0.555                        | -0.704                      | -3.266          |
| BRA  | -0.214                     | -0.219                        | -0.222                      | -0.196                        | -0.204                        | -0.219                      | -0.965          |
| KOR  | -0.252                     | -0.262                        | -0.263                      | -0.839                        | -0.866                        | -0.924                      | -1.924          |
| ROW  | -0.287                     | -0.295                        | -0.305                      | -0.824                        | -0.804                        | -0.798                      | -1.483          |
| TUR  | -0.164                     | -0.170                        | -0.178                      | 0.061                         | 0.059                         | 0.123                       | -3.742          |
| ESP  | -0.048                     | -0.050                        | -0.031                      | -0.201                        | -0.213                        | -0.249                      | -2.193          |
| CHE  | -0.269                     | -0.280                        | -0.281                      | -0.139                        | -0.141                        | -0.153                      | -3.554          |
| DEU  | -0.210                     | -0.211                        | -0.189                      | -0.435                        | -0.458                        | -0.530                      | -3.220          |
| JPN  | -0.235                     | -0.244                        | -0.246                      | -0.672                        | -0.710                        | -0.847                      | -1.519          |
| IND  | -0.076                     | -0.075                        | -0.073                      | -0.113                        | -0.111                        | -0.084                      | -0.686          |
| CHN  | -0.076                     | -0.076                        | -0.075                      | 0.571                         | 0.558                         | 0.438                       | -0.297          |
| ITA  | -0.125                     | -0.126                        | -0.117                      | -0.063                        | -0.042                        | 0.008                       | -1.705          |
| CAN  | -4.857                     | -4.787                        | -5.262                      | -0.555                        | -0.642                        | -0.845                      | -5.563          |
| RUS  | -0.014                     | -0.018                        | -0.021                      | -0.838                        | -1.190                        | -1.576                      | -2.914          |
| IDN  | -0.165                     | -0.191                        | -0.221                      | -0.352                        | -0.382                        | -0.388                      | -2.352          |
| MEX  | -4.829                     | -4.657                        | -5.279                      | -0.145                        | -0.140                        | -0.102                      | -5.424          |

# Welfare Changes

- Under the optimal policy, Home's welfare gains are greater than those using the endogenous *bilateral* trade share (overlooking cross-elasticities)
- Welfare gains are much lower when using "*observed* bilateral trade share", which shuts down the GE effect

# Conclusion

- Study optimal policies in a set of GE trade models
- Optimal policies and market powers
  - Explicit formula shows optimal tariffs depend on Home's endogenous buyer power
  - Interdependencies across sectors and countries affect the endogenous export supply curves and Home's market power, thereby influencing the interdependencies of optimal policies
- The explicit formulas enhance theoretical clarity and streamline the computation

# Appendix

# Model

A multi-country multi-sector model with a generic supply system

- $N$  countries and  $\bar{L}_n$  labor in country  $n$ ;  $J$  sectors
- Consumer preference in each country:  $C_n = \prod_{j \in J} (C_{n,j})^{\beta_{n,j}}$
- All goods are tradable, subject to bilateral iceberg trade costs  $d_{ni}$
- Sectoral level trade elasticity  $\epsilon_j$

# Generic Supply System

## ① Labor market specification $\Omega(\{w_{n,j}, L_{n,j}\})$

Example: Ricardo-Roy (imperfectly substitutable labor)

$$\Omega = \left\{ (w_{n,j}, L_{n,j}) : w_{n,j} L_{n,j} = \sum_{g=1}^G \left[ \frac{A_{n,j,g} (w_{n,j})^\kappa}{W_{n,g}^\kappa} W_{n,g} \bar{L}_{n,g} \right], W_{n,g} = \left[ \sum_{s=1}^J A_{n,s,g} (w_{n,s})^\kappa \right]^{\frac{1}{\kappa}} \right\}$$

Example: Perfectly mobile labor:  $\Omega = \{(w_{n,j}, L_{n,j}) : \sum_j L_{n,j} = \bar{L}_n, w_{n,j} = w_n\}$

## ② Supply-side assumption $\mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$

Example: generalized multi-sector Krugman model:

$$\mathbb{S} = \left\{ (p_{n,j}, w_{n,j}, L_{n,j}) : p_{n,j} = \bar{T}_{n,j}^{-1} w_{n,j} L_{n,j}^{1/(1-\sigma_j)} \right\}$$

# Unilateral Policies

- Home country (country 1) policy choices:
  - country-sector-specific tariffs  $\{\tau_{n,j}^m\}$
  - country-sector-specific export taxes  $\{\tau_{n,j}^x\}$
  - sector-specific domestic taxes  $\{\tau_j^d\}$
- Foreign governments are assumed to be passive

# World Market Equilibrium

Consists of  $\{L_{n,j}\}, \{C_n\}, \{x_n\}, \{P_n\}, \{p_{n,j}\}, \{w_{n,j}\}$  such that consumers maximize utility, firms maximize profits, market clearing conditions hold, taking as given Home government's policies  $\{\tau_{n,j}^m, \tau_{n,j}^x, \tau_j^d\}$ :

## 1 Expenditures

$$x_1 = \sum_{j=1}^J Y_{1,j} + \sum_{j=1}^J \sum_{i \neq 1}^N \beta_{i,j} \frac{\tau_{i,j}^x}{1 + \tau_{i,j}^x} \pi_{i1,j} x_i + \sum_{j=1}^J \beta_{1,j} \sum_{i \neq 1}^N \frac{\tau_{i,j}^m}{1 + \tau_{i,j}^m} \pi_{i1,j} x_1 + \sum_{j=1}^J \beta_{1,j} \frac{\tau_j^d}{1 + \tau_j^d} \pi_{11,j} x_1,$$

$$x_n = \sum_{j=1}^J Y_{n,j}, \quad n \neq 1$$

where  $Y_{n,j} = w_{n,j} L_{n,j}$  is the income in sector  $j$  country  $n$

## 2 Consumer prices

$$P_1 = \prod_{j=1}^J \left[ (p_{1,j}(1 + \tau_j^d))^{-\epsilon_j} + \sum_{i \neq 1}^N (p_{i,j}(1 + \tau_{i,j}^m) d_{1i})^{-\epsilon_j} \right]^{-\frac{\beta_{1,j}}{\epsilon_j}},$$

$$P_n = \prod_{j=1}^J \left[ (p_{1,j}(1 + \tau_{n,j}^x) d_{n1})^{-\epsilon_j} + \sum_{i \neq 1}^N (p_{i,j} d_{ni})^{-\epsilon_j} \right]^{-\frac{\beta_{n,j}}{\epsilon_j}}, \quad n \neq 1.$$

- ③ Trade shares satisfy, for each sector  $j$

$$\pi_{11,j} = \frac{(p_{1,j}(1 + \tau_j^d))^{-\epsilon_j}}{(p_{1,j}(1 + \tau_j^d))^{-\epsilon_j} + \sum_{i \neq 1}^N (p_{i,j}(1 + \tau_{i,j}^m)d_{1i})^{-\epsilon_j}},$$

$$\pi_{1n,j} = \frac{(p_{n,j}(1 + \tau_{n,j}^m)d_{1n})^{-\epsilon_j}}{(p_{1,j}(1 + \tau_j^d))^{-\epsilon_j} + \sum_{i \neq 1}^N (p_{i,j}(1 + \tau_{i,j}^m)d_{1i})^{-\epsilon_j}}, \quad n \neq 1,$$

$$\pi_{n1,j} = \frac{(p_{1,j}(1 + \tau_{n,j}^x)d_{n1})^{-\epsilon_j}}{(p_{1,j}(1 + \tau_{n,j}^x)d_{n1})^{-\epsilon_j} + \sum_{i \neq 1}^N (p_{i,j}d_{ni})^{-\epsilon_j}}, \quad n \neq 1,$$

$$\pi_{nm,j} = \frac{(p_{m,j}d_{nm})^{-\epsilon_j}}{(p_{1,j}(1 + \tau_{n,j}^x)d_{n1})^{-\epsilon_j} + \sum_{i \neq 1}^N (p_{i,j}d_{ni})^{-\epsilon_j}}, \quad n \neq 1.$$

- ④ Goods market clearing conditions, for each  $j$

$$\underbrace{Y_{1,j} - \sum_{i \neq 1}^N \beta_{i,j} \frac{1}{1 + \tau_{i,j}^x} \pi_{i1,j} x_i}_{E_{11,j}} = \beta_{1,j} \frac{1}{1 + \tau_j^d} \pi_{11,j} x_1, \quad (\gamma_{1,j})$$

$$\underbrace{Y_{n,j} - \sum_{i \neq 1}^N \beta_{i,j} \pi_{in,j} x_i}_{E_{1n,j}} = \beta_{1,j} \frac{1}{1 + \tau_{n,j}^m} \pi_{1n,j} x_1, \quad (\gamma_{n,j})$$

- ⑤ Supply-side assumptions  $\mathcal{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$  hold
- ⑥ Labor market specifications  $\Omega(\{w_{n,j}, L_{n,j}\})$  hold

