Distorted by Design: Size-Dependent Guarantees and Capital Misallocation

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Abstract

This paper studies the allocative and welfare consequences of government credit guarantees, focusing on Japan's uniquely large and persistent Credit Guarantee Scheme (CGS). Using a quasi-experimental design based on a 1999 policy reform that raised the paid-in capital thresholds for SME eligibility, I compare always-eligible and newly eligible firms to a control group of always-ineligible firms. I show that always-eligible firms expanded rapidly after the reform, while newly eligible firms exhibited little growth, consistent with strategic restraint to avoid crossing the eligibility cutoff. I also document that banks with lower equity ratios issue a disproportionately high share of guaranteed loans, indicating that bank fragility amplifies distortive effects. To quantify these patterns, I develop and calibrate a general equilibrium model in which size-dependent guarantees generate endogenous bunching and misallocation. The model maps reduced-form estimates into a structural borrowing cost wedge and computes counterfactual welfare effects. Results show that the guarantee program reduced welfare by 3% prior to the reform. Expanding the eligibility cutoff in 1999 further reduced welfare by 1%. These findings highlight the hidden costs of tying financial support to adjustable firm characteristics in settings with heterogeneous firm productivity and financial frictions.

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1 Introduction

Small and medium-sized enterprises (SMEs) constitute the backbone of most economies, representing over 90% of all businesses globally and employing more than half of the world's workforce (OECD, 2024). Despite their economic prominence, SMEs face disproportionately high financial constraints due to their limited collateral, short credit histories, and higher perceived risk by lenders. In response, governments across the world have adopted a variety of policy tools to promote SME development, with the goal of alleviating financing frictions and fostering inclusive economic growth.

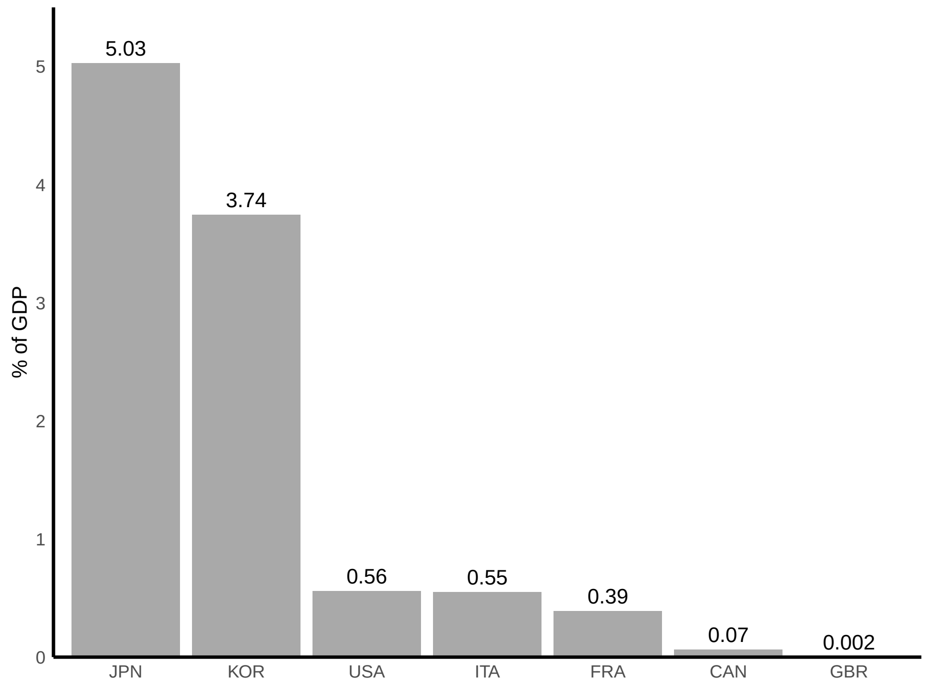
Credit guarantees have become an important public policy aimed at sustaining SME credit access, especially during episodes of macroeconomic stress. While most countries deploy such guarantees primarily in response to severe downturns, Japan is an outlier for its persistent and large-scale application of government-backed credit guarantees, even in normal economic conditions. Between 2011 and 2019, Japan's credit guarantees averaged nearly 5% of GDP—far exceeding the levels observed in other G7 countries, where the average remained below 0.6% (see Figure 1).[1]

This paper studies how size-dependent eligibility for public credit guarantees affects firm behavior, resource allocation, and welfare. In particular, it focuses on Japan's unique practice of defining SME status partly based on a firm's level of paid-in capital—a balance-sheet measure that can be strategically manipulated. Unlike employment or total assets, paid-in capital directly constrains a firm's ability to issue external equity. The capital-based eligibility rule introduces a direct and quantifiable distortion to firm financing choices. If firms bunch below the SME cutoff to remain eligible for credit subsidies, they may underinvest in productive inputs, leading to long-run misallocation.

To identify the impact of credit guarantee eligibility on firm outcomes, I exploit a major institutional reform: the 1999 revision of Japan's Basic Law on SMEs, which raised the capital thresholds for SME classification across industries. The unexpected reform generated discrete changes in eligibility for a well-defined set of firms, without directly altering underlying fundamentals. For

[1] Another country with relatively high credit guarantee coverage is South Korea. However, Japan's system differs significantly: it typically guarantees 80% or 100% of loan principal, whereas South Korea imposes a cap of 60%, implying greater risk-sharing by lenders such as banks or credit associations.

Figure 1: Government loan guarantees for SMEs, 2011–2019



Note: Country-level credit loan guarantee data from *Financing SMEs and Entrepreneurs 2024: An OECD Scoreboard*.

example, the paid-in capital ceiling for manufacturing SMEs rose from 100 million yen to 300 million yen, while the retail threshold increased from 10 to 50 million yen. These changes broadened access to guarantee subsidies for previously ineligible firms and created a natural experiment to test whether firms adjust their scale to maintain eligibility. Using a panel of over 427,000 firms from 1995 to 2022, I track how firm size and financing behavior evolved around the policy change.

The empirical analysis yields two main findings. First, I document persistent and sharp bunching in the distribution of paid-in capital just below the SME threshold—both before and after the reform—indicating strong behavioral responses to eligibility rules. Second, using an event-study difference-in-differences strategy, I show that always-eligible firms experienced rapid post-reform expansion relative to always-ineligible firms, reflecting the relaxation of a previously binding eligibility cutoff. Employment, physical capital, revenue, debt, and equity all increased following the reform. In contrast, newly eligible firms—those that gained SME status due to the reform—exhibited little change in real outcomes relative to always-ineligible firms, suggesting strategic restraint in response to the higher eligibility threshold. Together, these patterns indicate that firm behavior responds sharply to size-dependent guarantee rules rather than to intrinsic firm characteristics.

To interpret these findings and quantify aggregate welfare implications, I develop a general equilibrium model of entrepreneurial entry and firm investment with endogenous capital choice. Entrepreneurs differ in productivity and select their paid-in capital level before choosing scale subject to an exogenous leverage cap. A size-dependent guarantee program offers both a lump-sum subsidy and an interest rate discount for firms that remain below the statutory threshold. This institutional design induces bunching among intermediate-productivity firms that prefer to cap their growth rather than lose access to subsidized credit. By solving the firm's two-stage optimization problem and embedding it in a capital market clearing condition, the model delivers closed-form productivity thresholds and a unique general equilibrium interest rate that varies with guarantee generosity.

A central feature of the model is that it delivers a tractable mapping between the reduced-form treatment effect on log asset and the structural policy wedge induced by the credit guarantee. I show that the estimated coefficient from the difference-in-differences regression, combined with the structural first-order conditions, identifies the effective interest subsidy. Using this mapping, I estimate that the program lowers the effective cost of capital by approximately 1 to 1.3 percent for eligible firms, depending on the sector. This estimated wedge, combined with the lump-sum incentives identified from the bunching patterns, allows me to calibrate the full model and simulate counterfactual regimes.

Welfare is computed for each statutory sector (manufacturing, wholesale, retail, services) under the actual policy and under a benchmark without size-dependent guarantees. Results show that the guarantee program reduced welfare by 3.1% of output in the pre-reform period. After the 1999 policy change, which expanded eligibility, the welfare loss increased to 4.1%. While the reform allowed previously constrained firms to expand, these partial-equilibrium gains were outweighed by general equilibrium crowding-out effects.

Together, the results demonstrate that public credit guarantees, while successful in improving SME credit access, can generate significant misallocation when eligibility is tied to adjustable firm characteristics. The Japanese experience illustrates how persistent and generous support, in the absence of design safeguards, can entrench distorted firm behavior and limit the growth of productive enterprises. The paper highlights the importance of evaluating industrial policy not just by its reach, but by its long-run efficiency consequences. When eligibility thresholds become focal constraints in firm decision-making, well-intentioned support can produce sizable losses in aggregate output.

The remainder of the paper is structured as follows. Section 2 describes related literature. Section 3 introduces the theoretical framework of firm investment under size-dependent guarantees. Section 4 outlines the institutional background and presents the firm-level data. Section 5 provides empirical evidence on firm bunching and the effect of the 1999 SME reform. Section 6 calibrates the general equilibrium model, estimates policy wedges, and quantifies the welfare impact of the guarantee program. Section 7 concludes.

2 Related Literature

This paper contributes to three strands of literature. First, it builds on the growing literature examining the economic impact of government credit guarantee programs.[2] Evidence from Italy (Zecchini and Ventura, 2009), Japan (Uesugi et al., 2010; Ono et al., 2013), South Korea (Kang et al., 2008), the UK (Cowling, 2010), and the United States (Elenev et al., 2022) shows that CGSs have been effective in enhancing SME credit access, particularly during economic downturns. However, studies by Gropp et al. (2014), Elenev et al. (2016), and Tsuruta (2023) highlight the risk of moral hazard induced by such guarantees. Barrot et al. (2024), in the context of France, find that a crisis-era guarantee program reduced labor reallocation toward more productive firms. This paper contributes to the literature by providing direct evidence on how size-dependent credit guarantee program distorts capital allocation, particularly by inducing bunching in firms' paid-in capital around the SME threshold.

Second, this study relates to the literature on distortions created by size–dependent policies and bunching. A closely related paper is Garicano et al. (2016), which examines the welfare loss from bunching in response to France's employment size–based tax rules. Within this employment–size framework, the literature has explored firm responses through employment composition (Sollaci, 2018), occupational sorting (López and Torres, 2020), and entrepreneurial risk (Ando, 2021).[3] In

[2]This paper also relates to broader work on public credit interventions. Jiménez et al. (2018) study a targeted credit facility by a Spanish state-owned bank during the 2008–2009 crisis and find positive social value despite attracting riskier borrowers. Joaquim et al. (2022) analyze a large-scale lending expansion by Brazilian government-owned banks and document increased firm leverage and default with limited employment growth.

[3]Kleven (2016) provides a detailed review of the bunching literature.

the financial domain, Alvero et al. (2023) estimate the costs of bank regulation based on asset size. Distinct from these settings, this paper focuses on bunching in paid–in capital, a constraint that directly limits firms' ability to raise external financing. The 1999 reform serves as an exogenous policy shock, allowing for the identification of effects on both financial and real economic outcomes.

Third, this paper contributes to the broader literature on how policies and frictions contribute to misallocation of production resources. Policy channels studied include capital liberalization (Bau and Matray, 2023), firm subsidies (Jo and Senga, 2019; Rotemberg, 2019; Hughes and Majerovitz, 2023), taxation (Kaymak and Schott, 2023), and trade liberalization (Bai et al., 2019). Additional sources of misallocation emphasized in the literature include adjustment costs (Asker et al., 2014), financial frictions (Buera et al., 2011; Midrigan and Xu, 2014; Moll, 2014; Gopinath et al., 2017; Bai et al., 2018), information frictions (David et al., 2016), and risk (David et al., 2022).[4] In the Japanese context, much of this literature focuses on zombie lending. Peek and Rosengren (2005) and Caballero et al. (2008) show that evergreening by Japanese banks suppressed market efficiency and hindered job creation. A key challenge in this literature is isolating the effect of a single policy or friction from broader structural distortions. To address this, Sraer and Thesmar (2023) and Hughes and Majerovitz (2023) propose methods that map micro-level variation from quasi-experiments into macroeconomic misallocation measures. This paper contributes to this literature by focusing on a well-defined, size-dependent industrial policy: the SME credit guarantee program.

3 Theory

This section develops a static span–of–control economy in the spirit of Lucas (1978). Agents differ in productivity α, drawn from a distribution with density $f(\alpha)$ on $[\underline{\alpha}, \bar{\alpha}]$, with $\alpha f(\alpha)$ weakly decreasing. Agents can participate in the economy in one of two roles: as depositors, who supply funds to a competitive credit market at the equilibrium gross return r, or as entrepreneurs, who operate firms. Entrepreneurs choose paid–in capital (equity) E and debt D to finance assets $K = E + D$, and produce output using a decreasing–returns technology. The general equilibrium

[4] David and Venkateswaran (2019) develop a methodology for decomposing sources of capital misallocation, accounting for uncertainty, adjustment costs, and firm heterogeneity.

of the economy is governed by the allocation of financial resources between deposit supply and entrepreneurial asset demand.[5]

The policy environment is size dependent, following the organizing logic of Garicano et al. (2016), but the statutory threshold is *paid–in capital* rather than employment.[6] The key mechanism operates through financing incentives. A paid-in-capital threshold lowers the effective cost of capital for firms that remain below the cutoff, granting them access to both subsidized borrowing and a lump-sum transfer.

Environment

Technology. A firm operated by productivity type α with assets K produces[7]

$$Y = \alpha K^{\theta}, \qquad 0 < \theta < 1. \tag{1}$$

This formulation follows Whited and Zhao (2021) by treating financing resources—debt and equity—as direct inputs into production. The underlying idea is that both physical capital and labor ultimately require financing inputs.

Leverage cap. All borrowing is subject to a prudential leverage ceiling linked to paid–in capital:

$$D \;\leq\; (\bar{\lambda} - 1)\, E, \qquad \text{equivalently} \qquad K \in [E, \bar{\lambda} E], \qquad \bar{\lambda} \geq 1. \tag{2}$$

The knife–edge case $\bar{\lambda} = 1$ rules out debt finance, while $\bar{\lambda} > 1$ permits leverage. This leverage rule ties feasible firm scale directly to registered equity.

Guarantee program. Eligibility is determined by paid–in capital. If $E \leq \overline{E}$, the firm (i) borrows at a discounted interest rate σr with $0 < \sigma < 1$ and (ii) receives a lump–sum transfer $S \geq 0$. If

[5] In Lucas (1978), a span–of–control environment delivers an endogenous selection of managers versus workers. I use the same organizing idea—heterogeneous ability and a choice between operating a firm or supplying resources—but emphasize financial markets and credit allocation as the general–equilibrium backbone.

[6] Japan's SME Basic Act defines eligibility using paid-in capital and employment; My analysis focuses on the capital dimension that directly governs financing terms.

[7] An alternative formulation would endow agents with both labor and wealth and allow labor supply to enter the production function. I abstract from labor in order to isolate the effects of size-dependent financing incentives, so that paid-in capital maps directly into borrowing capacity and the cost of funds without confounding labor-input adjustments.

$E > \overline{E}$, the firm borrows at the market rate r and receives no transfer. A policy reform is modeled as an exogenous increase in the eligibility ceiling,

$$\overline{E}_0 \; \to \; \overline{E}_1 > \overline{E}_0, \tag{3}$$

and the economy is solved in each regime separately.

Financial market. Agents that do not enter as entrepreneurs act as depositors and supply funds to a competitive credit market at the gross return r. General equilibrium requires aggregate asset demand by firms to equal the supply of funds, with r adjusting to clear the market.

Firm optimization

An entrepreneur of type α makes two sequential choices: paid–in capital E (which controls eligibility and borrowing capacity), and then assets K (subject to the leverage cap).

Stage 0 (equity issuance). An entrant chooses paid–in capital $E \geq 0$ and pays an issuance cost $\psi(E)$, with $\psi(\cdot)$ weakly increasing and convex. The issuance cost captures frictions associated with raising and registering equity. In the baseline specification, I implement a linear issuance cost with constant marginal cost r', similar to the assumption in Guo et al. (2025).[8] And I further simplify the setting by assuming $r' = r$.

Stage 1 (scale choice under a leverage ceiling). Given (α, E), the firm chooses assets $K \in [E, \bar{\lambda}E]$. Let

$$\tilde{r}(E) = \begin{cases} \sigma r, & E \leq \overline{E} \quad \text{(eligible)}, \\ r, & E > \overline{E} \quad \text{(ineligible)}. \end{cases} \tag{4}$$

[8] While the baseline specification assumes a linear cost, an alternative formulation with convex issuance costs—such as a quadratic function—would capture increasing marginal frictions without altering the qualitative mechanisms of the model. These frictions may arise from dilution of control, informational asymmetries with outside investors, disclosure and governance requirements, all of which can increase with the scale of equity issuance.

Under the program, the interest discount applies to borrowing (the marginal unit of debt). With $D = K - E$, firms' profit function can be written as

$$\pi(\alpha, E; K) = \alpha K^{\theta} - \tilde{r}(E)\,(K - E), \qquad K \in [E, \bar{\lambda}E]. \tag{5}$$

The unconstrained (interior) $K^{\mathrm{FOC}}(\alpha, E)$ satisfies

$$\partial_K\big[\alpha K^{\theta}\big] = \tilde{r}(E) \qquad \Longleftrightarrow \qquad \theta\alpha K^{\theta-1} = \tilde{r}(E), \tag{6}$$

so that

$$K^{\mathrm{FOC}}(\alpha, E) = \left(\tfrac{\theta\alpha}{\tilde{r}(E)}\right)^{\frac{1}{1-\theta}}. \tag{7}$$

Imposing the leverage constraint (2), realized assets are

$$K^{*}(\alpha, E) = \min\{K^{\mathrm{FOC}}(\alpha, E),\, \bar{\lambda}E\}. \tag{8}$$

Notice that under this setting, firms with $E > \overline{E}$ face the same cost of debt and equity. As a result, their financing choice is neutral in the sense of Modigliani and Miller (1958).

Definition (Leverage and Scale Constraints). I distinguish between the firm's leverage and its scale constraints as follows:

i. *Leverage Constraint:* A firm's leverage capacity is exhausted when it operates at the maximum permissible debt-to-equity ratio, such that the leverage cap (2) holds with equality: $K^{*} = \bar{\lambda}E$. Due to the interest subsidy ($\sigma < 1$), all eligible entrepreneurs exhaust their leverage capacity to minimize financing costs.

ii. *Scale Constraint:* The policy environment restricts a firm's scale when the entrepreneur chooses to set equity exactly at the eligibility threshold ($E = \overline{E}$) to retain the policy benefits. I define the scale constraint binding when the firm's realized scale K^{*} is strictly less than its optimal asset level K^{FOC}. Otherwise, if the firm reaches its interior target ($K^{*} = K^{\mathrm{FOC}}$), the scale constraint is slack.

Solving the two stage problem backward, and consider the Stage 0 optimization problem. Given

(α, r), the entrepreneur chooses paid–in capital E, taking into account the optimal scale choice characterized in (8). The entrepreneur's payoff equals operating revenue net of debt costs, plus the transfer S if eligible, minus equity issuance costs. Formally, the Stage 0 objective is

$$\Pi(\alpha, E) = \alpha\,[K^*(\alpha, E)]^\theta - \tilde{r}(E)\big(K^*(\alpha, E) - E\big) \;+\; S\,\mathbf{1}_{\{E \le \overline{E}\}} \;-\; \psi(E). \tag{9}$$

The optimal paid–in capital choice is

$$E^*(\alpha) \in \arg\max_{E \ge 0} \; \Pi(\alpha, E). \tag{10}$$

Proposition 1 (Optimal paid–in capital)**.** For any productivity $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ and market return $r > 0$, the entrepreneur's Stage 0 problem (10) admits a solution. Define the first-best asset demands at the subsidized and market rates as:

$$K_\sigma(\alpha) := \left(\frac{\theta\alpha}{\sigma r}\right)^{\frac{1}{1-\theta}} \quad \text{and} \quad K_1(\alpha) := \left(\frac{\theta\alpha}{r}\right)^{\frac{1}{1-\theta}}.$$

The set of optimal equity choices $\mathcal{E}^*(\alpha)$ is characterized as follows:

i. If the entrepreneur chooses to be eligible ($E \le \overline{E}$), the optimal choice is unique and satisfies $E^* = \min\left\{\frac{K_\sigma(\alpha)}{\bar{\lambda}}, \overline{E}\right\}$.

ii. If the entrepreneur chooses to be ineligible ($E > \overline{E}$), any equity choice E in the interval $\left[\max\left\{\overline{E}, \frac{K_1(\alpha)}{\bar{\lambda}}\right\}, K_1(\alpha)\right]$ is optimal. In this case, the firm implements the scale $K_1(\alpha)$, and the specific capital structure is indeterminate.

The global optimum $E^*(\alpha)$ is the choice (or set of choices) that yields the higher payoff between these two regions.

Proof. Full proof can be found in Appendix A.1. □

Cutoffs and allocation

A static equilibrium allocation can be summarized by three productivity thresholds that partition agents into depositors, eligible firms that exhaust leverage capacity to reach their subsidized target, eligible firms that bunch at the statutory ceiling, and ineligible firms. This section defines these cutoffs and connects them to the policy mechanism under the assumption $r' = r$.

Entry cutoff: $\alpha_{\min}$**.** A depositor earns r from supplying her unit of wealth to the credit market. The marginal agent is indifferent between depositing and entering as an entrepreneur:

$$\Pi\big(\alpha_{\min}, E^*(\alpha_{\min})\big) = r. \tag{11}$$

Because $\Pi(\alpha, E)$ is increasing in α, the solution $\alpha_{\min}$ is unique. Types $\alpha < \alpha_{\min}$ act as depositors, while types $\alpha \geq \alpha_{\min}$ enter and operate a firm.

Bunching starts: α_c**.** For eligible firms, subsidized debt is cheaper than equity ($\sigma r < r$). Consequently, entrepreneurs choose the minimum equity $E = K_\sigma(\alpha)/\bar{\lambda}$ required to reach their subsidized target scale $K_\sigma(\alpha)$. The regulatory scale constraint begins to bind when this required equity reaches the statutory ceiling $\overline{E}$. Define α_c by:

$$\frac{K_\sigma(\alpha_c)}{\bar{\lambda}} = \overline{E} \implies \alpha_c = \left(\tfrac{\sigma r}{\theta}\right)^{1-\theta} (\bar{\lambda}\,\overline{E})^{1-\theta}. \tag{12}$$

For $\alpha \in [\alpha_{\min}, \alpha_c)$, firms exhaust leverage capacity to reach their interior target $K_\sigma(\alpha)$, and the regulatory scale constraint is slack. For $\alpha \geq \alpha_c$, the subsidized target exceeds $\bar{\lambda}\overline{E}$; firms choose to bunch at $E = \overline{E}$ to retain eligibility, making the regulatory scale constraint bind.

Exit from eligibility: α_b**.** High–productivity types face a choice: remain eligible, set $E = \overline{E}$, and bunch at $K = \bar{\lambda}\overline{E}$; or exit the program by choosing $E > \overline{E}$ and operate at the unconstrained market scale $K_1(\alpha)$. The cutoff α_b is defined by indifference between the optimized payoffs in the two regions:

$$\Pi(\alpha_b, \overline{E}) = \Pi(\alpha_b, E > \overline{E}). \tag{13}$$

Using the $r' = r$ setting, the ineligible payoff is independent of the specific equity choice E within the optimal interval. The indifference condition simplifies to:

$$\alpha_b\,(\bar{\lambda}\overline{E})^\theta - \sigma r\,(\bar{\lambda}\overline{E} - \overline{E}) - r\overline{E} + S = \alpha_b\, K_1(\alpha_b)^\theta - r\, K_1(\alpha_b). \tag{14}$$

The left–hand side represents the profit from bunching at the ceiling, while the right–hand side represents the profit from operating at the market scale with a neutral financing mix.

Allocation by productivity. Combining these thresholds, the equilibrium asset allocation is:

$$K^*(\alpha) = \begin{cases} 0, & \alpha < \alpha_{\min} \quad \text{(depositor)}, \\ \left(\frac{\theta\alpha}{\sigma r}\right)^{\frac{1}{1-\theta}}, & \alpha_{\min} \le \alpha < \alpha_c \quad \text{(leverage exhausted, scale unconstrained)}, \\ \bar{\lambda}\,\overline{E}, & \alpha_c \le \alpha < \alpha_b \quad \text{(regulatory scale constraint binds/bunching)}, \\ \left(\frac{\theta\alpha}{r}\right)^{\frac{1}{1-\theta}}, & \alpha \ge \alpha_b \quad \text{(ineligible, scale unconstrained)}. \end{cases} \tag{15}$$

Types below $\alpha_{\min}$ supply funds to the market. Between $\alpha_{\min}$ and α_c, firms utilize maximum leverage to reach their subsidized interior target. Between α_c and α_b, firms bunch at the ceiling to maintain eligibility. Above α_b, firms forgo the subsidy to reach a larger, undistorted scale.

General equilibrium

Let $S(r)$ denote the aggregate supply of deposits and $K^{\text{agg}}(r)$ aggregate demand for assets.

Aggregate supply of funds. Depositors supply

$$S(r) = \int_{\underline{\alpha}}^{\alpha_{\min}(r)} f(\alpha)\, d\alpha, \tag{16}$$

where $\alpha_{\min}(r)$ is pinned down by the entry condition (11).

Aggregate demand for funds. Entrepreneurs demand assets according to (15):

$$K^{\text{agg}}(r) = \int_{\alpha_{\min}(r)}^{\bar{\alpha}} K^*(\alpha; r)\, f(\alpha)\, d\alpha, \tag{17}$$

which is continuous and strictly decreasing in r.

Equilibrium. A static equilibrium is a tuple $\{r^*, \alpha_{\min}, \alpha_c, \alpha_b, K^*(\cdot)\}$ such that

i. the entry condition (11) holds;

ii. firms' asset choices follow (15);

iii. credit market clearing holds:

$$K^{\text{agg}}(r^*) = S(r^*). \tag{18}$$

Proposition 2 (Existence and uniqueness of equilibrium r^*)**.** Fix policy parameters $\sigma \in (0, 1]$, $S \geq 0$, $\overline{E} \geq 0$ and leverage cap $\bar{\lambda} \geq 1$. Under regularity conditions on $f(\alpha)$ and $\theta \in (0, 1)$, there exists a unique equilibrium interest rate $r^* > 0$ such that

$$K^{\text{agg}}(r^*) = S(r^*),$$

which in turn uniquely determines the cutoffs $\alpha_{\min}, \alpha_c, \alpha_b$ and the induced allocation $K^*(\cdot)$.

Proof. The formal proof is provided in Appendix A.2. □

Welfare

Welfare in this economy is determined by aggregate production net of the fiscal costs required to maintain the size-dependent subsidy. The policy environment generates a fundamental trade-off: the scale gains of small, subsidized firms are financed by the savings of depositors, which in turn influences the borrowing costs for all firms.

Aggregate output and welfare. Given the equilibrium allocation $K^*(\alpha)$, aggregate output Y is the sum of production across all active entrepreneurs:

$$Y = \int_{\alpha_{\min}}^{\bar{\alpha}} \alpha \, [K^*(\alpha)]^{\theta} \, f(\alpha) \, d\alpha. \tag{19}$$

The program's fiscal cost is determined by the mass of eligible firms. Let $M = \Pr[\alpha_{\min} \leq \alpha \leq \alpha_b]$ denote the share of agents who enter as eligible entrepreneurs. The total fiscal burden is SM. Static welfare W is defined as aggregate output minus these resource costs:

$$W = Y - S\,M. \tag{20}$$

Counterfactual 1: removing the program. To evaluate the program, we compare it to a benchmark economy in which the size–dependent program is absent ($\sigma = 1, S = 0$, and no eligibility threshold). Let Y^{N} and W^{N} denote the aggregate output and welfare in the benchmark economy, respectively. The change in output is defined as $\Delta Y = Y - Y^{\mathrm{N}}$, and the change in welfare is $\Delta W = W - W^{\mathrm{N}}$. The change in output can be decomposed into four parts:

$$\Delta Y = \underbrace{\int_{\alpha_{\min}}^{\alpha_{\min}^{\mathrm{N}}} \alpha[K^*(\alpha)]^\theta f(\alpha)d\alpha}_{\text{(i) Entry effect}} + \underbrace{\int_{\alpha_{\min}^{\mathrm{N}}}^{\alpha_c} \alpha\Big([K^*(\alpha)]^\theta - [K^{*\mathrm{N}}(\alpha)]^\theta\Big)f(\alpha)d\alpha}_{\text{(ii) Subsidized scale gains}}$$
$$+ \underbrace{\int_{\alpha_c}^{\alpha_b} \alpha\Big([K^*(\alpha)]^\theta - [K^{*\mathrm{N}}(\alpha)]^\theta\Big)f(\alpha)d\alpha}_{\text{(iii) Regulatory bunching loss}} + \underbrace{\int_{\alpha_b}^{\bar{\alpha}} \alpha\Big([K^*(\alpha)]^\theta - [K^{*\mathrm{N}}(\alpha)]^\theta\Big)f(\alpha)d\alpha}_{\text{(iv) GE crowding-out of large firms}}. \tag{21}$$

Term (i), the entry effect, captures the production added by new entrepreneurs who were previously depositors but are now induced to enter by the subsidy and cheaper credit. Term (ii) reflects the subsidized scale gains, representing the expansion of small eligible firms that take advantage of the discounted interest rate σr to reach a larger scale. Term (iii) isolates the "missing middle" by capturing the regulatory bunching loss. These are firms that choose to remain small specifically to keep their eligibility, despite having a higher scale potential. Finally, term (iv) captures the GE crowding-out of large firms. This represents the loss in production from the most productive, ineligible firms, which face a higher equilibrium interest rate r^* because the program's incentives increase the aggregate demand for limited loanable funds.

The change in welfare is then given by $\Delta W = \Delta Y - S\,M$. The program is welfare-improving only if the gains in the first two terms of (21) outweigh the distortions in the latter two and the fiscal cost.

Counterfactual 2: raising the statutory cutoff $\overline{E}_0 \to \overline{E}_1$. Consider a reform that raises the eligibility ceiling from $\overline{E}_0$ to $\overline{E}_1 > \overline{E}_0$. This policy change expands the set of firms that can qualify for subsidized borrowing and, for firms previously bunching at the old ceiling, relaxes the constraint on registered equity and thus feasible scale. Let (Y_j, W_j, M_j) denote equilibrium objects under ceiling $\overline{E}_j$ for $j \in \{0, 1\}$. The change in welfare is:

$$\Delta W_{\text{reform}} = (Y_1 - Y_0) - S(M_1 - M_0). \tag{22}$$

The output component $Y_1 - Y_0$ reflects (i) changes in entry, (ii) changes in scale for firms that become newly eligible, and (iii) de-bunching effects for firms previously constrained by the old cutoff. These forces raise aggregate demand for funds, which pushes up the equilibrium interest rate r^* and dampens investment for the most productive firms. The fiscal component $S(M_1 - M_0)$ captures the additional transfer burden from expanding eligibility.