# Home Government Problem

Home government's problem: choose country-sector-specific trade policies  $\{\tau_{n,j}^x, \tau_{n,j}^m\}$  and sector-specific domestic policies  $\{\tau_j^d\}$ , for  $\forall j, n \neq 1$ , to maximize domestic consumers' consumption

$$\max x_1/P_1$$

subject to the world market equilibrium conditions. [▶ Back](#)

## Two-Country Case: Home Government Optimal Policies

### Lemma (Optimal Policies and Multipliers)

*Irrespective of labor market specifications, optimal policies in sector  $j$  take the form of*

$$1 + \tau_j^d = -\gamma_{1,j}, \quad 1 + \tau_j^m = -\gamma_{2,j}, \quad 1 + \tau_j^x = \frac{1 + \tau_j^d}{1 + \tau_j^m} \left( 1 + \frac{1}{\epsilon_j \pi_{22,j}} \right)$$

- $\gamma_{1,j}$ : the Lagrange multipliers on the goods market clearing constraint for Home
- $\gamma_{2,j}$ : Foreign

# Imperfectly Substitutable Labor across Sector

- $L_n$ : total number of workers in country  $n$
- A worker in country  $n$  draws an efficiency unit  $z_j$  in sector  $j$  from a Fréchet distribution  $F_{n,j}$  with shape parameter  $\kappa$  and scale parameter  $A_{n,j}$
- A worker with  $\mathbf{z} = (z_1, z_2, \dots, z_J)$  chooses the sector that gives the highest income based on her productivity and wage in that sector
- $\Xi_{n,j}$ : set of workers choosing sector  $j$ ,  $\Xi_{n,j} \equiv \{\mathbf{z}: w_{n,j}z_j \geq w_{n,k}z_k \text{ for all } k\}$

# Imperfectly Substitutable Labor Across Sectors

- Share of workers that apply to sector  $j$  in country  $n$

$$\lambda_{n,j} = \frac{A_{n,j}(w_{n,j})^\kappa}{\sum_{s=1}^J A_{n,s}(w_{n,s})^\kappa}$$

- The sum of efficiency units supplied to sector  $j$  in country  $n$ :

$$L_{n,j} \equiv \bar{L}_n \int_{\Xi_{n,j}} z_j dF_n(\mathbf{z}) = \frac{\zeta [\sum_{s=1}^J A_{n,s}(w_{n,s})^\kappa]^{\frac{1}{\kappa}}}{w_{n,j}} \lambda_{n,j} \bar{L}_n, \text{ where } \zeta \equiv \Gamma(1 - 1/\kappa)$$

- Labor market clearing conditions:

$$\Omega = \left\{ (w_{n,j}, L_{n,j}) : w_{n,j} L_{n,j} = \frac{A_{n,j}(w_{n,j})^\kappa}{W_n^\kappa} W_n \bar{L}_n, W_n = \left[ \sum_{s=1}^J A_{n,s}(w_{n,s})^\kappa \right]^{\frac{1}{\kappa}} \right\}$$

The supply of sector  $j$  depends on the degree of labor substitutability across sectors

# Imperfectly Substitutable Labor across Sectors

Partial-supply-elasticity matrix  $\Lambda_n = \frac{\partial \ln Y_{n,i}}{\partial \ln w_{n,j}} \frac{Y_{n,i}}{Y_{n,j}}$ :

$$\Lambda_n = \begin{pmatrix} \kappa - (\kappa - 1)\lambda_{n,1} & \dots & -(\kappa - 1)\lambda_{n,j} & \dots & -(\kappa - 1)\lambda_{n,J} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa - 1)\lambda_{n,1} & \dots & \kappa - (\kappa - 1)\lambda_{n,j} & \dots & -(\kappa - 1)\lambda_{n,J} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa - 1)\lambda_{n,1} & \dots & -(\kappa - 1)\lambda_{n,j} & \dots & \kappa - (\kappa - 1)\lambda_{n,J} \end{pmatrix}$$

- Optimal tariff depends on Home's endogenous buyer power:

$$\tau_j^m - \tau_i^m = \frac{1}{\kappa} \left( \frac{\beta_{1,j}x_1 - Y_{1,j}}{Y_{2,j}} - \frac{\beta_{1,i}x_1 - Y_{1,i}}{Y_{2,i}} \right)$$

# Generalized Multi-sector Krugman

- With perfectly mobile labor, the labor market specification is  $w_{n,j} = w_n$  and  $\sum_j L_{n,j} = \bar{L}_n$
- The supply-side assumption is

$$p_{n,j} = \frac{w_n}{\bar{T}_{n,j}} L_{n,j}^{-\psi_j},$$

where  $\psi_j = 1/(\sigma_j - 1)$  is the scale elasticity

- This model is isomorphic to
  - Multi-sector Eaton-Kortum and Armington models with Marshallian economies of scale
  - Endogenous technology: steady-state of Bai, Jin, and Lu (2025) ▶ Endogenous T

# Generalized Multi-sector Krugman

The scaled partial supply elasticity matrix  $\Lambda_n$  of country  $n$

$$\Lambda_n = \begin{pmatrix} -\frac{1}{\psi_1} \left(1 - \frac{(1 + \frac{1}{\psi_1}) Y_{n,1}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}}\right) & \dots & \frac{1}{\psi_1} \frac{(1 + \frac{1}{\psi_j}) Y_{n,j}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}} & \dots & \frac{1}{\psi_1} \frac{(1 + \frac{1}{\psi_J}) Y_{n,J}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{1}{\psi_j} \frac{(1 + \frac{1}{\psi_1}) Y_{n,1}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}} & \dots & -\frac{1}{\psi_j} \left(1 - \frac{(1 + \frac{1}{\psi_j}) Y_{n,j}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}}\right) & \dots & \frac{1}{\psi_j} \frac{(1 + \frac{1}{\psi_J}) Y_{n,J}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{1}{\psi_J} \frac{(1 + \frac{1}{\psi_1}) Y_{n,1}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}} & \dots & \frac{1}{\psi_J} \frac{(1 + \frac{1}{\psi_j}) Y_{n,j}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}} & \dots & -\frac{1}{\psi_J} \left(1 - \frac{(1 + \frac{1}{\psi_J}) Y_{n,J}}{\sum_{s=1}^J \frac{1}{\psi_s} Y_{n,s}}\right) \end{pmatrix}.$$

Optimal policies

$$\frac{1 + \tau_j^d}{1 + \tau_k^d} = \frac{1 + \psi_k}{1 + \psi_j}, \quad \tau_j^m - \tau_i^m = -\psi_j \frac{\beta_{1,j} x_1 - Y_{1,j}}{Y_{2,j}} + \psi_i \frac{\beta_{1,i} x_1 - Y_{1,i}}{Y_{2,i}},$$

where  $\frac{1}{\eta_j} = -\psi_j$  [▶ Back](#)

# Differential Scale Elasticities and Imperfectly Mobile Labor

- With imperfectly mobile labor, the labor market specification is

$$\Omega = \left\{ (w_{n,j}, L_{n,j}) : w_{n,j}L_{n,j} = \frac{A_{n,j}(w_{n,j})^\kappa}{W_n^\kappa} W_n \bar{L}_n, W_n = \left[ \sum_{s=1}^J A_{n,s}(w_{n,s})^\kappa \right]^{\frac{1}{\kappa}} \right\}$$

- The supply-side assumption is

$$p_{n,j} = \frac{w_{n,j}}{\bar{T}_{n,j}} L_{n,j}^{-\psi_j},$$

where  $\psi_j$  is the scale elasticity

# Differential Scale Elasticities and Imperfectly Mobile Labor

The scaled partial supply elasticity matrix  $\Lambda_n$  of country  $n$

$$\Lambda_n = \begin{pmatrix} \frac{\kappa}{1-\psi_1(\kappa-1)} \left(1 + \frac{(1-\frac{\kappa}{1-\psi_1(\kappa-1)})Y_{n,1}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}}\right) & \cdots & \frac{\kappa}{1-\psi_1(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_j(\kappa-1)})Y_{n,j}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} & \cdots & \frac{\kappa}{1-\psi_1(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_J(\kappa-1)})Y_{n,J}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\kappa}{1-\psi_j(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_1(\kappa-1)})Y_{n,1}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} & \cdots & \frac{\kappa}{1-\psi_j(\kappa-1)} \left(1 + \frac{(1-\frac{\kappa}{1-\psi_j(\kappa-1)})Y_{n,j}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}}\right) & \cdots & \frac{\kappa}{1-\psi_j(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_J(\kappa-1)})Y_{n,J}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\kappa}{1-\psi_J(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_1(\kappa-1)})Y_{n,1}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} & \cdots & \frac{\kappa}{1-\psi_J(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_j(\kappa-1)})Y_{n,j}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} & \cdots & \frac{\kappa}{1-\psi_J(\kappa-1)} \left(1 + \frac{(1-\frac{\kappa}{1-\psi_J(\kappa-1)})Y_{n,J}}{\sum_{s=1}^J \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}}\right) \end{pmatrix}.$$

Optimal policies

$$\frac{1 + \tau_j^d}{1 + \tau_k^d} = \frac{1 + \psi_k}{1 + \psi_j},$$

$$\tau_j^m - \tau_i^m = \frac{1}{\eta_j} \frac{\beta_{1,j}x_1 - Y_{1,j}}{Y_{2,j}} - \frac{1}{\eta_i} \frac{\beta_{1,i}x_1 - Y_{1,i}}{Y_{2,i}},$$

where  $\frac{1}{\eta_j} = \frac{1}{\kappa} - \psi_j \frac{\kappa-1}{\kappa}$  [▶ Back](#)

# Endogenous Technology

- Technology:  $T_{n,j} \propto L_{n,j}^{v_j}$ 
  - $v_j$  governs the strength of economies of scale.
  - When  $v_j = 1$ , it returns to steady-state of Bai, Jin, and Lu (2025), who extend Eaton and Kortum (2001) endogenous technology to a multi-sector setting.
- Perfectly mobile labor
- Optimal tariffs

$$\tau_j^m - \tau_i^m = -\frac{v_j}{\epsilon_j} \left( \frac{\beta_{1,j}x_1 - Y_{1,j}}{Y_{2,j}} - \frac{\beta_{1,i}x_1 - Y_{1,i}}{Y_{2,i}} \right)$$

# Ricardo-Roy Models with Multi-factors

Galle, Rodríguez-Clare, and Yi (2023), which combines multi-sector EK and Roy model

- $G$  types of workers in each country,  $\bar{L}_{n,g}$
- A worker in type- $g$  of country  $n$  draws an efficiency unit  $z_j$  in sector  $j$  from a Fréchet distribution  $F_{n,j,g}$  with a shape parameter  $\kappa$  and a scale parameter  $A_{n,j,g}$
- A worker with  $\mathbf{z} = (z_1, z_2, \dots, z_J)$  chooses the sector that maximizes her income based on her productivity and wage in that sector
- $\Xi_{n,j}$ : set of workers choosing sector  $j$ ,  $\Xi_{n,j} \equiv \{\mathbf{z}: w_{n,j}z_j \geq w_{n,k}z_k \text{ for all } k\}$

## Ricardo-Roy Models with Multi-factors

- Share of workers in type- $g$  that enter sector  $j$  in country  $n$

$$\lambda_{n,j,g} = \frac{A_{n,j,g}(w_{n,j})^\kappa}{\sum_{s=1}^J A_{n,s,g}(w_{n,s})^\kappa}$$

- The sum of efficiency units supplied to sector  $j$  in country  $n$  by group  $g$ :

$$L_{n,j,g} \equiv L_{n,g} \int_{\Xi_{n,j}} z_j dF_{n,g}(\mathbf{z}) = \frac{\zeta [\sum_{s=1}^J A_{n,s,g}(w_{n,s})^\kappa]^{\frac{1}{\kappa}}}{w_{n,j}} \lambda_{n,j,g} \bar{L}_{n,g}, \text{ where } \zeta \equiv \Gamma(1 - 1/\kappa)$$

- $L_{n,j} = \sum_g L_{n,j,g}$ : total efficiency labor worked in sector  $j$

Labor market clearing conditions are

$$\Omega = \left\{ (w_{n,j}, L_{n,j}) : w_{n,j} L_{n,j} = \sum_{g=1}^G \left[ \frac{A_{n,j,g}(w_{n,j})^\kappa}{W_{n,g}^\kappa} W_{n,g} \bar{L}_{n,g} \right], W_{n,g} = \left[ \sum_{s=1}^J A_{n,s,g}(w_{n,s})^\kappa \right]^{\frac{1}{\kappa}} \right\}$$

# Ricardo-Roy Models with Multi-factors

Matrix of partial supply elasticity of Foreign country:

$$\Lambda_2 = \begin{pmatrix} \kappa - (\kappa - 1) \frac{\sum_g \lambda_{2,1,g} \lambda_{2,1,g} W_{2,g} \bar{L}_{2,g}}{Y_{21}} & \dots & -(\kappa - 1) \frac{\sum_g \lambda_{2,1,g} \lambda_{2,j,g} W_{2,g} \bar{L}_{2,g}}{Y_{21}} \\ \vdots & \vdots & \vdots \\ -(\kappa - 1) \frac{\sum_g \lambda_{2,j,g} \lambda_{2,1,g} W_{2,g} \bar{L}_{2,g}}{Y_{2j}} & \dots & -(\kappa - 1) \frac{\sum_g \lambda_{2,j,g} \lambda_{2,j,g} W_{2,g} \bar{L}_{2,g}}{Y_{2j}} \\ \vdots & \vdots & \vdots \\ -(\kappa - 1) \frac{\sum_g \lambda_{2,j,g} \lambda_{2,1,g} W_{2,g} \bar{L}_{2,g}}{Y_{2j}} & \dots & \kappa - (\kappa - 1) \frac{\sum_g \lambda_{2,j,g} \lambda_{2,j,g} W_{2,g} \bar{L}_{2,g}}{Y_{2j}} \end{pmatrix}$$

- $\lambda_{2,j,g} = Y_{2,j,g}/Y_{2,g}$ : share of type- $g$  workers that enter sector  $j$
- This does not satisfy the condition of the GIIA Restriction in Supply Space. Specifically, taking the difference between row  $i$  and  $j$  of  $\Lambda_2$  results in a vector  $d\Lambda_{2,ij}$  such that  $d\Lambda_{2,ij} \neq 0$

## Ricardo-Roy Models with Multi-factors

- Sector  $i$ 's  $w_i L_i$  is influenced by the wage in sector  $j$  as follows:

$$-(\kappa - 1) \frac{\sum_g \lambda_{2,j,g} \lambda_{2,i,g} W_{2,g} \bar{L}_{2,g}}{Y_{2,j}} = -(\kappa - 1) \sum_g \frac{\lambda_{2,j,g} Y_{2,g}}{Y_{2,j}} \lambda_{2,i,g}$$

- Depends on the covariance between what type of workers in sector  $j$ ,  $\frac{\lambda_{2,j,g} Y_{2,g}}{Y_{2,j}}$  and how likely they enter sector  $i$ ,  $\lambda_{2,i,g}$
- The covariance indicates how likely workers in type  $g$  within sector  $j$  apply to sector  $i$
- A higher covariance leads to higher elasticity and a greater impact of sector  $j$ 's wage on sector  $i$ 's income

# Proposition: Multi-Country Optimal Unilateral Policies

Irrespective of labor market and supply-side specifications, Home optimal policies satisfy

Domestic tax:

$$1 + \tau_j^d = -\gamma_{1,j}.$$

$\gamma_{1,j}$ : multiplier on goods market clearing condition of Home, sector  $j$

Import tariff:

$$1 + \tau_{n,j}^m = -\gamma_{n,j},$$

$\gamma_{n,j}$ : multiplier on the goods market clearing condition of country  $n$ , sector  $j$ ,

Export tax:

$$1 + \tau_{n,j}^x = (1 + \tau_j^d) \frac{1 + \epsilon_j(1 - \pi_{n1,j})}{\sum_{i \neq 1}^N (1 + \tau_{i,j}^m) \epsilon_j \pi_{ni,j}}.$$

## Diagonal Blocks of LHS

$$\frac{\partial E_{1n}}{\partial \ln w_n} = \begin{pmatrix} \frac{\partial E_{1n,1}}{\partial \ln w_{n,1}} & \dots & \frac{\partial E_{1n,J}}{\partial \ln w_{n,1}} \\ \dots & \dots & \dots \\ \frac{\partial E_{1n,1}}{\partial \ln w_{n,J}} & \dots & \frac{\partial E_{1n,J}}{\partial \ln w_{n,J}} \end{pmatrix} = \begin{pmatrix} Y_{n,1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & Y_{n,J} \end{pmatrix}$$

$$\begin{pmatrix} \kappa - (\kappa - 1)\lambda_{n,1} + \theta \sum_{h \neq 1}^N (1 - \pi_{hn,1})s_{hn,1} - \beta_{n,1}\pi_{nn,1} & \dots & -(\kappa - 1)\lambda_{n,J} - \beta_{n,J}\pi_{nn,J} \\ \vdots & \dots & \vdots \\ -(\kappa - 1)\lambda_{n,1} - \beta_{n,1}\pi_{nn,1} & \dots & \kappa - (\kappa - 1)\lambda_{n,J} + \theta \sum_{h \neq 1}^N (1 - \pi_{hn,J})s_{hn,J} - \beta_{n,J}\pi_{nn,J} \end{pmatrix}$$

where we define  $s_{hn,j} = \frac{\pi_{hn,j}\beta_{h,j}x_h}{Y_{n,j}}$  as the share of the exporter  $nj$ 's income from an importer  $h$ .