4 Institutional Setting and Data

4.1 History of Government Guarantee Scheme in Japan

The CGS is depicted in Figure 2. SMEs can apply for credit guarantees in two ways. The most common way is for the SME to first approach a bank, which then submits a guarantee application on the firm's behalf. Alternatively, SMEs may apply directly to the Credit Guarantee Corporations (CGCs). In either case, the CGC conducts a credit evaluation and decides whether to approve the guarantee for the specific loan. If the guarantee is approved, the CGC issues a guarantee certificate to the bank.[9] The bank is informed about the guarantee status before setting the interest rate and disbursing the loan. Both local and national governments provide oversight and financial support. If the SME defaults, the CGC reimburses the bank for the guaranteed portion of the loan.[10]

The evolution of Japan's Credit Guarantee System (CGS) can be traced back to its formal establishment in December 1950 under the Small and Medium-sized Enterprise Credit Insurance Act (Act No. 264 of 1950).[11]

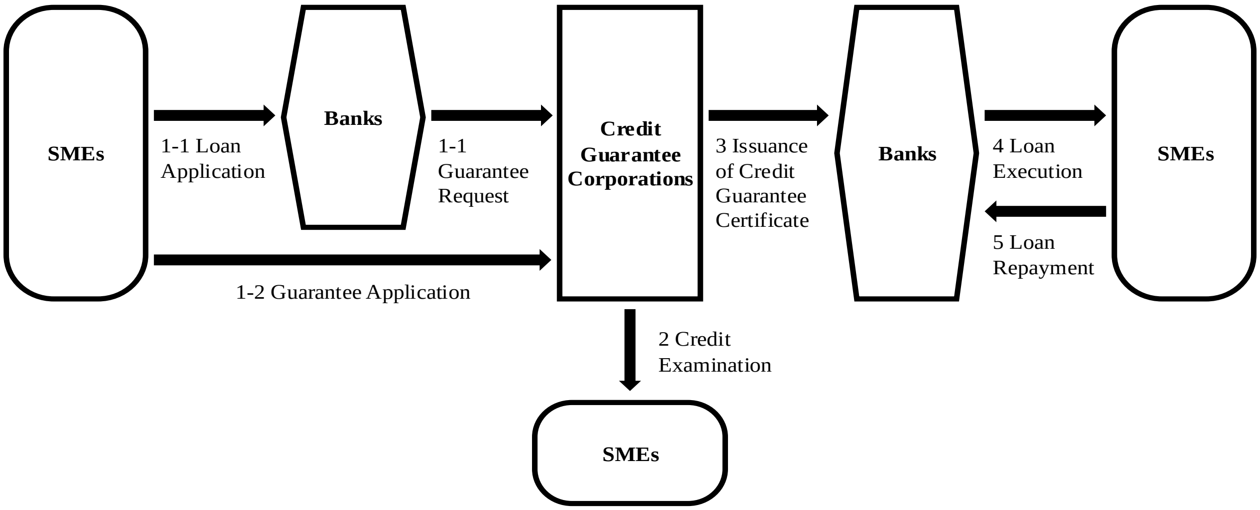
In December 1999, Japan revised the Basic Law on SMEs, creating the primary policy shock exploited in this paper. A central feature of Japan's SME classification is its reliance on paid-in capital as an eligibility criterion. Paid-in capital refers to the legally registered equity of the firm,

[9] If the SME applies directly to a CGC, the CGC will assign a partner bank to handle the loan.

[10] The Japan Federation of Credit Guarantee Corporations (JFG) compensates CGCs for losses arising from defaults on guaranteed loans. The Japan Finance Corporation (JFC) mitigates risks through its credit insurance program, aiming to distribute the financial burden faced by CGCs. Part of the credit guarantee fees, which are payments made by SMEs to CGCs for guarantees, are used to pay for credit insurance premiums.

[11] The Small and Medium-sized Enterprise Credit Insurance Act (Act No. 264 of 1950) was promulgated on December 14 and came into force on December 15, 1950, establishing the legal foundation for Japan's credit insurance system for SMEs. For details, see *Small and Medium-sized Enterprise Credit Insurance Act (Act No. 264 of 1950)* (1950) and Japan Federation of Credit Guarantee Corporations (2024).

Figure 2: Credit Guarantee Scheme in Japan



Note: This figure illustrates the CGS in Japan. A total of 51 CGCs serve as the direct guarantee providers for SMEs. If firms are unable to repay their loans, the local and national governments provide the ultimate backup.

which includes the owner's initial contribution at incorporation as well as part of the subsequent capital raised from investors. Under the Commercial Code in effect at the time (later codified in Article 445, Paragraph 2 of the Companies Act of 2005), when a firm issues new shares, at least 50% of the subscription proceeds must be recorded as paid-in capital.[12] This requirement implies that firms cannot raise outside equity without mechanically increasing their paid-in capital. As a result, privately held firms face a direct trade-off: accessing external financing risks pushing them above the SME threshold and forfeiting policy benefits.

The 1999 reform revised SME classification thresholds that had become outdated relative to the actual scale of Japanese firms. Before the change, the capital ceilings were set at relatively low levels, excluding many mid-sized firms that still faced significant financing constraints. By raising the paid-in capital thresholds, the reform expanded the scope of firms eligible for policy support, while keeping or only modestly adjusting the employee criteria. For example, the ceiling for manufacturing firms was raised from 100 million yen to 300 million yen, and that for wholesale

[12] Commercial Code, Article 168-2 (before 2005); Companies Act (Act No. 86 of 2005), Article 445, Paragraph 2. See Japanese Law Translation Database, Ministry of Justice, https://www.japaneselawtranslation.go.jp.

Table 1: SME Definition Change in 1999

Industry	**Before 1999**		**After 2000**	
	Capital	Employees	Capital	Employees
Manufacturing, etc.	100 million	300	300 million	300
Wholesale	30 million	100	100 million	100
Retail	10 million	50	50 million	50
Service	10 million	50	50 million	100

Note: This table summarizes the official criteria for defining SMEs in Japan before and after the 1999 revision to the Basic Law on SMEs, which came into effect in 2000. The classification thresholds are defined by industry and are based on maximum capital and number of employees. For example, under the new definition, the capital threshold for manufacturing firms increased from 100 million yen to 300 million yen, while the employee threshold remained unchanged. In contrast, service-sector firms saw both capital and employee thresholds increase. These revised criteria expanded the pool of firms eligible for credit guarantees. The data are sourced from the Small and Medium Enterprise Agency of Japan.

firms from 30 million yen to 100 million yen. These changes broadened the set of firms eligible for credit guarantees, making it possible to distinguish between those newly eligible after the reform and those that were always or never eligible. A detailed summary of the pre- and post-reform definitions is provided in Table 1.

Beginning in April 2006, Japan shifted from a uniform guarantee fee to a risk-based, nine-tier schedule calibrated with the Credit Risk Database (CRD), making fees sensitive to borrower risk (Goto, 2023).[13] In October 2007, the government introduced the responsibility-sharing system (partial guarantees). Until then, in principle, most guarantees were provided at 100 percent coverage. The 2007 reform made the standard coverage 80 percent, requiring banks to retain about 20 percent of credit risk, although some programs continued to operate at 100 percent (Goto, 2023).[14]

The global financial crisis led to the launch of the Emergency Credit Guarantee Program (ECGP) in October 2008, which temporarily restored full 100 percent guarantees on SME loans (Goto, 2023).[15] The ECGP also extended loan maturities to over ten years (longer than standard

[13] Before 2006, fees were broadly uniform (for example, around 1.35% for unsecured guarantees and 1.25% for secured guarantees); see Goto (2023, pp. 6–7).

[14] Operationally, under responsibility sharing, CGCs pay the bank upon subrogation and then collect a burden charge for the bank's 20 percent share; see Goto (2023, pp. 7–8) and the Japan Federation of Credit Guarantee Corporations' overview (Japan Federation of Credit Guarantee Corporations, 2023, pp. 10–11).

[15] The ECGP remained in effect until March 2011. Related safety-net lines also differ in coverage: Safety Net No. 4 (region-wide shocks) provides full guarantees, while No. 5 (industry-specific shocks) covers 80 percent; see Goto (2023, pp. 7–8).

programs) and suspended the risk-based fee system.

In 2018, another reform was enacted to rebalance the system. The coverage rate for ordinary loans was set at 80 percent, with the use of 100 percent guarantees limited to exceptional crisis situations and targeted schemes such as start-up support. At the same time, loan ceilings were raised to facilitate business succession and early-stage financing (Goto, 2023).[16] The goal of the 2018 reform was to reduce moral hazard and ensure that the CGS functioned as a mechanism for healthy business growth rather than as a permanent safety net for distressed firms.

Since the onset of the COVID-19 pandemic in January 2020, the Japanese government has implemented various financial support measures for SMEs to mitigate economic difficulties. These measures included establishing consultation desks, easing "Safety net loans" criteria, and urging financial institutions to offer leniency. In March 2020, the No.4 Safety Nets[17] for Financing Guarantee was designated as a nationwide program, providing a 100% guarantee and expanding industry coverage under the No.5 Safety Nets[18].

While these reforms in 2006, 2008, 2018, and 2020 were important for shaping the institutional evolution of Japan's CGS, they differ from the 1999 Basic Law revision in nature and scope. Unlike the 1999 reform—which created discrete variation in SME eligibility thresholds and serves as the central quasi-natural experiment in this paper—later reforms applied broadly to the entire system and are therefore not exploited for causal identification in the empirical analysis. Instead, they provide important historical context for understanding how the CGS has evolved in response to crises and policy priorities.

4.2 Data

This study uses several complementary datasets to examine how government credit guarantees interact with bank capitalization and influence lending behavior. The analysis leverages firm-

[16] Start-ups are defined by the Small and Medium Enterprise Agency as firms within five years of establishment; micro businesses are those with fewer than five employees in commerce and services, or fewer than twenty in manufacturing.

[17] No.4 Safety Nets: If an SME operates in a designated area where various industries are impacted by the disease and experiences a sales decline of 20% or more compared to the previous year, it qualifies for a full loan amount financing guarantee. This guarantee is provided under a special framework that is distinct from the general financing guarantee.

[18] No.5 Safety Nets: If an SME operates in an industry severely impacted by the disease and experiences a sales decline of 5% or more compared to the previous year, it qualifies for a financing guarantee that covers 80% of the loan amount. This guarantee is provided under a special framework separate from the general financing guarantee.

level, bank-level, and policy-level data spanning the period from 1995 to 2022.

The primary source of firm-level data is the ORBIS database provided by Bureau van Dijk. The sample covers Japanese firms across all industries except agriculture, finance, and public administration. In Japan, legal requirements mandate firms to submit their financial reports to industry-specific regulatory authorities. As a result, the ORBIS database offers a comprehensive representation of companies across various sizes. Listed firms represent less than 1 percent of the sample.

ORBIS also includes information on the main financial institutions from which each firm borrows, enabling the construction of firm-bank-year panels. The key financial variables used in the analysis include the paid capital, the number of employees used to define SMEs; total liabilities, shareholders' equity, and operating earnings which are used to evaluate the financial quality of the firm. I exclude firm-year observations with missing values for these key balance sheet information. I also divide the firm sample into two periods: 1995-2010 and 2011-2022 where the first sample is used to do the event study analysis and the second sample is used to be paired with bank-level guarantee information and do bank-firm analysis. The whole sample comprises 2,332,420 observations (96.54% of which are SME observations). This dataset includes 427,953 firms.

Table 2 summarizes the financial disparities between SMEs and non-SMEs from 1995 to 2010. A persistent feature throughout the sample is the higher debt-to-equity ratio among SMEs relative to non-SMEs, implying a heavier reliance on debt financing by smaller firms. This contrasts with patterns observed in other economies—for instance, Whited and Zhao (2021) find that in both China and the United States, larger firms tend to carry higher debt-to-equity ratios. One notable discontinuity appears in the revenue figures around 1999 and 2000: average revenue for non-SMEs drops sharply in 1999, while SME revenue rises abruptly. This pattern reflects the SME eligibility reform enacted in late 1999.

To capture bank characteristics, I use the Nikkei Financial Institutions Database, available through Nikkei Financial Quest (Nikkei Inc., various years). This dataset provides annual balance sheet information for all major categories of Japanese banks, including city banks, regional banks, and shinkin banks. Reported items include total assets, liabilities, and shareholders' equity, which allow for the construction of time-varying measures of bank capitalization such as the equity ratio.

Table 2: Summary Statistics by Year and Size

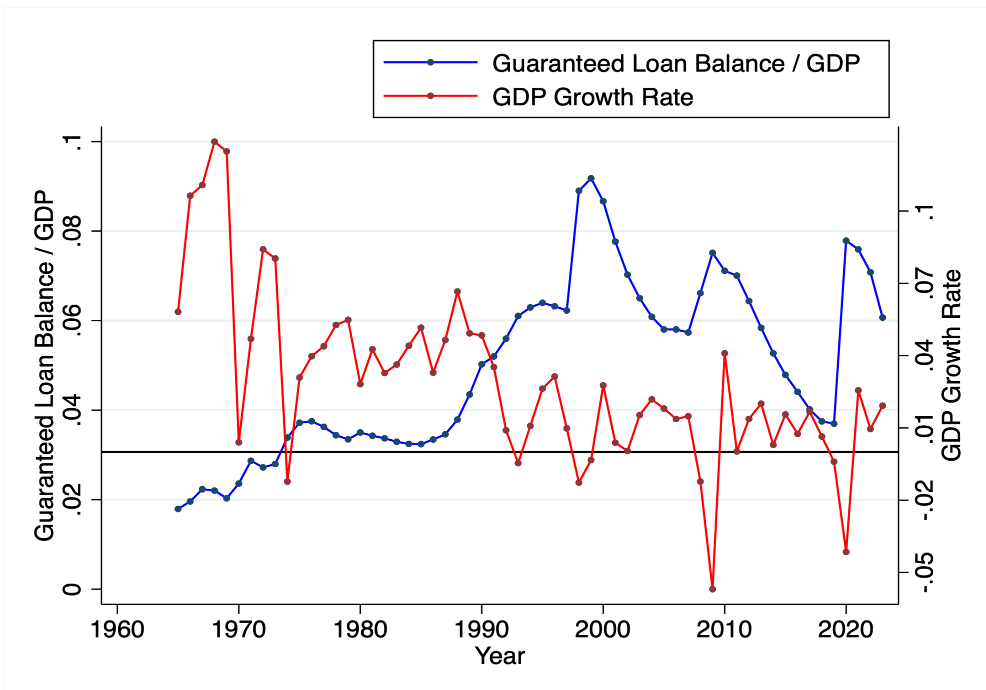
	Non-SME					SME				
Year	Observations	Debt	Equity	D/E	Revenue	Observations	Debt	Equity	D/E	Revenue
1995	508	134.0	61.7	2.2	162.1	11280	0.4	0.1	2.9	0.7
1996	608	138.0	59.7	2.3	183.3	15683	0.4	0.1	2.9	0.6
1997	717	130.0	52.5	2.5	168.4	21029	0.3	0.1	2.7	0.6
1998	1178	109.0	44.4	2.5	143.5	25739	0.3	0.1	2.5	0.6
1999	2650	69.3	29.6	2.3	95.4	38363	0.5	0.2	2.4	0.9
2000	3469	64.5	33.3	1.9	96.8	57611	0.9	0.4	2.5	1.7
2001	3643	61.5	32.5	1.9	90.7	61942	0.9	0.4	2.2	1.6
2002	3937	55.2	30.5	1.8	84.4	72399	0.8	0.4	2.0	1.6
2003	4118	53.8	32.3	1.7	84.8	80111	0.8	0.4	1.9	1.5
2004	4191	56.1	34.5	1.6	89.8	85225	0.8	0.4	1.9	1.5
2005	4282	57.7	37.9	1.5	95.2	87887	0.8	0.4	1.8	1.5
2006	4440	59.0	38.7	1.5	98.2	91564	0.8	0.4	1.8	1.6
2007	4544	67.8	41.6	1.6	112.4	93668	0.7	0.4	1.8	1.5
2008	4554	69.4	38.0	1.8	111.4	96342	0.6	0.4	1.7	1.3
2009	4618	67.4	41.2	1.6	100.6	101406	0.6	0.4	1.6	1.2
2010	4961	64.4	40.4	1.6	102.5	112541	0.6	0.4	1.6	1.2

Note: All financial values in this table are measured in billions of 2010 Japanese yen. The "Observation" column documents the number of firms (Non-SME or SME) in each year in the data sample. Debt, equity, and revenue are summarized as the mean of all firms (Non-SME or SME) within each year. The D/E ratio measures the ratio of debt to equity.

For the current analysis, I merge these indicators with administrative records on credit guarantee issuance at the bank level, enabling a direct link between banks' balance sheet strength and their reliance on government guarantees.

Finally, I use administrative data on public credit guarantee programs from 2011 to 2022.[19] These data are compiled by the National Federation of Credit Guarantee Corporations based on information submitted by financial institutions during the guarantee application process. The dataset covers both the bank and prefecture levels and records the total annual volume of credit-guaranteed loans issued by each bank. This information is used to compute the share of guaranteed loans in total lending at the bank-year level. The disaggregated structure of the data enables analysis of regional and institutional variation in guarantee usage.