## Off-Diagonal Blocks of LHS

$$\begin{aligned}
 \frac{\partial E_{1i}}{\partial \ln w_n} &= \begin{pmatrix} \frac{\partial E_{1i,1}}{\partial \ln w_{n,1}} & \cdots & \frac{\partial E_{1i,J}}{\partial \ln w_{n,1}} \\ \cdots & & \cdots \\ \frac{\partial E_{1i,1}}{\partial \ln w_{n,J}} & \cdots & \frac{\partial E_{1i,J}}{\partial \ln w_{n,J}} \end{pmatrix} \\
 &= \begin{pmatrix} Y_{n,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Y_{n,J} \end{pmatrix} \begin{pmatrix} -\theta \sum_{h \neq 1}^N \pi_{hi,1} s_{hn,1} - \beta_{n,1} \pi_{ni,1} & \cdots & -\beta_{n,J} \pi_{ni,J} \\ \vdots & & \vdots \\ -\beta_{n,1} \pi_{ni,1} & \cdots & -\theta \sum_{h \neq 1}^N \pi_{hi,J} s_{hn,J} - \beta_{n,J} \pi_{ni,J} \end{pmatrix}
 \end{aligned}$$

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# Parameters

$\theta = 4, \kappa = 1.1, \bar{L}_n = 1, \forall n. T_1 = [50, \dots, 1], T_2 = [1, \dots, 50], T_3 = T_1 \circ r$ , where  $r$  represents values ranging from 2 to 0.5, consisting of 10 elements.  $A_{n,j} = 1, \forall n, j$ .

$$d = \begin{pmatrix} 1 & 1.01 & 2 \\ 2 & 1 & 1.01 \\ 1.01 & 1.01 & 1 \end{pmatrix}$$

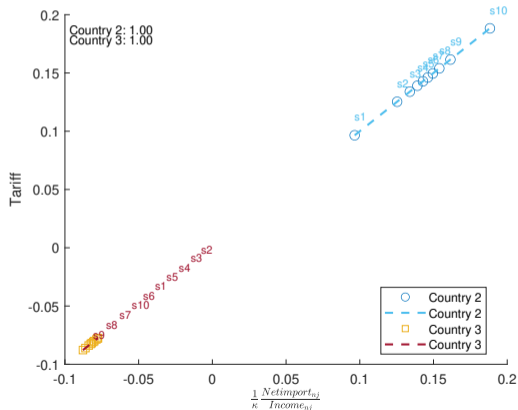
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# Home Treats Each Foreign Country Independently

Country 2 has a comparative advantage in s10 relative to Home

Country 3 has a comparative advantage in s1 relative to Home

Asymmetric trade costs



- Endogenous Bilateral Share: Home imposes policies

$$\tau_{n,j}^m = \frac{1}{\kappa} \frac{\text{Net import}_{n,j}}{\text{Income}_{n,j}}$$

$$1 + \tau_{n,j}^x = \frac{1}{1 + \tau_{n,j}^m} \left( 1 + \frac{1}{\epsilon_j(1 - \pi_{n1,j})} \right)$$

- Within a country  $n$ , the slope of tariff over  $\frac{1}{\kappa} \frac{\text{Net import}_{n,j}}{\text{Income}_{n,j}}$  across sectors is 1

# Effects of Interdependencies Between Foreign Countries

- Trade networks
  - generates larger self-elasticity
  - negative cross elasticities, resulting in smaller differential tariffs
  - these supply elasticities and imports are endogenous, response to Home's policies
- Welfare

| Parameters   | Optimal<br>(1) | Endogenous bilateral<br>trade share<br>(2) | Pre-tax bilateral<br>trade share<br>(3) |
|--|----------------|--|---|
| $N = 3, J = 10, d = \begin{pmatrix} 1 & 1.01 & 2 \\ 2 & 1 & 1.01 \\ 1.01 & 1.01 & 1 \end{pmatrix}$ | 3.31           | 2.51 (76%)                                 | -5.09                                   |

# Role of Heterogeneity and Interdependency

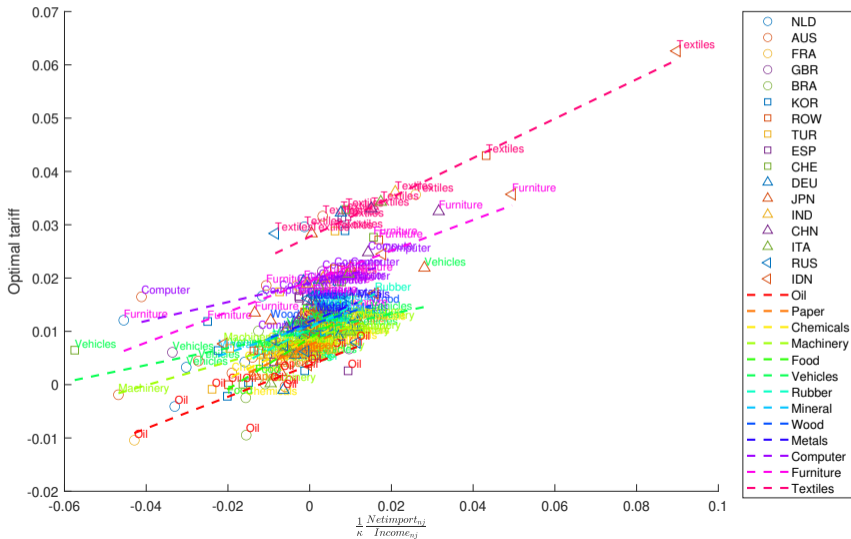
Table 1: Welfare Implications: Optimal versus Alternative Policies

| Parameters |  | Optimal | Endogenous<br>bilateral share | Pre-tax<br>bilateral share |
|------------|--|---------|-------------------------------|----------------------------|
|            |  | (1)     | (2)                           | (3)                        |
| Case 1     | $N = 2, J = 2, d = \begin{pmatrix} 1 & 1.2 \\ 1.2 & 1 \end{pmatrix}$                               | 4.85    | 4.85                          | 0.52                       |
| Case 1a    | $N = 2, J = 10, d = \begin{pmatrix} 1 & 1.2 \\ 1.2 & 1 \end{pmatrix}$                              | 2.89    | 2.89                          | 1.54                       |
| Case 2     | $N = 3, J = 10, d = \begin{pmatrix} 1 & 1.2 & 1.2 \\ 1.2 & 1 & 1.2 \\ 1.2 & 1.2 & 1 \end{pmatrix}$ | 2.67    | 2.61                          | 0.96                       |
| Case 3     | $N = 3, J = 10, d = \begin{pmatrix} 1 & 1.2 & 1.2 \\ 1.2 & 1 & 10 \\ 1.2 & 10 & 1 \end{pmatrix}$   | 4.84    | 4.84                          | -1.61                      |
| Case 4     | $N = 3, J = 10, d = \begin{pmatrix} 1 & 1.01 & 2 \\ 2 & 1 & 1.01 \\ 1.01 & 1.01 & 1 \end{pmatrix}$ | 3.31    | 2.51                          | -5.09                      |

# Quantitative Optimal Policies

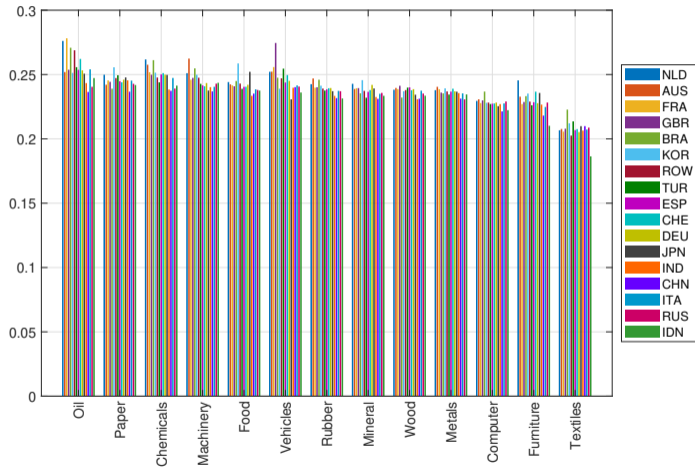
- Data (year 2014)
  - Domestic and international production and expenditure from the WIOD
  - Aggregate the 56 WIOD sectors  $\Rightarrow J = 13$  manufacturing sectors
  - Top 19 countries by GDP in 2014, based on rankings from the World Bank's WDI
- Parameters
  - $\theta = 4$ , from Simonovska and Waugh (2014)
  - Imperfectly substitutable labor with  $\eta = \kappa = 1.5$ , from Galle, Rodríguez-Clare, and Yi (2023)