Figure 3: Total Guaranteed Loan Balance as a Share of GDP



Note: This plot displays the time series of guaranteed loan balance as a share of annual GDP from 1964 to 2023. The guaranteed loan balance data is adjusted to 2015 Japanese yen. Data for the period from 1964 to 2004 is derived from the "gyoumu yoran" (business directory) issued by the Japan Federation of Credit Guarantee Corporations. Data from 2005 to 2023 is obtained from the Credit Guarantee Performance Report published by the National Federation of Credit Guarantee Associations. GDP data is sourced from the World Bank's dataset on GDP (constant local currency).

4.3 Stylized Facts about CGCs

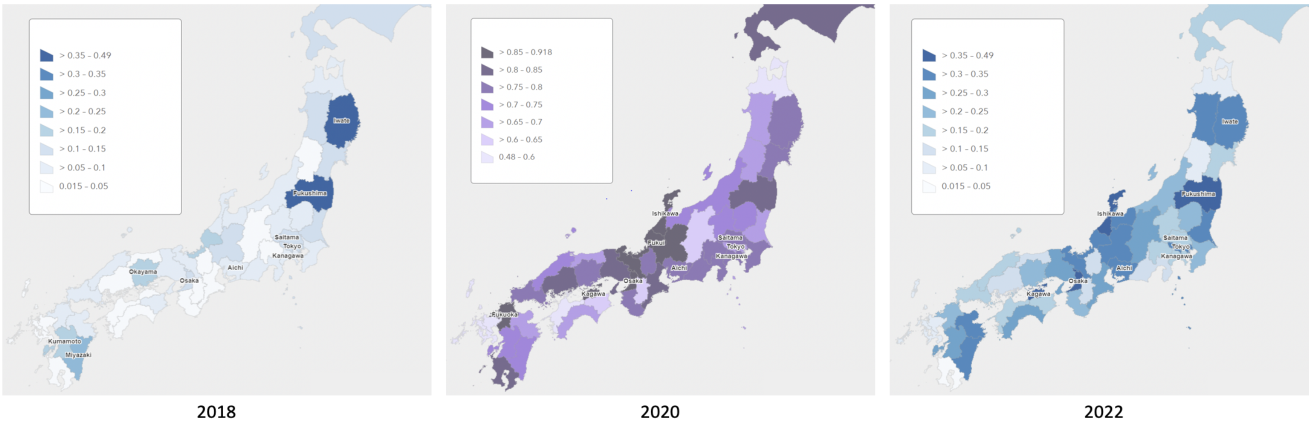
First, Figure 3 shows the evolution of the guaranteed loan balance relative to GDP since the 1960s. The ratio rose steadily through the 1970s and 1980s, before increasing sharply in the 1990s in response to the collapse of the asset price bubble and the Asian financial crisis. It reached a peak of about 6–7 percent of GDP around 2000, a surge also reinforced by the 1999 SME Basic Law reform, which broadened firm eligibility for guarantees. After 2000, the ratio gradually declined, reflecting the introduction of responsibility-sharing reforms in 2006–2007 that reduced guarantee coverage to 80 percent. Temporary spikes reappeared during the global financial crisis of 2008, when the Emergency Credit Guarantee Program reinstated full coverage, and again in 2020, when COVID-19 emergency measures caused the ratio to nearly double from 4 to 8 percent of GDP. These patterns underscore how the guarantee system expands most aggressively during periods of financial crisis or major policy reforms, and contracts in more stable periods.

Second, the COVID-19 pandemic triggered a sharp policy reversal in Japan's guarantee system. The 2018 reform had deliberately reduced the coverage of ordinary loans from 100% to 80%, in order to strengthen banks' incentives for screening and risk management. Yet in 2020, the Emergency Credit Guarantee Program reinstated full (100%) coverage across all prefectures, and Figure 4 shows that the share of fully guaranteed loans surged nationwide. Strikingly, this surge was not short-lived: the elevated reliance on full guarantees persisted through 2022, even after the peak of the pandemic had passed. This persistence underscores how temporary emergency measures can entrench lasting shifts in financial intermediation, as banks and firms adapt their behavior to the expectation of government backstops, with fiscal and allocative consequences that extend well beyond the crisis itself.

Third, I examine how the use of credit guarantees varies with banks' own financial health. Table 3 shows a strong inverse relationship between bank capitalization and reliance on guarantees: banks with lower equity ratios consistently extend a higher share of guaranteed loans. Across specifications, the estimated coefficients on the equity ratio are negative and statistically significant, ranging from -0.086 to -0.215. The relationship remains robust after introducing controls

[19] Prefecture-level data from 1968 to 2005 is also available from the *Gyoumu Yoran* (Business Directory) published by the Japan Federation of Credit Guarantee Corporations. This historical series is used in Figure 3.

Figure 4: Ratio of Fully Guaranteed Loan



Note: This figure presents the ratio of fully guaranteed loans as a share of all guaranteed loans across prefectures. The legends for the years 2018 and 2022 are identical but differ from that of 2020. The loan guarantee amounts for the four cities with their own CGCs are aggregated with the prefectures to which they belong. The share of fully guaranteed loans increased for all prefectures in 2020 due to the emergency lending program. The data source is the online publication of the Small and Medium Enterprises Agency.

for bank type (city, regional, shinkin) and prefecture fixed effects to absorb regional heterogeneity. This pattern suggests that less-capitalized banks—those with limited ability to absorb credit risk—lean more heavily on the public guarantee system when extending loans. While guarantees are often justified as a policy to support small and medium-sized enterprises, these results raise the possibility that the program also functions as a de facto support mechanism for weaker banks, allowing them to shift risk onto the government balance sheet. This dual role of the policy—relieving financing constraints for SMEs while simultaneously backstopping fragile lenders—clarifies why guarantee usage persists at high levels in Japan relative to other advanced economies.

5 Empirical Methodology and Reduced-Form Results

5.1 Bunching Around the SME Capital Cutoff

To test how firms respond to SME eligibility thresholds in practice, I first analyze the distribution of paid-in capital across firms in the ORBIS dataset, focusing on the last available observation for each firm before 2000 and after 2010.

Figure 5 Panel (a) presents the firm size distribution by paid-in capital before the 1999 SME

Table 3: Bank Equity and Share of Guaranteed Lending

	Guaranteed Loan/Total Loan		
	(1)	(2)	(3)
Equity Ratio	-0.086**	-0.285***	-0.215***
	(0.043)	(0.038)	(0.037)
N	4,165	4,165	4,165
Bank Controls	yes	yes	yes
Time FE	yes	yes	yes
Bank Type FE	no	yes	yes
Prefecture FE	no	no	yes

Note: This table reports the results from regressions of the share of guaranteed loans in total loans on the bank equity ratio. The dependent variable in all columns is the ratio of guaranteed loans to total loans at the bank-year level. The key explanatory variable is the bank equity ratio, measured as equity over total assets. A control for the (log) total loan volume of the bank is included in all regressions to account for size-related differences in guarantee usage. Column (1) includes time fixed effects only. Column (2) additionally includes fixed effects for bank type (City Banks, Regional Banks, and Shinkin Banks). Column (3) further includes prefecture fixed effects to account for regional variation in guarantee take-up. Standard errors are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

definition reform. Across all sectors, there is visible bunching just below the pre-reform capital thresholds: 100 million yen for manufacturing, 30 million yen for wholesale, and 10 million yen for retail and service. This excess mass near the eligibility cutoff suggests strategic behavior by firms to remain classified as SMEs and retain access to subsidized credit. The pattern is strongest in the retail and service sectors, where a significant portion of firms cluster immediately below the 10 million yen mark.

Panel (b) shows the distribution using each firm's latest post-2010 observation, after the capital thresholds were raised. Bunching behavior persists under the new regime. For wholesale, retail, and service firms, the distributions exhibit a marked shift upward, with pronounced spikes just below the new eligibility thresholds (100 million yen for wholesale; 50 million yen for retail and service). This shift suggests that many firms expanded their capital base following the reform but still maintained levels just under the new thresholds to remain eligible for SME classification. The sharp discontinuities indicate that firms continue to respond to policy incentives embedded in the capital-based SME definition.

In contrast, manufacturing firms display less dramatic changes in bunching behavior. Even after the post-reform eligibility threshold increased to 300 million yen, most firms remain clustered below 100 million yen, similar to the pre-2000 distribution. One reason may be that corporate tax benefits in Japan apply universally to firms with paid-in capital below 100 million yen[20], independent of credit guarantee criteria; hence manufacturing firms have a strong incentive to stay under that threshold. A further possibility is structural: the employment share of manufacturing has declined over decades, reducing opportunities for expansion within the sector. Fewer new entrants or expanding firms in manufacturing may exist to respond to the raised SME cutoff, thereby muting the bunching response in this sector.

Overall, the distributional evidence supports the view that SME classification thresholds shape firm behavior in capital registration. The persistent bunching around eligibility cutoffs reinforces concerns that policy-induced incentives distort firm growth trajectories and capital allocation.

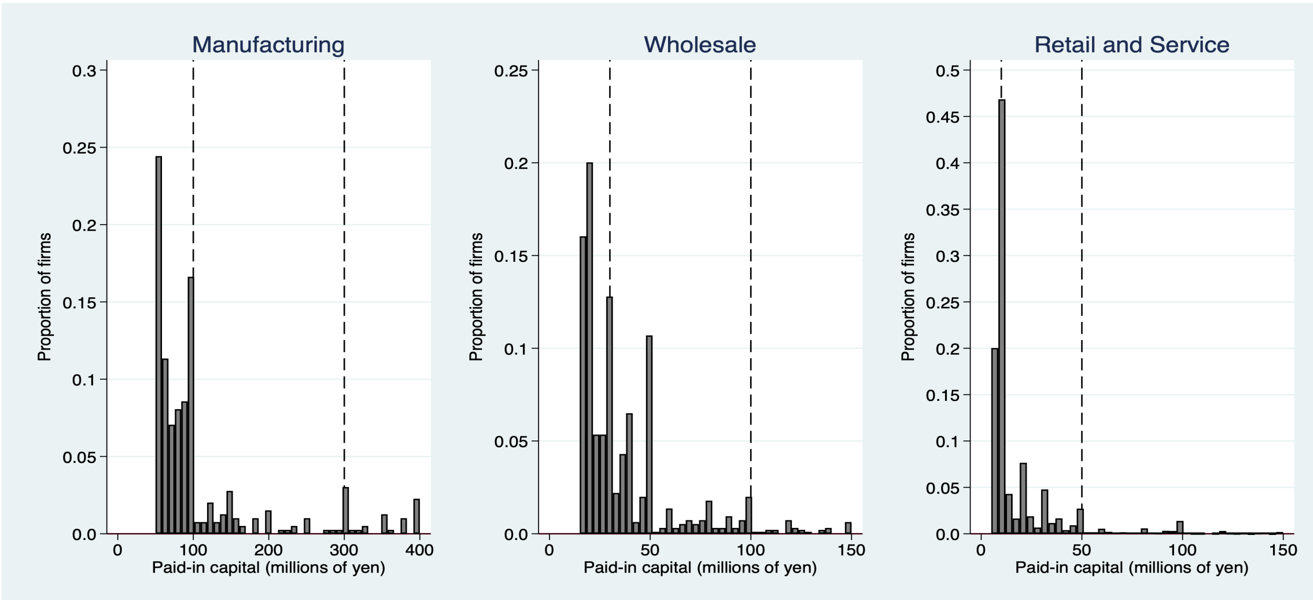
Appendix Figure B1 illustrates how firms respond to employment-based SME eligibility thresholds. In both periods, there is clear evidence of bunching near the eligibility cutoffs across all sectors, although some of the bunching occurs just above the cutoffs. In the service sector, where the employment threshold was raised from 50 to 100 employees, a strong new mode appears just below the higher threshold after 2010, while some firms continue to cluster below the original 50-employee cutoff. The retail sector similarly shows persistent clustering just below the 50-employee threshold.

5.2 Dynamic Effect of SME Policy Change in 1999

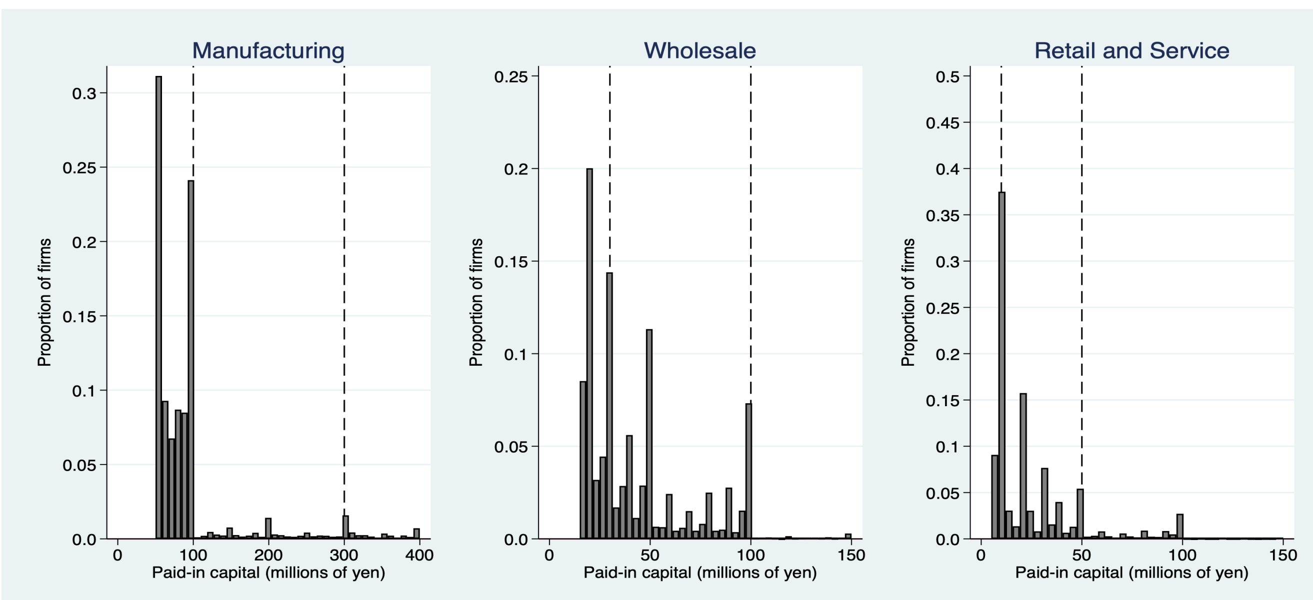
To quantify the effects of Japan's 1999 revision to the Basic Law on SMEs, I exploit variation in firm eligibility generated by the policy change. The empirical strategy focuses on identifying relative changes in firm outcomes—such as revenue, employment, capital, debt, and equity—across firms differentially exposed to the reform. I distinguish between two treated groups: firms that became newly eligible following the expansion of the size thresholds and firms that were always eligible for SME classification. The "newly eligible" firms are defined as those that exceeded the

[20] Under Japan's corporate tax system, firms with paid-in capital below 100 million yen are classified as SMEs for tax purposes and are eligible for preferential tax treatment. As of the 2000s, these benefits include a reduced corporate income tax rate on a portion of taxable income (e.g., 15% on income up to 8 million yen, versus 23.2% for standard corporations), accelerated depreciation schemes, and exemptions from certain local enterprise taxes. These tax advantages are codified in Japan's Special Taxation Measures Law and continue to apply irrespective of the SME definition used in credit or industrial policy.

Figure 5: Firm Bunching Around SME Capital Eligibility Threshold



(a) Before the 1999 SME Definition Reform



(b) After the 1999 SME Definition Reform

Note: This figure shows the distribution of firms by paid-in capital around the SME eligibility threshold. Panel (a) displays the firm size distribution before the 1999 reform, when the capital thresholds for SME classification were 100 million yen for manufacturing, 30 million yen for wholesale, and 10 million yen for both retail and service sectors. Panel (b) shows the distribution after the reform took effect in 1999, which raised the capital thresholds to 300 million yen for manufacturing, 100 million yen for wholesale, and 50 million yen for retail and service industries.

SME size threshold prior to the reform but fell below the expanded thresholds afterward. The "always eligible" firms also constitute a treated group, as they were previously constrained by the old eligibility cutoff, which was relaxed by the policy reform. The control group consists of firms that were always ineligible for SME classification (i.e., always large) and were therefore unaffected by the change in eligibility rules.

The main empirical specification follows an event-study difference-in-differences framework:

$$\ln(y_{ft}) = \sum_{t=1995}^{2007} \beta_t \cdot \text{Treated}_{ft} + \alpha_f + \gamma_{pt} + \delta_{it} + \varepsilon_{ft} \tag{23}$$

where $\ln(y_{ft})$ denotes the log of the outcome variable for firm f in year t, and Treated_{ft} is an indicator equal to one if firm f belongs to either the always-eligible or newly eligible group in year t. The coefficient vector $\{\beta_t\}$ captures the differential change in the outcome of treated firms relative to always-ineligible firms in each year.

The identifying assumption is that, in the absence of the policy reform, treated firms would have followed parallel trends to the control group. The specification includes firm fixed effects (α_f) to absorb all time-invariant firm characteristics, as well as prefecture-year fixed effects (γ_{pt}) to flexibly control for local economic shocks and region-specific trends, such as variation in credit conditions or local policy environments. Industry-year fixed effects (δ_{it}) are included to capture sector-level shocks, including those arising from international demand fluctuations or regulatory changes. To address serial correlation and heteroskedasticity in the error term, standard errors are clustered at both the firm and industry-year levels.

To improve comparability around the SME eligibility threshold, I restrict the sample to firms of similar size prior to the 1999 policy change. In the baseline analysis, I identify firms whose size in their last pre-2000 observation exceeded twice the post-reform SME threshold or fell below half of the pre-reform threshold, and remove all observations for these firms from the panel. Firms are kept in the sample if they fell within the feasible size range prior to 2000, regardless of subsequent growth or contraction. This trimming strategy focuses the analysis on firms near the eligibility margin, ensuring that treated and control groups are drawn from a comparable portion of the size distribution and reducing concerns about compositional effects.

The coefficients of interest, β_t, are plotted in Figure 6 to illustrate the dynamic effects of the reform for always-eligible firms relative to firms that were never eligible for SME classification. Prior to the 1999 policy change, the estimated coefficients are statistically indifferent from zero, and display no systematic pre-trend across outcomes. Following the reform, always-eligible firms experience a significant and persistent expansion along both economic and financial dimensions. Revenue rises steadily, reaching an increase of 12 percentage points within six years after the reform. Employment and physical capital exhibit an even larger response, increasing by 20–30 percentage points.

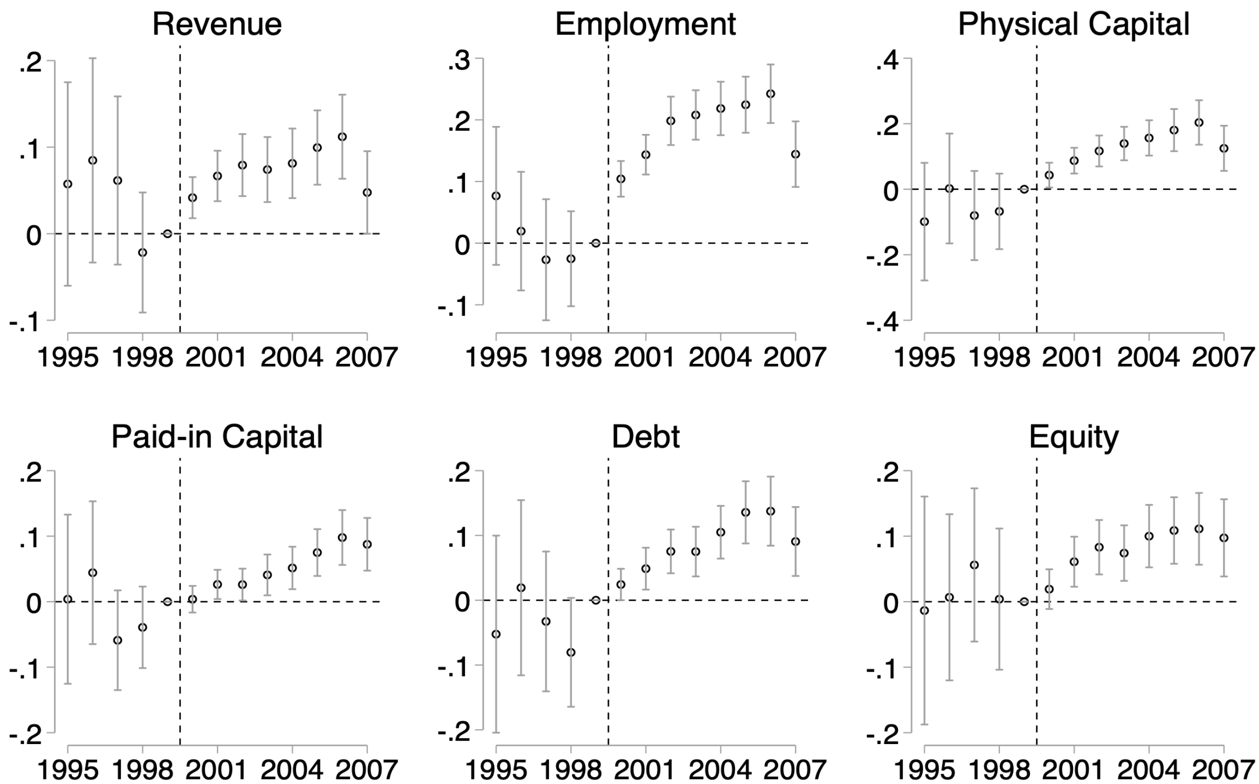
Balance-sheet adjustments closely mirror the economic responses. Paid-in capital increases gradually after the reform, while debt expands sharply, with effects on the order of 10–15 percentage points relative to never-eligible firms. Equity also grows persistently, rising by roughly 10 percentage points in the post-reform period. Importantly, these effects are not transitory: across all outcomes, the post-reform coefficients remain positive and statistically significant throughout the sample window.

Figure 7 compares newly eligible firms to firms that were never eligible for SME classification. Across revenue, employment, and physical capital, the estimated post-reform effects are generally small and not statistically different from zero. There is even some indication of a modest decline in employment beginning in the mid-2000s. On the financial side, debt exhibits a positive post-2000 trend, consistent with the expansion of credit guarantees; however, this effect becomes only marginally statistically significant about four years after the policy change. Paid-in capital also increases among newly eligible firms relative to never-eligible firms.

Appendix Figure B2 further decomposes the newly eligible firms into two separate treated groups by paid-in capital size. The black line represents firms with 1999 paid-in capital below the midpoint between the pre-reform and post-reform SME cutoffs, while the gray line represents firms with 1999 paid-in capital above that midpoint.[21] The results indicate that the increase in paid-in capital is driven entirely by the smaller newly eligible firms, which actively raise external equity financing. In contrast, larger newly eligible firms exhibit little change in paid-in capital, consistent with concerns about crossing the new eligibility threshold. The observed increase in

[21] For example, in the manufacturing sector, the black lines represent firms with 1999 paid-in capital between 100 million yen and 200 million yen, while the gray lines represent firms with paid-in capital between 200 million yen and 300 million yen.

Figure 6: Dynamic Effects: Always-Eligible vs. Never-Eligible Firms



Notes: This figure plots estimated event-study coefficients β_t from a difference-in-differences specification comparing firms that were always below the SME classification (treated group) to those that were always above the SME threshold (control group). The outcome variables include revenue, employment, physical capital, paid-in capital, debt, and equity. The specification includes firm fixed effects, prefecture-year fixed effects, and industry-year fixed effects. Standard errors are clustered at the firm level and industry-year level. To improve comparability near the eligibility threshold, the sample is restricted to firms with size larger than 0.5 times the pre-reform SME cutoff before 2000. The coefficients are normalized to zero in 1999, the year of the policy change.

debt is likewise driven by the smaller group; despite facing a lower effective cost of borrowing, larger firms do not significantly expand their debt.

The interpretation of the contrasting trends observed across Figure 6 and 7 is that the SME size threshold itself acted as a growth barrier. Prior to the reform, firms in the always-treated group have strategically constrained their growth to remain below the original cutoff and retain access to generous government-backed credit guarantees. The 1999 policy change effectively relaxed this constraint for them by raising the eligibility threshold, thereby allowing these firms to expand without jeopardizing access to subsidized credit. In contrast, firms that became newly eligible after the reform have begun to face similar incentives to limit their scale in order to avoid crossing the new threshold. Since the incentive directly influences firms' external financing, it also indirectly affects their bank borrowing and real economic behavior, including employment, investment, and sales.

5.3 Estimating the Direct Effect of SME Policy Change in 1999

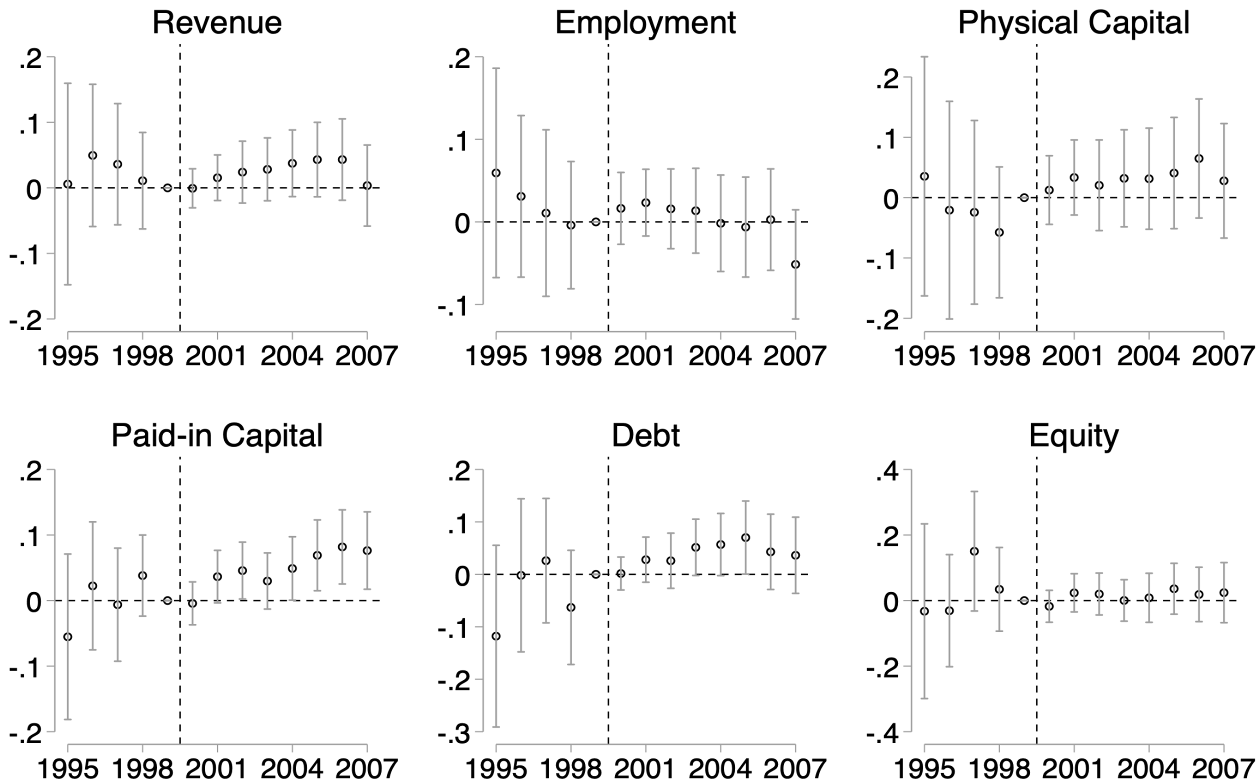
To estimate the direct effects of the SME reclassification policy, I implement the following difference-in-differences specification:

$$\ln(y_{ft}) = \beta \text{Post} \times \text{Treated} + \alpha_f + \gamma_{pt} + \delta_{it} + \varepsilon_{ft} \tag{24}$$

Similar to the dynamic specification, to improve comparability around the thresholds, firms with extremely small or large values of paid-in capital and employment before the policy change are excluded from the estimation sample. All regressions include firm fixed effects, prefecture-year fixed effects, and industry-year fixed effects. Standard errors are clustered at the firm and industry-year levels.

Table 4 reports difference-in-differences estimates comparing always-eligible firms to firms that were never eligible for SME classification. Across all outcomes, the coefficients on the interaction term *Post* × *Treated* are positive and statistically significant, confirming a broad-based expansion among always-eligible firms following the reform. Column (1) shows that revenue increased by 4.5% relative to never-eligible firms. Column (2) reveals a particularly large response in employment,

Figure 7: Dynamic Effects: Newly-Eligible vs. Never-Eligible Firms



Notes: This figure plots estimated event-study coefficients β_t from a difference-in-differences specification comparing firms that became newly eligible for SME classification following the 1999 policy change (treated group) to firms that were always classified as large and thus never eligible (control group). The outcome variables include revenue, employment, physical capital, paid-in capital, debt, and equity. The specification includes firm fixed effects, prefecture-year fixed effects, and industry-year fixed effects. Standard errors are clustered at the firm level and industry-year level. To improve comparability near the eligibility threshold, the sample is restricted to firms with size lower than 2 times the post-reform cutoff before 2000. The coefficients are normalized to zero in 1999, the year of the policy change.

Table 4: Direct Effect of SME Redefinition (Always-Treated vs. Never-Treated)

	Revenue (1)	Employment (2)	Physical Capital (3)	Paid-in Capital (4)	Debt (5)	Equity (6)
Post×Treated	0.045***	0.170***	0.092***	0.039***	0.070***	0.025*
	(0.012)	(0.014)	(0.020)	(0.009)	(0.015)	(0.014)
Firm FE	yes	yes	yes	yes	yes	yes
Prefecture-Time FE	yes	yes	yes	yes	yes	yes
Industry-Time FE	yes	yes	yes	yes	yes	yes
Firms	22,023	22,023	21,925	22,023	22,023	22,023
Firm-Year Obs	134,768	134,768	134,324	134,768	134,768	134,768

Note: This table reports estimates from difference-in-differences regressions evaluating the impact of the 1999 SME Basic Law reform on firm outcomes. The dependent variables in columns (1) through (6) are the log of operating revenue, employment, physical capital, paid-in capital, debt, and equity, respectively. The treatment group consists of firms that were always eligible for credit guarantee, while the control group includes never-eligible firms. Firms with extreme values of paid-in capital and employment size (very small) are excluded from the sample. All regressions include firm fixed effects, prefecture-year fixed effects, and industry-year fixed effects. Standard errors are clustered at the firm and industry-year level. Standard errors are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

which rose by 17%. Physical capital also increased substantially, by 9.2% (column 3). Financial outcomes display a similar pattern: paid-in capital rose by 3.9%, debt expanded by 7%, and equity increased by 2.5% (columns 4–6). The magnitude of the employment and asset responses suggests that these firms had previously been constrained by the old cutoff and were able to expand once this constraint was relaxed.

Table 5 reports estimates comparing newly eligible firms to firms that were never classified as SMEs. In contrast to the strong responses observed for always-eligible firms, revenue and employment exhibit small and statistically insignificant changes relative to never-eligible firms (columns 1 and 2). Physical capital increases by 7.5% (column 3). Paid-in capital rises by 5.7% and debt increases by 7.1% (columns 4 and 5), while equity remains statistically unchanged (column 6). These results suggest that the reform induced limited real expansion among newly eligible firms relative to larger peers, despite granting them formal access to SME status. The concentration of responses on balance-sheet variables, rather than on employment or revenue, is consistent with newly eligible firms adjusting cautiously to the reform, reflecting concerns about crossing the expanded eligibility threshold and losing access to guarantees.

Table 5: Direct Effect of SME Redefinition (Newly-Eligible vs. Never-Treated)

	Revenue (1)	Employment (2)	Physical Capital (3)	Paid-in Capital (4)	Debt (5)	Equity (6)
Post×Treated	0.033	0.006	0.075**	0.057***	0.071**	0.008
	(0.024)	(0.027)	(0.036)	(0.018)	(0.032)	(0.030)
Firm FE	yes	yes	yes	yes	yes	yes
Prefecture-Time FE	yes	yes	yes	yes	yes	yes
Industry-Time FE	yes	yes	yes	yes	yes	yes
Firms	2,195	2,195	2,193	2,195	2,195	2,195
Firm-Year Obs	15,813	15,813	15,796	15,813	15,813	15,813

Note: This table reports estimates from difference-in-differences regressions evaluating the impact of the 1999 SME Basic Law reform on firm outcomes. The dependent variables in columns (1) through (6) are the log of operating revenue, employment, physical capital, paid-in capital, debt, and equity, respectively. The treatment group consists of firms that became newly eligible for SME classification after the reform, while the control group includes never-eligible firms. Firms with extreme values of paid-in capital or employment size (very large) are excluded from the sample. All regressions include firm fixed effects, prefecture-year fixed effects, and industry-year fixed effects. Standard errors are clustered at the firm and industry-year level. Standard errors are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

6 Quantitative Implementation

This section takes the model in Section 3 to the data and evaluates welfare under the observed credit–guarantee regime and counterfactual policies. The quantification workflow proceeds in four steps: (i) calibrate production technology and financial constraint parameters to matched moments from the micro–data; (ii) identify the variable interest rate subsidy σ; (iii) identify the lump–sum subsidy S by matching the excess mass of firms bunching at the eligibility thresholds; and (iv) compute the general equilibrium and welfare outcomes.