# U.S. Unilateral Optimal Tariffs and Net Import Share over Foreign Income





# U.S. Optimal Policies: Export Tax



- Export taxes are higher in its exporting sectors

Table 3: U.S. Policies: Optimal versus Using Endogenous Bilateral Share versus Nash

| Sectors   | Optimal    |      |               |      | Endogenous Bilateral Share |      |               |      | Nash       |      |               |      |
|-----------|------------|------|---------------|------|----------------------------|------|---------------|------|------------|------|---------------|------|
|           | Tariff (%) |      | Export tax(%) |      | Tariff (%)                 |      | Export tax(%) |      | Tariff (%) |      | Export tax(%) |      |
|           | Mean       | Std  | Mean          | Std  | Mean                       | Std  | Mean          | Std  | Mean       | Std  | Mean          | Std  |
|           | (1)        | (2)  | (3)           | (4)  | (5)                        | (6)  | (7)           | (8)  | (9)        | (10) | (11)          | (12) |
| Oil       | 0.03       | 1.67 | 26.05         | 2.77 | -0.86                      | 2.04 | 27.24         | 3.37 | 0.07       | 1.49 | 25.92         | 2.73 |
| Paper     | 0.51       | 1.14 | 25.11         | 2.27 | -0.41                      | 1.52 | 26.33         | 2.74 | 0.43       | 1.30 | 25.05         | 2.43 |
| Chemicals | 0.68       | 1.15 | 25.61         | 2.28 | -0.52                      | 1.48 | 27.17         | 2.86 | 0.42       | 1.15 | 25.71         | 2.25 |
| Machinery | 1.72       | 3.59 | 25.71         | 3.68 | 0.08                       | 2.28 | 27.05         | 3.31 | 1.64       | 4.50 | 26.13         | 4.58 |
| Food      | 0.75       | 0.42 | 24.46         | 0.98 | -0.14                      | 0.56 | 25.59         | 1.14 | 0.59       | 0.53 | 24.53         | 0.83 |
| Vehicles  | 1.54       | 2.01 | 24.79         | 1.96 | 0.21                       | 3.11 | 26.51         | 3.46 | 1.14       | 1.64 | 24.77         | 2.11 |
| Rubber    | 1.35       | 1.49 | 24.25         | 1.26 | 0.28                       | 1.80 | 25.58         | 1.71 | 0.88       | 1.27 | 24.51         | 1.49 |
| Mineral   | 1.28       | 0.57 | 23.75         | 0.55 | 0.28                       | 1.00 | 25.02         | 1.14 | 0.88       | 0.54 | 24.05         | 0.68 |
| Wood      | 1.22       | 1.15 | 23.82         | 1.13 | 0.27                       | 1.91 | 25.10         | 1.81 | 0.98       | 1.46 | 23.85         | 1.92 |
| Metals    | 1.50       | 0.85 | 23.83         | 0.74 | 0.60                       | 1.36 | 24.94         | 0.95 | 1.16       | 1.03 | 23.85         | 1.10 |
| Computer  | 2.77       | 2.82 | 23.23         | 1.38 | 0.96                       | 3.79 | 25.24         | 2.72 | 2.01       | 2.52 | 23.78         | 1.12 |
| Furniture | 2.49       | 1.54 | 22.71         | 0.92 | 1.16                       | 2.85 | 24.53         | 3.00 | 1.89       | 1.50 | 23.03         | 1.74 |
| Textiles  | 4.17       | 2.58 | 20.62         | 1.00 | 2.47                       | 3.27 | 22.43         | 3.16 | 3.13       | 1.93 | 21.19         | 1.73 |
| Avg.      | 1.54       | 1.61 | 24.15         | 1.61 | 0.34                       | 2.07 | 25.60         | 2.41 | 1.17       | 1.60 | 24.34         | 1.90 |

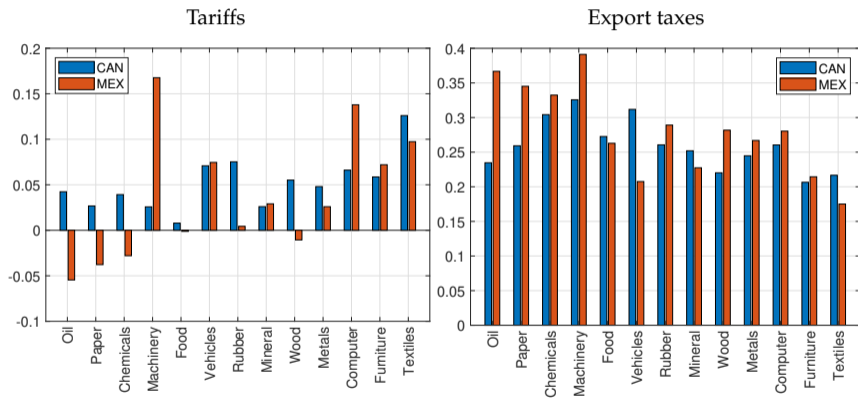
**Note:** This table summarizes the U.S. policies (mean and standard deviation) for 13 sectors under three scenarios: unilateral optimal policies, unilateral policies considering endogenous bilateral trade share, and Nash optimal policies. The sectors are ranked in ascending order based on the U.S. sectoral net imports relative to world sectoral income before the policy. The last row represents the average of the entire column.

Table 4: U.S. Policies: Optimal versus Using Endogenous Bilateral Share versus Nash

| Cty  | Optimal    |      |              |          |      | Endogenous Bilateral Share |      |              |          |      | Nash       |      |              |          |      |
|------|------------|------|--------------|----------|------|----------------------------|------|--------------|----------|------|------------|------|--------------|----------|------|
|      | Tariff (%) |      |              | Extax(%) |      | Tariff (%)                 |      |              | Extax(%) |      | Tariff (%) |      |              | Extax(%) |      |
|      | Mean       | Std  | <i>Slope</i> | Mean     | Std  | Mean                       | Std  | <i>Slope</i> | Mean     | Std  | Mean       | Std  | <i>Slope</i> | Mean     | Std  |
| (1)  | (2)        | (3)  | (4)          | (5)      | (6)  | (7)                        | (8)  | (9)          | (10)     | (11) | (12)       | (13) | (14)         | (15)     |      |
| NLD  | 0.85       | 0.77 | 0.18         | 24.45    | 1.56 | -1.13                      | 1.10 | 1            | 27.34    | 2.11 | 0.30       | 0.65 | 0.23         | 25.09    | 1.47 |
| AUS  | 0.94       | 0.85 | 0.16         | 24.19    | 1.33 | -0.89                      | 1.39 | 1            | 26.79    | 2.24 | 0.34       | 0.66 | 0.19         | 24.81    | 1.28 |
| FRA  | 1.00       | 1.00 | 0.60         | 24.12    | 1.62 | -0.18                      | 1.31 | 1            | 25.83    | 2.18 | 0.60       | 1.02 | 0.50         | 24.64    | 1.45 |
| GBR  | 1.22       | 0.71 | 0.22         | 24.11    | 1.47 | 0.18                       | 0.86 | 1            | 25.50    | 1.82 | 0.73       | 0.51 | 0.28         | 24.43    | 1.25 |
| BRA  | 0.75       | 0.79 | 0.61         | 24.30    | 1.26 | 0.02                       | 1.04 | 1            | 25.38    | 1.64 | 0.25       | 0.63 | 0.60         | 24.75    | 1.19 |
| KOR  | 0.94       | 0.79 | 0.42         | 24.18    | 1.21 | -0.60                      | 1.09 | 1            | 26.31    | 1.56 | 0.49       | 0.65 | 0.43         | 24.68    | 1.18 |
| ROW  | 1.32       | 1.09 | 0.70         | 23.94    | 1.45 | 0.15                       | 1.45 | 1            | 25.44    | 2.05 | 1.00       | 0.81 | 0.59         | 24.09    | 1.32 |
| TUR  | 1.19       | 0.72 | 0.53         | 23.81    | 1.14 | -0.32                      | 0.95 | 1            | 25.82    | 1.50 | 1.01       | 0.59 | 0.37         | 24.09    | 1.15 |
| ESP  | 1.05       | 0.72 | 0.44         | 23.76    | 1.13 | 0.07                       | 0.42 | 1            | 25.15    | 0.63 | 0.74       | 0.49 | 0.45         | 24.13    | 0.87 |
| CHE  | 1.24       | 0.86 | 0.25         | 23.95    | 1.22 | -0.17                      | 1.41 | 1            | 25.73    | 2.00 | 0.89       | 0.78 | 0.20         | 24.42    | 1.15 |
| DEU  | 1.15       | 0.79 | 0.84         | 23.83    | 1.18 | 0.09                       | 0.48 | 1            | 25.28    | 0.61 | 0.82       | 0.61 | 0.65         | 24.20    | 0.92 |
| JPN  | 1.22       | 0.77 | 0.42         | 23.74    | 1.11 | -0.03                      | 1.04 | 1            | 25.43    | 1.33 | 0.93       | 0.58 | 0.42         | 24.00    | 1.01 |
| IND  | 1.49       | 0.72 | 0.91         | 23.33    | 0.95 | 0.20                       | 0.65 | 1            | 24.92    | 0.89 | 1.43       | 0.59 | 0.74         | 23.36    | 0.95 |
| CHN  | 1.74       | 0.74 | 0.69         | 23.05    | 0.85 | 0.40                       | 0.97 | 1            | 24.66    | 1.14 | 1.50       | 0.49 | 0.60         | 23.25    | 0.72 |
| ITA  | 1.29       | 0.79 | 0.87         | 23.63    | 1.12 | 0.30                       | 0.73 | 1            | 24.86    | 1.00 | 0.99       | 0.58 | 0.62         | 23.93    | 0.97 |
| CAN  | 5.14       | 2.89 | 0.32         | 25.92    | 3.55 | 3.94                       | 3.08 | 1            | 26.16    | 5.40 | 4.19       | 1.94 | 0.23         | 24.09    | 3.83 |
| RUS  | 1.26       | 0.56 | -0.16        | 23.45    | 0.88 | 0.07                       | 0.61 | 1            | 25.02    | 0.80 | 1.17       | 0.32 | 0.08         | 23.58    | 0.62 |
| IDN  | 1.81       | 1.50 | 0.54         | 23.07    | 1.58 | 1.16                       | 2.51 | 1            | 23.80    | 3.01 | 1.87       | 1.51 | 0.45         | 22.83    | 2.06 |
| MEX  | 3.67       | 6.60 | 0.68         | 28.01    | 6.25 | 3.13                       | 7.10 | 1            | 26.93    | 8.10 | 2.97       | 7.03 | 0.65         | 27.99    | 7.64 |
| Avg. | 1.54       | 1.25 | 0.49         | 24.15    | 1.62 | 0.34                       | 1.48 | 1            | 25.60    | 2.11 | 1.17       | 1.08 | 0.44         | 24.34    | 1.63 |

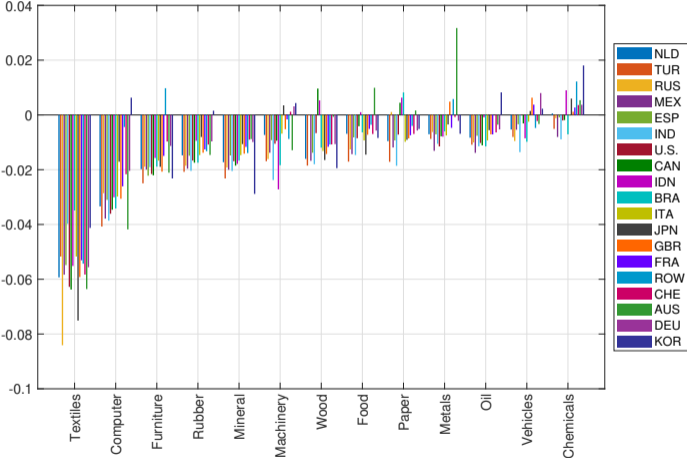
**Note:** This table summarizes the U.S. import tariffs (mean, standard deviation, and *tariff-import slope*) and export taxes (mean and standard deviation) for the other 19 countries under three scenarios: unilateral

# U.S. Optimal Policies on Canada and Mexico



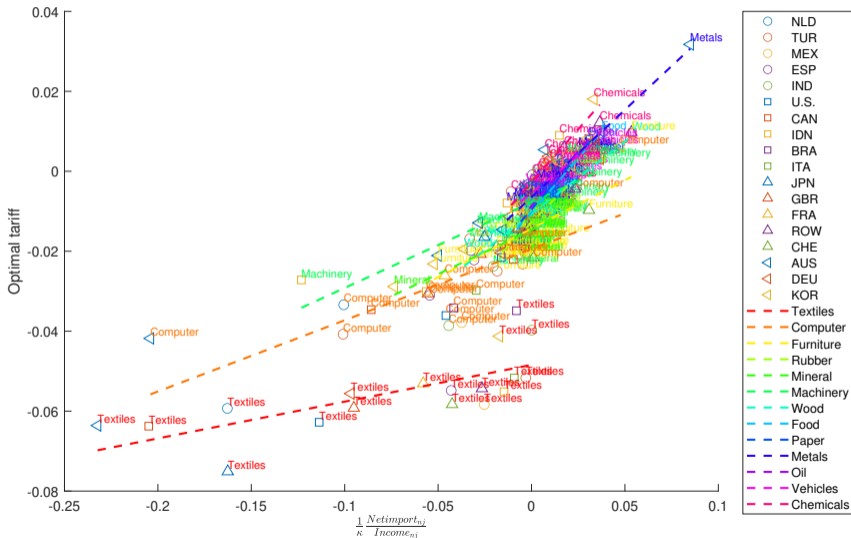
**Note:** Sectors are ordered by U.S. net exports relative to world output before policy.

# China's Optimal Policies: Import Tariffs



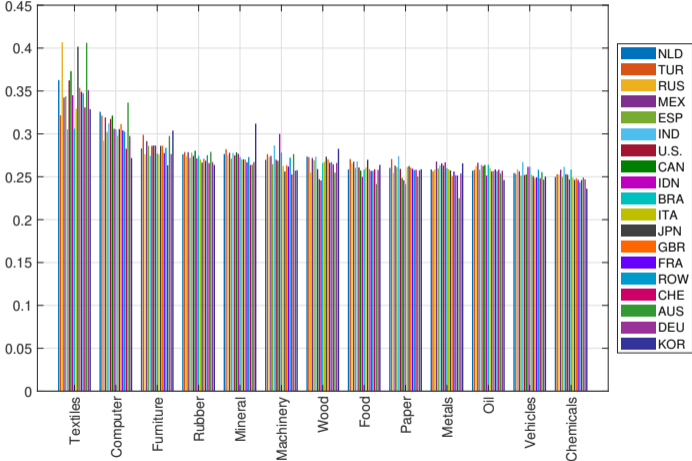
**Note:** Sectors are ordered by China's net exports relative to world output before policy.

# China's Unilateral Optimal Tariffs and Net Import Share





# China's Optimal Policies: Export Tax



**Note:** Sectors are ordered by China's net exports relative to world output before policy.

Table 5: China's Policies: Optimal versus Using Endogenous Bilateral Share versus Nash

| Sectors   | Optimal    |      |               |      | Endogenous Bilateral Share |      |               |       | Nash       |      |               |      |
|-----------|------------|------|---------------|------|----------------------------|------|---------------|-------|------------|------|---------------|------|
|           | Tariff (%) |      | Export tax(%) |      | Tariff (%)                 |      | Export tax(%) |       | Tariff (%) |      | Export tax(%) |      |
|           | Mean       | Std  | Mean          | Std  | Mean                       | Std  | Mean          | Std   | Mean       | Std  | Mean          | Std  |
|           | (1)        | (2)  | (3)           | (4)  | (5)                        | (6)  | (7)           | (8)   | (9)        | (10) | (11)          | (12) |
| Textiles  | -5.67      | 1.10 | 35.09         | 2.90 | -6.92                      | 6.27 | 37.64         | 11.30 | -4.95      | 1.34 | 34.91         | 3.80 |
| Computer  | -2.79      | 1.20 | 30.70         | 1.49 | -3.52                      | 3.52 | 32.42         | 5.23  | -2.35      | 1.36 | 30.35         | 2.32 |
| Furniture | -1.74      | 0.74 | 28.41         | 0.93 | -0.95                      | 1.97 | 27.75         | 2.74  | -1.22      | 0.73 | 27.60         | 1.09 |
| Rubber    | -1.40      | 0.51 | 27.29         | 0.48 | -0.19                      | 0.82 | 25.77         | 1.11  | -1.07      | 0.58 | 26.71         | 0.65 |
| Mineral   | -1.61      | 0.50 | 27.48         | 1.00 | -0.47                      | 1.27 | 26.07         | 2.24  | -1.29      | 0.70 | 27.02         | 1.47 |
| Machinery | -0.93      | 0.87 | 26.96         | 1.11 | -0.30                      | 2.67 | 26.47         | 3.91  | -0.59      | 1.07 | 26.39         | 1.72 |
| Wood      | -1.02      | 0.80 | 26.56         | 0.92 | 0.14                       | 1.88 | 25.22         | 2.46  | -0.65      | 1.16 | 25.86         | 1.70 |
| Food      | -0.75      | 0.60 | 26.01         | 0.67 | 0.55                       | 0.93 | 24.42         | 1.16  | -0.62      | 0.85 | 25.68         | 1.13 |
| Paper     | -0.57      | 0.71 | 25.82         | 0.75 | 0.73                       | 1.21 | 24.25         | 1.46  | -0.40      | 0.95 | 25.34         | 1.29 |
| Metals    | -0.36      | 0.96 | 25.74         | 0.91 | 0.69                       | 1.93 | 24.57         | 2.25  | -0.23      | 1.41 | 25.25         | 1.86 |
| Oil       | -0.69      | 0.50 | 25.85         | 0.47 | 0.36                       | 0.77 | 24.62         | 0.89  | -0.66      | 0.55 | 25.70         | 0.57 |
| Vehicles  | -0.30      | 0.55 | 25.42         | 0.52 | 0.68                       | 1.27 | 24.40         | 1.60  | -0.17      | 0.73 | 25.04         | 0.85 |
| Chemicals | 0.14       | 0.67 | 24.99         | 0.56 | 0.87                       | 1.12 | 24.28         | 1.33  | -0.02      | 0.76 | 24.94         | 0.84 |
| Avg.      | -1.36      | 0.75 | 27.41         | 0.98 | -0.64                      | 1.97 | 26.76         | 2.90  | -1.09      | 0.94 | 26.98         | 1.48 |

**Note:** This table summarizes China's policies (mean and standard deviation) for 13 sectors under three scenarios: unilateral optimal policies, unilateral policies considering endogenous bilateral trade share, and Nash optimal policies. The sectors are ranked in ascending order based on China's sectoral net imports relative to world sectoral income before the policy. The last row represents the average of the entire column.