6.1 Calibration

Scale parameter θ_s. I estimate the returns to scale parameter, θ_s, separately for each sector to capture substantial heterogeneity in production technologies. Using the full panel of firms in the Orbis database, I estimate a log–linear production function of sales on total assets, controlling for year fixed effects to absorb aggregate shocks. The resulting estimates are $\theta_M = 0.79$ for Manufacturing, $\theta_S = 0.80$ for Services, $\theta_R = 0.90$ for Retail, and $\theta_W = 0.93$ for Wholesale.

The higher estimates for the Wholesale and Retail sectors ($\theta \approx 0.9$) reflect the nature of trade industries, where revenue scales nearly linearly with inventory and floor space, exhibiting closer to constant returns to scale.

Statutory cutoffs $\overline{E}_0$ and $\overline{E}_1$. To capture the sector–specific heterogeneity of the policy, I use the distinct paid–in capital thresholds established by the SME Basic Act. The 1999 amendment significantly raised these eligibility limits. I calibrate the threshold vector for the four main sectors (Manufacturing, Wholesale, Retail, and Services) to match these statutory values. Specifically, the cutoffs shift from a pre–2000 baseline of $\overline{E}_0 = (100, 30, 10, 10)$ million yen to a post–2000 regime of $\overline{E}_1 = (300, 100, 50, 50)$ million yen.

Leverage Constraint $\bar{\lambda}$. A key parameter in the model is the leverage cap $\bar{\lambda}$, which dictates the maximum asset scale a firm can support given its paid-in capital. In the data, mature firms often exhibit very high asset-to-paid-in-capital ratios (median ≈ 15 in manufacturing) due to accumulated retained earnings. However, these firms are typically unconstrained by the SME policy. The welfare distortion is driven by growing, credit-dependent firms that lack internal wealth and must rely on external financing collateralized by their equity base.

To capture the binding constraint for these marginal agents, I calibrate $\bar{\lambda}_s$ using the median asset-to-paid-in-capital ratio of "constrained" firms—defined as those with a total equity to paid-in capital ratio of less than 1.5. This subsample represents approximately 18% of the universe and proxies for young or growing firms with limited retained earnings. The resulting calibrated values are $\bar{\lambda}_M = 3.00$ for Manufacturing, $\bar{\lambda}_S = 3.21$ for Services, $\bar{\lambda}_R = 5.07$ for Retail, and $\bar{\lambda}_W = 6.31$ for Wholesale. These values reflect the effective borrowing capacity for firms that are actually sensitive to the eligibility thresholds.

Productivity distribution. The underlying productivity α is assumed to follow a Pareto distribution with CDF $F(\alpha) = 1 - (\alpha/\underline{\alpha})^{-k}$. I estimate the shape parameter k_s for each sector structurally by exploiting the mapping between the firm size distribution and productivity. In the model, the optimal capital stock for unconstrained firms is proportional to $\alpha^{1/(1-\theta)}$. This implies that if assets follow a power law with tail index ζ, fundamental productivity follows a power law with index $k = \zeta/(1-\theta)$.

To avoid bias from the policy–induced distortions at the lower end of the size distribution, I identify the productivity tail using only unconstrained firms (defined as those with paid–in capital at least 10% above the eligibility threshold). Using the log–log rank regression method on the upper tail of the asset distribution, I estimate the size tail index ζ_s for each sector. Combining these estimates with the sector–specific returns to scale θ_s, I recover the structural productivity parameters: $k_M = 3.62$ for Manufacturing, $k_S = 2.83$ for Services, $k_R = 5.78$ for Retail, and $k_W = 10.19$ for Wholesale. These estimates satisfy the condition $k > 1$, ensuring a well–defined mean for the productivity distribution, and reflect the intuition that sectors with higher returns to scale (such as Wholesale) require less dispersion in fundamental productivity to generate the observed dispersion in firm size.

I calibrate $\underline{\alpha}_s$ separately for each sector to ensure that the model generates a firm size distribution consistent with the actual Japanese data. Specifically, I require the model to match the observed fraction of firms falling below the statutory paid–in capital thresholds.

Let Φ_s^{data} denote the empirical share of firms in sector s with paid–in capital $E \leq \overline{E}_s$, where $\overline{E}_s$ represents the statutory limits (300 million yen for Manufacturing, 100 million for Wholesale, and 50 million for Retail and Services). I first identify the productivity level $\alpha^*_{s,cut}$ at which an unconstrained firm would optimally choose a capital stock exactly equal to the maximum leverage capacity at the cutoff ($K^*(\alpha^*_{s,cut}) = \bar{\lambda}_s \overline{E}_s$). Inverting the firm's first–order condition yields $\alpha^*_{s,cut} = (r/\theta_s)(\bar{\lambda}_s \overline{E}_s)^{1-\theta_s}$. Given the Pareto assumption, I solve for the calibrated lower bound $\underline{\alpha}_s$ by equating the theoretical probability of falling below this cutoff to the empirical share:

$$1 - \left(\frac{\alpha^*_{s,cut}}{\underline{\alpha}_s} \right)^{-k_s} = \Phi_s^{data} \quad \Longrightarrow \quad \underline{\alpha}_s = \alpha^*_{s,cut} \cdot (1 - \Phi_s^{data})^{1/k_s}.$$

This procedure ensures that when the nominal statutory thresholds are applied to the simulated economy, the mass of firms subject to the subsidy perfectly matches the data.

The resulting calibrated values are $\underline{\alpha}_M = 0.129$ for Manufacturing, $\underline{\alpha}_W = 0.041$ for Wholesale, $\underline{\alpha}_R = 0.053$ for Retail, and $\underline{\alpha}_S = 0.120$ for Services. These values ensure that when the nominal statutory thresholds are applied to the simulated economy, the mass of firms subject to the subsidy matches the data.

6.2 Identifying the interest rate subsidy σ

Proposition 3 (Mapping from DID estimates to the structural subsidy)**.** Let K denote total assets. Define the Difference-in-Differences estimator on log assets as:

$$\widehat{\beta}_K \;:=\; \Delta^{DID} \ln K \;=\; \big[\ln K_{i1} - \ln K_{i0}\big] - \big[\ln K_{c1} - \ln K_{c0}\big]$$

where i indexes newly eligible firms ($\overline{E}_0 < E^*_{i0} \leq \overline{E}_1$) and c indexes always–large controls ($E^*_{c0} > \overline{E}_1$). Assuming the leverage constraint $\bar{\lambda}$ binds for eligible firms, the structural interest subsidy σ is identified by:

$$\sigma \;=\; \frac{\exp\big[-(1-\theta)\widehat{\beta}_K\big] - (1/\bar{\lambda})}{1 - (1/\bar{\lambda})}.$$

Proof. Full proof can be found in Appendix A.3. □

Based on Proposition 3, I obtain the structural estimate of the subsidy σ by inverting the relationship between the observed asset response $\widehat{\beta}_K$ and the firm's cost of capital. Intuitively, the mapping identifies how large the interest rate discount must have been to induce the observed growth in eligible firms, conditional on their technology and capital structure. The implied magnitude of σ depends on three key forces: the elasticity of firm scale (determined by θ), the transmission of the subsidy through the balance sheet (determined by $\bar{\lambda}$), and the magnitude of the reduced-form response itself ($\widehat{\beta}_K$).

First, the estimate is decreasing in $\widehat{\beta}_K$. A larger reduced-form coefficient implies that eligible firms expanded aggressively relative to the control group. In the model, since scale is inversely proportional to the user cost of capital, a larger expansion reveals a sharper reduction in financing costs, requiring a deeper discount (a lower σ). Second, the estimate is increasing in the returns to scale θ. As θ approaches unity, the production function exhibits near-constant returns to scale, making firm size extremely sensitive to factor prices. In this region, even a miniscule reduction in the cost of capital would generate explosive growth. Consequently, if we observe only a modest expansion $\widehat{\beta}_K$ in a high-θ sector, the model infers that the underlying cost shock must have been very small, implying a σ close to one. Third, the estimate is increasing in the leverage cap $\bar{\lambda}$. This dependence arises because the subsidy applies only to the debt portion of the firm's capital stack, while the equity portion continues to command the un-subsidized market rate. A tighter leverage constraint (lower $\bar{\lambda}$) forces the firm to hold a higher share of expensive equity, diluting the impact of the cheap debt. To achieve the same reduction in the weighted average cost of capital necessary to explain $\widehat{\beta}_K$, the subsidy on the small debt fraction must be massive. As leverage increases, the firm becomes mostly debt-financed, and the subsidized rate passes through more directly to the user cost, implying a σ closer to the effective cost reduction.

Applying this logic to the data, I combine the estimated asset response $\widehat{\beta}_K = 0.043$ with the sector-specific parameters calibrated in the previous section. For the Manufacturing sector, with decreasing returns $\theta_M = 0.79$ and a leverage constraint $\bar{\lambda}_M = 3.00$, the structural mapping yields:

$$\widehat{\sigma}_M = \frac{\exp\left[-(1-0.79) \times 0.043\right] - (1/3.00)}{1-(1/3.00)} \approx 0.987.$$

This estimate implies that the policy effectively lowers the marginal borrowing rate to 98.7% of the market rate for manufacturing firms. The estimates for the other sectors reflect their higher returns to scale and leverage capacities. For Services ($\theta_S = 0.80, \bar{\lambda}_S = 3.21$), the estimated subsidy is similar at $\widehat{\sigma}_S \approx 0.988$. In the trade sectors, where returns to scale are high and inventory facilitates higher leverage, the implied subsidy is smaller: $\widehat{\sigma}_R \approx 0.995$ for Retail ($\theta_R = 0.90, \bar{\lambda}_R = 5.07$) and $\widehat{\sigma}_W \approx 0.996$ for Wholesale ($\theta_W = 0.93, \bar{\lambda}_W = 6.31$). In these high-$\theta$ sectors, firms are so sensitive to costs that the moderate observed growth ($\widehat{\beta}_K$) is consistent with only a very slight reduction in interest expenses.

6.3 Identifying the lump–sum subsidy S

The lump–sum subsidy S captures the combined value of non–interest benefits associated with eligibility, such as statutory fee rebates, tax incentives, and administrative exemptions. I estimate S_s structurally for each sector by matching the excess mass of firms bunching at the eligibility threshold.

I assume the "marginal buncher" (with productivity α^*) is indifferent between staying at the threshold $\overline{E}$ to receive the subsidy S (and the subsidized rate σr) versus jumping to their optimal unconstrained scale $E^*(\alpha^*)$ and paying market rates. The indifference condition is:

$$\pi(\overline{E}; \sigma r) + S \;=\; \pi(E^*(\alpha^*); r).$$

I recover α^* from the observed excess mass of firms (B) at the cutoff using the estimated Pareto distribution of productivity: $B \approx F(\alpha^*) - F(\alpha_{cut})$.

Current Regime (Post-2000). I estimate S_{new} using the current statutory thresholds (300m for Manufacturing, 100m for Wholesale, 50m for Retail/Services). The results indicate significant heterogeneity driven by the interaction between the lending cutoffs and the broader tax code.

For the Wholesale sector, the estimated lump–sum subsidy is large and positive ($S \approx$ 822k ¥). This result is driven by the fact that the sector's new eligibility threshold (100 million yen) coincides with Japan's most critical corporate tax cliff. Since the interest rate subsidy σ_W alone is insufficient to account for the substantial bunching at this threshold, the estimation identifies a large S to capture the value of this tax avoidance.

In contrast, for Manufacturing, the lump–sum estimate is negative ($S \approx -124$k ¥). At the 300 million yen cutoff, firms are still eligible for credit guarantees, and the variable interest rate subsidy σ_M provides a substantial theoretical incentive to bunch. This negative result is similar to the "negative fixed cost" finding in Garicano et al. (2016).

Old Regime (Pre-2000). To simulate the counterfactual policy change, I estimate S_{old} using the old eligibility thresholds (100m for Manufacturing, 30m for Wholesale, 10m for Retail/Services). Identification exploits a "difference-in-bunching" design, isolating the excess mass of "legacy firms" (established pre-2000) relative to "modern firms" (established post-2000).

The results, summarized in the last row of Table 6, reveal that distortions were concentrated at the lower end of the size distribution. For Wholesale and Retail, S_{old} is negligible (-11k and -4k ¥), suggesting that the interest rate subsidy alone is sufficient to explain the modest bunching at these lower cutoffs. However, the Service sector exhibits a massive lump–sum subsidy ($S_{old} \approx 414$k ¥) at the 10 million yen cutoff.

6.4 Welfare analysis

I evaluate the aggregate implications of the credit guarantee system by comparing the equilibrium allocations under the Old (pre-2000) and New (post-2000) regimes against a counterfactual "No

Table 6: Calibrated Structural Parameters

Parameter	Manuf.	Wholesale	Retail	Services
A. Technology & Preferences				
Returns to Scale (θ_s)	0.79	0.93	0.90	0.80
Prod. Tail Parameter (k_s)	3.62	10.19	5.78	2.83
Prod. Lower Bound ($\underline{\alpha}_s$)	0.129	0.041	0.053	0.120
B. Financial Frictions				
Leverage Constraint ($\bar{\lambda}_s$)	3.00	6.31	5.07	3.21
Current Cutoff ($\overline{E}_{new}$, mn ¥)	300	100	50	50
Old Cutoff ($\overline{E}_{old}$, mn ¥)	100	30	10	10
C. Policy Distortions				
Interest Rate Subsidy (σ_s)	0.987	0.996	0.995	0.988
Current Lump-sum (S_{new}, k¥)	-124	822	120	47
Old Lump-sum (S_{old}, k¥)	161	-11	-4	414

Note: S_{new} and S_{old} represent the implied annual lump-sum benefit of maintaining SME status (in thousands of Yen). S_{old} is identified using the net excess mass of pre-2000 firms at the old cutoffs relative to post-2000 firms. Negative values imply the interest rate subsidy alone over-predicts the observed bunching.

Policy" friction–free economy. Aggregate welfare is defined as total output net of the lump–sum transfers (including tax expenditures and administrative costs) absorbed by eligible firms.

Distortions arise from two channels: the misallocation of capital among subsidized firms (who bunch at $\overline{E}$), and the general equilibrium price effects that distort the scale of large firms.

Table 7 reports the estimated welfare effects. The results contradict the simple intuition that "relaxing constraints" improves welfare; instead, I find that the expansion of the credit guarantee system has exacerbated aggregate misallocation.

Table 7: Welfare Effects of the Credit Guarantee Reform

	Welfare Loss vs. First Best		**Impact of Reform**
Sector	Old Regime	New Regime	(Old → New)
Manufacturing	−0.3%	−0.5%	−0.2%
Wholesale	−5.5%	−7.2%	−1.7%
Retail	−2.8%	−3.5%	−0.7%
Services	−3.2%	−4.1%	−0.9%
Aggregate	−3.1%	−4.1%	−1.0%

Note: Welfare is measured as aggregate output equivalent variation relative to the friction-free first best. Negative values indicate a welfare loss. The "Impact of Reform" column shows that transitioning from the Old (narrow) to the New (broad) eligibility criteria deepened the aggregate welfare loss.

The results highlight a policy paradox: expanding the scope of the subsidy increased aggregate misallocation. While the reform raised the eligibility ceiling, allowing some constrained mid–sized firms to grow, it did not eliminate the distortion. Instead, many firms simply migrated to the new, higher threshold (e.g., 50 million yen) to retain eligibility, shifting the "bunching mass" further up the distribution without resolving the underlying inefficiency. Crucially, the partial–equilibrium gains for these firms were outweighed by severe general equilibrium losses.