Table 6: China's Policies: Optimal versus Using Endogenous Bilateral Share versus Nash

| Cty  | Optimal    |      |              |          |      | Endogenous Bilateral Share |      |              |          |       | Nash       |      |              |          |      |
|------|------------|------|--------------|----------|------|----------------------------|------|--------------|----------|-------|------------|------|--------------|----------|------|
|      | Tariff (%) |      |              | Extax(%) |      | Tariff (%)                 |      |              | Extax(%) |       | Tariff (%) |      |              | Extax(%) |      |
|      | Mean       | Std  | <i>Slope</i> | Mean     | Std  | Mean                       | Std  | <i>Slope</i> | Mean     | Std   | Mean       | Std  | <i>Slope</i> | Mean     | Std  |
| (1)  | (2)        | (3)  | (4)          | (5)      | (6)  | (7)                        | (8)  | (9)          | (10)     | (11)  | (12)       | (13) | (14)         | (15)     |      |
| NLD  | -1.57      | 1.50 | 0.31         | 27.74    | 3.09 | -2.03                      | 3.48 | 1            | 28.89    | 6.01  | -1.53      | 1.52 | 0.24         | 27.68    | 3.26 |
| TUR  | -2.03      | 1.26 | 0.20         | 27.80    | 2.23 | -1.31                      | 1.99 | 1            | 27.58    | 3.63  | -2.10      | 1.07 | 0.20         | 27.80    | 2.11 |
| RUS  | -1.72      | 2.10 | 0.12         | 27.73    | 3.89 | -1.67                      | 6.88 | 1            | 28.58    | 12.46 | -1.42      | 2.06 | 0.12         | 27.21    | 4.64 |
| MEX  | -1.95      | 1.35 | 0.74         | 27.96    | 2.40 | -0.93                      | 1.16 | 1            | 26.99    | 2.40  | -1.78      | 1.13 | 0.09         | 27.85    | 2.24 |
| ESP  | -1.52      | 1.38 | 0.64         | 27.30    | 2.45 | -0.98                      | 1.68 | 1            | 26.89    | 2.93  | -1.34      | 1.17 | 0.59         | 27.08    | 2.37 |
| IND  | -1.98      | 0.93 | 0.44         | 27.73    | 1.51 | -0.66                      | 1.11 | 1            | 26.28    | 1.95  | -2.09      | 0.65 | 0.43         | 27.84    | 1.32 |
| U.S. | -1.62      | 1.61 | 0.48         | 27.67    | 2.97 | -0.85                      | 2.82 | 1            | 26.90    | 4.77  | -1.33      | 1.37 | 0.43         | 27.22    | 3.08 |
| CAN  | -1.38      | 1.82 | 0.28         | 27.64    | 3.40 | -0.91                      | 4.52 | 1            | 27.30    | 7.34  | -0.86      | 2.00 | 0.24         | 26.86    | 4.14 |
| IDN  | -1.16      | 1.72 | 0.25         | 27.33    | 2.85 | -0.44                      | 3.04 | 1            | 26.73    | 4.82  | -1.06      | 1.56 | 0.26         | 26.97    | 2.99 |
| BRA  | -1.42      | 1.11 | 0.61         | 27.10    | 1.75 | -0.25                      | 1.47 | 1            | 25.74    | 2.26  | -0.95      | 0.88 | 0.60         | 26.40    | 1.62 |
| ITA  | -1.43      | 1.29 | 0.87         | 27.05    | 2.06 | -0.16                      | 0.94 | 1            | 25.63    | 1.59  | -1.23      | 1.04 | 0.70         | 26.75    | 1.91 |
| JPN  | -1.29      | 1.95 | 0.41         | 27.71    | 3.90 | -0.62                      | 3.60 | 1            | 27.12    | 6.36  | -0.98      | 1.83 | 0.40         | 27.15    | 4.30 |
| GBR  | -1.30      | 1.65 | 0.48         | 27.31    | 2.85 | -0.57                      | 2.93 | 1            | 26.60    | 4.75  | -0.91      | 1.48 | 0.43         | 26.77    | 2.98 |
| FRA  | -1.15      | 1.42 | 0.56         | 27.09    | 2.65 | -0.42                      | 2.12 | 1            | 26.28    | 3.63  | -0.89      | 1.43 | 0.37         | 26.81    | 2.88 |
| ROW  | -0.69      | 1.58 | 0.64         | 27.28    | 2.61 | 0.90                       | 1.89 | 1            | 25.10    | 2.61  | -0.56      | 1.23 | 0.57         | 26.53    | 2.25 |
| CHE  | -1.07      | 1.51 | 0.73         | 26.50    | 2.10 | 0.34                       | 1.69 | 1            | 24.84    | 2.33  | -0.59      | 1.34 | 0.47         | 26.01    | 2.03 |
| AUS  | -0.94      | 2.27 | 0.25         | 27.61    | 4.62 | -1.48                      | 5.79 | 1            | 28.73    | 9.74  | -0.27      | 2.67 | 0.28         | 26.73    | 5.80 |
| DEU  | -0.94      | 1.52 | 0.48         | 26.94    | 2.68 | -0.10                      | 2.69 | 1            | 25.93    | 4.29  | -0.59      | 1.48 | 0.32         | 26.46    | 2.91 |
| KOR  | -0.71      | 1.62 | 0.39         | 27.24    | 2.62 | -0.01                      | 2.88 | 1            | 26.35    | 4.25  | -0.32      | 1.47 | 0.42         | 26.56    | 2.87 |
| Avg. | -1.36      | 1.56 | 0.47         | 27.41    | 2.77 | -0.64                      | 2.77 | 1            | 26.76    | 4.64  | -1.09      | 1.44 | 0.38         | 26.98    | 2.93 |

**Note:** This table summarizes China's tariffs (mean, standard deviation, and tariff-import slope) and export

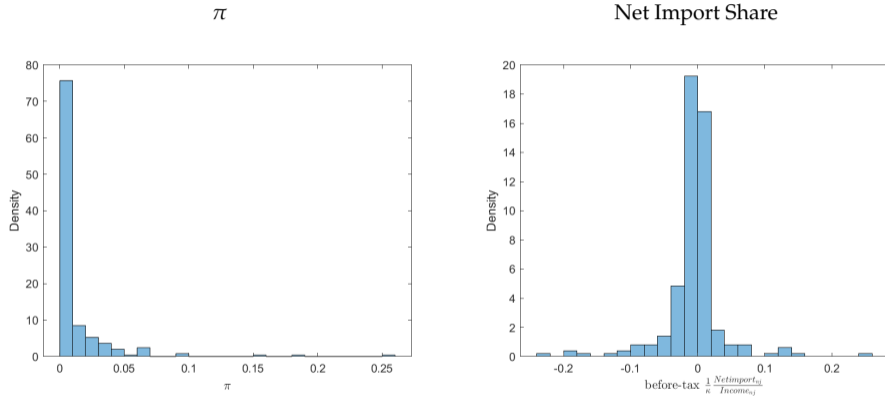
# Quantify Trade Network Effect

Alternative Policies: Using Bilateral Trade Shares

$$\tau_{n,j}^m - \tau_{n,i}^m = \frac{1}{\kappa} \left[ \frac{\pi_{1n,j} \beta_{1,j} x_1 - \frac{1}{1+\tau_{n,j}^x} \pi_{n1,j} \beta_{n,j} x_n}{Y_{n,j}} - \frac{\pi_{1n,i} \beta_{1,i} x_1 - \frac{1}{1+\tau_{n,i}^x} \pi_{n1,i} \beta_{n,i} x_n}{Y_{n,i}} \right]$$
$$1 + \tau_{n,j}^x = \frac{1}{1 + \tau_{n,j}^m} \left( 1 + \frac{1}{\epsilon_j (1 - \pi_{n1,j})} \right)$$

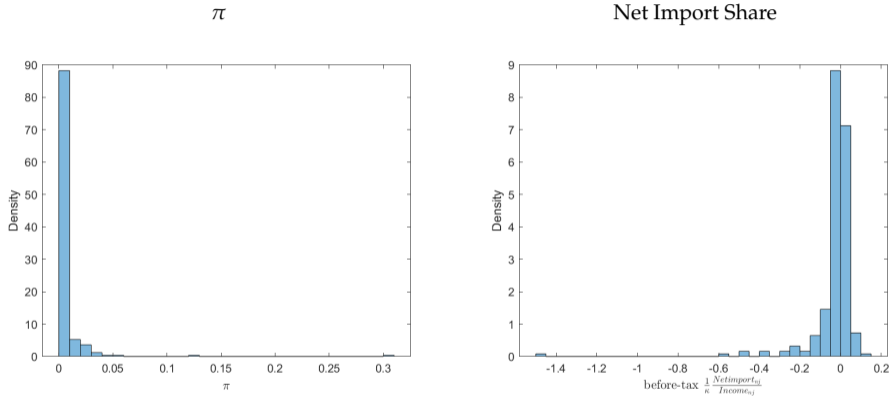
- **Endogenous bilateral shares** under these policies
  - Not optimal formula, but with GE effect
- **Observed bilateral shares** as sufficient statistics
  - Not optimal formula, and missing GE effects

# Distribution of U.S. Trade and Bilateral Net Import Share



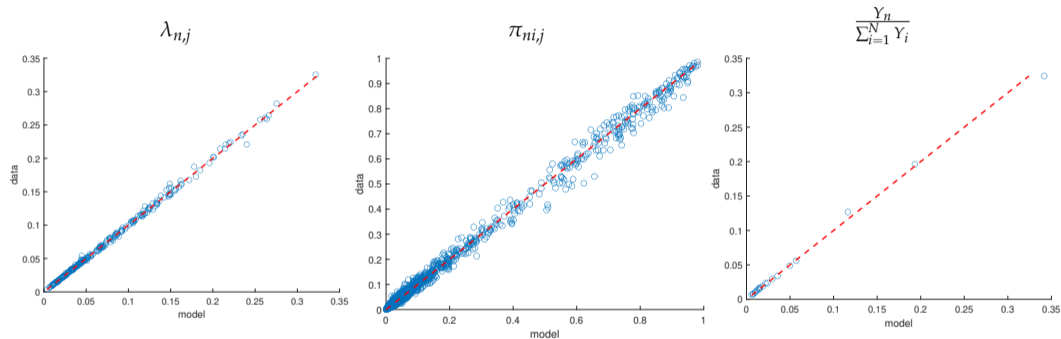
**Note:** Panel (a) shows the distribution of U.S. before-tax import share for all country-sector. Panel (b) shows the distribution of U.S. before-tax net import share in foreign income, normalized by  $\kappa$ , for all country-sector.

# Distribution of China Trade and Bilateral Net Import Share



**Note:** Panel (a) shows the distribution of China's before-tax import share for all country-sector. Panel (b) shows the distribution of China's before-tax net import share in foreign income, normalized by  $\kappa$ , for all country-sector.

# Model and Data Comparison



**Note:** This figure shows the relationship between the data and the model's income share, trade share, and the income of a country in global income. The red dashed line represents the 45-degree line.

# Sectors

Table 2: List of manufacturing sectors in the World Input-Output Database

| Sector Code | Short Name | Description   |
|-------------|------------|---|
| 1           | Food       | Food, Beverages and Tobacco   |
| 2           | Textiles   | Textiles, wearing apparel and leather products                                  |
| 3           | Wood       | Wood and of products of wood and cork; articles of straw and plaiting materials |
| 4           | Paper      | Paper and paper products, Printing and reproduction of recorded media           |
| 5           | Oil        | Coke and refined petroleum products   |
| 6           | Chemicals  | Chemicals and chemical products   |
| 7           | Rubber     | Rubber and plastics   |
| 8           | Mineral    | Other Non-metallic mineral  |
| 9           | Metals     | Basic metals and fabricated metal   |
| 10          | Computer   | Computer, electronic, optical products and electrical equipment                 |
| 11          | Machinery  | Machinery and equipment n.e.c.  |
| 12          | Vehicles   | Vehicles and other transport equipment  |
| 13          | Furniture  | Furniture and other manufacturing   |

# Counterfactual Equilibrium

The labor market clearing conditions become

$$\lambda'_{n,j} = \frac{\lambda_{n,j} \hat{w}_{n,j}^\kappa}{\sum_k \lambda_{n,k} \hat{w}_{n,k}^\kappa}$$
$$W'_n = W_n \left( \sum_k \lambda_{n,k} \hat{w}_{n,k}^\kappa \right)^{\frac{1}{\kappa}}$$

Thus,

$$W'_n \bar{L}_n = W_n \bar{L}_n \left( \sum_k \lambda_{n,k} \hat{w}_{n,k}^\kappa \right)^{\frac{1}{\kappa}}$$
$$\Rightarrow \frac{w'_{n,j} L'_{n,j}}{\lambda'_{n,j}} = \frac{w_{n,j} L_{n,j}}{\lambda_{n,j}} \left( \sum_k \lambda_{n,k} \hat{w}_{n,k}^\kappa \right)^{\frac{1}{\kappa}}$$
$$\Rightarrow \hat{L}_{n,j} = \frac{\hat{w}_{n,j}^{\kappa-1}}{\left( \sum_k \lambda_{n,k} \hat{w}_{n,k}^\kappa \right)^{\frac{\kappa-1}{\kappa}}}$$

# Multi-Country Nash Optimal Policy Formula

Given the policies of countries  $n \neq 1$ , the optimal policy of country 1's government satisfies, for any country  $n \neq 1$  and sector  $\forall j$ ,

$$\text{Import tariff: } 1 + \tau_{1n,j}^m = \frac{(\gamma_{xn} - \gamma_{n,j}) \frac{1}{1 + \tau_{1n,j}^x} - \gamma_{xn}}{u_{c1} / P_1},$$

Export tax:

$$\frac{1}{1 + \tau_{n1,j}^x} = \frac{\sum_{i \neq 1, n}^N (\gamma_{xi} - \gamma_{i,j}) \frac{1}{1 + \tau_{ni,j}^x} \frac{1}{1 + \tau_{ni,j}^m} \epsilon_j \pi_{ni,j} + (\gamma_{xn} - \gamma_{n,j}) \epsilon_j \pi_{nn,j} + \sum_{i \neq 1, n}^N (\gamma_{xn} - \gamma_{xi}) \frac{1}{1 + \tau_{ni,j}^m} \epsilon_j \pi_{ni,j} - \gamma_{xn} \frac{\epsilon_j}{1 + \tau_{n1,j}^m} (1 - \pi_{n1,j})}{\frac{u_{c1}}{P_1} \frac{1}{1 + \tau_{n1,j}^m} (1 + \epsilon_j (1 - \pi_{n1,j}))}.$$

► Back

# Perfectly Mobile Labor across Sectors

- With mobile labor, we can sum over the labor market clearing conditions across sectors

▶ marketclearing

$$\underbrace{w_1 \bar{L}_1 - \sum_{j=1}^J \beta_{2,j} \frac{1}{1 + \tau_j^x} \pi_{21,j} x_2}_{E_{11}} = \sum_{j=1}^J \beta_{1,j} \frac{1}{1 + \tau_j^d} \pi_{11,j} x_1, \quad (\gamma_1)$$

$$\underbrace{w_2 \bar{L}_2 - \sum_{j=1}^J \beta_{2,j} \pi_{22,j} x_2}_{E_{12}} = \sum_{j=1}^J \beta_{1,j} \frac{1}{1 + \tau_j^m} \pi_{12,j} x_1, \quad (\gamma_2)$$

- Optimal policies
  - Domestic taxes and tariffs are not sector-specific

$$\tau_j^d = -\gamma_1 = 0, \quad \tau_j^m = -\gamma_2 = 0$$

- Export taxes:  $\tau_j^x = \frac{1}{\epsilon_j \pi_{22,j}}$

## Immobile/Sector-specific Labor

- Market clearing conditions

$$\underbrace{w_{1,j}\bar{L}_{1,j} - \beta_{2,j} \frac{1}{1 + \tau_j^x} \pi_{21,j} x_2}_{E_{11,j}} = \beta_{1,j} \frac{1}{1 + \tau_j^d} \pi_{11,j} x_1 \quad (\gamma_{1,j}, J)$$

$$\underbrace{w_{2,j}\bar{L}_{2,j} - \beta_{2,j} \pi_{22,j} x_2}_{E_{12,j}} = \beta_{1,j} \frac{1}{1 + \tau_j^m} \pi_{12,j} x_1 \quad (\gamma_{2,j}, J)$$

- Optimal policies

$$\tau_j^d = 0$$

$$\tau_j^m - \tau_i^m = \frac{\beta_{1,j} x_1 - w_{1,j} \bar{L}_{1,j}}{w_{2,j} \bar{L}_{2,j}} - \frac{\beta_{1,i} x_1 - w_{1,i} \bar{L}_{1,i}}{w_{2,i} \bar{L}_{2,i}}$$

$$1 + \tau_j^x = \frac{1}{1 + \tau_j^m} \left( 1 + \frac{1}{\epsilon_j \pi_{22,j}} \right)$$

## Dixit Optimal Policies

- Lagrangian:

$$L = W(U^1(c^1), U^1(c^1), \dots) + \pi \cdot \left( \sum_i e^i + \sum_j x^j + m - g - \sum_i c^i \right) \\ - \sum_j \phi_j F^j(x^j) - \gamma G(m^t)$$

$G(m^t)$  foreign offer curve

- Dixit Optimum tariffs

$$\frac{\pi^t - r^t}{r^t} = R'_m(m^t) \frac{m^t}{r^t}$$

( $m^t$  imports,  $r^t$  prices, hence inverse of export supply elasticities. But cannot apply this formula directly)