The primary driver of the welfare loss is the crowding out of high–productivity firms. By raising the eligibility cutoffs, the reform dramatically increased the aggregate demand for subsidized credit. To clear the capital market, the equilibrium interest rate r rose. Because the distribution of firm productivity is fat–tailed (Pareto), aggregate output is disproportionately generated by a small number of very large, high–productivity firms (the "unconstrained giants") that are ineligible for the subsidy. The rise in the market interest rate acts as a tax on these firms, causing them to contract. Since these giants account for roughly half of the economy's output, even a small contraction in their scale generates deadweight losses that dwarf the efficiency gains reaped by the smaller subsidized firms. Consequently, the reform effectively reallocated capital from the most productive giants to moderately productive SMEs, deepening the aggregate TFP loss.

Table 7 reveals a striking dichotomy: Manufacturing exhibits minimal welfare losses (-0.5%), whereas the Wholesale and Service sectors face severe distortions (-7.2% and -4.1%, respectively). This divergence is partly driven by the estimated bunching incentives (S), but the result for Manufacturing requires careful interpretation regarding the identification strategy.

Recall that the structural parameters were identified using the change in the bunching mass following the 1999 reform. For Manufacturing, the old credit eligibility cutoff (100 million yen) coincides exactly with a major corporate tax threshold. Because firms continued to cluster at 100 million yen after the reform—likely to retain the tax benefit rather than the credit guarantee—the difference–in–bunching estimator infers that manufacturing firms are relatively insensitive to the credit policy change. This results in a low estimated distortion for the credit program itself. Consequently, the true aggregate distortion at this threshold is likely larger than estimated, and the reported welfare loss for the Manufacturing sector should be interpreted as a conservative lower bound.

7 Conclusion

This paper evaluates the aggregate consequences of size-dependent financial frictions using Japan's Credit Guarantee Scheme. Leveraging the 1999 eligibility expansion as a natural experiment, I identify the causal effects of the policy by comparing two treated groups—"always eligible" and "newly eligible" firms—relative to a control group of "always–large" firms. I document a divergent response to the reform. "Always eligible" firms, which had been constrained by the previous strict cutoffs, responded to the relaxation with significant financial and economic expansion. In contrast, "newly eligible" firms exhibited reluctance to grow; rather than scaling up, they largely clustered below the new, higher threshold to preserve their access to guaranteed credit.

To interpret these findings, I develop a two-period general equilibrium model in which firms choose equity and scale under a leverage constraint. The model generates endogenous bunching in firm size. Calibrating the model to match estimated behavioral responses, I find that Japan's size-based guarantee program imposes a welfare loss of 3.1% of output under the original regime. The 1999 reform, by relaxing eligibility thresholds, resulted in a further welfare loss of approximately 1.0% of value added, as the crowding out of high-productivity firms outweighed the benefits of relaxation.

As a complementary exercise, I develop an extended general equilibrium model in the Appendix C that explores how optimal credit guarantee policies should be designed when banks are heterogeneous in capitalization and firms differ in default risk. This framework introduces endogenous bank sorting and general equilibrium credit market clearing. It highlights a key policy trade-off: relaxing eligibility thresholds may support marginal firms but can also encourage undercapitalized banks to lend excessively to riskier borrowers. The model shows that a guarantee scheme that conditions more tightly on bank capitalization—offering lower guarantees to well-capitalized banks—can improve credit allocation and maximize value added. The results underscore the importance of tailoring guarantee intensity to institutional quality and reinforce the empirical evidence of misallocation due to uniform guarantee policies.

Future research could deepen this analysis by extending the theoretical framework along several important dimensions. One promising direction is to embed a fully articulated banking sector into the size-dependent policy model, allowing for endogenous determination of bank balance sheet risk, deposit pricing, and regulatory constraints. This would enable richer analysis of how guarantee design interacts with bank incentives, particularly in contexts where capital regulation and monetary policy jointly influence lending behavior. Another avenue is to introduce firm dynamics—such as multi-period investment and exit decisions—into the bunching environment, which would capture long-run distortions in innovation and reallocation. These extensions would help build a more comprehensive theory of financial intermediation under targeted government support.

A Proof of Propositions

A.1 Proof of Proposition 1

To establish the existence and characterization of $E^*(\alpha)$, we analyze the Stage 0 payoff $\Pi(\alpha, E)$ separately on the eligible domain $D_e = [0, \overline{E}]$ and the ineligible domain $D_i = (\overline{E}, \infty)$.

1. Optimization in the Eligible Region. On D_e, the firm faces the borrowing rate $\sigma r < r$. Substituting the optimal scale choice $K^*(\alpha, E) = \min\{K_\sigma(\alpha), \bar{\lambda}E\}$ into the profit function:

- **Binding Case:** For $E < K_\sigma(\alpha)/\bar{\lambda}$, the firm is leverage-constrained ($K^* = \bar{\lambda}E$). The payoff is $\Pi = \alpha(\bar{\lambda}E)^\theta - \sigma r(\bar{\lambda} - 1)E - rE + S$. The marginal profit is:

$$\frac{\partial \Pi}{\partial E} = \theta\alpha(\bar{\lambda}E)^{\theta-1}\bar{\lambda} - \sigma r(\bar{\lambda} - 1) - r$$

  By the definition of $K_\sigma(\alpha)$, we have $\theta\alpha K_\sigma(\alpha)^{\theta-1} = \sigma r$. Since $E < K_\sigma(\alpha)/\bar{\lambda}$, it follows that $\bar{\lambda}E < K_\sigma(\alpha)$, which implies $\theta\alpha(\bar{\lambda}E)^{\theta-1} > \sigma r$. Thus, $\frac{\partial \Pi}{\partial E} > \sigma r\bar{\lambda} - \sigma r\bar{\lambda} + \sigma r - r = (\sigma - 1)r$. While $(\sigma - 1)r$ is negative, the first term dominates for sufficiently small E, and the objective is concave, leading to an increasing profile until the scale K_σ is reached.

- **Slack Case:** For $E \geq K_\sigma(\alpha)/\bar{\lambda}$, the firm reaches $K_\sigma(\alpha)$. The payoff becomes $\Pi = \alpha K_\sigma(\alpha)^\theta - \sigma r(K_\sigma(\alpha) - E) - rE + S$. Here, $\frac{\partial \Pi}{\partial E} = \sigma r - r < 0$.

Thus, $\Pi(\alpha, E)$ is unimodal on D_e, and the local maximum is $E_e = \min\{\frac{K_\sigma(\alpha)}{\bar{\lambda}}, \overline{E}\}$.

2. Optimization in the Ineligible Region. On D_i, the interest rate is r. Because the marginal cost of equity is assumed to be $r' = r$, the payoff simplifies to:

$$\Pi(\alpha, E) = \alpha(K^*)^\theta - r(K^* - E) - rE = \alpha(K^*)^\theta - rK^*$$

- If $E \in (\overline{E}, K_1(\alpha)/\bar{\lambda})$, the leverage cap binds ($K^* = \bar{\lambda}E$). Profit is strictly increasing because $K^* < K_1(\alpha)$, and the marginal product of capital exceeds r.

- If $E \in [K_1(\alpha)/\bar{\lambda}, K_1(\alpha)]$, the firm reaches the interior scale $K^* = K_1(\alpha)$. The payoff is constant at $\alpha K_1(\alpha)^\theta - rK_1(\alpha)$. The specific value of E does not affect profits as long as it supports the scale $K_1(\alpha)$ and does not exceed it.

- If $E > K_1(\alpha)$, the firm would be forced to operate at $K^* = E$ (all-equity) because $K \geq E$. Since $E > K_1(\alpha)$, the marginal product is less than r, and profit is strictly decreasing.

Therefore, any $E \in [\max\{\overline{E}, \frac{K_1(\alpha)}{\bar{\lambda}}\}, K_1(\alpha)]$ yields the maximum profit for the ineligible regime.

3. Global Existence. The global payoff function $\Pi(\alpha, E)$ is upper semi-continuous. At the eligibility threshold $E = \overline{E}$, the function is defined by the eligible branch, which includes the subsidy $S \geq 0$ and the lower rate $\sigma r < r$, ensuring $\Pi(\alpha, \overline{E}) \geq \lim_{E \to \overline{E}^+} \Pi(\alpha, E)$. Because the function is strictly decreasing for all $E > K_1(\alpha)$, the global maximum must lie within the compact interval $[0, K_1(\alpha)]$. By the Extreme Value Theorem, a maximum exists.

A.2 Proof of Proposition 2

Proof. To establish the existence and uniqueness of the equilibrium interest rate r^*, we define the excess–demand function for credit as $\Phi(r) := D^{\text{agg}}(r) - S(r)$, where $D^{\text{agg}}(r)$ is the aggregate debt demanded by entrepreneurs and $S(r)$ is the aggregate wealth supplied by depositors.

1. Monotonicity. We show that $\Phi(r)$ is strictly decreasing in r:

i. *Credit Supply:* The entry cutoff $\alpha_{\min}(r)$ is defined by $\Pi(\alpha_{\min}, E^*(\alpha_{\min}); r) = r$. An increase in r raises the opportunity cost of entrepreneurship (the right–hand side) and lowers operating profits via higher financing costs (the left–hand side). Thus, $\alpha_{\min}(r)$ is strictly increasing in r. Since $S(r) = \int_{\underline{\alpha}}^{\alpha_{\min}(r)} f(\alpha) d\alpha$, the supply of funds is continuous and strictly increasing in r.

ii. *Credit Demand:* For any active entrepreneur, debt demand is $D^*(\alpha) = K^*(\alpha) - E^*(\alpha)$.

- For eligible firms ($\alpha < \alpha_b$), entrepreneurs exhaust leverage capacity ($K = \bar{\lambda}E$), so $D^*(\alpha) = (\bar{\lambda} - 1)E^*(\alpha)$. Since $E^*(\alpha) = \min\{K_\sigma(\alpha)/\bar{\lambda}, \overline{E}\}$ and $K_\sigma(\alpha)$ is strictly decreasing in r, debt demand is weakly decreasing in r for these types.

- For ineligible firms ($\alpha \geq \alpha_b$), the interior scale $K_1(\alpha)$ is strictly decreasing in r. While the financing mix is indeterminate in the interval $[\frac{K_1}{\lambda}, K_1]$, any consistent selection rule (e.g., constant leverage) ensures D^* is strictly decreasing in r.

By the Leibniz Rule, $D^{\text{agg}}(r) = \int_{\alpha_{\min}(r)}^{\bar{\alpha}} D^*(\alpha; r) f(\alpha) d\alpha$ is strictly decreasing in r because the intensive margin (D^*) decreases and the extensive margin ($\alpha_{\min}$) increases (reducing the number of borrowers).

Therefore, $\Phi(r)$ is the difference between a strictly decreasing and a strictly increasing function, making it strictly decreasing.

2. Boundary Conditions.

- As $r \downarrow 0$, the interior asset demands K_σ and K_1 explode. The entry threshold $\alpha_{\min} \to \underline{\alpha}$ as the return on deposits vanishes. Thus, $D^{\text{agg}}(r) \to \infty$ and $S(r) \to 0$, implying $\Phi(r) \to \infty$.
- As $r \uparrow \infty$, interior scales shrink to zero. The entry threshold $\alpha_{\min} \to \bar{\alpha}$ as the depositor's return dominates any possible entrepreneurial profit. Thus, $D^{\text{agg}}(r) \to 0$ and $S(r) \to 1$ (the total wealth of the economy), implying $\Phi(r) \to -1$.

3. Existence and Uniqueness. Since $\Phi(r)$ is continuous and strictly decreasing on $(0, \infty)$, and its range spans $(\infty, -1)$, the Intermediate Value Theorem guarantees a unique $r^* > 0$ such that $\Phi(r^*) = 0$. This unique interest rate then determines the unique cutoffs $\alpha_{\min}, \alpha_c, \alpha_b$ and the resulting allocation $K^*(\cdot)$. □

A.3 Proof of Proposition 3

This section derives the mapping between the reduced-form Difference-in-Differences (DiD) coefficient $\widehat{\beta}_K$ and the structural interest subsidy σ.

User Cost of Capital. Consider a firm with productivity α and production technology $Y = \alpha K^\theta$. Firms finance capital K using a mix of equity E and debt D, such that $K = E + D$.

i. **Control Group (Ineligible):** Ineligible firms pay the market interest rate r on debt. Assuming the opportunity cost of equity is also r (standard in Modigliani-Miller environments without tax distortions on equity), the user cost of capital for a control firm is simply r.

ii. **Treatment Group (Eligible):** Eligible firms receive a subsidized interest rate σr on debt, where $\sigma < 1$. They pay the market rate r on equity. We assume the leverage constraint $K \leq \bar{\lambda} E$ binds for these growing firms, fixing their capital structure at:

$$E = \frac{1}{\bar{\lambda}} K \quad \text{and} \quad D = \frac{\bar{\lambda} - 1}{\bar{\lambda}} K.$$

The weighted average cost of capital (user cost) for a treated firm is:

$$\begin{aligned} \tilde{r} &= r \cdot \frac{E}{K} + \sigma r \cdot \frac{D}{K} \\ &= r \left[\frac{1}{\bar{\lambda}} + \sigma \left(\frac{\bar{\lambda} - 1}{\bar{\lambda}} \right) \right]. \end{aligned}$$

Optimal Scale. The firm maximizes profit $\pi = \alpha K^{\theta} - \text{UserCost} \cdot K$. The first-order condition with respect to K is:

$$\theta \alpha K^{\theta - 1} = \text{UserCost} \implies K^* = \left(\frac{\theta \alpha}{\text{UserCost}} \right)^{\frac{1}{1-\theta}}.$$

Taking logs, we obtain the linear equation for firm size:

$$\ln K^* = \frac{1}{1-\theta} \ln(\theta) + \frac{1}{1-\theta} \ln \alpha - \frac{1}{1-\theta} \ln(\text{UserCost}).$$

Difference-in-Differences Estimator. The DiD estimator compares the change in log assets for treated firms (i) versus control firms (c) between period 0 (pre-reform) and period 1 (post-reform).

- **Period 0:** Both groups face the market user cost r_0.
- **Period 1:** The control group faces market cost r_1. The treated group becomes eligible and faces the subsidized cost $\tilde{r}_1$.

Assuming common trends in productivity ($\Delta \ln \alpha_i = \Delta \ln \alpha_c$) and aggregate shocks, the DiD estimator isolates the change in the user cost wedge:

$$\begin{aligned}
\widehat{\beta}_K &= \Delta \ln K_i - \Delta \ln K_c \\
&= \left[-\frac{1}{1-\theta}(\ln \tilde{r}_1 - \ln r_0)\right] - \left[-\frac{1}{1-\theta}(\ln r_1 - \ln r_0)\right] \\
&= -\frac{1}{1-\theta}\left(\ln \tilde{r}_1 - \ln r_1\right) \\
&= -\frac{1}{1-\theta}\ln\left(\frac{\tilde{r}_1}{r_1}\right).
\end{aligned}$$

Substituting the definition of the subsidized cost $\tilde{r}_1$:

$$\frac{\tilde{r}_1}{r_1} = \frac{r_1\left[\frac{1}{\bar{\lambda}} + \sigma\left(\frac{\bar{\lambda}-1}{\bar{\lambda}}\right)\right]}{r_1} = \frac{1}{\bar{\lambda}} + \sigma\left(\frac{\bar{\lambda}-1}{\bar{\lambda}}\right).$$

Substituting this back into the expression for $\widehat{\beta}_K$:

$$\widehat{\beta}_K = -\frac{1}{1-\theta}\ln\left[\frac{1}{\bar{\lambda}} + \sigma\left(1-\frac{1}{\bar{\lambda}}\right)\right].$$

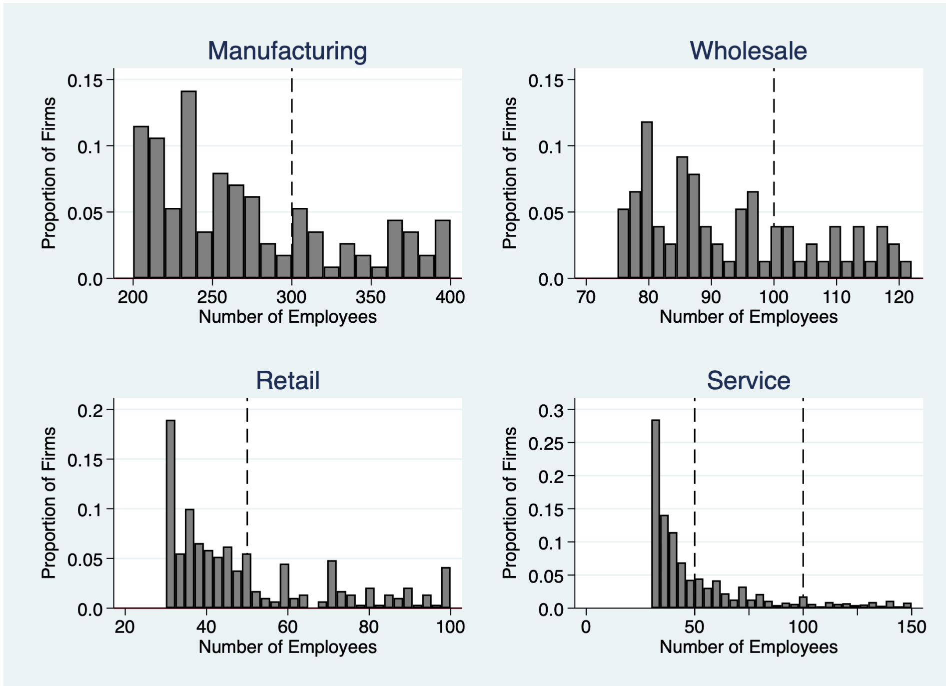
Inversion. Finally, we solve for σ:

$$\begin{aligned}
\exp\bigl(-(1-\theta)\widehat{\beta}_K\bigr) &= \frac{1}{\bar{\lambda}} + \sigma\left(1-\frac{1}{\bar{\lambda}}\right) \\
\exp\bigl(-(1-\theta)\widehat{\beta}_K\bigr) - \frac{1}{\bar{\lambda}} &= \sigma\left(1-\frac{1}{\bar{\lambda}}\right) \\
\sigma &= \frac{\exp\bigl[-(1-\theta)\widehat{\beta}_K\bigr] - (1/\bar{\lambda})}{1-(1/\bar{\lambda})}.
\end{aligned}$$

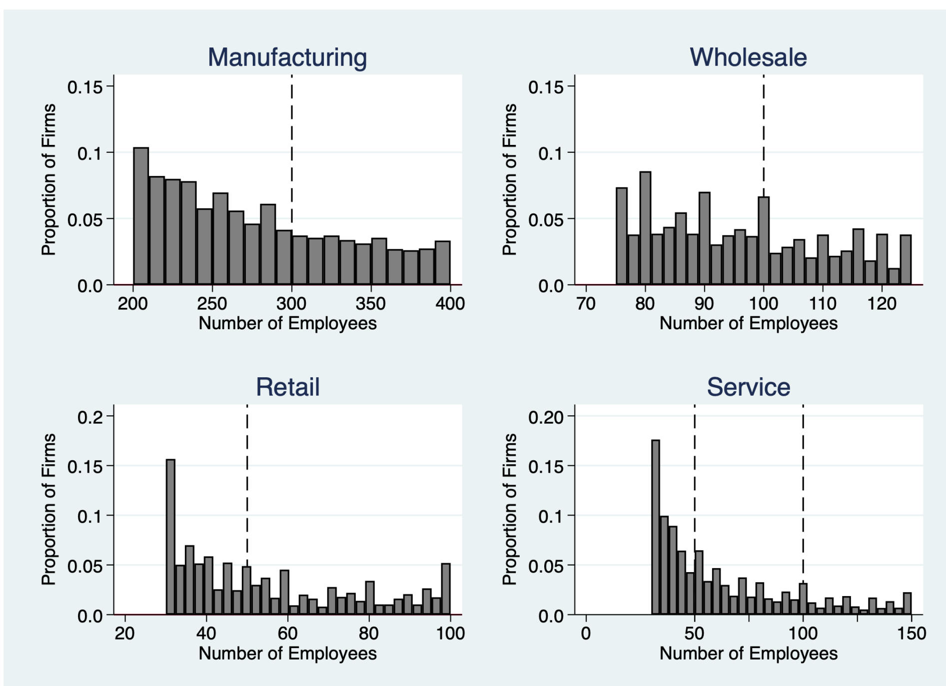
This completes the proof. □

B Additional Tables and Figures

Figure B1: Firm Bunching Around SME Employment Eligibility Threshold



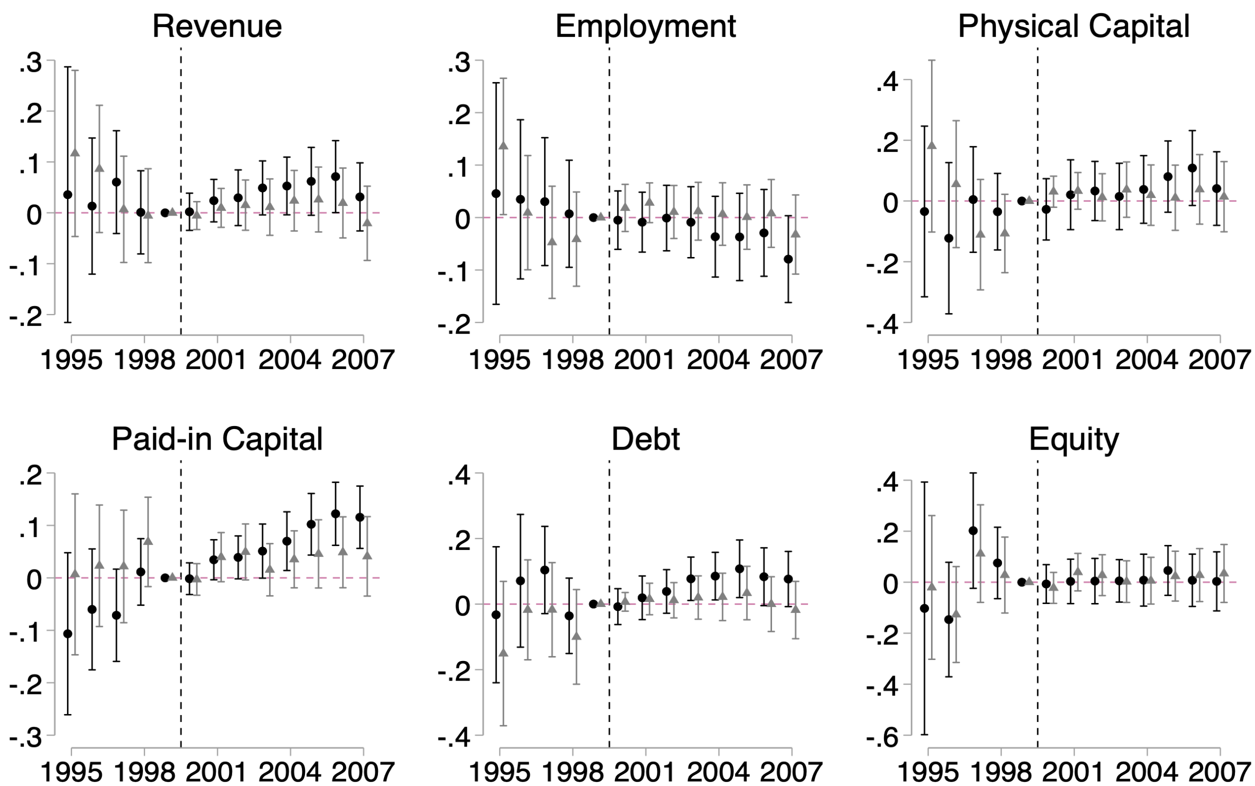
(a) Before the 1999 SME Definition Reform



(b) After the 1999 SME Definition Reform

Note: This figure shows the distribution of firms by employment around the SME eligibility threshold. Panel (a) displays the firm size distribution before the 1999 reform. Panel (b) shows the distribution after the reform took effect in 1999, which raised the employment thresholds only for the service industries.

Figure B2: Dynamic Effects: Newly-Eligible (Two Groups) vs. Never-Eligible Firms



Note: This figure plots event-study coefficients β_t from a difference-in-differences specification that compares firms that became newly eligible for SME classification after the 1999 reform to firms that were never eligible. Newly eligible firms are split into two groups based on paid-in capital. The black lines correspond to firms with paid-in capital below the midpoint between the pre-1999 and post-1999 SME cutoffs, while the gray lines correspond to firms with paid-in capital above that midpoint. The specification includes firm fixed effects, prefecture-by-year fixed effects, and industry-by-year fixed effects. Standard errors are clustered at the firm level and at the industry-year level. All coefficients are normalized to zero in 1999, the year of the policy change.

C A Model of Optimal Credit Guarantee Policy

This section introduces a general equilibrium model to analyze how government credit guarantees interact with firm heterogeneity, bank capital structure, and credit allocation. The purpose of the model is to quantify distortions arising from equity-dependent guarantees and identify the optimal credit guarantee policy.

C.1 Environment: Firms, Banks, and Government Policy

The economy comprises a continuum of incumbent firms, normalized to a mass of one. These firms differ in their default probabilities and productivity. Specifically, there are two firm types: healthy (h) firms and zombie (z) firms. Healthy firms constitute a fraction $(1-\lambda)$ of incumbents, while zombie firms account for the remaining fraction λ. Additionally, potential entrants with a mass of λ are all healthy firms.

Each firm requires financing of one monetary unit to implement a single, indivisible project. Projects yield returns contingent on success or failure. A type-i firm (with $i \in \{h, z\}$) successfully produces an output y_i with probability $(1-p_i)$, and defaults with probability p_i, where it produces zero output. All firms draw an idiosyncratic operating cost shock, ε, from a common distribution $F(\varepsilon)$.

Given these project characteristics, a firm of type i decides to operate if and only if its realized operating cost is below a threshold determined by its expected net revenue, which equals the difference between expected output and the loan repayment obligation, weighted by the probability of success. Specifically, firm i operates if:

$$\varepsilon \leq \varepsilon_i^* \equiv (1-p_i)(y_i - R_i),$$

where R_i is the loan rate charged by the bank to the firm. Consequently, the equilibrium measure of operating firms for each type is:

$$m_h = (1-\lambda+\lambda)F(\varepsilon_h^*) = F(\varepsilon_h^*), \quad m_z = \lambda F(\varepsilon_z^*).$$

Banks are characterized by their equity ratios $e \in [e_{\min}, e_{\max}]$, with the distribution of banks across equity ratios given by $\mu(e)$. Each bank has a normalized balance sheet size of one monetary unit and decides among three possible investment options: lending to healthy firms, lending to zombie firms, or investing in a safe asset yielding a risk-free return R_f. I assume competitive credit markets, implying that loan interest rates R_i are determined in general equilibrium and taken as given by both banks and firms. On the liability side, banks raise $1-e$ units from depositors, who must always be compensated at the risk-free rate R_f, regardless of the project's outcome.

The government sets two key policy instruments: the risk-free rate R_f and a bank-specific credit guarantee rate $\theta(e)$. In the Japanese context, I assume that R_f has already reached its effective lower bound, meaning the government cannot further reduce interest rates in response to negative economic shocks. I also assume the guarantee rate decreases with a bank's equity ratio e, formally expressed as $\theta'(e) < 0$, an assumption supported by the empirical findings.

C.2 Bank Optimization

Banks evaluate three investment options by maximizing their expected return on equity (ROE), given by:

$$\max_{i\in\{h,z,f\}} \mathrm{ROE}_i = \frac{(1-p_i)R_i + p_i\theta(e) - R_f(1-e)}{e}$$

When investing in the safe asset, banks' ROE equals the risk-free rate, $\mathrm{ROE}_f = R_f$.

Proposition 4. Consider a credit guarantee policy defined by $\theta(e) = \bar{\theta}(1-\kappa e)$, where $\kappa > 0$ governs the negative sensitivity of the guarantee rate to a bank's equity ratio e. Then, there exist two equity thresholds:

$$e^* = \frac{1}{\kappa}\left[1 - \frac{(1-p_z)R_z - (1-p_h)R_h}{\bar{\theta}(p_h - p_z)}\right], \quad e^{**} = \frac{1}{\kappa}\left[1 - \frac{R_f - (1-p_h)R_h}{p_h\bar{\theta}}\right],$$

which segment banks into three different equilibrium lending strategies:

$$\text{Bank investment choice} = \begin{cases} \text{lend to zombie firms,} & \text{if } e < e^*, \\ \text{lend to healthy firms,} & \text{if } e \in (e^*, e^{**}), \\ \text{invest in safe asset,} & \text{if } e > e^{**}. \end{cases}$$

The intuition underlying Proposition 4 is straightforward. Under an equity-sensitive guarantee policy, less-capitalized banks benefit disproportionately from generous guarantees, thereby incentivizing them to finance riskier, zombie firms. Conversely, well-capitalized banks receive less favorable guarantees and thus prefer safer investments or lending to healthier firms. This sorting mechanism illustrates how government guarantees can unintentionally encourage fragile banks to engage in riskier lending practices, exacerbating financial misallocation and inefficiencies.

C.3 General Equilibrium and Optimal Policy

General Equilibrium. A general equilibrium in this economy consists of loan rates (R_h, R_z), a government guarantee function $\theta(e)$, and the risk-free rate R_f satisfying the following conditions simultaneously:

i. *Bank Optimization:* Given loan rates (R_h, R_z), the guarantee schedule $\theta(e)$, and risk-free rate R_f, each bank characterized by equity ratio e chooses investment $i(e) \in \{h, z, f\}$ to maximize its expected return on equity:

$$i(e) = \arg\max_{i\in\{h,z,f\}} \frac{(1-p_i)R_i + p_i\theta(e) - R_f(1-e)}{e},$$

with the optimal choices inducing thresholds (e^*, e^{**}) that partition banks' lending behavior.

ii. *Firm Entry and Exit Conditions:* Given the equilibrium loan rates (R_h, R_z), each type-i firm operates if and only if its realized operating cost is below the profitability threshold:

$$\varepsilon \le \varepsilon_i^* = (1-p_i)(y_i - R_i), \quad i \in \{h, z\}.$$

Thus, equilibrium masses of operating firms are:

$$m_z = \lambda F(\varepsilon_z^*), \quad m_h = (1-\lambda+\lambda)F(\varepsilon_h^*) = F(\varepsilon_h^*).$$

iii. *Loan Market Clearing Conditions:* Aggregate credit supply by banks to each type of firm equals the respective aggregate demand from operating firms. Formally, this requires:

$$\int_{e_{\min}}^{e^*} \mu(e)\, de = m_z, \quad \int_{e^*}^{e^{**}} \mu(e)\, de = m_h.$$

An equilibrium therefore simultaneously satisfies the bank optimization conditions, firm entry and exit thresholds, and market clearing equations, given the policy choices $(R_f, \theta(e))$ set by the government.

Total Value Added. Total output net of operating costs (i.e., total value added) in this economy is defined as the sum of expected output across all operating firms, minus their respective operating costs. Let Y denote total value added:

$$Y = \lambda \left[y_z F(\varepsilon_z^*) - \int_0^{\varepsilon_z^*} \varepsilon f(\varepsilon)\, d\varepsilon \right] + \left[y_h F(\varepsilon_h^*) - \int_0^{\varepsilon_h^*} \varepsilon f(\varepsilon)\, d\varepsilon \right].$$

The first term captures the contribution of zombie firms weighted by their measure λ, while the second term reflects the contribution of healthy firms, including both incumbents and entrants.

Optimal Credit Guarantee Policy. Given that the risk-free rate R_f is constrained by its effective lower bound, when facing negative shocks, the government cannot stimulate credit further by lowering interest rates. Instead, it chooses the guarantee schedule $\theta(e)$ to maximize total value added Y, while minimizing the fiscal cost of providing guarantees. Formally, the government solves:

$$\min_{\theta(e)} \int_{e_{\min}}^{e_{\max}} \theta(e)\, de \quad \text{subject to} \quad Y[\theta(e)] = \sup_{\theta'(e)} Y[\theta'(e)].$$

This problem characterizes the optimal trade-off between allocative efficiency and fiscal discipline. The government aims to implement a guarantee policy that achieves the highest possible aggregate output net of costs, while committing the least amount of public resources.

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